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Numerical studies of the effect of residual imperfection on the mechanical behavior of heat-corrected steel plates, and analysis of a further repair method

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Abstract. Heating correction, through heating and flattening a structure with a pressing machine, is the in-situ method used to repair buckled steel structures. The primary purpose of this investigation is to develop an FEM model which can predict the mechanical response of heat-corrected plates accurately. Our model clarifies several unsolved problems. In previous research, the location of the imperfection was limited to the center of the specimen although the mechanical behavior is strongly affected by the location of the imperfection. Our research clarifies the relationship between the location of the imperfection and the mechanical behavior. In addition, we propose further reinforcement methods and validate their effectiveness. Our research concludes that the strength of a buckled specimen can be recovered by heating correction and the use of an adequate stiffener.

Keywords : heating correction; residual imperfection; buckling analysis; finite element analysis.

1. Introduction

The 1995 Great Hanshin Earthquake in Japan measured 7.3 on the Richter magnitude scale and it took a drastic toll on the infrastructure (Kawata 1995). Since it was desired to restore the infrastructure as soon as possible from the viewpoint of disaster assistance and post-disaster rehabilitation, various emergency repair methods were adopted. One of these methods was heating correction. Heating correction, through heating and flattening a structure with a pressing machine, is the in-situ method used to repair steel structures which have been buckled by an earthquake. Although the heating correction approach is often used to repair buckled structures (Fig. 1-left), the strength of a repaired specimen is always lower than that of the original specimen because it is difficult to make repairs without leaving residual imperfections in the structure (Fig. 1-right). There have been a number of reports (e.g. Komatsu and Kitada 1983) on the behavior of plates with initial imperfections; however, these results cannot be used

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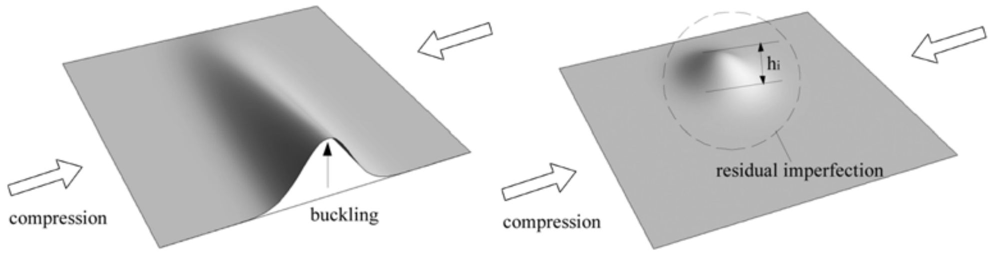

Fig. 1 Simplified schematic diagram of buckled plate (left figure) and heat-corrected plate (right figure)

for our purpose because most of them have considered imperfections of which the height is one hundredth of the plate breadth, although the height of imperfection of heat-corrected specimens (h_i in Fig. 1-right) is one tenth of plate breadth. Since very few studies have reported on the mechanical behavior of heat-corrected specimens, it is important to clarify their mechanical responses in post-heating correction.

In Japan, epicentral earthquakes often occur, and it is predicted that an ocean-trench earthquake will occur in the present century. The 1995 Great Hanshin earthquake and the 2007 Noto Hanto earthquake are examples of epicentral earthquakes. Of course, great earthquakes occur not only in Japan but worldwide, like the 1994 Northridge earthquake and the 2004 Indian Ocean earthquake; therefore it is a pressing issue to evaluate heating correction as an emergency repair.

The primary purpose of this investigation is to develop a numerical model by elasto-plastic finite element analysis which can predict the mechanical response of heat-corrected specimens accurately. The model is validated by compression test results. Our model clarifies several unsolved problems. In previous research, the location of the residual imperfection was limited to the center of the specimen, although the mechanical behavior is strongly affected by the location of the imperfection. Our research clarifies the relationship between the location of the residual imperfection and the mechanical behavior. In addition, we propose further reinforcement methods and validate their effectiveness. Our research concludes that the strength of a buckled specimen can be recovered by heating correction and the use of an adequate stiffener.

