Steel and Composite Structures, Vol. 9, No. 6 (2009) 535-555 535
DOI: http://dx.doi.org/10.12989/scs.2009.9.6.535

Optimum design of geometrically non-linear steel frames with semi-rigid connections using a harmony search algorithm

S.O. Degertekin*, M.S. Hayalioglu and H. Gorgun

Department of Civil Engineering, Dicle University, 21280, Diyarbakir, Turkey

(Received June 8, 2009, Accepted October 21, 2009)

Abstract. The harmony search method based optimum design algorithm is presented for geometrically non-linear semi-rigid steel frames. Harmony search method is recently developed metaheuristic algorithm which simulates the process of producing a musical performance. The optimum design algorithm aims at obtaining minimum weight steel frames by selecting from standard set of steel sections such as European wide flange beams (HE sections). Strength constraints of Turkish Building Code for Steel Structures (TS648) specification and displacement constraints were used in the optimum design formulation. The optimum design algorithm takes into account both the geometric non-linearity of the frame members and the semi-rigid behaviour of the beam-to-column connections. The Frye-Morris polynomial model is used to calculate the moment-rotation relation of beam-to-column connections. The robustness of harmony search algorithm, in comparison with genetic algorithms, is verified with two benchmark examples. The comparisons revealed that the harmony search algorithm yielded not only minimum weight steel frames but also required less computational effort for the presented examples.

Keywords : optimum design; harmony search; steel frames; semi-rigid connections.

1. Introduction

In the analysis of steel frames the real behaviour of connections are generally idealized either pinned or fully rigid. The rigid connection idealization indicates that relative rotation of the connection does not exist and the end moment of the beam is entirely transferred to the column. In contrast to the rigid connection assumption, the pinned connection idealization indicates that any restraint does exist for rotation of the connection and the connection moment is zero. Although these idealizations simplify the analysis and design process, the predicted response of the frame may be different from its real behaviour. Numerous experimental studies proved that all beam-to-column connections posses some flexural stiffness between these two extreme assumptions. The term semi-rigid is used to express the real connection behaviour. Therefore, beam-to-column connections in the analysis/design of steel frames should be described as semi-rigid connections. American Institute of Steel Construction-Allowable Stress Design AISC-ASD (1989) specification defines three types of steel constructions: rigid-frame, simple framing (unrestrained) and semi-rigid framing (partially restrained). This specification requires that the connections of the type of partially restrained construction have a flexibility intermediate

* Associate Professor, Corresponding Author, Email: sozgur@dicle.edu.tr

in degree between the rigidity of Type 1 and the flexibility of Type 2, and this type of construction may necessitate non-elastic (non-linear) deformations of structural steel parts. However, no guideline is available for the design of semi-rigid steel frames in this specification.

Analysis and design of steel frames with semi-rigid connections have been the subject of a lot of works (Monforton and Wu 1963, Frye and Morris 1975, Lui and Chen 1986, Cunningham 1990, King 1994, King and Chen 1994, Dhillon and O'Malley 1999, Sekulovic and Salatic 2001, Kaveh and Moez 2006, Wang 2008, Ihaddoudene, *et al.* 2009). Optimum design of steel frames with semi-rigid connections has also been studied by several researchers. The early studies on the subject use mathematical programming techniques (Xu and Grierson 1993, Almusallam 1995, Alsalloum and Almusallam 1995, Simoes 1996). These methods utilizes gradient of functions to search the design space and they tend to reach local optimum solutions. Furthermore, they are largely suitable for optimization problems with continuous design variables and are not good enough for problems with discrete design variables. However, the availability of standard steel sections and their limitations for construction and manufacturing reasons necessitate that design variables selections should be made from standard steel section lists recommended by design codes.

In order to tackle with these deficiencies of classical techniques, heuristic search methods emerged in the first half of 1990s. All heuristic search algorithms are inspired from natural phenomenon. The name of each heuristic method is indicative of its underlying principle. One of the most popular heuristic search techniques is genetic algorithms (GAs). They are based on evolution theory of Darwin's which are proposed by Holland (1975). The main principle of GAs is the survival of robust ones and the elimination of the others in a population. On the contrary to mathematical programming methods, GAs cope with discrete optimum design problems and do not need derivatives of functions. In addition to its different engineering applications, GAs are also applied to optimum design of steel frames with semi-rigid connections (Kameshki and Saka 2001, 2003; Hayalioglu and Degertekin 2004a, 2004b, 2005; Csebfalvi 2007). From the literature survey, it should be said that optimum design of semi-rigid steel frames has drawn much less attention than their analysis and design.

Harmony search (HS) algorithm is another heuristic method developed by Geem, *et al.* (2001) solving combinatorial optimization problems. HS bases on the analogy between the performance process of natural music and searching for solutions to optimization problems. HS has been applied to a diverse range of optimization problems. One of these problems is the optimization of structural systems. The application of HS to the optimization of structural systems could be summarized as follows: Lee and Geem (2004) used HS algorithm for optimization of planar/space truss structures with continuous design variables, Lee, *et al.* (2005) optimized planar/space truss structures with discrete design variables, Degertekin (2008a, 2008b) employed HS to the optimum design of planar/space steel frames. Saka (2009) reported optimized designs for planar steel frame structures using HS algorithm.

In comparison with GAs, HS has the following superiorities: (i) HS generates a new design considering all existing designs whereas GAs generate a new design from a couple of chosen parents by exchanging the artificial genes; (ii) HS takes into account each design variable independently. On the other hand, GAs consider design variables dependently with building block theory (Goldberg 1989). (iii) HS has memory facility which preserves the better designs in the search process, while GAs has not any memory facility.

The literature survey revealed that optimization of structural systems using HS are about either ideally pinned truss structures or fully rigid steel frames. The main goal of this study is to introduce a HS-based optimum design algorithm for geometrically non-linear steel frames with semi-rigid connections. The objective of optimum design problem is to minimize the weight of steel frames with semi-rigid connections under the actual design constraints. The performance of HS is verified by using two steel frames with semi-rigid connections which are available in the literature.

2. Optimum design problem

Formulation of an optimum design problem involves transcribing a verbal description of the problem into a well-defined mathematical statement Arora (1989). A set of variables, called design variables, are given in the formulation to describe the design. All steel designs have to satisfy a given set of constraints such as serviceability and strength requirements specified in the design codes. If a design satisfies all constraints, it is accepted as a feasible design. A criterion is needed to decide whether a design is better than the other. This criterion is called the objective function.

The discrete optimum design problem of steel frames with semi-rigid connections, where the minimum weight considered as the objective and the steel sections taken from standard set of steel sections of European wide flange beams (HE sections) treated as design variables, subjected to the strength constraints of TS648 (1980) and displacement constraints, is formulated as follows

$$\text{Minimize } W(x) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i \tag{1}$$

where mk = total number of members in group k, ρ_i = density of member i, L_i = length of member i, A_k = cross-sectional area of the members belonging to group k, and ng = total number of member groups in the frame. It should be noted that this study consists of two benchmark design examples taken from previous article of authors' (Hayalioglu and Degertekin 2004a). Therefore, the optimum design formulation and analysis of steel frames with semi-rigid connections are the same as the ones of this article. The unconstrained objective function $\varphi(x)$ is then written as

$$\varphi(x) = W(x)(1 + c \times K) \tag{2}$$

where K = constraint violation function, c = penalty constant selected depend on the problem. K is calculated as:

$$K = \sum_{i=1}^{N_s} K_i^d + \sum_{i=1}^{N_c} K_i^s \tag{3}$$

where K_i^d = constraint violation for displacement, K_i^s = constraint violation for strength requirements of the TS648 specification. N_s = total number of restricted displacements, N_c= number of members. The penalty may be expressed as

$$K_i = \begin{cases} 0 & if\ g_i \le 0 \\ g_i & if\ g_i > 0 \end{cases} \tag{4}$$

The displacement constraint is:

$$g_i^d = \frac{\delta_i}{\delta_{iu}} - 1.0 \tag{5}$$

where δ_i is the displacement of the i-th degree of freedom, δ_{iu} is its upper limit.

