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Snap-through buckling of single-layer squarely-reticulated shallow spherical shells continuously supported on springs

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Abstract. An asymptotic solution for snap-through buckling of single-layer squarely-reticulated shallow spherical shells continuously supported on springs is developed in this paper. Based on the fundamental governing equations and boundary conditions, a nondimensional analytical expression associated with the external load, stiffness of spring and central transverse displacement (deflection) is derived with the aid of asymptotic iteration method. The effects of stiffness of spring and characteristic geometrical parameter on buckling of the structures are given by the analyses of numerical examples. In a special case, for reticulated circular plates, the influence of stiffness of spring on the characteristic relation between load and deflection is also demonstrated.

Key words: reticulated spherical shells; characteristic geometrical parameter; stiffness of spring; snap-through buckling; asymptotic iteration method.

1. Introduction

Owing to many mechanical and structural properties of space reticulated shell structures formed by beam members, there have been wide applications of them in civil, mechanical, aeronautical and astronautical engineering in recent years.

Reticulated shell structures have common features with both discrete (lattice) structures and continuous (solid) shells, such that characteristic difficulties of both types of structures are cumulated and amplified. Although the research development of such structures has been made rapidly recently, one important problem has not yet been solved satisfactorily, and that is the problem of buckling. Furthermore, for single-layer reticulated shell structures, the geometrical nonlinearity is stronger while the material nonlinearity appears mainly for double-layer structures. In addition, an increase use of soft filaments in aerospace structures, e.g., large-sized space shell-type antennas continuously supported on springs, etc., has intensified the need for a better understanding of buckling behaviors of such structures continuously supported by elastic media.

So far there exist some investigations on static and dynamic response and buckling analyses of various (solid) shallow shell structures on elastic Winkler and Pasternak foundation, including shallow cylindrical panels (Ramachandran and Murthy 1976, Massalas and Kafousias 1979, Chia 1990), shallow spherical shells (Nath and Jain 1983, Dumir 1985, Paliwal *et al.* 1986) and doubly curved shallow shells (Nath and Mahrenholtz 1987, Chia 1988, Librescu and Lin 1997). However,

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little attention has been paid to analysis of non-linear buckling of single-layer reticulated shell structures with large-number of beam members continuously supported by elastic media, e.g., supported on springs (in principle, i.e., Winkler foundation), etc (Nie 1994).

On the basis of the analyses of the internal forces and deformations of single-layer reticulated shallow shell structures formed by beam members placed in two orthogonal directions, Nie *et al.* (1991, 1994, 1995) propose a continuum shell model to perform non-linear analyses of such structures with and without geometrical imperfections. The model has been examined by numerical computations and experimental studies, and comparisons of results show exactness and effectiveness of use of the proposed model (Nie and Cheung 1995, Nie and Liu 1995).

This paper is devoted to an asymptotic analysis of the snap-through buckling of the above-mentioned structures subjected to uniform vertical load by solving the axisymmetrical fundamental governing equations (coupled equilibrium equation and compatibility equation) in terms of the nondimensional transverse displacement (deflection) and the membrane force with the help of an asymptotic iteration method. For the resulting linear ordinary differential equations in the process of iteration, the power series with rapid convergence has been adopted to obtain an analytical expression for the non-linear characteristic relation related to the load, stiffness of spring and central transverse displacement of the structure. Numerical examples are given to demonstrate the effects of stiffness of spring and characteristic geometrical parameter on buckling of the structures for immovable simply-supported (hinged) and clamped edge conditions. In a special case, for reticulated circular plates, the influence of stiffness of spring on the characteristic relation between load and deflection is also considered.

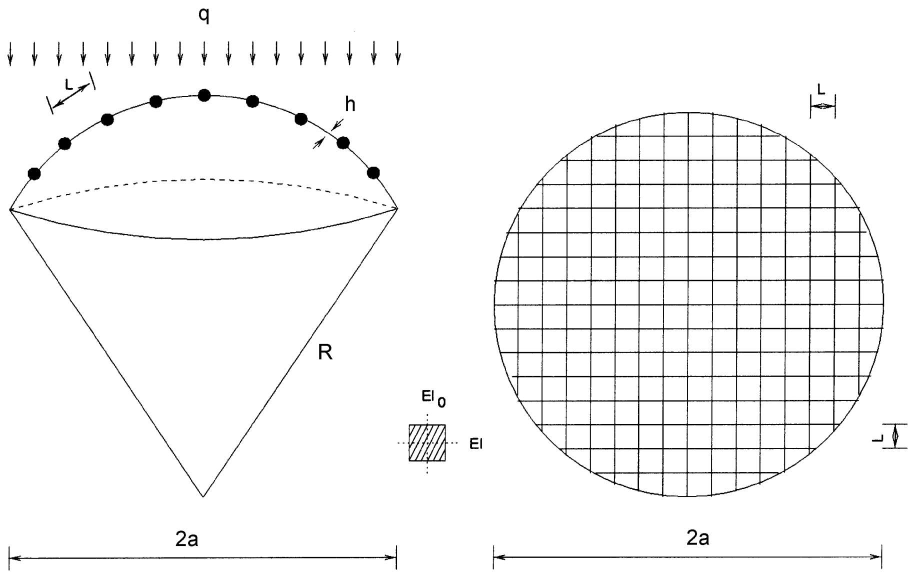
2. The mathematical formulation of problem