2. Numerical model and its validation

2.1 Compression test of cruciform column

In this chapter, a numerical model is developed by elasto-plastic finite element analysis and the validity is ascertained by the compression experiment conducted by Kim, *et al*. (2004). In the experiment, the cruciform columns shown in Fig. 2 were tested. The material properties of the specimen are shown in Table 1.

Compression tests of the original cruciform column and the heat-corrected column were conducted and relationships between compressive load and out-of-plane displacement at the center of the free end of the projection panel (Fig. 2 point D) were obtained (Fig. 3). The procedure used for heat correction is as follows: (I) heating the buckled steel plate by a burner; (II) temperature of the plate is controlled with an upper limit of 550~650°C, which is the transformation point of steel; (III) the buckled plate is then flattened by a pressing machine. The strength of the heat-corrected specimen is always lower than that

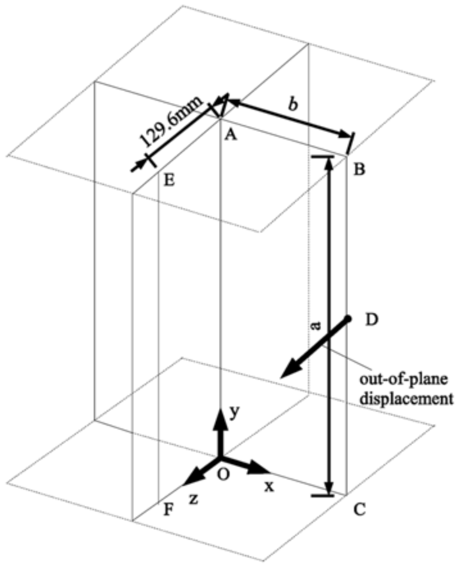

Fig. 2 Compression test of cruciform column

Table 1. Material properties of the specimen

Material	SM490YA
Young's Modulus (GPa)	200
Yield Stress (MPa)	412
Tensile Strength (MPa)	539
Poisson's Ratio	0.3
Length a (mm)	700
Breadth b (mm)	162
Thickness t (mm)	9

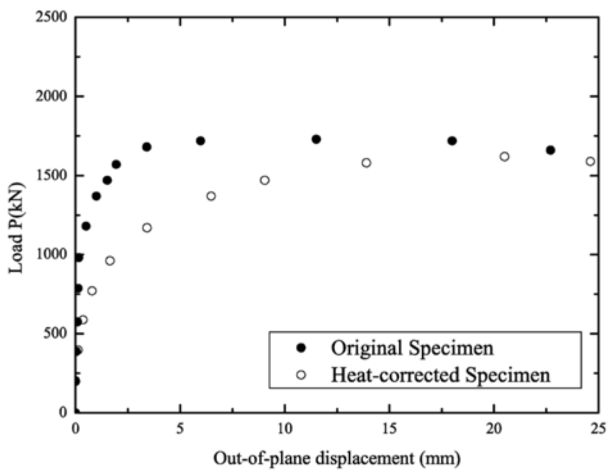

Fig. 3 Comparison of experiment result of original specimen and heat-corrected specimen

of the original specimen because it is difficult to make repairs without leaving a residual imperfection (Fig. 1-left) when the buckling deformation is large before the repair is carried out (Fig. 1-right).

In addition, it was observed that local plate buckling governs the mechanical behavior of both the original specimen and the heat-corrected specimen. This observation seems to be reasonable because there is an initial deflection even in an undamaged specimen and therefore local plate buckling occurs easier than global buckling or squash failure. The results obtained from the compression tests are employed to validate our finite-element method (FEM) model.

