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The applications and conduct of vibration equations for constrained layered damped plates with impact

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Abstract Visco-elastic material and thin metals were adhered to plate structures, forming the composite components that are similar to the sandwich plates, called constrained layered damped (CLD) plates. Constrained layer damping has been utilized for years to reduce vibration, and advances in computation and finite element analysis software have enabled various problems to be solved by computer. However, some problems consume much calculation time. The vibration equation for a constrained layered damped plate with simple supports and an impact force is obtained theoretically herein. Then, the results of the vibration equation are compared with those obtained using the finite element method (FEM) software, ABAQUS, to verify the accuracy of the theory. Finally, the 3 M constrained layer damper SJ-2052 was attached to plates to form constrained layered damped plates, and the vibration equation was used to elucidate the damping effects and vibration characteristics.

Key words: visco-elastic; constrained layered damped (CLD); finite element method (FEM).

1. Introduction

Most visco-elastic materials are macromolecular polymers with long chains; the friction between the molecules can be exploited for damping, they consume and storage energy associated with dynamic loading.

The most common method for settling the vibrations of plate is to bond visco-elastic material and thin metal to the plates, to form a constrained layered damped component that is similar to a sandwich plate. This composite component is called a constrained layered damped (CLD) plate. The first layer of the CLD plate is the based layer; the second layer is the damping layer, and the third layer is the constrained layer. The constrained layer is typically a thin metal layer with low stiffness, and its main purpose is to constrain the damping layer. When the base layer is loaded, the base layer and constrained layer bend, the damping layer is constrained and undergoes a shear deformation, consuming energy (Ditaranto 1965, Agbasiere and Grootenhuis 1968).

In other work on damped composite components, Oberst use the vibration difference equation to discuss the loss factor and other parameters that govern free layer damping (Oberst 1952). In 1959, Ross, Unger and Kerwin examined constrained layer damping (Ross *et al* 1959). In 1969, Ditaranto and McGraw began to discuss the bending of damped composite plates (Ditaranto and McGraw 1969), and M. J. Yan and E. H. Dowell further adopted the governing equations of constrained layer damped beam and plate (Yan and Dowell 1972). In 1982, Johnson and Kienholz employed finite element analysis

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software to solve CLD problems (Conor *et al.* 1982), suggesting the use of the modal strain energy method and finite element method (FEM) software NASTRAN to obtain the damping ratio and loss factor in various modes. In 1999, Int. J. Numer (Int. J. Numer. Meth. Eng. 1999) used FEM to simulate the transient response of visco-elastic materials, and used the complex model to simulate visco-elastic behavior. Evgeny Barkanov adopted FEM package to simulate the transient response of CLD cantilever beam with different impact forms (Evgeny Barkanov 1999).

In recent years, the applications of CLD were still developed to control the vibration of thin-wall structures, and the finite element methods were used to simulate the dynamic response increasingly. Chantalakhana and Stanway (Chantalakhana and Stanway 2000) used FEM and experiments to realize the vibration of CLD plate, and Yi Sung *et al.* (Sung and Sze 2000) simulated the forced oscillations by FEM. Ho Sung Kim *et al.* (Kim *et al.* 2001) discussed the relationship between impact energy and impact force. In 2002, T. X. Liu and H. X. Hua (Liu and Hua 2002) employed the FEM combined with the GHM (Golla-Hughes-Mctavish) model of visco-elastic materials to get over the vibrations of the visco-elastic materials that vary with frequency and temperature, and G. Wang (Wereley *et al.* 2002) used 2-D plate modes to calculate the nature frequency, mode shape and loss factor of CLD plate. T. Y. Wang *et al.* (Wang *et al.* 2004) utilized basic elastic equations and Tailor serious to discuss the thickness effects of constrained layer and visco-elastic layer. J. Y. Yeh (Yeh and Chen 2005) utilized FEM to calculate the instability regions of the CLD plate and illustrated the CLD that is effective in dynamic stability.

This study utilizes linear elastic theory to elucidate the force and moment balance of a CLD plate, and to consider shear deformation from the damping layer to conduct the forced vibration equation of CLD plate with simply support. Then the forced vibration equation for the CLD plate with impact is adopted to determine the form of vibration for various structural sizes and impact loads. Finally, the FEM package ABAQUS is utilized to simulate the various vibration cases of CLD plate, and compare with the results calculated from the forced vibration equation.

The plate components are always used to construct the vehicle structure, like aircraft, boat and car. The purpose of this study is wanted to find the method to strengthen the plate-structure and absorb the energy under impact loading. The theory conduct of CLD plates present invaluable means for designing and predicting the performance of the CLD plates under the impact load that can be used in many engineering applications.

2. Theory for three-layer damped plate structures

Before conduct the theory of a constrained layered damped plate can be developed, some parameters must be defined. The thickness of the base layer is H_3, that of the damping layer is H_2, and that of the constrained layer is H_1. The material properties of three layers are isotropic and homogeneous. Fig. 1 presents the constrained layered damped plate, and the base layer thickness H_3 typically exceeds H_2 and H_1. Therefore, some reasonable assumptions are made to simplify the theory-conduct process efficiently. In this work, the following five assumptions are made.

a. The stiffness in the x-y directions of the damping layer is lower then that of the base layer, and the bending stress and axial force of the damping layer is negligible can be ignored.
b. Under loading, the deformations w (z direction) of the three layers are the same.
c. The shear deformations of based layer and constrained layer are negligible, and can be ignored.
d. An infinitesimal deformation is assumed, and the relationship between stress and strain is linear elastic.
e. The boundary condition is simply support for four-edge.