Let us consider a squarely-reticulated shallow spherical shell subjected to uniform vertical load q and continuously supported on springs. All the beam members have the same material properties and sizes, and are placed in the same spherical surface and in two orthogonal directions, the middle surface of the shell is defined as the surface interwoven by the centroids of all the cross-sections of the beam members, as shown in Fig. 1. Based on the non-linear analysis of internal forces and deformations of a typical latticed element from the discrete structure, a continuum shell model is adopted and the application of the principle of virtual work leads to the corresponding non-linear governing equations. Taking into account the effect of stiffness of spring k_m, non-linear governing equations are expressed by the nondimensional transverse displacement (deflection) W and internal force T as follows (Nie and Cheung 1995).

$$\pounds(W)+K_f W=\frac{1}{\rho}\frac{d}{d\rho}\left[T\left(M_1\rho+\frac{dW}{d\rho}\right)\right]+2Q \tag{1}$$

$$\hbar(\rho T)=M_2\hbar\left(\rho\frac{dW}{d\rho}\right)-\frac{dW}{d\rho}\left(M_1\rho+\frac{1}{2}\frac{dW}{d\rho}\right)+M_5\left[\left(\frac{d^2W}{d\rho^2}\right)^2-\left(\frac{dW}{\rho d\rho}\right)^2\right]$$
$$+\frac{M_5}{2}\rho\frac{d}{d\rho}\left[\left(\frac{d^2W}{d\rho^2}\right)^2-\left(\frac{dW}{\rho d\rho}\right)^2\right]-M_3Q\rho^2 \tag{2}$$

in which



(a) (b)

Fig. 1 The geometry of the reticulated spherical shell

$$\rho=\frac{r}{a},\quad K_f=\frac{4La^4k_m}{3EI+GJ}$$

$$W=\frac{4w}{\sqrt{(3EI+GJ)\left(\frac{3}{EA}+\frac{L^2}{12EI_0}\right)}}$$

$$Q=\frac{8La^4}{(3EI+GJ)\sqrt{(3EI+GJ)\left(\frac{3}{EA}+\frac{L^2}{12EI_0}\right)}}q$$

$$T=\frac{4La^2\rho}{3EI+GJ}N_r,\quad N_r=\frac{1}{r}\frac{d\phi}{dr}-F,\quad F=\frac{r^2}{2R}q \tag{3}$$

and M_1, M_2, M_3, M_4, M_5 are nondimensional material constants, and listed in 'Notation'. The differential operators $\mathcal{L}$, $\hbar$ are defined by

$$\mathcal{L}(\ldots)=\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}\rho\frac{d}{d\rho}(\ldots)$$

$$\hbar(\ldots)=\rho\frac{d}{d\rho}\frac{1}{\rho}\frac{d}{d\rho}(\ldots)$$

For the case of a hinged structure, boundary conditions can be expressed by (Liu *et al.* 1991, Nie and Cheung 1995)

$$W=0,\ \frac{d^2W}{d\rho^2}+m\frac{dW}{d\rho}=0,\ \frac{dT}{d\rho}-nT+M_4Q=0,\ \rho=1 \tag{4}$$

$$\frac{dW}{d\rho}=0,\ T=0,\ \ \rho=0 \tag{5}$$

where $m=\dfrac{EI-GJ}{3EI+GJ}$, $n=\dfrac{\sqrt{2}}{4+\sqrt{2}-\dfrac{8}{\pi}}$. It can be observed that, if $m\to\infty$ in Eq. (4), then the above conditions will change to the ones for the case of an immovable clamped edge. Hence, in the following section only the case of a hinged edge will be considered in seeking the solutions for W, T by using an asymptotic method.