2.2 Development of an FEM model

In this section, an FEM model is established and the validity is ascertained by the experimental result in the previous section. Due to symmetry, only one projection panel is considered in this model. It has been shown that the initial out-of-plane deflection of the original cruciform specimen $w_0(x,y)$ and the heat-corrected cruciform specimen $w_{res}(x,y)$ are given by the following equation (Kim, *et al*. 2006):

$$w_0(x,y) = A_{0z}\sin\frac{\pi x}{b} + \sum_{m,n} A_{0mn}\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{a} \tag{1}$$

$$w_{res}(x,y) = A_{rz}\sin\frac{\pi x}{b} + \sum_{m,n} A_{rmn}\sin\frac{m\pi x}{b}\sin\frac{n\pi y}{a} + w \tag{2}$$

$$w = \begin{cases} e\sin\dfrac{\pi(x-x_1)}{d}\sin\dfrac{\pi(y-y_1)}{k} \\ (x_1 \le x \le x_1 + d \cap y_1 \le y \le y_1 + k) \\ 0 \\ (\text{otherwise}) \end{cases}$$

where A_{0z}, A_{rz}, A_{0mn}, A_{rmn}, e, d, k, x1, y1 are determined by the measurement of the specimen and the values are in the following Table 2 as given in Kim's previous compression test. e represents out-of-plane deflection and d, k represent x and y directional length of residual imperfection, respectively (Fig. 2). m = 1, n = 1,3 is employed in Eq. (1) and m = 1, n = 1 is employed in Eq. (2).

Stress-strain relationship of steel used in the analysis is shown in Fig. 4 and the mesh and loading information are given in Fig. 5. In this research, ABAQUS/Standard is employed to conduct finite element analysis. In the analysis, four node doubly curved shell elements are used, and the panel is divided into 1760 elements and 1869 nodes.

Since the supporting condition of the steel product is of an intermediate level between the simply-

Table 2. Coefficients in Eqs. (1) and (2) (unit: mm)

A_{0z}	A_{011}	A_{013}	A_{rz}	A_{r11}	e	d	k	x1	y1
0.5	0.1	0.1	0.5	4.0	11.1	110.0	158.7	2.0	270.65