The strength constraints taken from TS648 (1980) are given in the following interaction equations. For members subjected to both axial compression and bending stress:

$$g_i^s = \left[\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{C_m \sigma_{bx}}{\left(1 - \frac{\sigma_{eb}}{\sigma'_{ex}}\right)\sigma_{Bx}}\right]_i - 1.0 \tag{6}$$

$$g_i^s = \left[\frac{\sigma_{eb}}{0.6\sigma_a} + \frac{\sigma_{bx}}{\sigma_{Bx}}\right]_i - 1.0 \tag{7}$$

If $\sigma_{eb}/\sigma_{bem} \le 0.15$, Eq.(8) is used in lieu of Eqs.(6) and (7),

$$g_i^s = \left[\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}}\right]_i - 1.0 \tag{8}$$

and,

$$g_i^s = \left[\frac{\sigma_{ec}}{0.60\sigma_a} + \frac{\sigma_{cx}}{\sigma_{cem}}\right]_i - 1.0 \tag{9}$$

where the subscript x, combined with subscripts b, B and e indicates the axis of bending about which a particular stress or design property applies. σ_{eb} = required axial compressive stress, σ_{bem} = axial compressive stress that would be permitted in the existence of axial force alone, σ_{bx} = computed compressive bending stress, σ_{Bx} = compressive bending stress that would be permitted in the existence of bending moment alone, σ'_{ex} = Euler stress divided by a factor of safety, C_m = a coefficient whose value is taken as 0.85 for compression members in unbraced frames, σ_a = the yield stress of steel, σ_{ec} = the computed axial tensile stress, σ_{cx} = the computed bending tensile stress and σ_{cem} = allowable bending stress which is equal to $0.6\sigma_a$.

The strength equations used in TS648 are the same as the equations used in AISC-ASD (1989), except the permitted axial compressive stress and Euler stress divided by a factor of safety. It should be noted that both the permitted axial compressive stress and Euler stress are calculated more conservative in TS 648 (1980) (Deren, *et al.* 2008).

The effective length factor K is required to determine the nominal compressive stress σ_{bem} and Euler stress σ'_{ex} in the design of frame members. K-factor of columns must be calculated to evaluate the stability of columns in frames with rigid and semi-rigid connections. The effective length factor K for the columns in an unbraced frame is determined from the following interaction equation (Kishi, *et al.* 1997):

$$\frac{G_A G_B(\pi/K)^2 - 36}{6(G_A + G_B)} = \frac{\pi/K}{\tan(\pi/K)} \tag{10}$$

where G_A and G_B are modified relative stiffness factors for A-th and B-th ends of columns and expressed as:

$$G = \frac{\sum I_c/L_c}{\sum I_g/L_g} \tag{11}$$

where the summation is taken over all members connected to the joint, and where I_c = moment of inertia of column section corresponding to plane of buckling, L_c = unbraced length of column, I_g = moment of inertia of beam/girder corresponding to plane of bending, and L_g = unbraced length of beam/girder.

It should be noted that Eq.(10) is derived from the assumption of the beams and girders are rigidly connected to columns at the joints. Therefore, the beam/girder stiffness I_g / L_g iven in Eq.(11) is multiplied by the following factors to consider for different end connections (Dhillon and O'Malley 1999):

For unbraced frames, the factor is 0.5 for far ends fixed, 0.67 for pinned, and $1/(1 + 6EI/Lk)$ for flexibly connected, where k is rotational spring stiffness of corresponding end.

3. Connection modelling and analysis of steel frames with semi-rigid connections

The stiffness or flexibility of a beam-to-column connection varies depending on its geometric parameters of the component parts such as the bolt diameter, the gauge in the vertical leg of the flange angle, thickness of the flange angle and the beam depth. The nonlinearity of connection behaviour is due to a number of factors such as material discontinuity of the connection subassemblage, local yielding of some component part, local buckling of a plate element, and so on (Chen, *et al*. 1996).

The moment-rotation relationship is the most important factor for the semi-rigid connection behaviour. The modelling of beam-to-column connections and predicting the real behaviour of them have been demonstrated by a number of experimental and numerical works (Lui and Chen 1986, Chen and Kishi 1989, Abdalla and Chen 1995, Ivanyi 2000, Lee and Moon 2002, Prabha, *et al*. 2008, Pirmoz, *et al*. 2009). Moreover, experimental studies proved that moment-rotation curves of semi-rigid connections are non-linear. The moment-rotation behaviour of semi-rigid connections used in this study is depicted in Fig. 1.

Several mathematical models are developed to curve fit the experimental data of beam-to-column connections (Frye and Morris 1975, Jones, *et al*. 1980, Lui and Chen 1986, Yee and Melchers 1986, Wu and Chen 1990, Kishi and Chen 1990). These models vary from a linear model to polynomial and exponential models. In this study, the semi-rigid connections are modelled with the Frye-Morris polynomial model because of its easy implementation. This model is expressed in the following form (Frye and Morris 1975):

$$\theta_r = c_1(\kappa M)^1 + c_2(\kappa M)^3 + c_3(\kappa M)^5 \tag{12}$$

where κ = standardization constant depends upon connection type and geometry; c_1, c_2, c_3 = curve fitting constants. The values of standardization constants and curve fitting constants for semi-rigid

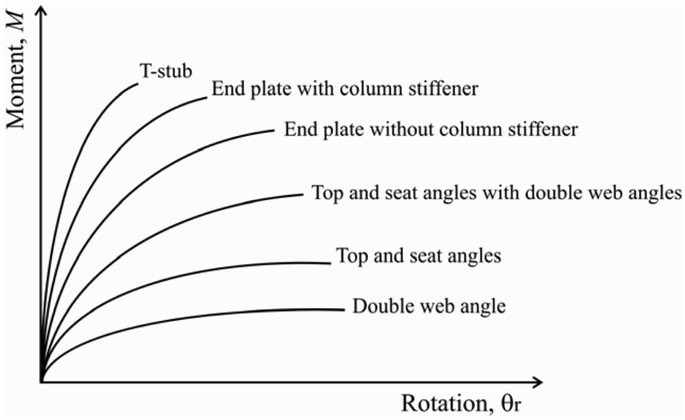

Fig. 1 Connection moment-rotation curves

connections used in this study are given in Table 1 (Chen, *et al*. 1996). The geometry and size parameters of these connections are also shown in Fig. 2.

The non-linear analysis of steel frames with semi-rigid connections includes both the geometrical non-linearity of beam-column members and non-linearity due to end connection flexibility of beam members. The columns of frames are continuous and do not have any internal flexible connections. However, the beams possess semi-rigid end connections, but have small axial forces with a geometric non-linearity of little importance. Based on these considerations, two types of members are defined to design of steel frames with semi-rigid connections. These are beam-column member and beam member with semi-rigid end connections.