3. The solution by asymptotic iteration method

Following the same procedures used in Liu *et al.* (1991) and Nie and Cheung (1995), first, from Eqs. (1) and (2) in conjunction with conditions (4) and (5), a linear boundary-value problem containing only $W(\rho)$ is expressed by

$$\pounds[W^{(1)}]+K_fW^{(1)}=2Q \tag{6}$$

$$\hbar(\rho T^{(1)})=M_2\hbar\left(\rho\frac{dW^{(1)}}{d\rho}\right)-\frac{dW^{(1)}}{d\rho}\left(M_1\rho+\frac{1}{2}\frac{dW^{(1)}}{d\rho}\right)$$

$$+M_5\left[\left(\frac{d^2W^{(1)}}{d\rho^2}\right)^2-\left(\frac{dW^{(1)}}{\rho d\rho}\right)^2\right]+\frac{M_5}{2}\rho\frac{d}{d\rho}\left[\left(\frac{d^2W^{(1)}}{d\rho^2}\right)^2-\left(\frac{dW^{(1)}}{\rho d\rho}\right)^2\right]-M_3Q\rho^2 \tag{7}$$

$$W^{(1)}=0,\ \ \frac{d^2W^{(1)}}{d\rho^2}+m\frac{dW^{(1)}}{d\rho}=0,\ \ \frac{dT^{(1)}}{d\rho}-nT^{(1)}+M_4Q=0,\ \ \rho=1 \tag{8}$$

$$\frac{dW^{(1)}}{d\rho}=0,\ \ \ T^{(1)}=0,\ \ \ \rho=0 \tag{9}$$

and denote

$$W^{(1)}(\rho)|_{\rho=0}=W_m \tag{10}$$

in which $W^{(1)}$, $T^{(1)}$ correspond to the solutions for the first iteration. The solution for Eq. (6) is expressed by using two power series and a particular solution as follows (Nie 1999)

$$W^{(1)}(\rho)=A_1\sum_{k=0}^{\infty}b_k\rho^{4k}+A_2\sum_{k=0}^{\infty}c_k\rho^{4k+2}+\frac{2Q}{K_f} \tag{11}$$

where A_1, A_2 are unknown constants, the coefficients b_k, c_k (k =1, 2, ...) depend on K_f and

$$b_0=c_0=1$$

$$b_k=(-K_f)^kg_k,\ \ c_k=(-K_f)^kf_k,\ b_k=(2k+1)^2c_k \qquad (k=1, 2, \ldots)$$

$$f_k = \frac{1}{16^k[(2k+1)!]^2},\quad g_k = \frac{1}{16^k[(2k)!]^2} \qquad (k=1, 2, \ldots) \tag{12}$$

Using Eqs. (8) and (9), the following equation is obtained

$$W^{(1)}(\rho) = \frac{2Q}{K_f\delta_1}\left(\sum_{k=0}^{\infty} b_k\rho^{4k} - \delta_2\sum_{k=0}^{\infty} c_k\rho^{4k+2} + \delta_1\right) \tag{13}$$

in which coefficients δ_1, δ_2 are expressed in 'Notation'.

Introducing Eq. (10) to Eq. (13), a linear relation between Q and W_m can be derived

$$Q = \frac{K_f\delta_1}{2(\delta_1+1)}W_m \tag{14}$$

Accordingly, $W^{(1)}$ can be rewritten by

$$W^{(1)}(\rho) = W_0\left(\sum_{k=0}^{\infty} b_k\rho^{4k} - \delta_2\sum_{k=0}^{\infty} c_k\rho^{4k+2} + \delta_1\right) \tag{15}$$

where

$$W_0 = \frac{1}{\delta_1+1}W_m \tag{16}$$

Substitution of Eq. (15) into Eq. (7), $T^{(1)}$ can be solved by applying the corresponding conditions (8) and (9), its result is

$$\begin{aligned}
T^{(1)} = &\left\{c_1^0\rho - \frac{M_3\delta_1}{16}K_f\rho^3 + M_2\left[\sum_{k=1}^{\infty} 4kb_k\rho^{4k-1} - \delta_2\sum_{k=0}^{\infty}(4k+2)c_k\rho^{4k+1}\right]\right.\\
&\left.-M_1\left[\sum_{k=1}^{\infty}\frac{1}{4k+2}b_k\rho^{4k+1} - \delta_2\sum_{k=0}^{\infty}\frac{1}{4k+4}c_k\rho^{4k+3}\right]\right\}W_0 +\\
&\left\{c_2^0\rho - \sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{8kj}{[4(k+j)-2][4(k+j)]}b_kb_j\rho^{4(k+j)-1}\right.\\
&-\delta_2^2\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}\frac{(4k+2)(4j+2)}{2[4(k+j)+2][4(k+j)+4]}c_kc_j\rho^{4(k+j)+3}\\
&+\delta_2\sum_{k=1}^{\infty}\sum_{j=0}^{\infty}\frac{4k(4j+2)}{[4(k+j)][4(k+j)+2]}b_kc_j\rho^{4(k+j)+1}\\
&+M_5\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{4k(4j)[(4k-1)(4j-1)-1]}{8(k+j-1)}b_kb_j\rho^{4(k+j)-3}\\
&+\delta_2^2M_5\sum_{k=1}^{\infty}\sum_{j=1}^{\infty}\frac{(4k+2)(4j+2)[(4k+1)(4j+1)-1]}{8(k+j)}c_kc_j\rho^{4(k+j)+1}
\end{aligned}$$