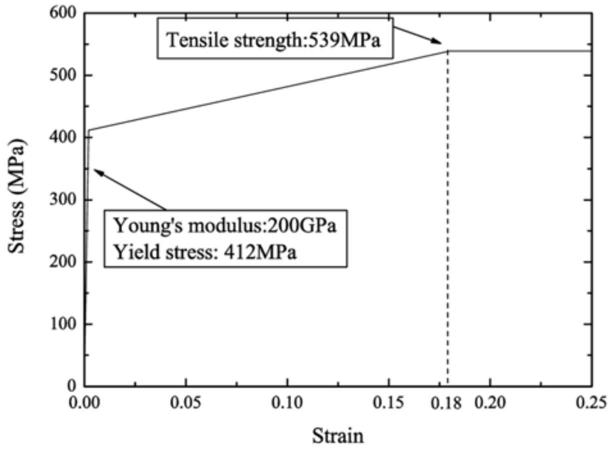

Fig. 4 Stress-strain relationship of steel used in the finite element analysis

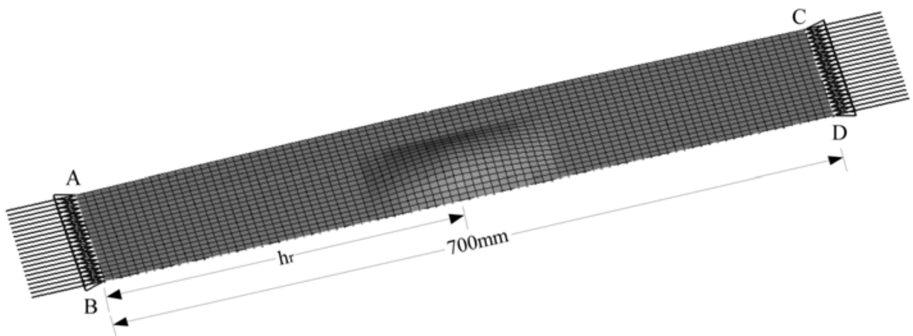

Fig. 5 Mesh used in FEM analysis

supported condition and the clamped condition, FEM analyses are conducted under two boundary conditions: (I) -free on one unloaded edge (AC in Fig. 5), simply-supported on three edges (AB, BD, and DC in Fig. 5) (II) - free on one unloaded edge (AC in Fig. 5), clamped on three edges (AB, BD, and DC in Fig. 5). In both cases, AB, BD, and DC are kept straight. The experimental result is expected to be between these two FEM results. In order to analyze buckling behavior, the load increment method is widely used (Lam and Zou 2000). In this research, the arc-length method (modified Riks algorithm) is applied to simulate buckling and collapse of plates (Riks 1979).

In this FEM analysis, we shall assume that buckling distortion occurs only in the projection panel (OABC in Fig. 2), not on the center axis (OA in Fig. 2). This assumption is considered to be reasonable from the experimental result.

Fig. 6 shows the comparison between experimental and FEM results (Fig. 6 left: original specimen Fig.6 right: heat-corrected specimen). It was found that the experimental result was between two FEM results in both cases. In addition, we found that the FEM results reproduced the strength degradation phenomena of the heat-corrected specimen compared to the original specimen and reproduced the buckling mode of both the original specimen and the heat-corrected specimen. From these results, it can be said that our FEM model was well validated.

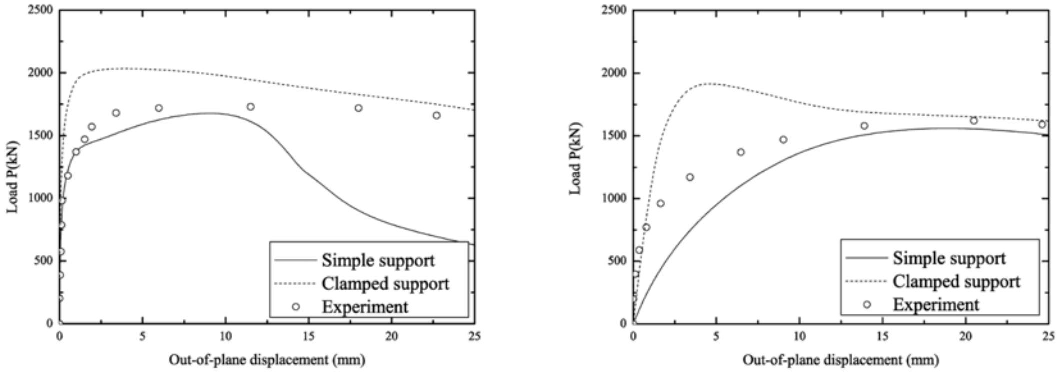

Fig. 6 Comparison of experiment and FEM result (left figure: original specimen, right figure: heat-corrected specimen)

3. Effect of the location of residual imperfection on strength

The dependence of the strength on the location of the residual imperfection is discussed in this section. Kim, *et al*. (2006) concluded that buckling strength and ultimate strength are governed only by the dimensions of residual imperfection, and the location of residual imperfection does not affect the strength. Kim, *et al*. (2004) defined the buckling strength as the strength when buckling deformation starts. Note that this definition is different from our definition, shown in chapter 3.2. In addition, Komatsu and Kitada (1983) found that the location of an imperfection does not affect the strength when out-of-plane deformation is small enough (nearly hundredth part of plate breadth). However, this paper shows, from an analytical and numerical point of view, that the location of a residual imperfection which is bigger than an ordinary initial imperfection (nearly one tenth of the plate breadth) affects the strength of the plate.

Hereinafter, the boundary condition of (I) –free on one unloaded edge, simply-supported on three edges described in the previous chapter, which is the lower strength, is applied to estimate the strength on the conservatively. This boundary condition can be applied not only to the cruciform column but also to the compressive flange of a plate girder (Fukumoto 1987). In this chapter, the analytical discussion is given in advance of the numerical discussion.