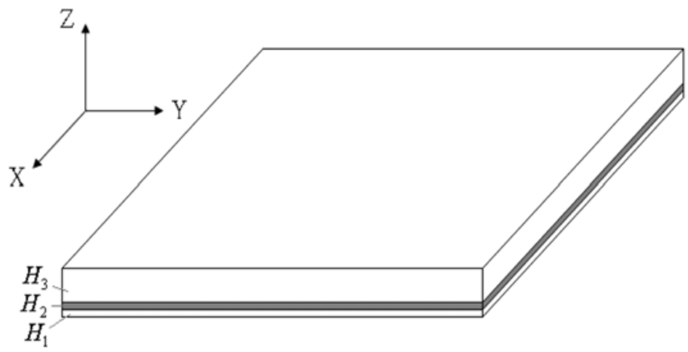


Fig. 1 Constrained layered damped (CLD) plate

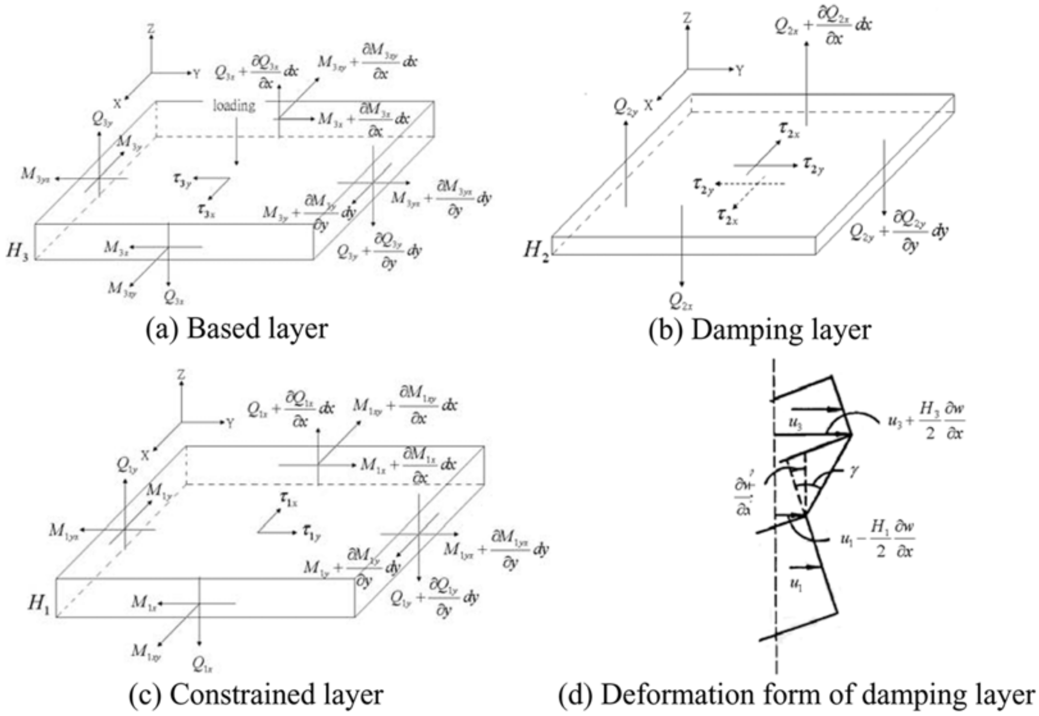


Fig. 2 The distributions of forces and moments of each layer and deformation form of damping layer

Based on these five assumptions, moment M, section shear force Q, and shear stress τ of each layer are defined. Fig. 2 plots the distributions of forces and moments of each layer and the form of the deformation of the damping layer.

The balance of forces and moments yields 11 balance equations. The transformations of stress and strain are applied to yield relationships between vertical deflection w and impact force $F(x,y,t)$, see Eq. (1); D_1 and D_3 are the bending rigidities of the constrained layer and the base layer respectively. Then, the damped layer is assumed to be a pure shear layer and the shear stress of the damped layer can thus be determined Eqs. (2) and (3). Fig. 2(d) presents the shear form. Finally, the effects of shear strain on the base layer and the constrained layer are considered; the relationship between the x-y deflections of the constrained layer and the base layer are obtained, and Eqs. (4) and (5) yield the relationships.

$$(D_1+D_3)\left[\frac{\partial^4 w}{\partial x^4}+2\frac{\partial^2 w}{\partial x^2\partial y^2}+\frac{\partial^4 w}{\partial y^4}\right]-\left(\frac{\partial \tau_{2x}}{\partial x}+\frac{\partial \tau_{2y}}{\partial y}\right)\left(H_2+\frac{H_1}{2}+\frac{H_3}{2}\right)\mu\frac{\partial^2 w}{\partial t^2} = F(x,y,t) \tag{1}$$

$$\tau_{2x} = \frac{\partial N_{3x}}{\partial x} = \frac{EH_3}{1-\nu^2}\left[\frac{\partial^2 u_3}{\partial x^2}+\nu\frac{\partial^2 v_3}{\partial x\partial y}+\left(\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}+\nu\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial y}\right)\right] = -\frac{\partial N_{1x}}{\partial x} \tag{2}$$

$$\tau_{2y} = \frac{\partial N_{3y}}{\partial y} = \frac{EH_3}{1-\nu^2}\left[\frac{\partial^2 v_3}{\partial y^2}+\nu\frac{\partial^2 u_3}{\partial x\partial y}+\left(\frac{\partial^2 w}{\partial y^2}\frac{\partial w}{\partial y}+\nu\frac{\partial^2 w}{\partial x^2}\frac{\partial w}{\partial x}\right)\right] = -\frac{\partial N_{1y}}{\partial y} \tag{3}$$

$$\left(1+\frac{E_3H_3}{E_1H_1}\right)\frac{\partial u_3}{\partial x} = \frac{\partial}{\partial x}\left(\tau_{2x}\frac{H_2}{G}\right)-\frac{\partial^2 w}{\partial x^2}\left(\frac{H_1}{2}+H_2+\frac{H_3}{2}\right)+\frac{1}{2}\left(1+\frac{E_3H_3}{E_1H_1}\right)\left(\frac{\partial w}{\partial y}\right)^2 \tag{4}$$

$$\left(1+\frac{E_3H_3}{E_1H_1}\right)\frac{\partial v_3}{\partial y} = \frac{\partial}{\partial y}\left(\tau_{2x}\frac{H_2}{G}\right)-\frac{\partial^2 w}{\partial y^2}\left(\frac{H_1}{2}+H_2+\frac{H_3}{2}\right)+\frac{1}{2}\left(1+\frac{E_3H_3}{E_1H_1}\right)\left(\frac{\partial w}{\partial y}\right)^2 \tag{5}$$