3.1 Beam-column member

The stiffness matrix of a beam-column element incorporating P-Δ effect can be expressed as:

$$[k]_i = [k_e]_i + [k_g]_i \tag{13}$$

Table 1 Curve fitting constants and standardization constants for Frye-Morris polynomial model (All size parameters are in inches*)

Connection types	Curve fitting constants			Standardization constants
	C_1	C_2	C_3	
1	3.66×10^{-4}	1.15×10^{-6}	4.57×10^{-8}	$\kappa = d_a^{-2.4} t_a^{-1.81} g^{0.15}$
2	2.23×10^{-5}	1.85×10^{-8}	3.19×10^{-12}	$\kappa = d^{-1.287} t^{-1.128} t_c^{-0.415} l_a^{-0.694} g^{1.35}$
3	8.46×10^{-4}	1.01×10^{-4}	1.24×10^{-8}	$\kappa = d^{-1.5} t^{-0.5} l_a^{-0.7} d_b^{-1.5}$
4	1.83×10^{-3}	1.04×10^{-4}	6.38×10^{-6}	$\kappa = d_g^{-2.4} t_p^{-0.4} d_b^{-1.5}$
5	1.79×10^{-3}	1.76×10^{-4}	2.04×10^{-4}	$\kappa = d_g^{-2.4} t_p^{-0.6}$
6	2.10×10^{-4}	6.20×10^{-6}	-7.60×10^{-9}	$\kappa = d^{-1.5} t^{-0.5} l_t^{-0.7} d_b^{-1.1}$

(*1 inch = 2.54 cm)

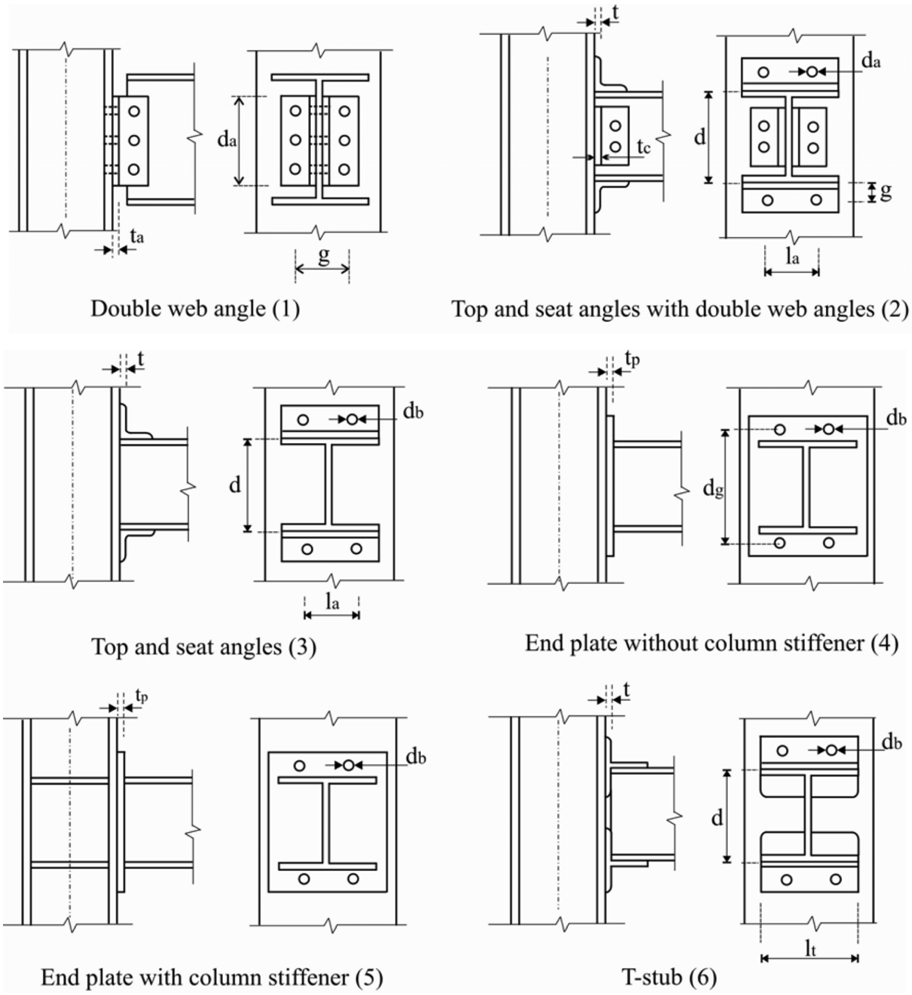

Fig. 2 Connection types and size parameters

where $[k_e]_i$ = conventional linear-elastic stiffness matrix and $[k_g]_i$ = geometrical stiffness matrix (Dhillon and O'Malley 1999). End forces and end displacements of typical plane-frame element in member coordinates are depicted in Fig. 3.

3.2 Beam member with semi-rigid end connections

Fig. 4 illustrates beam member with semi-rigid end connections which is modelled with rotational springs at its both ends.

θ_{rA} and θ_{rB} are the relative spring rotations of both ends and k_A and k_B are the corresponding spring stiffness expressed as:

$$k_A = \frac{M_A}{\theta_{rA}} \tag{14}$$

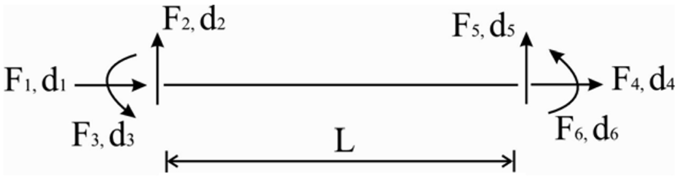

Fig. 3 Plane-frame element with end forces and end displacements

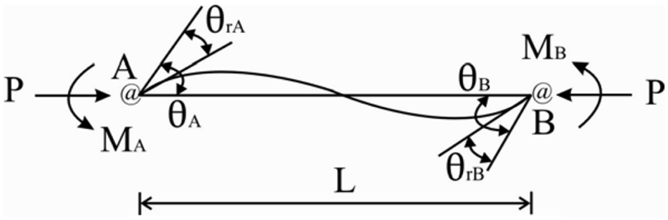

Fig. 4 Beam member with semi-rigid end connections

$$k_B = \frac{M_B}{\theta_{rB}} \tag{15}$$

The relationship between end-moments and end-rotations of a beam can be written by replacing the end-rotations θ_A and θ_B by $(\theta_A - \theta_{rA})$ and $(\theta_B - \theta_{rB})$ respectively, as follows (Hayalioglu and Degertekin, 2005):

$$M_A = \frac{EI}{L}\left[4\left(\theta_A - \frac{M_A}{k_A}\right) + 2\left(\theta_B - \frac{M_B}{k_B}\right)\right] \tag{16a}$$

$$M_B = \frac{EI}{L}\left[4\left(\theta_B - \frac{M_B}{k_B}\right) + 2\left(\theta_A - \frac{M_A}{k_A}\right)\right] \tag{16b}$$

where E is the modulus of elasticity. I and L are the moment of inertia and the length of the member, respectively. Eqs. (16a) and (16b) can be expressed in the following form:

$$M_A = \frac{EI}{L}(r_{ii}\theta_A + r_{ij}\theta_B) \tag{17a}$$

$$M_B = \frac{EI}{L}(r_{ij}\theta_A + r_{jj}\theta_B) \tag{17b}$$

$$r_{ii} = \frac{1}{k_R}\left(4 + \frac{12EI}{Lk_B}\right) \tag{18a}$$

$$r_{jj} = \frac{1}{k_R}\left(4 + \frac{12EI}{Lk_A}\right) \tag{18b}$$

$$r_{ij} = \frac{2}{k_R} \tag{18c}$$

$$k_R = \left(1 + \frac{4EI}{Lk_A}\right)\left(1 + \frac{4EI}{Lk_B}\right) - \left(\frac{EI}{L}\right)^2\left(\frac{4}{k_A k_B}\right) \tag{18d}$$

Eqs. (18) are converted to the following stiffness matrix of a semi-rigid beam member with 6 degrees of freedom in local coordinates (Chen and Lui 1991).

$$[K]_i = \begin{bmatrix} \frac{AE}{L} & & & & & \\ 0 & (r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & & & & \\ 0 & (r_{ii}+r_{ij})\frac{EI}{L^2} & r_{ii}\frac{EI}{L} & & & \\ \frac{-AE}{L} & 0 & 0 & \frac{AE}{L} & & \\ 0 & -(r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & -(r_{ii}+r_{ij})\frac{EI}{L^2} & 0 & (r_{ii}+2r_{ij}+r_{jj})\frac{EI}{L^3} & \\ 0 & (r_{ij}+r_{jj})\frac{EI}{L^2} & r_{ij}\frac{EI}{L} & 0 & -(r_{ij}+r_{jj})\frac{EI}{L^2} & r_{jj}\frac{EI}{L} \end{bmatrix} \tag{19}$$

where A is the cross-sectional area of the member. Applying the known steps of the matrix displacement method, this matrix is obtained in global or structure coordinates for each member and structure stiffness matrix is constituted. The relationships between end-forces and end-displacements are also constructed according to the method (Hayalioglu and Degertekin 2005). In this study, fixed-end forces which are derived in Dhillon and O'Malley (1999) are used for the beam members with semi-rigid end connections.