3.1 Analytical Discussion

There is much literature on the analytical approach which clarifies the buckling behavior of plates, accompanied with a large deflection. The fundamental equations for the large deflection of thin flat plates have been derived by Von Karman, *et al*. (1932), and the extension to a plate with a small initial curvature has been achieved by Marguerre. Since their studies, much additional research has been conducted using their fundamental equation (Yamaki 1959). In this paper, the dependence of buckling behavior on the location of the residual imperfection is shown by using Marguerre's fundamental equation.

It is assumed that the residual imperfection $w_0(x, y)$ and additional deflections $w(x, y)$ of the plate shown in Fig. 7 can be expressed as in the following Eqs. (3) and (4), respectively (Fukumoto 1982). p_i is determined from the initial residual imperfection of the plate and $i = 1,2,\ldots,10$ is employed. In this paper, plates which have the largest imperfection at $y = a/2$ (Case A) and $y = 3a/10$ (Case B) are investigated.

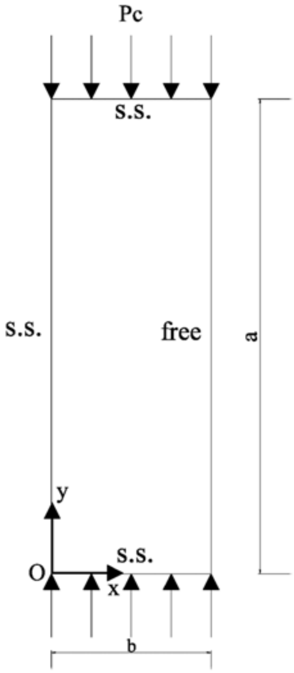

Fig. 7 SSSF rectangular plate

p_i for each case is shown in Table 3 and each shape is shown in Fig. 8.

$$w_0(x,y) = h\sum_{i=1} p_i x \sin\frac{i\pi y}{a} \tag{3}$$

$$w(x,y) = h\sum_{i=1} q_i x \sin\frac{i\pi y}{a} \tag{4}$$

where h is the thickness of the plate.

The following compatibility equation is obtained by introducing the Airy stress function Φ.

$$\nabla^4\Phi = E(w_{,xy}{}^2 - w_{,xx}w_{,yy} + 2w_{0,xy}w_{,xy} - w_{0,xx}w_{,yy} - w_{0,yy}w_{,xx}) \tag{5}$$

where E is Young's modulus. Φ is derived from Eq. (5) as the function of the compressive stress P_c, x,

Table 3. Values of p_i for Case A and Case B

	p_1	p_2	p_3	p_4	p_5	p_6	p_7	p_8	p_9	p_{10}
Case A	0.263	0	−0.233	0	0.182	0	−0.123	0	0.068	0
Case B	0.176	0.250	0.189	0.044	−0.091	−0.145	−0.112	-0.038	0.021	0.039

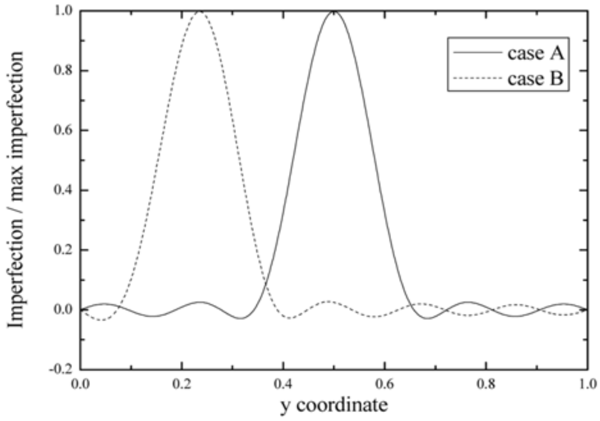

Fig. 8 Imperfection shape of Case A (maximum imperfection is at y = a/2) and Case B (maximum imperfection is at y = 3a/10)

and y. Homogeneous solution of Φ can be given as following Eq. (6).