The conduct of Eqs. (2) and (3) were based on the elasticity mechanics, and the relations between plane stress of isotropic plate and deformation can be written below.

$$\sigma_x-\nu\sigma_y = E\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2\right], \qquad \sigma_y-\nu\sigma_x = E\left[\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2\right] \tag{6}$$

According to the relationship between stress and deformation, the stress indications of x-direction and y-direction can be written as Eqs. (7) and (8).

$$\sigma_x = \frac{E}{1-\nu^2}\left[\frac{\partial u}{\partial x}+\frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2+\nu\frac{\partial v}{\partial y}+\frac{1}{2}\nu\left(\frac{\partial w}{\partial y}\right)^2\right] \tag{7}$$

$$\sigma_y = \frac{E}{1-\nu^2}\left[\frac{\partial v}{\partial y}+\frac{1}{2}\left(\frac{\partial w}{\partial y}\right)^2+\nu\frac{\partial u}{\partial x}+\frac{1}{2}\nu\left(\frac{\partial w}{\partial x}\right)^2\right] \tag{8}$$

Assuming the damping layer is pure-shear layer, the section force of damping layer is neglected, and only the based layer and constrained layer have section force N_1 and N_3. From the force equilibriums of x-direction and y-direction, the section force can be presented below.

$$N_{3x} = -N_{1x} = \sigma_{3x}H_3 = \frac{EH_3}{1-\nu^2}\left[\frac{\partial u_3}{\partial x} + \nu\frac{\partial v_3}{\partial y} + \frac{1}{2}\left(\left(\frac{\partial w}{\partial x}\right)^2 + \nu\left(\frac{\partial w}{\partial y}\right)^2\right)\right] = -\sigma_{1x}H_1$$

$$N_{3y} = -N_{1y} = \sigma_{3y}H_3 = \frac{EH_3}{1-\nu^2}\left[\frac{\partial v_3}{\partial y} + \nu\frac{\partial u_3}{\partial x} + \frac{1}{2}\left(\left(\frac{\partial w}{\partial y}\right)^2 + \nu\left(\frac{\partial w}{\partial x}\right)^2\right)\right] = -\sigma_1 H_1 \tag{9}$$

Then the section force of x-direction and y-direction can presented using H_1 or H_3. In this paper, the parameter H_3 is chosen. Finally, the shear-stress of the damping layer can be written as Eq. (2) and (3).

Rearranging Eqs. (1) ~ (5) and iterating cannot eliminate the term $(\partial^3 v_3/\partial x^2\partial y) + (\partial^3 u_3/\partial y^2\partial x)$, which refers to in-plane shear deflection. In this work, the deformation of rectangle plate is considered to be symmetric and to deform linearly elastic. Accordingly, the effects of in-plane shear is negligible and compatibility equations can be applied Eq. (10) (Fenner 1987).

$$\frac{\partial^3 v_3}{\partial x^2\partial y} + \frac{\partial^3 u_3}{\partial y^2\partial x} = \frac{\partial^2\gamma_{xy}}{\partial x\partial y} = 0 \tag{10}$$

The compatibility equation can be used to eliminate the item $(\partial^3 v_3/\partial x^2\partial y) + (\partial^3 u_3/\partial y^2\partial x)$. Combining Eqs. (1) ~ (5) yields the relationship between deflection w and impact force, given by Eq. (11).

$$\frac{-E_3H_3H_2(D_1+D_3)}{1-\nu^2 G\left(1+\frac{E_3H_3}{E_1H_1}\right)}\nabla^6 w + \left(D_1 + D_3 + \frac{E_3H_3}{1-\nu^2}\frac{\left(H_2+\frac{H_1}{2}+\frac{H_3}{2}\right)}{\left(1+\frac{E_3H_3}{E_1H_1}\right)}\right)\nabla^4 w$$

$$+\mu\left[\frac{\partial^2 w}{\partial t^2} - \frac{E_3H_3}{1-\nu^2}\frac{H_2}{G\left(1+\frac{E_3H_3}{E_1H_1}\right)}\left(\frac{\partial^4 w}{\partial x^2\partial t^2} + \frac{\partial^4 w}{\partial y^2\partial t^2}\right)\right]$$

$$-\left[\frac{E_3H_3}{1-\nu^2}\frac{H_2}{G\left(1+\frac{E_3H_3}{E_1H_1}\right)}\right]\left[\begin{array}{c}\left(\frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial y^2}\right)\\ +\nu\left(\frac{\partial^3 w}{\partial x\partial y^2}\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial x\partial y} + \frac{\partial^3 w}{\partial x\partial y^2}\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x\partial y}\right)\end{array}\right]$$

$$-\frac{E_3H_3}{1-\nu^2}\left[\begin{array}{c}\left(\frac{\partial^3 w}{\partial y^3}\frac{\partial w}{\partial y} + \frac{\partial^2 w}{\partial y^2}\frac{\partial^2 w}{\partial y^2} + \frac{\partial^3 w}{\partial y^2\partial x}\frac{\partial w}{\partial x}\right) + 2\frac{\partial^2 w}{\partial x\partial y^2}\frac{\partial^2 w}{\partial x\partial y}\\ +\frac{\partial^3 w}{\partial x^3}\frac{\partial w}{\partial x} + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial x^2} + \frac{\partial^3 w}{\partial x^2\partial y}\frac{\partial w}{\partial x}\end{array}\right]\left(H_2 + \frac{H_1}{2} + \frac{H_3}{2}\right)$$

$$= F(x, y, t) - \frac{E_3H_3}{1-\nu^2}\frac{H_2}{G\left(1+\frac{E_3H_3}{E_1H_1}\right)}\left(\frac{\partial^2 F}{\partial x^2}+\frac{\partial^2 F}{\partial y^2}\right) \tag{11}$$

From the basic assumption that boundary condition is one of simple support, the vertical deformation *w(x,y,t)* and the impact force *F(x,y,t)* can both be expanded using an eigen-function, see Eqs. (12) and (13). *a* and *b* are the width in x and y direction individually.