The iterative numerical analysis procedure used in this study is the same as the authors' previous article (Hayalioglu and Degertekin 2004a). It is originally taken from the study of Dhillon and O'Malley (1999). Therefore, the same definitions and procedures are not repeated here.

4. Optimum design of steel frames with semi-rigid connections using harmony search algorithm

The optimum design algorithm using HS could be explained in the following subsections.

4.1 Initialize the harmony search parameters

The HS algorithm parameters are chosen in this subsection. These parameters are; harmony memory size (HMS), harmony memory consideration rate (HMCR), pitch adjusting rate (PAR) and terminating criteria (number of improvisation). They are selected depending on the problem type.

4.2 Initialize harmony memory

The harmony memory (HM) matrix is filled with randomly generated designs as the size of the harmony memory (HMS).

$$HM = \begin{bmatrix} x_1^1 & x_2^1 & \cdots & x_{ng-1}^1 & x_{ng}^1 \\ x_1^2 & x_2^2 & \cdots & x_{ng-1}^2 & x_{ng}^2 \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ x_1^{HMS-1} & x_2^{HMS-1} & \cdots & x_{ng-1}^{HMS-1} & x_{ng}^{HMS-1} \\ x_1^{HMS} & x_2^{HMS} & \cdots & x_{ng-1}^{HMS} & x_{ng}^{HMS} \end{bmatrix} \begin{matrix} \rightarrow & \varphi(x^1) \\ \rightarrow & \varphi(x^2) \\ \rightarrow & \vdots \\ \rightarrow & \vdots \\ \rightarrow & \varphi(x^{HMS-1}) \\ \rightarrow & \varphi(x^{HMS}) \end{matrix} \tag{20}$$

Each row represents a steel design in the HM. x^1, x^2, ….., x^{HMS-1}, x^{HMS} and $\varphi(x^1)$, $\varphi(x^2)$,…, $\varphi(x^{HMS-1})$, $\varphi(x^{HMS})$ are designs and the corresponding unconstrained objective function values, respectively. The steel designs in the HM are sorted by the unconstrained objective function values (i.e. $\varphi(x^1) < \varphi(x^2) < \ldots\ldots < \varphi(x^{HMS})$) which are calculated by using Eq. (2). It should be noted that HM could not only consist of feasible designs it could also contain infeasible ones. If an infeasible design is obtained in the search process, it is violated by using Eq. (3). Therefore, the probability of existence a feasible design in the vicinity of an infeasible design is considered and premature convergence is also prevented in HS. The aim of using HM is to preserve better designs (i.e. feasible or less violated designs) in the search process. This process is repeated as many as %5×predetermined total number of searches were made. If any design in this process is a feasible one and better than the previous optimum (i.e. lower than $\varphi(x)_{\mathrm{opt}}$), it is assigned as current optimum design. The steel designs are sorted according to their $\varphi(x)$ values until harmony memory matrix is filled completely. The other designs out-of-HM are discarded. The best design with the lowest $\varphi(x)$ one is denoted by $\varphi(x^{best})$ and placed in the first row of HM matrix and the worst design is denoted by $\varphi(x^{worst})$ and placed in the last row of HM matrix. The aim of generating %5×predetermined total number searches is to increase the probability of obtaining feasible or less violated initial designs for HM. Therefore, HS could be started more appropriate initial designs.

4.3 Improvise a new harmony

A new harmony $[x^{nh}] = \lfloor x_1^{nh}, x_2^{nh}, \ldots, x_{ng}^{nh} \rfloor$ is improvised from either the HM or entire section list. Three rules, HM consideration, pitch adjustment and random generation, are applied for the generation of the new harmony.

In the HM consideration, the value of first design variable x_1^{nh} for the new harmony is chosen from any value of the first design variables in the HM (i.e. $x_1^1, x_1^2, ..., x_1^{HMS-1}, x_1^{HMS}$) or entire section list $[X_{sl}]$. $[X_{sl}]$ represents the section list. The other design variables of new harmony $[x_2^{nh}, \ldots, x_{ng-1}^{nh}, x_{ng}^{nh}]$ are chosen by the same rationale. HMCR is applied as follows

$$\begin{cases} x_i^{nh} \in \{x_i^1, x_i^2, \ldots, x_i^{HMS-1}, x_i^{HMS}\} & if \quad rn \le HMCR \\ x_i^{nh} \in X_{sl} & if \quad rn > HMCR \end{cases} \tag{21}$$

At first, a random number (rn) uniformly distributed over the interval [0,1] is generated. If this random number is equal or less than the HMCR value, i-th design variable of new design $[x^{nh}]$ is selected from the current values stored in the i-th column of HM. If rn is greater than HMCR, i-th design variable of new design $[x^{nh}]$ is selected from the entire section list $[X_{sl}]$. For example, an HMCR of 0.80 shows that the algorithm will choose the i-th design variable (i.e. steel section) from the current stored steel sections in the i-th column of the HM with a 80% probability or from the entire section list with a 20% probability. A value of 1.0 for HMCR is not appropriate because of the possibility that the new design may be improved by values not stored in the HM (Lee and Geem 2005).

Any design variable of the new harmony, $[x^{nh}] = \lfloor x_1^{nh}, x_2^{nh}, \ldots, x_{ng}^{nh} \rfloor$, obtained by the memory consideration is examined to determine whether it is pitch-adjusted or not. Pitch adjustment is made by pitch adjustment ratio (PAR) which investigates better design in the neighbouring of the current design. PAR is applied as follows

$$\text{Pitch adjusting decision for } x_i^{nh} \leftarrow \begin{cases} yes & if \quad rna \le PAR \\ no & if \quad rna > PAR \end{cases} \tag{22}$$

A random number (rna) uniformly distributed over the interval [0,1] is generated for x_i^{nh}. If this random number is less than the PAR, x_i^{nh} is replaced with its neighbour steel section in the section list. If this random number is not less than PAR, x_i^{nh} remains the same. The selection of neighbour section is determined by neighbouring index. A PAR of 0.35 indicates that the algorithm chooses a neighbour section with a 35%×HMCR probability. For example, if x_i^{nh} is HE 450AA, neighbouring index is -1 or 1 and the neighbour of this section is [HE 320AA, HE450AA, HE 280B], the algorithm will choose a neighbour section (HE 320AA or HE 280B) with a 35%×HMCR probability, or remain the same section (HE450AA,) with a (1-35%×HMCR) probability. HMCR and PAR parameters are introduced to allow the solution to escape from local optima and to improve the global optimum prediction of the HS algorithm (Lee and Geem 2004). This step is repeated until all design variables selected only once, and thus, the new design vector is obtained for the steel frame. The frame is analyzed for the new design vector and its response is obtained. The value of the objective function $\varphi(x^{new})$ using Eq. (2) is calculated.

4.4 Update the harmony memory

If the new harmony $[x^{nh}] = \lfloor x_1^{nh}, x_2^{nh}, \ldots, x_{ng}^{nh} \rfloor$ is better than the worst design in the HM (i.e. the last row of the HM), the new design is included in the HM and the existing worst harmony is excluded from the HM. In this process, it should be noted that HM matrix is sorted again by unconstrained objective function (i.e. Eq. (2)) and the same design is not permitted in the HM more than once.

4.5 Termination criteria

4.3 and 4.4 steps are repeated until the termination criterion is satisfied. In this study, two termination criteria were used for HS. The first one stops the algorithm when a predetermined total number of searches (number of frame analyses) are performed. The second criterion stops the process before reaching the maximum search number, if more economical design (lighter frame) is not found during a definite number of searches in HS. If one of these criteria is satisfied, the algorithm is terminated and the current optimum is defined as the final optimum design.