$$\Phi_h = \sum_n r_n \cos\frac{n\pi y}{a} \tag{6}$$

Particular solution of Φ can be determined by following conditions.

$$\int_0^a \Phi_{,yy} dy = 0 \tag{7a}$$

$$\int_0^b \Phi_{,xx} dx = -P_c b \tag{7b}$$

$$\Phi_{,xy} = 0 \tag{7c}$$

In order to satisfy the above conditions, the particular solution can be determined as following Eq. (8).

$$\Phi_p = r_p x^4 - 2a^2 r_p x^2 - p_c y^2/2 \tag{8}$$

From homogeneous and particular solution shown in Eqs. (6) and (8), general solution of Φ is described as follows:

$$\Phi = \Phi_h + \Phi_p \tag{9}$$

where r_n and r_p are the function of p_i and q_i.

The equilibrium equation in the z-direction, taking account of the initial deflection, becomes

$$\nabla^4 w = \frac{h}{D}[\Phi,_{yy}(w+w_0),_{xx} + \Phi,_{xx}(w+w_0),_{yy} - 2\Phi,_{xy}(w+w_0),_{xy}] \tag{10}$$

where D is the flexural rigidity of the plate. Eq. (10) is Marguerre's fundamental equation governing the finite deflection of the initially deflected thin plate. For the determination of w, Galerkin's method is applied to the Eq. (10), which gives:

$$\int_0^a \int_0^b \left\{ \nabla^4\Phi - \frac{h}{D}[\Phi,_{yy}(w+w_0),_{xx} + \Phi,_{xx}(w+w_0),_{yy} - 2\Phi,_{xy}(w+w_0),_{xy}] \right\} x \sin\frac{i\pi y}{a} dx dy = 0 \tag{11}$$

where number of terms i is the same as the number of terms in Eq. (4). We have i equations with i unknowns (q_i) and therefore can solve.

From the w determined from Eq. (11), the relation between the compressive load and the maximum out-of-plane deflection is obtained (Fig.9). The solid line and the dashed line indicate the imperfection at x = a/2 and x = 3a/10, respectively. In the figure, λ is the non-dimensional load factor defined as $\lambda = P_c a^2/\pi^2 E h^2$. This result demonstrates that the location of the residual imperfection affects the buckling behavior. Buckling and ultimate strength is lower when the residual imperfection is located at the center, as is consistent with FEA result shown in the next section.

3.2 Numerical Discussion

In the previous section, it was shown analytically that the location of the residual imperfection affects the mechanical behavior of the plate. However, it is very difficult to obtain precise mechanical behavior analytically when material nonlinearity has to be considered. In this section, the relationship between the location of the imperfection and the ultimate strength is investigated numerically by the FEM model developed

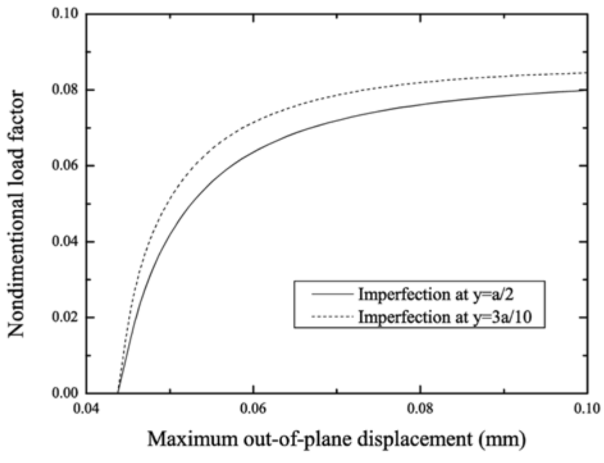

Fig. 9 Relation between stress and deflection for y = a/2 and y = 3a/10

in the previous chapter. In addition to that, eigenvalue analysis is conducted to obtain buckling strength.