$$w(x, y, t) = \sum\sum q(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{12}$$

$$w(x, y, t) = \sum\sum q(t)\sin\frac{m\pi x}{a}\sin\frac{n\pi y}{b} \tag{13}$$

Eq. (11) is substituted for the above eigen-functions, and the mode shape of the vibration with simple support exhibits orthogonality. Mode shape orthogonality is used to determine the vibration function, considering that the second layer is under pure-shear.

$$\mu\frac{d^2q(t)}{dt^2} + C\cdot A^2\cdot q(t) = f(t)$$

$$and \quad C = (D_1+D_3) + \frac{E_3H_3G\left(H_2+\frac{H_1}{2}+\frac{H_3}{2}\right)^2}{G(1-\nu^2)\left(1+\frac{E_3H_3}{E_1H_1}\right)+E_3H_3H_2A}$$

$$A = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2$$

$$D_i = \frac{E_iH_i^3}{12(1-\nu^2)} \qquad i = 1, 3$$

$$f(t) = \frac{4}{ab}\int_0^a\int_0^b F(x, y, t)\sin\frac{m\pi x}{a}\sin\frac{m\pi x}{b}dxdy \tag{14}$$

In Eq. (14), A is the shape parameter of constrained layer damping (CLD) plate, and the D is the bending stiffness (rigidity) of plate.

Through the Eq. (14), the natural frequency of this sandwich-like plate can be presented as $\omega = \sqrt{K/m} = \sqrt{C\cdot A^2/\mu}$. Conor D. Johnson (Conor *et al.* 1982) and Gao. Jianxin *et al.* (Jianxin and Yapeng 1999) had accomplished the evaluation of natural frequency of CLD plate using theory and finite element simulation. The natural frequency comparisons between Eq. (14) and the results of Conor D. Johnson

Table 1. The natural frequency comparisons with other references

	Coenor D. Johnson (Conor *et al.* 1982)		Gao. Jianxin (Jianxin and Yapeng 1999)	Equation (10)
	Theory	FEM	Theory	$\omega = \sqrt{K/m} = \sqrt{C \cdot A/\mu}$
Natural frequency	60.3	57.4	60.2	59.097

and Gao. Jianxin are showed in Table 1.

The visco-elastic characteristics of the damped layer must be considered. Now, the relationship between shear stress and shear strain is no longer $\tau = G\gamma$. In fact, if damping is presented in the vibration systems, the complex form is always used to obtain the solution. Therefore, the shear stiffness G is given by,

$$\tau = G\gamma + \frac{G_{real} \cdot \eta_G}{\omega}\frac{\partial \gamma}{\partial t} \tag{15}$$

In the CLD plate system, the damping effect is considered. So, the shear stiffness G is always expressed as complex form, $G = G_{real}(1 + i\eta_G)$, and η_G is the loss factor of damped layer. Then, the stiffness term C in Eq. (14) can be written as C', and C' is presented as,

$$C' = (D_1 + D_3) + \frac{E_3H_3\left(H_2 + \frac{H_1}{2} + \frac{H_3}{2}\right)}{(1-\nu^2)\left(1 + \frac{E_3H_3}{E_1H_1}\right)} - \frac{E_3^2H_3^2H_2\,A\left(H_2 + \frac{H_1}{2} + \frac{H_3}{2}\right)^2(1 - i\eta_G)}{2G(1-\nu^2)\left(1 + \frac{E_3H_3}{E_1H_1}\right)^2} \tag{16}$$

C' is the new stiffness term when just the second layer is considered to be the damped layer. C' is substituted for C in Eq. (14), and just the real part is considered, yielding Eq. (17).

$$Re\left[\mu\frac{d^2}{dt^2}q_{real} \cdot e^{i\omega t} + \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 C'q_{real} \cdot e^{i\omega t}\right] = Re[f(t)] \tag{17}$$

Eq. (17) can be written as:

$$Re\left[\mu\frac{d^2}{dt^2}q_{real}(\cos\omega t + i\sin\omega t) + \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 C'q_{real}(\cos\omega t + i\sin\omega t)\right] = f(t) \tag{18}$$

Expanding Eq. (18) yield the vibration equation of the CLD plate.

$$\mu\frac{d^2}{dt^2}q_{real} + \left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 B_1 \cdot q_{real} + \eta\left(\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2\right)^2 \frac{B_2}{\omega}\frac{dq_{real}}{dt} = f(t)$$

$$\text{and } B_1 = (D_1 + D_3) + \frac{E_3H_3\left(H_2 + \frac{H_1}{2} + \frac{H_3}{2}\right)^2}{(1-\nu^2)\left(1 + \frac{E_3H_3}{E_1H_1}\right)} - B_2 \qquad B_2 = \frac{E_3^2H_3^2A\left(H_2 + \frac{H_1}{2} + \frac{H_3}{2}\right)^2}{2G(1-\nu^2)\left(1 + \frac{E_3H_3}{E_1H_1}\right)^2} \tag{19}$$

In Eq. (19), B_2 is the damping coefficient, and the damping effect is considered to be exist only in the damped layer, not through the CLD system.B_1 is the stiffness coefficient for the entire CLD plate.

From Eq. (19), the coefficients B_1 and B_2 are indicated stiffness and damping individually. The coefficient B_1 influences the deformation after impact and natural frequency, and the coefficient B_2 influences the vibration decay rate. The thickness variations of the damping layer and constrained layer will change the coefficient B_1 and B_2 directly. For example, the variation of damping layer thickness H_2 will vary the B_2 obviously, and will change the damping effect without influence the vibration characters. But the variation of constrained layer thickness H_1 will vary the B_1 and B_2 at the same time. Then the variations of stiffness and damping of the CLD plate are become complicatedly.