5. Benchmark examples

In this section, two steel frames are used to verify the effectiveness and robustness of HS. These frames are previously optimized using GA (Hayalioglu and Degertekin 2004a). Therefore; the same material properties and design constraints are considered. The Young's modulus and weight density of the frame members are taken as E = 205940 MPa and ρ = 7850 kg/m^3, respectively. Yield stress of steel is 235.4 MPa. European wide flange beams (i.e. HE sections) in accordance with Euronorm (1993) are used in the optimum design of the frames. The maximum drift is restricted to $H/250$ (H=total height of the frame) for the frames with rigid and semi-rigid connections.

The HS algorithm, programmed in Fortran, is executed with the following tuning parameters: The harmony memory size (HMS) was selected as 40 in the design examples that the algorithm is sensitive to its value. When HMS was selected greater than 40, HS does not improve the optimal solutions. For HMS<40, HS resulted in premature convergence. Another tuning parameter affecting the results is the harmony memory consideration rate (HMCR), which was selected as 0.8. The higher values of HMCR tended to reach local optima, while the lower values of HMCR caused the non-optimal solutions. HS is also influenced by the value of pitch adjusting rate (PAR) which was taken as 0.4. Using higher values for PAR caused non-optimal designs, while lower values for it resulted in local optima. The neighbouring index used in the pitch-adjustment selected as ±1. Using higher values of ±1 do not improve the optimal solutions. The maximum number of searches is another important parameter in the HS algorithm. Computational experience gained after a number of independent runs shown that if the optimum design remains the same during the execution of 20% of the maximum search number, additional improvement is not made in the HS process afterwards. The penalty constant (c) was assigned as 10, which was the same value as the GAs-based optimum designs in the benchmark examples.

5.1 Design of three-storey, two-bay frame

The three-storey two-bay frame illustrated in Fig. 5 is the first benchmark example. This frame was optimized using GAs (Hayalioglu and Degertekin 2004a). The top storey drift was limited to 4.38 cm for the frames with rigid and semi-rigid connections. The applied loads shown in Fig. 5 were divided into ten equal parts to carry out the non-linear analysis. Based on the results of a number of independent runs, it was observed that HS converged to the optimum designs between 2000 and 4000 number of searches. Therefore, the first and second termination criteria, explained in Section 4.5, were selected as

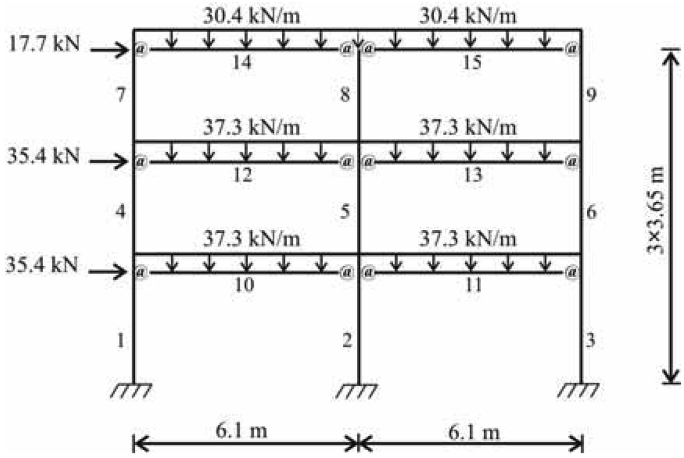

Fig. 5 Three-storey, two-bay planar frame

Table 2 The connection size parameters for three-storey, two-bay frame

Connection type	Connection size parameters (cm)		
1	$t_a = 2.4$	$g = 31.0$	
2	$t = 2.0$	$t_c = 2.0$	$g = 10.5$
3	$t = 2.8$	$d_b = 2.8$	
4	$t_p = 2.8$	$d_b = 2.8$	
5	$t_p = 2.8$		
6	$t = 2.0$	$d_b = 2.0$	

7000 and 1400 in this example, respectively. The connection size parameters which remain fixed during the optimum design process were given in Table 2 depending on the connection types (Hayalioglu and Degertekin 2004a).

Since HS is stochastic method in nature, 10 independent runs were made to optimize frame weight for each connection. The lightest ones of those were reported in Table 3. Maximum interaction ratio, number of frame analyses required by HS and the standard deviation of 10 different runs for each connection were given in Table 3. Furthermore, the optimum weight of frames for each connection is also compared in Fig. 6.

Fig. 6 clearly demonstrates that HS obtained lighter designs than the ones obtained by GAs. It yielded 1.7%-9.6% lighter frames compared with GAs-based design results. Maximum interaction ratios are calculated between 0.90 and 1.0 in all designs while the top storey drifts are far below from the boundary value as given in Table 3. These indicate that the optimum designs are controlled by strength constraints. In this case, semi-rigid frames may sometimes result in a design that weighs less than rigid frames due to the redistribution of the internal member forces in the beams with semi-rigid end connections.

The minimum weight with a value of 4542 kg was produced for the second semi-rigid connection. The design history of the optimum frame weight of this connection was depicted in Fig. 7.

As shown in this figure, HS obtained the optimum design at the 2602-th analysis and it did not change during 1400 frame analyses afterwards, and thus, HS terminated the search process after 4002 frame

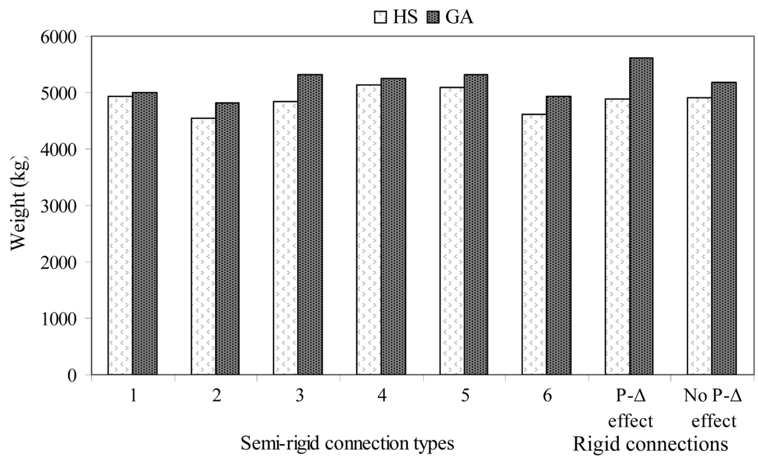

Fig. 6 Comparison of optimum weights for three-storey, two-bay planar frame

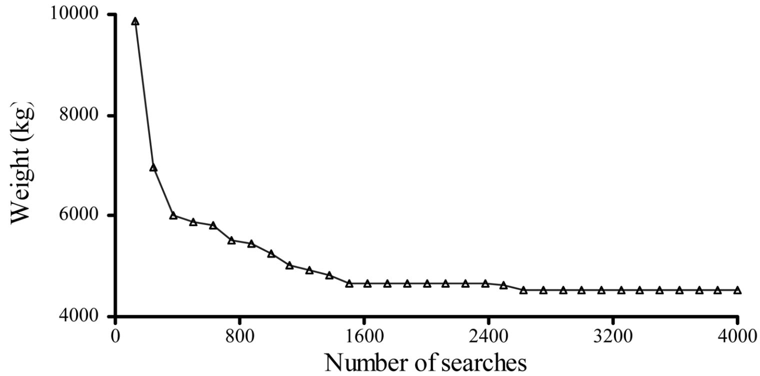

Fig. 7 Design history of three-storey, two-bay frame for the second semi-rigid connection type