$$+4\delta_2^2 M_5 \sum_{j=1}^{\infty} (2j+1)c_j \rho^{4j+1}$$

$$-2\delta_2 M_5 \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} \frac{4k(4j+2)[(4k-1)(4j+1)-1]}{8(k+j)-4} b_k c_j \rho^{4(k+j)-1} \Bigg\} W_0^2 \tag{17}$$

in which

$$c_1^0 = \frac{1}{n-1} \Bigg\{ M_2 \left[\sum_{k=1}^{\infty} 4k(4k-1-n)b_k - \delta_2 \sum_{k=0}^{\infty} (4k+2)(4k+1-n)c_k \right]$$

$$-M_1 \left(\sum_{k=1}^{\infty} \frac{4k+1-n}{4k+2} b_k - \delta_2 \sum_{k=0}^{\infty} \frac{4k+3-n}{4k+4} c_k \right) - \frac{K_f \delta_1}{2} \left[\frac{(3-n)M_3}{8} - M_4 \right] \Bigg\} \tag{18}$$

and

$$c_2^0 = \frac{1}{n-1} \Bigg\{ -\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{8kj[4(k+j)-1-n]}{[4(k+j)-2][4(k+j)]} b_k b_j$$

$$-\delta_2^2 \sum_{k=0}^{\infty} \sum_{j=0}^{\infty} \frac{(4k+2)(4j+2)[4(k+j)+3-n]}{2[4(k+j)+2][4(k+j)+4]} c_k c_j$$

$$+\delta_2 \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} \frac{4k(4j+2)[4(k+j)+1-n]}{[4(k+j)][4(k+j)+2]} b_k c_j$$

$$+M_5 \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{2kj[(4k-1)(4j-1)-1][4(k+j-1)+1-n]}{(k+j-1)} b_k b_j$$

$$+\delta_2^2 M_5 \sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \frac{(4k+2)(4j+2)[(4k+1)(4j+1)-1][4(k+j)+1-n]}{8(k+j)} c_k c_j$$

$$+4\delta_2^2 M_5 \sum_{j=1}^{\infty} (2j+1)(4j+1-n)c_j$$

$$-2\delta_2 M_5 \sum_{k=1}^{\infty} \sum_{j=0}^{\infty} \frac{4k(4j+2)[(4k-1)(4j+1)-1][4(k+j)-1-n]}{8(k+j)-4} b_k c_j \Bigg\} \tag{19}$$

Next, adopting the solutions resulting from the first iteration as reference variables, the corresponding differential equation for $W^{(2)}$, i.e., the solution for the second iteration, is formulated as follows

$$\mathcal{L}[W^{(2)}(\rho)] + K_f W^{(2)}(\rho) = \frac{1}{\rho} \frac{d}{d\rho} \left\{ T^{(1)}(\rho) \left[M_1 \rho + \frac{dW^{(1)}}{d\rho} \right] \right\} + 2Q \tag{20}$$

The boundary conditions are

$$W^{(2)}=0,\ \frac{d^2W^{(2)}}{d\rho^2}+m\frac{dW^{(2)}}{d\rho}=0,\quad \rho=1 \tag{21}$$

$$\frac{dW^{(2)}}{d\rho}=0\quad \rho=0 \tag{22}$$

and

$$W^{(2)}(\rho)|_{\rho=0}=W_m \tag{23}$$

The solution for the above differential equation has the following form

$$W^{(2)}=A_3\sum_{k=0}^{\infty}b_k\rho^{4k}+A_4\sum_{k=0}^{\infty}c_k\rho^{4k+2}+\frac{2Q}{K_f}+W^*(\rho)$$

in which A_3, A_4 are also two unknown constants, and the main part of particular solution resulting from $T^{(1)}$ and $W^{(1)}$, denoted by $W^*(\rho)$, is

$$W^*(\rho)=\frac{1}{K_f}[V_1(\rho)W_0+V_2(\rho)W_0^2+V_3(\rho)W_0^3]$$

where the mathematical formulae for $V_1(\rho)$, $V_2(\rho)$ and $V_3(\rho)$ are omitted due to their lengthy algebraic formulations. Utilizing the conditions (21)-(23), and considering Eq. (16), a nonlinear relation between load Q and central transverse displacement W_m is established as follows