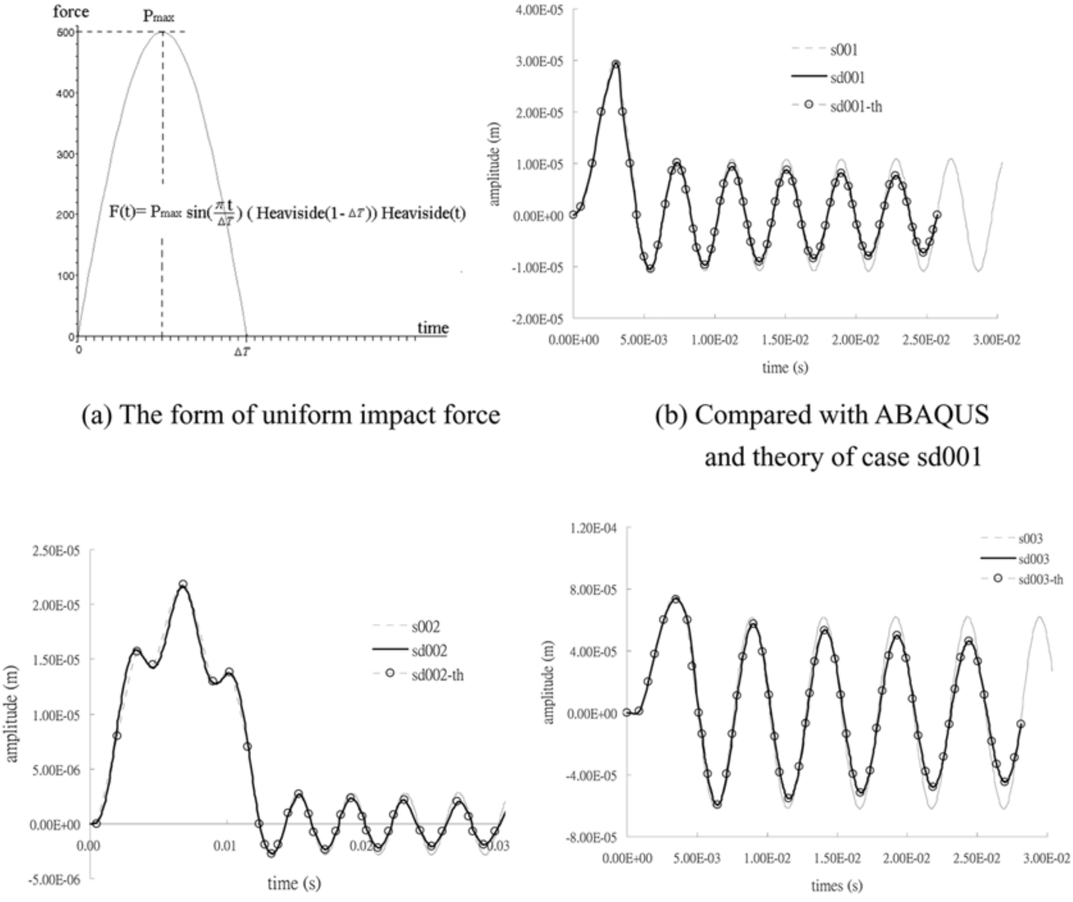
3. Results of analysis and discussion

The deformation of the CLD plate with four-edge simple support and impact can be obtained from the vibration Eq. (19). To verify the accuracy of the vibration equation, three examples are chosen and the parameters changed; then, use both the finite element method and Eq. (19) are employed to solve the examples and the results are compared. The software used to solve Eq. (19) is MAPLE and the finite element method (FEM) software used herein is ABAQUS.

FEM was adopted to solve dynamic questions that typically take too much time to solve-especially for the constrained layered damped plate-because the shear effect of the damped layer must be considered. In this regard, shell elements cannot be used in the analysis, rather, solid elements must be used. In FEM analysis, the shear effect can be represented using solid elements, but the calculation time is higher. Therefore, if Eq. (19) can correctly solve the problem of the CLD plate with simple support and impact, then Eq. (19) can always be used, and helping to elucidate the characteristics of various CLD plates quickly.

The three compared examples are named sd001, sd002 and sd003. In these examples, the base layer material is steel; the damped layer material is visco-elastic polymer; the constrained layer material is aluminum, and the loss factor η_G of the visco-elastic polymer is 0.5. The impact force form is assumed to be a half-sine wave, with a max. impact of 500 N/m^2, and variation in impact that continued for time ΔT. In MAPLE, the form of impact force can be written as $F(t) = P_{max} \times sin\left(\frac{\pi t}{\Delta T}\right) \times Heaviside(1 - \Delta T) \times Heaviside(t)$, please see Fig. 3(a).

Table 2 presses the related parameters of the three examples. Table 3 and Fig. 3 present the results after computation. In Fig. 3, the notation 'sd' means the CLD cases which have three layers to indicate the different materials. The notation 's' means the cases without CLD, and only the based layer thickness H_3 is considered. The cases of s001 and sd001 are computed using ABAQUS; sd001-th curve in Fig. 3(b) is obtained using Eq. (19). The curves are similar to those in Fig. 3(c) and Fig. 3(d). Fig. 3 shows that despite the change in the thickness of the base layer and the impact for time ΔT, the agreement between numerical analysis and theory is very good-including with respect to the vibration mode, the frequency and the vibration decay rate. Fig. 4 presents the finite element model and the form



(a) The form of uniform impact force

(b) Compared with ABAQUS and theory of case sd001

(c) Compared with ABAQUS and theory of case sd002

(d) Compared with ABAQUS and theory of case sd003

Fig. 3 The form of impact force and analysis results compared with numerical and theory

Table 2 Basic parameter of analysis examples

	sd001	sd002	sd003	s001	s002	s003
H3(m)	0.002	0.002	0.0015	0.002	0.002	0.0015
H2(m)	0.0001	0.0001	0.00008	--	--	--
H1(m)	0.0003	0.0003	0.00022	--	--	--
L × B(m)	0.2 × 0.2	0.2 × 0.2	0.2 × 0.2	0.2 × 0.2	0.2 × 0.2	0.2 × 0.2
ΔT (s)	0.005	0.013	0.005	0.005	0.013	0.005

of deformation. Table 3 concerns the vibration frequency and the decay rate, but the decay ratio must be first defined. Following impact, the CLD plate undergoes vibration, and if damping is included, the vibration amplitude decays with the number of pure cycle, so the decay ratio is defined as the amplitude

Table 3 Compared with theory and ABAQUS solutions

	Nature frequency (1/s)		Decay ratio	
	Theory	ABAQUS	Theory	ABAQUS
sd001	256.014	256.093	0.0720	0.0722
sd002	256.016	256.093	0.0719	0.0722
sd003	192.390	193.734	0.0695	0.0697