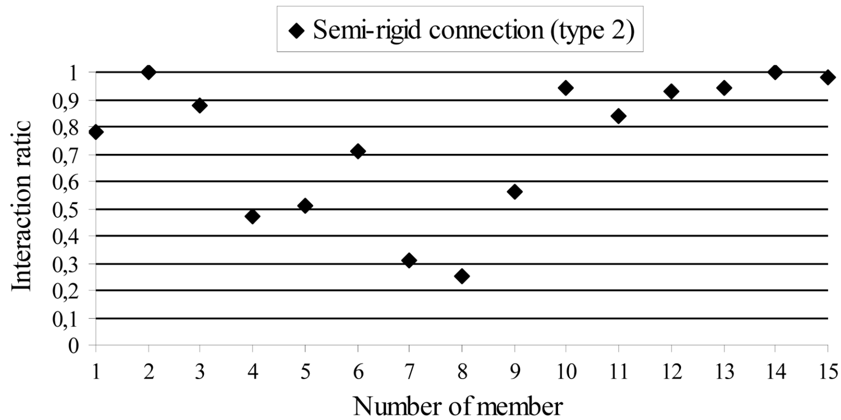

Fig. 8 Interaction ratios for three-storey, two-bay planar frame for the second semi-rigid connection type

Table 3 Optimum design results of three-storey, two-bay frame

Member number	Genetic algorithm (Hayalioglu and Degertekin 2004a)							
	HE sections							
	Semi-rigid connection types						Rigid connection	
	(1)	(2)	(3)	(4)	(5)	(6)	P-Δ effect	no P-Δ effect
1,4,7	200AA	180AA	200AA	180AA	180A	180AA	180AA	320A
2,5,8	450AA	500AA	300A	300A	500AA	280A	280A	260A
3,6,9	200AA	200AA	320AA	340AA	200A	340AA	340AA	320AA
10-13	400AA	340AA	400AA	400A	320A	400AA	400AA	300A
14,15	320AA	240B	320AA	300AA	320AA	320AA	320AA	260A
Weight (kg)	5011	4810	5319	5251	5317	4925	5615	5174
Top storey drift (cm)	2.04	2.20	2.11	1.90	2.58	1.86	1.83	0.88
Maximum interaction ratio	*	*	*	*	*	*	*	*
Number of analyses	*	7500	*	*	*	*	*	*
Standard deviation (kg)	*	*	*	*	*	*	*	*

Member number	Harmony search algorithm							
	HE sections							
	Semi-rigid connection types						Rigid connection	
	(1)	(2)	(3)	(4)	(5)	(6)	P-Δ effect	no P-Δ effect
1,4,7	220AA	200AA	180AA	220AA	200AA	180A	180AA	180AA
2,5,8	450AA	450AA	500AA	450AA	500AA	260B	340AA	280A
3,6,9	220AA	240AA	200AA	220AA	200AA	300AA	320AA	340AA
10-13	360AA	320AA	360AA	400AA	400AA	300AA	360AA	360AA
14,15	320AA	280AA	280AA	320AA	320AA	280AA	300AA	300AA
Weight (kg)	4925	4542	4843	5137	5096	4621	4884	4907
Top storey drift (cm)	2.16	2.14	2.36	2.23	2.19	2.36	0.93	0.98
Maximum interaction ratio	0.99	1.0	0.99	0.90	0.90	0.99	0.99	0.99
Number of analyses	5071	4002	5303	5837	6065	5627	4948	6445
Standard deviation (kg)	56.87	107.24	63.65	93.03	60.50	80.13	51.6	49.3

*Not available

analyses. It was less than the 7500 frame analyses required by GAs as reported in Table 3. GAs also developed a design weighing 4626 kg with a population size of 120 and required 24000 frame analyses (Hayalioglu and Degertekin 2004a). In this case, HS yielded both lighter frame designs and required less computational effort than GAs.

Interaction ratios of frame designs for the second semi-rigid were illustrated in Fig. 8. It is worth mentioning that interaction ratio of five beam members is within 90% of maximum interaction ratio in the semi-rigid frame design as shown in Fig. 8.

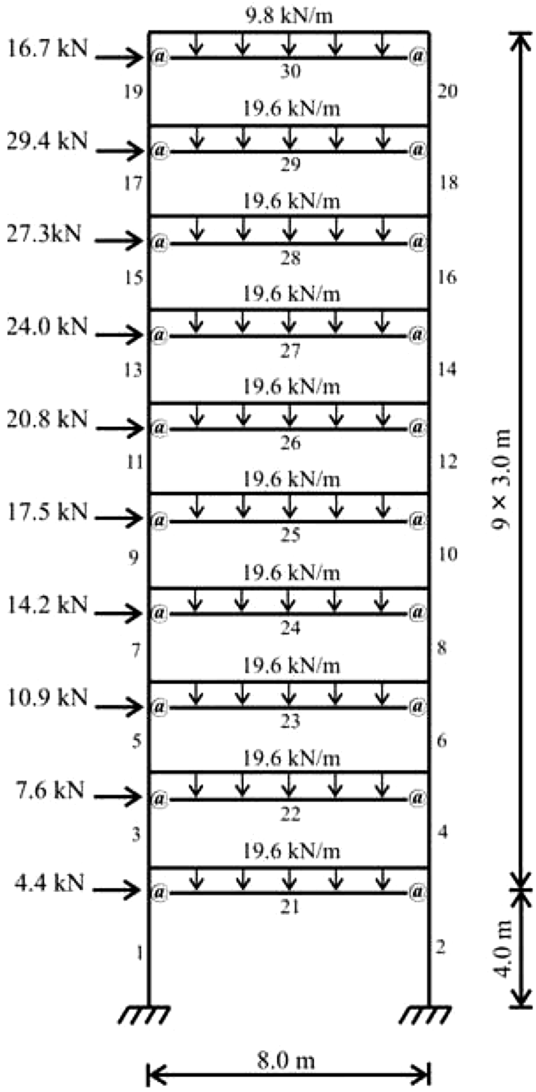

Fig. 9 Ten-storey, single-bay frame

5.2 Design of ten-storey, single-bay frame

The ten-storey, single-bay frame was previously designed by Hayalioglu and Degertekin (2004a) using GAs in accordance with TS648. Fig. 9 shows configuration, dimensions, loading and numbering of members.

The top storey drift was restricted to 12.4 cm for the frames with rigid and semi-rigid connections. The fixed connection size parameters for semi-rigid connections are given in Table 4.

10 independent frames were obtained generated from randomly selected 10 different initial designs and the lightest ones of those were reported in Table 5. It was observed from several independent runs that HS converged to the optimum designs between 6000 and 8000 number of searches. Therefore, the

Table 4 The connection size parameters for ten-storey, single-bay frame

Connection type	Connection size parameters (cm)		
1	$t_a = 2.0$	$g = 22.0$	
2	$t = 1.6$	$t_c = 1.6$	$g = 10.5$
3	$t = 2.4$	$d_b = 2.8$	
4	$t_p = 2.0$	$d_b = 2.0$	
5	$t_p = 2.0$		
6	$t = 2.0$	$d_b = 2.0$	

Table 5 Optimum design results of ten-storey, single-bay frame

Member number	Genetic algorithm (Hayalioglu and Degertekin 2004a)							
	HE sections							
	Semi-rigid connection types						Rigid connection	
	(1)	(2)	(3)	(4)	(5)	(6)	P-Δ effect	no P-Δ effect
1-6	450B	650A	600A	650A	500B	450B	450×312	400×299
7-12	550AA	500AA	500AA	360B	500A	550AA	500AA	450AA
13-18	340AA	340AA	340AA	450AA	360A	320AA	340AA	550AA
19,20	450AA	360AA	320AA	320AA	320AA	320AA	400B	320AA
21-23	550AA	500AA	600AA	500AA	600AA	650AA	550AA	550AA
24-26	450AA	650AA	500AA	650AA	450AA	360A	650AA	500AA
27-29	450AA	360AA	600AA	400AA	450AA	340A	400AA	450AA
30	320AA	400AA	320AA	320AA	320AA	320AA	320AA	320AA
Weight (kg)	15862	16288	16712	17289	17577	16475	18818	19520
Drift (cm)	12.38	12.39	12.39	12.32	12.31	12.35	6.72	6.18
Maximum interaction ratio	*	*	*	*	*	*	*	*
Numberof analyses	*	*	*	11460	*	*	*	*
Standard deviation (kg)	*	*	*	*	*	*	*	*