$$Q=\alpha_1W_m+\alpha_2W_m^2+\alpha_3W_m^3 \tag{24}$$

in which

$$\alpha_i=\frac{1}{2(\delta_1+1)}\left\{\delta_1\left[D(i)K_f-V_i(0)\left(\frac{1}{\delta_1+1}\right)^i\right]+\delta_3[V_i''(1)+mV_i'(1)]\left(\frac{1}{\delta_1+1}\right)^i-V_i(1)\left(\frac{1}{\delta_1+1}\right)^i\right\}$$
$$(i=1,\ 2,\ 3) \tag{25}$$

where the function $D(i)$ is defined by

$$D(i)=\begin{cases}1 & i=1\\ 0 & i\neq 1\end{cases}$$

the expression for δ_3 can be found in 'Notation', and

$$V_i(0)=V_i(\rho)|_{\rho=0},\ V_i(1)=V_i(\rho)|_{\rho=1}$$
$$V_i'(1)=\frac{dV_i(\rho)}{d\rho}|_{\rho=1},\ V_i''(1)=\frac{d^2V_i(\rho)}{d\rho^2}|_{\rho=1} \tag{26}$$

The coefficients α_i (i=1, 2, 3) are related to nondimensional stiffness of spring K_f and characteristic geometrical parameter, namely M_1. Using the extremum condition $(dQ/dW_m)=0$ in Eq. (24), the critical loads can be obtained by determining the corresponding values of W_m expressed by

$$W_m=\frac{-\alpha_2\pm\sqrt{\alpha_2^2-3\alpha_1\alpha_3}}{3\alpha_3} \tag{27}$$

Especially, for a given value of K_f, $\alpha_2^2 - 3\alpha_1\alpha_3 = 0$ determines the critical value of characteristic geometrical parameter denoted by M_{1cr}, corresponding to the case that snap-through buckling just occurs. For the case, the critical load has the following result

$$Q_{cr} = -\frac{\alpha_1 \alpha_2}{9\alpha_3} \tag{28}$$

4. Numerical results and analysis

For the purpose of checking the derived analytic expressions, based on present model and finite element method (FEM), respectively, some comparisons are made for two clamped reticulated structures with specific geometrical sizes and material constants, i.e., their spans $2a$=$2m$, each beam member having length L=0.2m and the same circular cross-section with 5.8 mm diameter. The curvature radii are R=15m and $R=\infty$, respectively. The latter corresponds to a reticulated circular plate structure. Poisson's ratio is 0.3, and elastic modulus E=1.99×10^{10} kg/m^2. The stiffness of supporting spring k_m=104.17 kg/m^2. The results for the relation between central transverse displacement and the load are presented in Figs. 2 and 3, respectively. From the two figures, a good agreement between present and numerical models can be observed. Especially, for reticulated structures without supporting springs, the present model has been compared with numerical model by using a versatile program for structural analysis-COSMOS (Nie and Cheung 1995). The computational results indicate that, even if the present structural model behaves as anisotropic one, the structure has approximately an axisymmetrical deformation when beam elements are placed densely, e.g. the length of each beam member does not exceed one-eighth of the span of the shell. Accordingly, the present problem can be simplified by considering the case of axisymmetry.

In the present paper all nondimensional numerical computations have been carried out for the given nondimensional parameters h/L=0.029 and L/a=0.2. According to Eq. (12), the values of f_k and g_k(k=1–6) are listed in Table 1. It can be noted that they rapidly decrease with the increase of value of k. For a given value of K_f, the values of coefficients b_k, c_k have similar changes with k, this leads to fast convergence for all undetermined variables and quantities expressed in series.

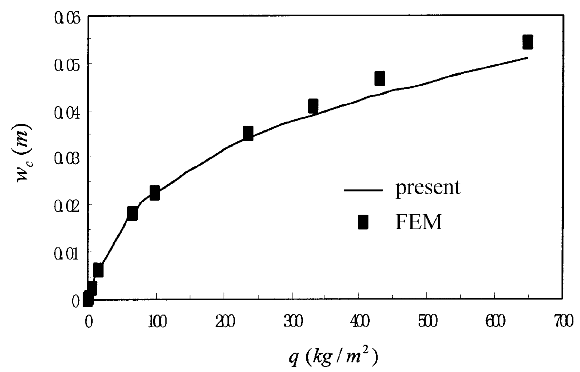


Fig. 2 A comparison of the present results with corresponding data by finite element calculations for the clamped reticulated circular plate

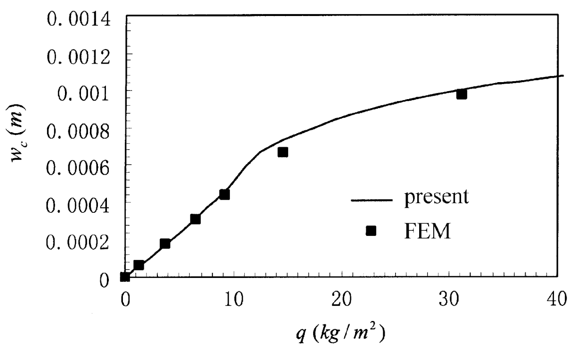


Fig. 3 A comparison of the present results with corresponding data by finite element calculations for the clamped reticulated spherical shell