Specifically, FEM analysis is conducted from $h_r = 350$ mm to 500 mm at 25 mm intervals. Fig.10-left shows the dependence of the ultimate and buckling strength ratio to the strength of $h_r = 350$ mm specimen on the location of the imperfection. Buckling strength is defined as the critical bifurcation load obtained from the eigenvalue analysis. Fig.10-right shows the dependence of the strength ratio to the original specimen on the location of the imperfection. If the strength ratio of a heat-corrected structure is low, a rapid repair is required. From this viewpoint, determining the dependence of the strength ratio on the location of the residual imperfection has a significant practical use. From the result obtained in this section, it is found that the ultimate and buckling strength monotonically decrease as the location of the imperfection approaches the center ($h_r = 350$ mm).

4. Reinforcement by stiffened plate

In this section, a reinforcement scheme using a stiffened plate is proposed and its effectiveness is estimated. As stated above, since the mechanical behavior is governed by the local plate buckling, a stiffened plate attached by welding at the EF in Fig. 2 may increase buckling strength. In FEM analysis, the stiffened plate is considered as the 3-D Timoshenko beam and its tensile stiffness, flexural stiffness, and torsional stiffness are considered (Lee and Lee 1995). In this chapter, the case of $h_r = 350$ mm, in which strength is the lowest of all the imperfection location, is considered. FEM analysis is conducted by changing the height of the 3-D Timoshenko beam which represents the stiffened plate. Mesh used in the analysis is shown in Fig. 11. The relationship between ultimate strength and the height of the stiffened plate is shown in Fig. 12. The ultimate strength of the undamaged specimen is also shown for comparison by the solid line in the figure. From this result, it is found that the ultimate strength of a reinforced plate is higher than the original undamaged specimen when the height of the stiffened plate is higher than 40 mm. The load-displacement curve of the plate reinforced by the stiffened plate, of which the height is 40mm, is shown in Fig.13.

An important finding here is that the heat-corrected ultimate strength can be greater than the ultimate strength of the obtained specimen with the use of a properly designed stiffened plate.