Member number	Harmony search algorithm							
	HE sections							
	Semi-rigid connection types						Rigid connection	
	(1)	(2)	(3)	(4)	(5)	(6)	P-Δ effect	no P-Δ effect
1-6	700AA	700A	600A	550A	550A	550A	700AA	650AA
7-12	550AA	550AA	550AA	450AA	550AA	550AA	450AA	500AA
13-18	320AA	320AA	340AA	340AA	340AA	360AA	360AA	320AA
19,20	360AA	340AA	320AA	360AA	400AA	600B	360AA	340AA
21-23	600AA	360AA	600AA	600AA	550AA	500AA	550AA	650AA
24-26	400AA	500AA	500AA	550AA	550AA	450AA	500AA	500AA
27-29	400AA	400AA	450AA	500AA	550AA	340AA	400AA	400AA
30	320AA	340AA	340AA	340AA	550AA	340AA	320AA	320AA
Weight (kg)	15115	15495	16277	16224	17055	15761	15071	15207
Drift(cm)	12.21	12.36	12.28	12.32	10.42	12.38	7.67	7.29
Maximum interaction ratio	0.98	1.0	0.99	0.98	0.93	0.99	0.99	0.99
Numberof analyses	9915	8927	9523	9876	10000	7766	7295	9006
Standard deviation (kg)	616.74	416.73	653.18	462.63	211.22	600.76	224.3	191.2

*Not available

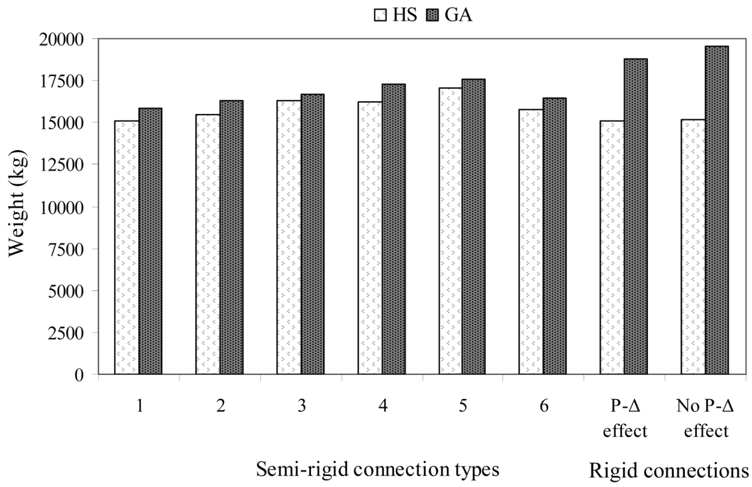

Fig. 10 Comparison of optimum weights for ten-storey, single-bay frame

first and second termination criteria, explained in Section 4.5, were selected as 10000 and 2000 in this example, respectively.

The comparison of the optimum designs for each connection is also depicted in Fig. 10. HS resulted in lighter frames than the ones with GAs as shown in this figure. It is noticed from Table 5 that HS yielded 2.6%-11.6% lighter designs than GAs. The lightest design was obtained for the rigid connection with P-Δ effect with a weight of 15071 kg.

The convergence history of the optimum frame weight for the rigid connection with P-Δ effect was shown in Fig. 11. As illustrated in this figure, HS developed the optimum design at the 5295-th analysis and it did not change during 2000 frame analyses afterwards and terminated the search process after 7295 frame analyses.

As regards to number of analyses required by HS algorithm, it obtained the optimum design with a

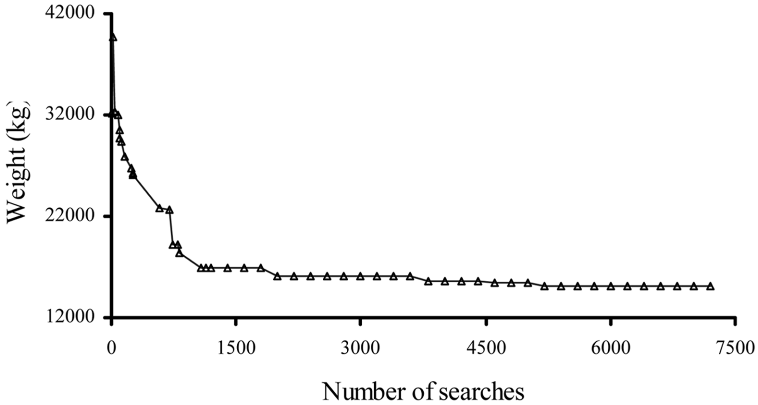

Fig. 11 Design history of the ten-storey, single-bay frame for the rigid frame with P-Δ effect

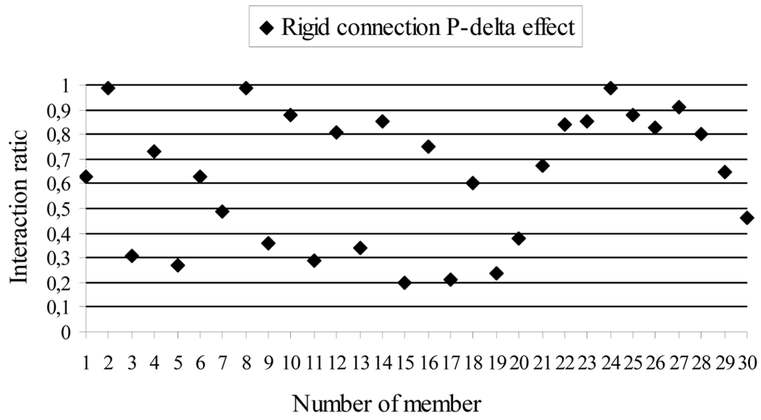

Fig. 12 Interaction ratios for ten-storey, single-bay frame for the rigid frame with P-Δ effect

weight of 16224 kg after 9876 frame analyses for the forth semi-rigid connection type whereas GAs developed the optimum design with a weight of 17289 kg after 11460 frame analyses. Furthermore, GAs was also executed with a population size of 96 and the optimum frame with a weight of 17035 kg was produced after 26496 frame analyses (Hayalioglu and Degertekin 2004a). In this case, HS not only yielded lighter designs but also it consumed significantly less computing effort than GAs.

Maximum interaction ratios for all connections are above 0.90. However, the drifts were also near their boundary value for the semi-rigid frames. Semi-rigid connections increase the displacements of frames and these displacements are adjusted to their restrictions by the optimization process assigning larger sections to the members. Hence, semi-rigid frame designs weighted greater than rigid frame designs. Interaction ratios of frame design for the rigid connection with P-Δ effect were also depicted in Fig. 12.

6. Conclusions

HS algorithm is introduced on the optimum design of steel frames with semi-rigid connections. The effectiveness and robustness of HS are verified by using two design examples and the following conclusions are drawn from this study.

I. HS obtained 1.7%-22% lighter frames when compared to GAs. It seems from the presented examples that HS gives more suitable designs than GAs.

II. In addition to obtaining lighter frames, HS required significantly less computational effort than GAs as indicated in the sections 6.1 and 6.2.

III. Standard deviations of the frame weights given in Table 3 and Table 5 were quite small in comparison with the frame weights, which was less than 5% in the examples. These indicate that HS is able to converge to the global or near global optimum.

IV. Since the first design example is a low-rise frame, geometric non-linearity does not play important role in the optimum designs and the drift constraint become passive. On the other hand, the second design example is a slender frame and the geometric non-linearity becomes important. In this example, drift constraint becomes active with strength constraints.

References

Abdalla, K.M. and Chen, W.F. (1995), "Expanded database of semi-rigid steel connections", *Comput. Struct.* **56**(4), 553-564.

Almusallam, T.H. (1995), "Effect of connection flexibility on the optimum design of steel frames", *Proc. Int. Conf. on developments in computational techniques for civil engineering*, Edinburgh.