Table 1 Values of f_k and g_k (k=1–6)

f_1	f_2	f_3	f_4	f_5	f_6
1.74×10^{-3}	2.71×10^{-7}	9.61×10^{-12}	1.16×10^{-16}	5.99×10^{-22}	1.54×10^{-27}

g_1	g_2	g_3	g_4	g_5	g_6
1.56×10^{-2}	6.78×10^{-6}	4.71×10^{-10}	9.39×10^{-15}	7.24×10^{-20}	2.60×10^{-25}

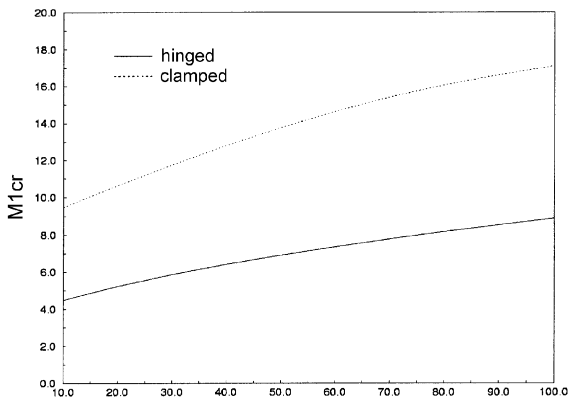


Fig. 4 The relation between the critical characteristic geometrical parameter of the reticulated spherical shells and the stiffness of spring

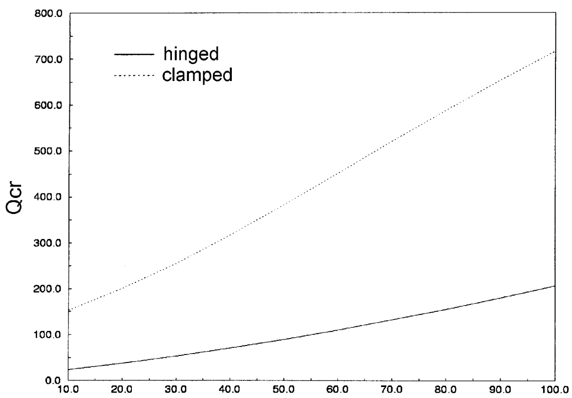


Fig. 5 The relation between the critical load of the reticulated spherical shell and the stiffness of spring

The changes of critical values of characteristic geometrical parameter M_{1cr} and load Q_{cr} with the value of stiffness of spring K_f are displayed in Figs. 4 and 5 respectively. When K_f increases, M_{1cr} will become larger which corresponds to "deeper" reticulated shallow spherical shell. Simultaneously, the corresponding critical load is also larger for a harder spring. This coincides with the behaviors of reticulated shallow shell structures on elastic Winkler foundation and with rectangular boundaries (Nie 1994).

Figs. 6-8 illustrate the effects of stiffness of spring on buckling of hinged reticulated shallow spherical shells for M_1=6.482179 and M_1=17.825993 respectively and clamped one for M_1= 17.825993. It can be observed that loop shapes always appear for small values of K_f, which indicates that there is a large increase in the value of the transverse displacement when the value of the load increases to a critical value, i.e., the snapping phenomena will happen. Softer springs with a smaller value of K_f correspond to a lower critical load. Meanwhile, Fig. 7 presents an phenomenon of having negative Q values for K_f<100.00. It states clearly that, after snapping phenomena occur, during resilience residual deformation remains until an opposite vertical load is exerted on the shell. When the value of the opposite load increases to a critical value, the shell bounces back. Then the residual deformation will disappear when the opposite load decreases to zero. For enough hard supporting springs, when K_f> 41.46 and K_f> 326.99 for the two hinged structures, and K_f> 121.34 for the clamped one, the corresponding zones of negative rigidity disappear and show that snapping phenomena no longer exist. Also, for the same structure, there is a bigger value of K_f corresponding

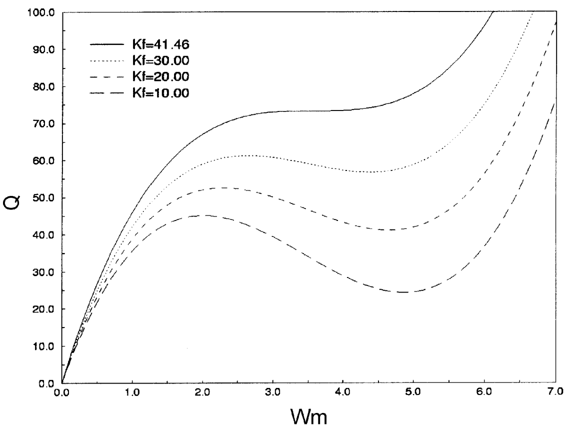


Fig. 6 The effect of stiffness of spring on buckling of the hinged reticulated spherical shell for M_1=6.482179