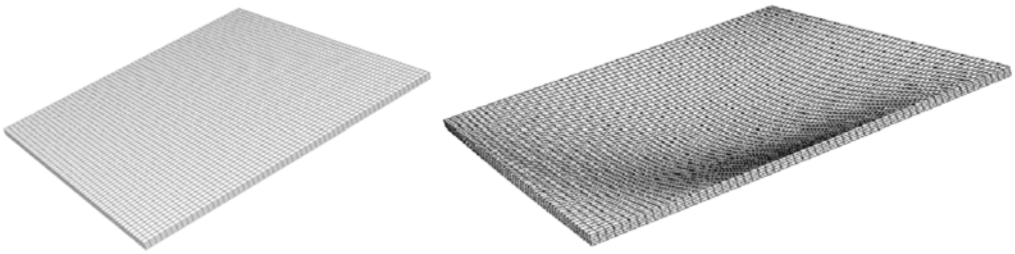


Fig. 4 The finite element model and deformation after impact force

reduction ratio per pure cycle. Comparing the vibration frequency determined numerically using ABAQUS with the theoretical results from Eq. (19) reveals that the numerical and theoretical vibration frequencies and decay ratios agree closely. This fact reveals the accuracy of Eq. (19), and Eq. (19) is adopted to solve the dynamic question of the two-dimensional CLD plates.

After the mutual consistency of theory and FEM analysis has been confirmed, the characteristics of the CLD plate are theoretically discussed including the effects of the based layer thickness and the impact continued time.

A 3M constrained layered damper SJ-2052 is adopted to simulate the damped layer and the constrained layer, Table 4 presents basic information about SJ-2052. SJ-2052 is a composite component that formed from visco-elastic glue and aluminum; it has a great capacity for shear deformation; the visco-elastic glue performs the damping; when it adheres to any structure, this glue can be regarded as a damping layer and aluminum can be treated as a constrained layer.

The form of the impact is assumed to be the half-sine wave and the maximum impact is assumed to be 500 N/m^2, changing the thickness of based layer H_3; the impact continues for time ΔT; then the vibration forms are discussed. Two examples are used to elucidate the effects on the CLD plate of change of changing H_3 and ΔT. These two examples are called case-1 and case-2.

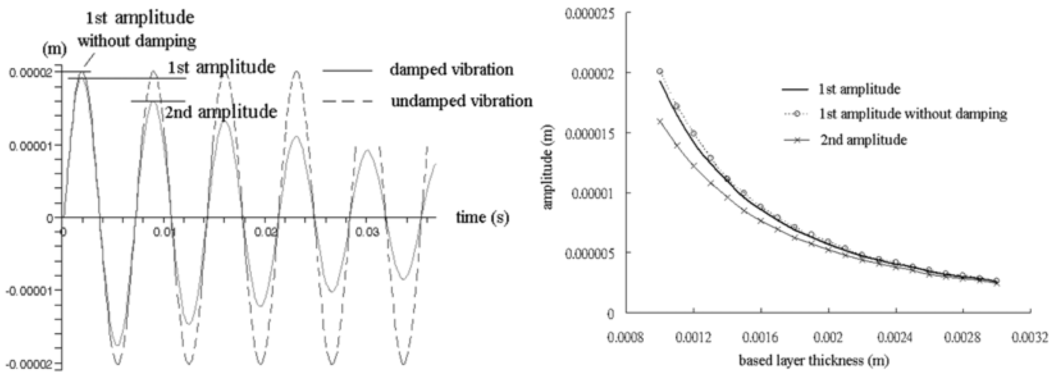
In case-1, the thickness of the based layer is assumed to increase from 0.001(m) to 0.003(m), and the impact continued time is 0.0003(s). In case-2, the thickness of the based layer increases from 0.001(m)

Table 4 Material parameters of constrained layer damping, (http://www.mmm.com, 3MTM Constrained Layer Damper SJ-2052)

Material	3M ISD-112 Damping Foil (Damping Layer)	Aluminum (Constrained Layer)
Thickness	0.127 mm	0.254 mm
Density	980 Kg/m^3	2,688 Kg/m^3
Loss factor	0.90	--

Table 5 Parameters of theory for case-1 & case-2

H3 (m)	Natural frequency (1/s)	B1	B2
0.0010	141.147	28.063	0.2867
0.0012	163.130	44.287	0.3773
0.0014	185.907	66.149	0.4805
0.0016	208.881	94.531	0.5951
0.0018	232.087	130.313	0.7219
0.0020	255.461	174.734	0.8599
0.0022	278.959	227.593	1.0122
0.0024	302.549	290.849	1.1722
0.0026	326.214	365.023	1.3462
0.0028	349.935	450.992	1.5320
0.0030	373.702	549.637	1.7297



(a) H_3=0.001 m (b) amplitude with different based layer thickness

Fig 5 Vibration form and amplitude with different based layer thickness of case-1

to 0.004(m), and the duration of impact is 0.005(s). The very difference between case-1and case-2 is the duration of impact; impact continues for a very short time in case-1, so the CLD plate exhibits free vibration when the impact is over. However, the impact frequency in case-2 is close to the natural frequency of the CLD plate, so the impact duration affects the form of the vibration. In Table 5, the theoretical parameters are calculated using Eq. (19), yielding the relationship between the plate deflection w and time.

Fig. 5 plots the form of vibration and the decay ratio in case-1. Since the impact duration is very short, the vibration in case-1 is like a free vibration. In Fig. 5(a), the decay of the vibration of the CLD plate per pure cycle clearly exceeds that of the general plate. Therefore, the constrained layer is useful in reducing vibration when the impact duration is very short. Eq. (19) yields the vibration amplitude for various based layer thickness, which is plotted in Fig. 5(b).