Alsalloum, Y.A. and Almusallam, T.H. (1995), "Optimality and safety of rigidly-jointed and flexibly-jointed steel frames", *J. Constr. Steel Res.*, **35**(2), 189-215.

American Institute of Steel Construction (1989), *Manual of steel construction: allowable stress design,* Chicago, Illionis.

Arora, J.S. (1989), *Introduction to Optimum Design*, Mc Graw-Hill, New York.

Chen, W.F. and Kishi, N. (1989), "Semirigid steel beam-to-column connections: data base and modeling", *J. Struct. Eng.* ASCE, **115**(1), 105-119.

Chen, W.F. and Lui, E.M. (1991), *Stability design of steel frames*, Baco Raton, CRC Press, Florida.

Chen, W.F., Goto, Y. and Liew, J.Y.R. (1996), *Stability design of semi-rigid frames*, John Wiley & Sons Inc., New York.

Csebfalvi, A. (2007), "Optimal design of frame structures with semi-rigid joints", *Periodica Polytechnica-Civil Eng.*, **51**(1), 9-15.

Cunningham, R.J. (1990), "Some aspects of semi-rigid connections in structural steelwork", *Struct. Eng.*, **68**(5), 85-92.

Degertekin, S.O. (2008a), "Harmony search algorithm for optimum design of steel frame structures: a comparative study with other optimization methods", *Struct. Eng. Mech.*, **29**(4), 391-410.

Degertekin, S.O. (2008b), "Optimum design of steel frames using harmony search algorithm", *Struct. Multidiscip. O.*, **36**(4), 393-401.

Deren, H., Uzgider, E., Piroglu, F. and Caglayan, B. (2008), "Steel structures",(In Turkish), Caglayan Kitabevi, Istanbul.

Dhillon, B.S. and O'Malley, J.W. (1999), "Interactive design of semirigid steel frames", *J. Struct. Eng.* ASCE, **125**(5), 556-564.

Euronorm (1993), "*European Wide Flange Beams*", 53-62, CEN, Brussels.

Frye, M.J. and Morris, G.A. (1975), "Analysis of flexibly connected steel frames", *Can. J. Civil Eng.*, **2**(3), 280-291.

Geem, Z.W., Kim, J.H. and Loganathan, G.V. (2001), "A new heuristic optimization algorithm: harmony search", *Simul.-T. Soc. Mod. Sim.*, **76**(2), 60-68.

Goldberg, D.E. (1989), *Genetic Algorithms in Search, Optimization and Machine Learning*, Addisson-Wesley, Reading.

Hayalioglu, M.S. and Degertekin, S.O. (2004a), "Genetic algorithm based optimum design of non-linear steel frames with semi-rigid connections", *Steel Compos. Struct.*, **4**(6), 453-469.

Hayalioglu, M.S. and Degertekin, S.O. (2004b), "Design of non-linear steel frames for stress and displacement constraints with semi-rigid connections via genetic optimization", *Struct. Multidiscip. O.*, **27**(4), 259-271.

Hayalioglu, M.S. and Degertekin, S.O. (2005), "Minimum cost design of steel frames with semi-rigid connections and column bases via genetic optimization", *Comput. Struct.*, **83**(21-22), 1849-1863.

Holland, J.H. (1975), *Adaption in Natural and Artificial Systems*, The Universtiy of Michigan Press, Ann Arbor.

Ihaddoudene, A.N.T., Saidani, M. and Chemrouk, M. (2009), "Mechanical model for the analysis of steel frames with semi rigid joints", *J. Constr. Steel Res.,* **65**(3), 631-640.

Ivanyi, M. (2000), "Full-scale tests of steel frames with semi-rigid connections", *Eng. Struct.,* **22**(2), 168-179.

Jones, S.W., Kirby, P.A. and Nethercot, D.A. (1980), "Effect of semi-rigid connections on steel column strength", *J. Constr. Steel Res.,* **1**(1), 38-46.

Kameshki, E.S. and Saka, M.P. (2001), "Optimum design of nonlinear steel frames with semi rigid connections using a genetic algorithms", *Comput. Struct.,* **79**(17), 1593-1604.

Kameski, E.S. and Saka, M.P. (2003), "Genetic algorithm based optimum design of nonlinear planar steel frames with various semirigid connections", *J. Constr. Steel Res.,* **59**(1), 109-134.

Kaveh, A. and Moez, H. (2006), "Analysis of frames with semi-rigid joints: A graph-theoretical approach", *Eng.*

Struct., **28**(6), 829-836.
King, W.S. (1994), "The limit loads of steel semi-rigid frames analyzed with different methods", *Comput. Struct.*, **51**(5), 475-487.
King, W.S. and Chen, W.F. (1994), "Practical 2nd-order inelastic analysis of semirigid frames", *J. Struct. Eng.* ASCE, **120**(7), 2156-2175.
Kishi, N. and Chen, W.F. (1990), "Moment-rotation relations of semi-rigid connections with angles", *J. Struct. Eng.* ASCE, **116**(7), 1813-1834.
Kishi, N., Chen, W.F. and Goto, Y. (1997), "Effective length factor of columns in semirigid and unbraced frames", *J. Struct. Eng.* ASCE, **123**(3), 313-320.
Lee, K.S. and Geem, Z.W. (2004), "A new structural optimization method based on the harmony search algorithm", *Comput. Struct.,* **82**(9-10), 781-798.
Lee, K.S. and Geem, Z.W. (2005), "A new meta-heuristic algorithm for continuous engineering optimization: harmony search theory and practice", *Comput. Method Appl. M.,* **194**(36-38), 3902-3933.
Lee, K.S., Geem, Z.W., Lee, S.H. and Bae, K.W. (2005), "The harmony search heuristic algorithm for discrete structural optimization", *Eng. Optimiz.,* **37**(7), 663-684.
Lee, S.S. and Moon, T.S. (2002), "Moment-rotation model of semi-rigid connections with angles", *Eng. Struct.,* **24**(2), 227-237.
Lui, E.M. and Chen, W.F. (1986), "Analysis and behaviour of flexibly-jointed frames", *Eng. Struct*., **8**(2), 107-118.
Monforton, G.R. and Wu, T.S. (1963), "Matrix analysis of semi-rigidly connected steel frames", *J. Struct. Div.* ASCE, **89**(6), 13-42.
Pirmoz, A., Khoei, A.S., Mohammadrezapour, E. and Daryan, A.S. (2009), "Moment-rotation behavior of bolted top-seat angle connections", *J. Constr. Steel Res.,* **65**(4), 973-984.
Prabha, P., Marimuthu, V., Jayachandran, S.A. and Seetharaman, S. (2008), "An improved polynomial model for top-and seat-angle connection", *Steel Compos. Struct.*, **8**(5), 403-421.
Saka, M.P. (2009), "Optimum design of steel sway frames to BS5950 using Harmony search algorithm", *J. Constr. Steel Res.,* **65**(1), 36-43.
Sekulovic, M. and Salatic, R. (2001), "Nonlinear analysis of frames with flexible connections", *Comput. Struct*., **79**(11), 1097-1107.
Simoes, L.M.C. (1996), "Optimization of frames with semi-rigid connections", *Comput. Struct*., **60**(4), 531-539.
TS648 (1980), *Building Code for Steel Structures, Turkish Institute of Standards*, Ankara.
Wang, X.W. (2008), "Nonlinear finite element analysis on the steel frame with semi-rigid connections", *7th WSEAS Int. Conf. on Applied Computer and Applied Computational Science*, Hangzhou.
Wu, F.S. and Chen, W.F. (1990), "A design model for semi-rigid connections", *Eng. Struct*., **12**(2), 88-97.
Xu, L. and Grierson, D.E. (1993), "Computer automated design of semi-rigid steel frameworks", *J. Struct. Eng.* ASCE, **119**(6), 1740-1760.
Yee, Y.L. and Melchers, R.E. (1986), "Moment-rotation curves for bolted connections, *J. Struct. Eng.* ASCE, **112**(3), 615-635.

CC