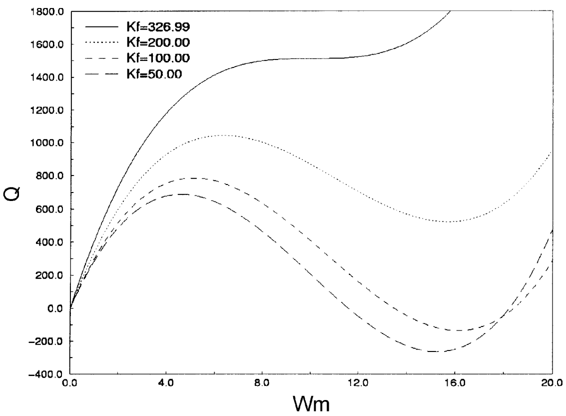


Fig. 7 The effect of stiffness of spring on buckling of the hinged reticulated spherical shell for M_1=17.825993

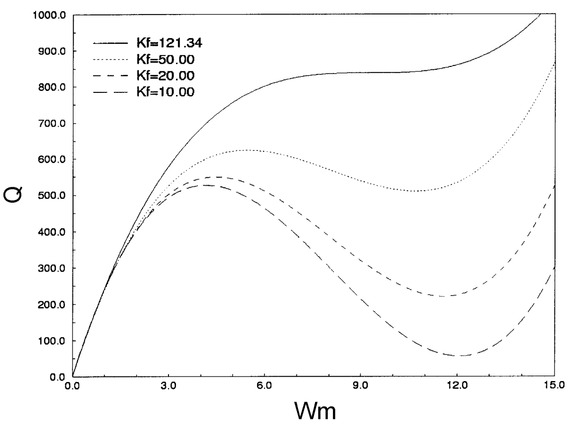


Fig. 8 The effect of stiffness of spring on buckling of the clamped reticulated spherical shell for M_1=17.825993

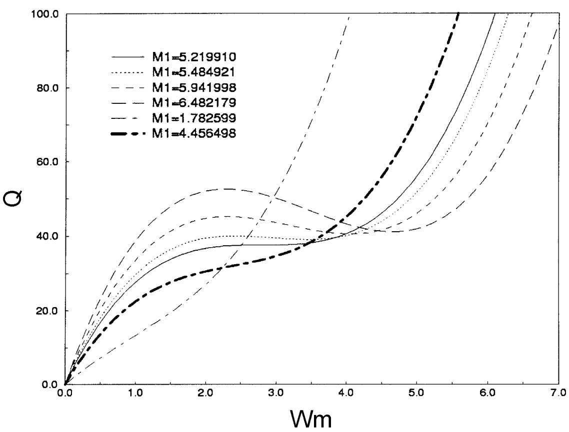


Fig. 9 The effect of characteristic geometrical parameter on buckling of the hinged reticulated spherical shells for K_f=20.00

to which snap-through buckling becomes possible for the case of hinged edge (see Figs. 7 and 8).

For hinged and clamped reticulated spherical shells with various curvature radii determined by nondimensional characteristic geometrical parameter M_1, supported on the same springs with K_f= 20.00, the effects of this geometrical parameter on buckling of the structures are shown graphically in Figs. 9 and 10, respectively. For "flatter" shallow shells with smaller values of M_1 or larger curvature radii R, e.g., when $M_1 < 5.219910$ for hinged structures while $M_1<10.642385$ for clamped ones, the values of of the transverse displacement increase with the increasing load, and there are no loop shapes appearing. However, for "deeper" shallow shells, snapping phenomenon will happen when the values of M_1 exceed the above given values for the two boundary conditions. A bigger value of M_1 results in a larger critical load.

In a special case, for the reticulated plates, i.e., M_1=0, the curves for the relation between the load and central deflection are plotted in Fig. 11. It is clear that central deflection monotonically

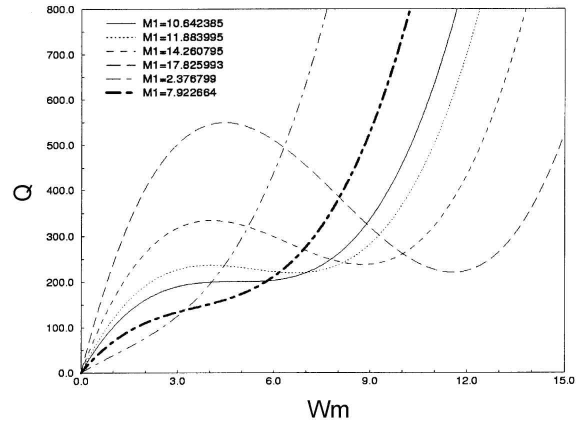


Fig. 10 The effect of characteristic geometrical parameter on buckling of the clamped reticulated spherical shells for K_f=20.00