When the vibration amplitude for any thickness and any period is known, the vibration decay value

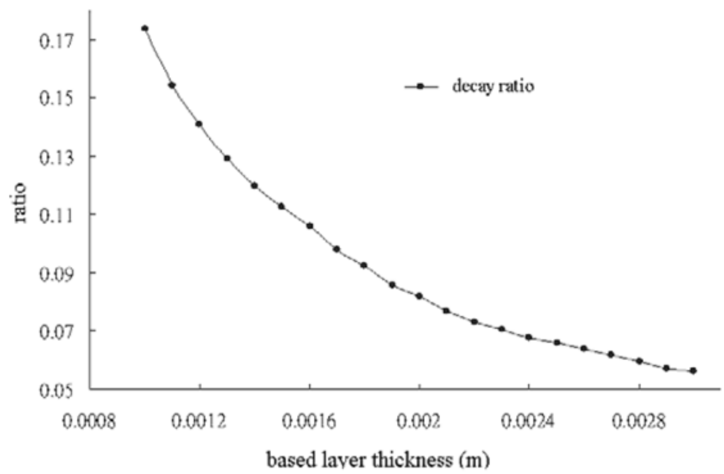


Fig 6 Decay ratio for different thickness of case-1 (calculated from theory)

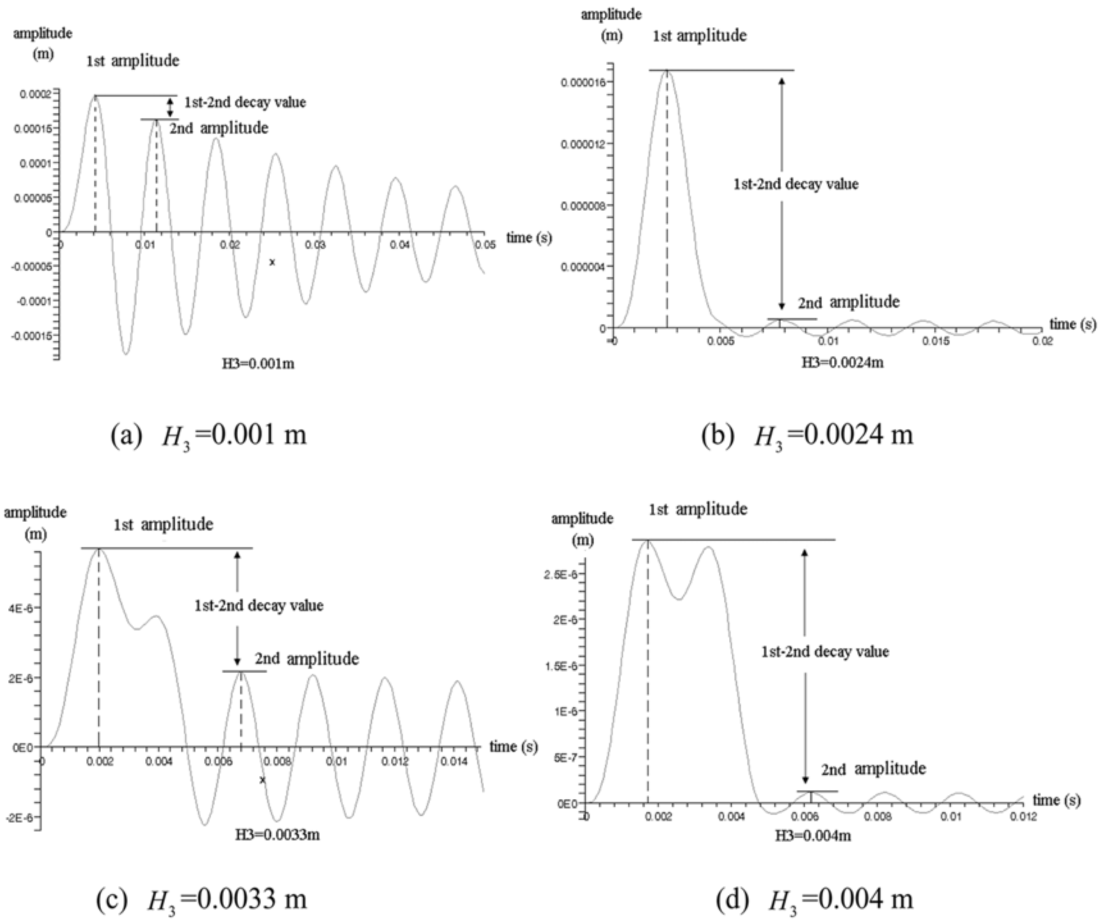
with pure cycle can easily be determined. The decay ratio is assumed to be the decay value with pure cycle as a fraction of the previous amplitude. Figure 6 plots the decay ratio of SJ-2052 that adhesive on the steel plate as a function of based layer thickness, and the decay ratio does not vary with form of the impact and duration.

In case-2, the impact continued time ΔT is extended to 0.005(s). At this time, the impact continued time and the vibration period of CLD plate are closely. When the vibration frequency is a multiple of the impact frequency, the free vibration of the plate will be occurred something different. Fig. 7 plots the forms of vibration for various based layer thickness H_3. At an H_3 of between 0.001(m) and 0.0024(m), the 1st amplitude and 2nd amplitude are both decrease when the based layer thickness H_3 increases, as in case-1. Then, as H_3 increases, the 1st amplitude decreases, but the 2nd amplitude does not. For instance, the 2nd amplitude of H_3 is 0.0033(m), which exceeds H_3 is 0.0024(m) and H_3 is 0.004 (m).

Eq. (19) yields the 1st and 2nd amplitudes in case-2, which can both be determined quickly. Then, the variation in amplitude with thickness H_3 can also be quickly determined. Fig. 8 plots the relationship between H_3 and amplitude. In case-1, the 1st and 2nd amplitudes decay as H_3 increases, but something different occurs in case-2. Since the impact frequency is close to the natural frequency of the plates, if the natural frequency is a multiple of the impact frequency, then the 2nd amplitude will become irregularly.