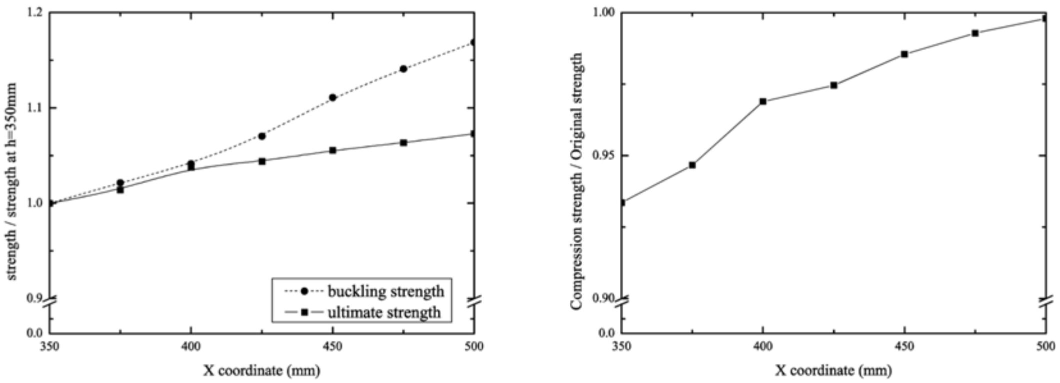

Fig. 10 Dependence of the strength ratio on the location of the imperfection

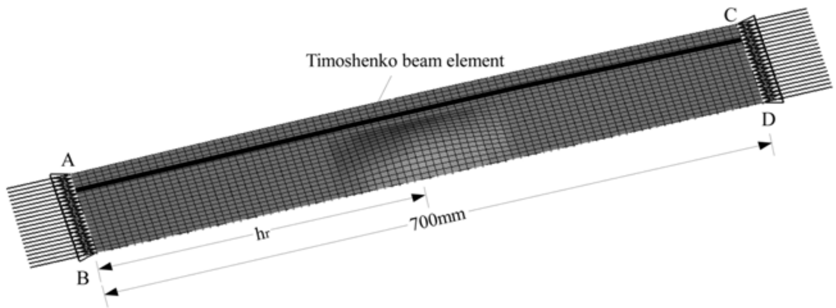

Fig. 11 Mesh used in FEM analysis of stiffened specimen

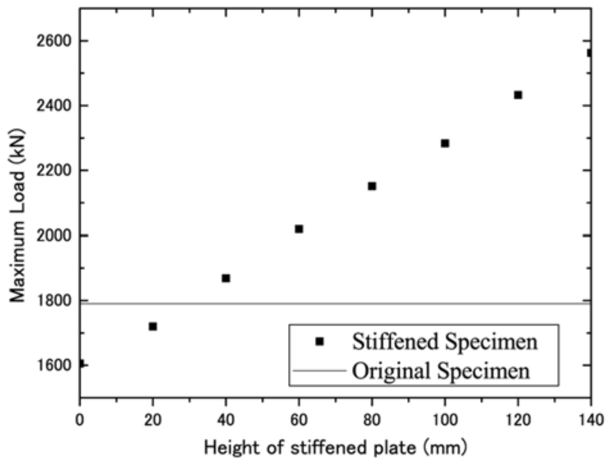

Fig. 12 Comparison of original specimen and stiffened specimen

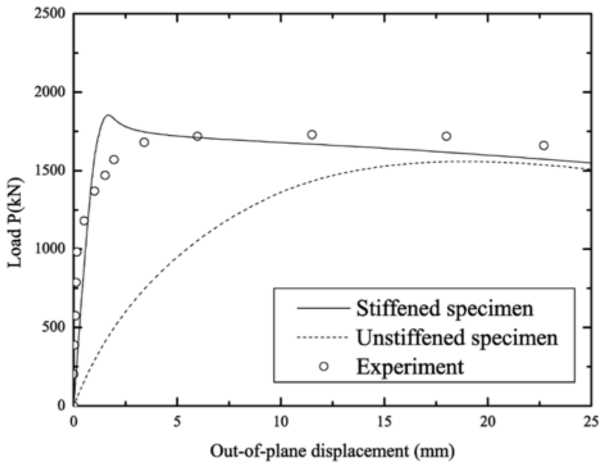

Fig. 13 Comparison of original specimen and stiffened specimen

5. Conclusions

In the present work a numerical model using elasto-plastic finite element analysis which can predict the mechanical response of a heat-corrected specimen was developed and the validity was ascertained by a compression experiment. In addition, several numerical studies were conducted using the developed model. First of all, it was clarified for the first time that the location of the residual imperfection affects the mechanical behavior of the plate. The work revealed the trend that ultimate strength of the heat-corrected specimen depends on the location of the residual imperfection, and this finding has significant practical use, for example, from the viewpoint of emergency repairs. Secondly, it was shown numerically that the heat-corrected ultimate strength can be larger than the ultimate strength of the original structure with the use of a properly designed stiffened plate.

As mentioned above, this study produced important findings; however, more detailed work is desired in order to understand the behavior more. First, since limited boundary conditions (free on one unloaded edge, simply-supported on three edges and free on one unloaded edge, clamped on three edges) were investigated in this research, it is necessary to investigate another boundary condition, for example, simply-supported on every edge. Second, since the present investigation dealt only with compression force, it is important to understand the behavior under another loading condition. This warrants future work on experimental and numerical studies. Third, it is required to investigate the change in material property during heating correction process.

Nonetheless, the success in developing the numerical model and finding new results will allow us to understand the behavior of heat-corrected structures and enhance the reliability of using heat-correction on buckling structures.

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