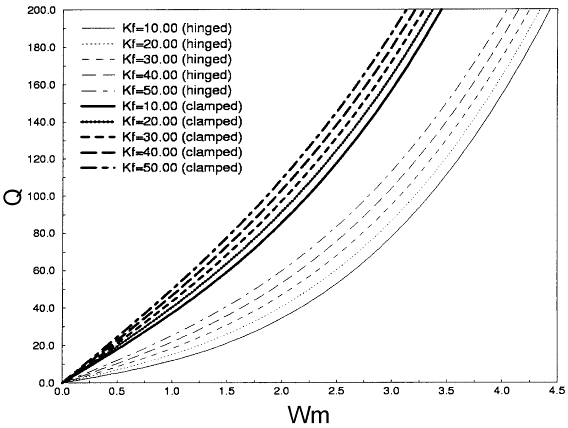


Fig. 11 The relation between the load and central deflection of the reticulated circular plate

increases when the load becomes large, there is never snapping phenomenon. It can be also noted that, for an identical load, the value of transverse displacement is larger for a smaller value of K_f corresponding to softer springs. The conclusion agrees with available results for the same kind of structures and solid anisotropic plates (Chia 1980, Nie 1994).

5. Conclusions

In the present work an asymptotic analysis of snap-through buckling of reticulated shallow spherical shells continuously supported on springs is given.

The analysis and computation results show that stiffness of spring plays an important role in buckling behaviors of such structures while the geometries of the structures also influence these behaviors. The action of springs can result in an increase in the load-bearing capacity of the structures and prevent them from buckling. In special, for the reticulated plates, application of springs can decrease the deformation and enhance the safety of use of the structures.

It should be noted that the present shell model is based on the use of shallow shell theory, the model is thus applicable to shallow shell structures only. Meanwhile, beam elements must be placed densely, as has already been stated, the length of each beam member does not exceed one-eighth of the span of the shell. In addition, in the process of solution resulting from power series, stiffness of spring, k_m or K_f, can not be taken zero. However, when k_m=0 corresponding to the shell without supporting springs, the fundamental governing equations will change to the known ones and the corresponding solution has been obtained by the same asymptotic iteration method (Nie and Cheung 1995).

It may be also concluded that the developed and used method in this paper may be extended to non-linear buckling analysis of anisotropic and composite shell structures on elastic Winkler foundation.

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Notation

L	: length of each beam member
A	: area of cross-section of each beam member
R	: radius of curvature of the shell
h	: thickness of the shell
$2a$	: span of the shell
EI	: transverse bending stiffness
EI_0	: lateral bending stiffness
GJ	: twisting stiffness
q	: uniform vertical load
Q	: nondimensional uniform vertical load
k_m	: stiffness of spring
K_f	: nondimensional stiffness of spring
M_1	: $\dfrac{4a^2}{R\sqrt{(3EI+GJ)\left(\dfrac{3}{EA}+\dfrac{L^2}{12EI_0}\right)}}$
M_2	: $\dfrac{GJL^2}{8EI_0R\sqrt{(3EI+GJ)\left(\dfrac{3}{EA}+\dfrac{L^2}{12EI_0}\right)}}$
M_3	: $\dfrac{1}{R}\left(\dfrac{4}{EA}+\dfrac{L^2}{6EI_0}\right)\sqrt{\dfrac{3EI+GJ}{\dfrac{3}{EA}+\dfrac{L^2}{12EI_0}}}$
M_4	: $\dfrac{1}{2R}\sqrt{(3EI+GJ)\left(\dfrac{3}{EA}+\dfrac{L^2}{12EI_0}\right)}$
M_5	: $\dfrac{(GJ-EI)L^2}{32EI_0a^2}$
b_k, c_k	: coefficients in power series
δ_1	: $\dfrac{\sum_{k=0}^{\infty}\sum_{j=0}^{\infty}(4k-4j-2)(4k+4j+1+m)b_kc_j}{\sum_{k=0}^{\infty}(4k+2)(4k+1+m)c_k}$
δ_2	: $\dfrac{\sum_{k=0}^{\infty}4k(4k-1+m)b_k}{\sum_{k=0}^{\infty}(4k+2)(4k+1+m)c_k}$
δ_3	: $\dfrac{\sum_{k=0}^{\infty}c_k}{\sum_{k=0}^{\infty}(4k+2)(4k+1+m)c_k}$
w	: transverse displacement (deflection)
W	: nondimensional transverse displacement (deflection)
W_m	: nondimensional central transverse displacement (deflection)