Eq. (19) not only yields the amplitude decay with time, but also can be differentiated to determine the plate velocity. Figure 9 plots the plate position u and the velocity v after impact. In Fig. 9, when the natural frequency of the plate is a multiple of the impact force, the plate returns to its original position following impact. Restated the deformation of the plate and the impact force are in phase. Then velocity of the plate is given by determined using Eq. (19) and plotted in Fig. 9. When the vibration frequency of plate is 300 Hz, the plate velocity following impact is smaller than at other frequencies. Furthermore, when the vibration frequency of the plate is 400 Hz, the plate velocity after impact is higher than at 300Hz, this trend is the same with the 2nd amplitude in figure 8, so it is considered that the plate velocity influences the 2nd amplitude. If the plate has a higher velocity when the impact has ended, the 2nd amplitude also increases. The 2nd amplitude of the plate seems to be correlated with the plate velocity when the natural frequency of impact force and CLD plate are closely.

Furthermore, the vibration equation of the CLD plate is very useful for vibration attenuation design. In this paper, the decay ratio of SJ-2052 adhering to steel plate has been obtained. Seeing Fig. 10, if the



(a) H_3=0.001 m (b) H_3=0.0024 m

(c) H_3=0.0033 m (d) H_3=0.004 m

Fig 7. Damped vibration form of case-2 with different based layer thickness

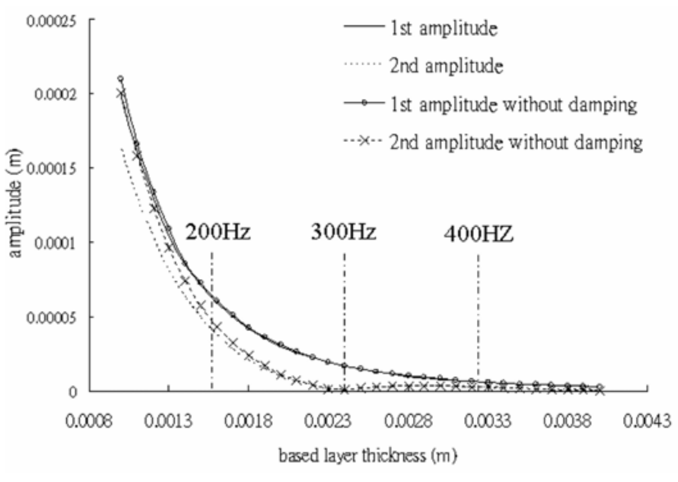


Fig. 8 Relationship with H_3 and amplitude in different frequency of case-2

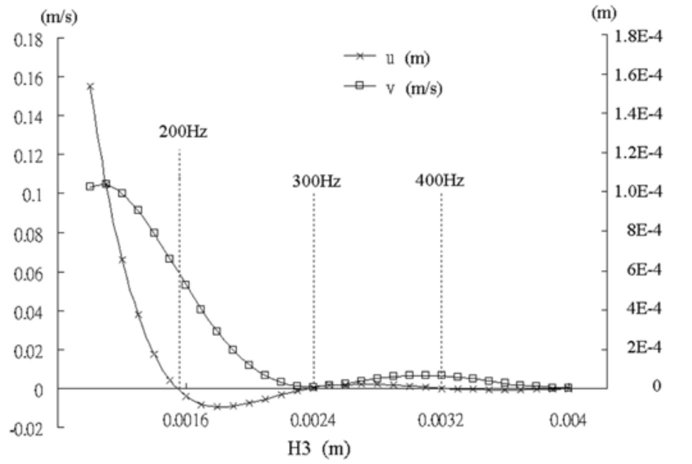


Fig 9 The plate position (u) and velocity (v) of plate when impact is over

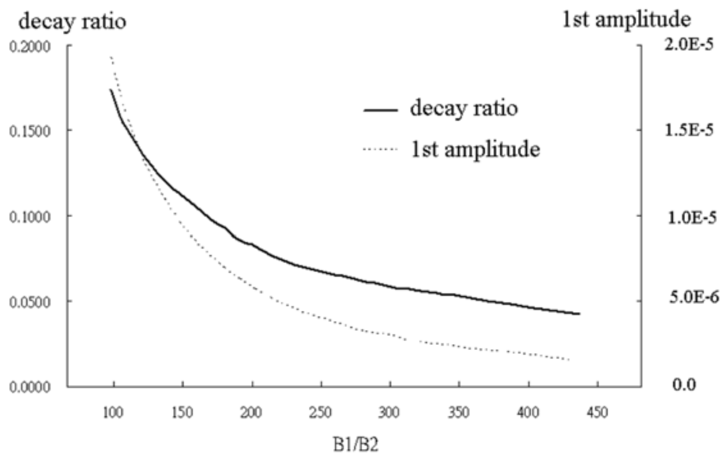


Fig 10 The variations of 1st amplitude and vibration decay ratio are changed by B_1 / B_2

value B_1 / B_2 from Eq. (19) is known, the decay ratio and 1st amplitude of CLD plate can be got easily.

4. Conclusions

The main propose in this study is to conduct the forced vibration equation of the CLD plate with impact and a simply support boundary condition. Calculating the parameters and solving the equation can easily yields the vibration characteristics of CLD plate, and the CLD plate deformation with time can be obtained quickly form the vibration equation not via FEM.

The vibration equation of CLD plate with four-edge simple support under impact was applied, and the results were consistent with those obtained using ABAQUS. So, solving the questions of CLD plate vibration, the FEM software, ABAQUS, is not required, as it takes much more time.

About this approach, the vibration mode shape after impact must be presumed first. Then, use the eigen-function to expand the deformation. The conduct results of this approach can not be easily used for other boundary conditions. However, for other boundary conditions, if the vibration modes are regular and the eigen-function are confirmed. Changing the eigen-function of equation (8), and the mode shape orthogonality is existent. Following the method of this approach, the new forced vibration of CLD plate can be easily got.

In this study, SJ-2052 is adheres to the plate to form the CLD plate. Under impact, the first amplitude of the CLD plate is nearly that of the original plate, and the first amplitude of the CLD plate is just only 2% ~ 4% lower. Therefore, SJ-2052 should be used to reduce vibration and noise, and the vibration attenuation effect of SJ-2052 is obviously.

The theoretical solution for the CLD plate not only yields the natural frequency, the variation of the amplitude with time, and the velocity of the plate, but also quickly yields the vibration characteristics of the CLD plate, like B_1, B_2 and natural frequency. In this study, the impact of a CLD plate with simple support boundary conditions was theoretically analyzed, and the accuracy of the approach was verified.

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