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# Vibration analysis of steel frames with semi-rigid connections on an elastic foundation

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**Abstract.** An investigation on the combined effect of foundation type, foundation flexibility, axial load and PR (semi-rigid) connections on the natural frequencies of steel frames is presented. These effects were investigated using a suitable modified FE program for cases where the foundation flexibility, foundation connectivity, and semi-rigid connections could be treated as equivalent linear springs. The effect of axial load on the natural frequency of a structure was found to be significant for slender structures subjected to high axial loads. In general, if columns of medium slenderness are designed without consideration of axial load effects, the frequency of the structure will be overestimated. Studies on the 3-story Los Angeles PR SAC frame indicate that the assumption of rigid connections at beam-column and column-base interfaces, as well as the assumption of a rigid foundation, can lead to significant errors if simplified design procedures are used. These errors in an equivalent static analysis are expected to lead to even more serious problems when considering the effect of higher modes under a non-linear dynamic analysis.

## 1. Introduction

For low-rise frames with flexible connections, the effect of foundation and column base flexibility can have a pronounced effect on the dynamic characteristics of the structure. In such cases, a substantial portion of the lateral stiffness stems from whatever assumptions are made for both the column base connection and vertical foundation stiffnesses. Numerical studies have shown that the column base connections contribute significantly to the overall stability for frames with flexible beam-column connections, with column base restraint becoming less significant for frames with more rigid beam-to-column connections (Liew *et al*. 1997). Taking foundation and connection flexibility effects into account can result in significant cost savings linked to reduction of the manpower and materials necessary to obtain relatively rigid column bases/connections and/or to reduction of column and beam sizes in the case of nominally pinned column bases (Jaspart and Vandegans 1998). For both the cases of column bases and beam-to-column connections, the effects are far more pronounced for sway than non-sway frames (Ivanyi 2000). Extensive descriptions of available models and analysis techniques for structures with semi-rigid or partially restrained (PR) connections can be found in [Chen *et al*, 1996] and [Chan and Chiu 2000].

In low-rise frames with flexible connections the effectiveness of the columns in limiting drift is dependent not only on the end restraint conditions but also on any stiffness reduction in the member. The latter depends primarily on the interaction between axial and bending forces and can result in

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substantial stiffness reductions for slender members such as those used in low-rise steel construction. The end restraint also is dependent on the beam moments, as the flexural stiffness of PR connections is non-linear and decreases drastically as the ultimate connection strength is reached. For low-rise, partially-restrained (PR or semi-rigid) frames, these effects are currently not properly accounted for in design codes which considers only rigid connections and a deflected shape based on first mode deformations. The fundamental period used for design of these frames is limited to a relatively small multiple (1.2 to 1.4) of that of an equivalent fully rigid frame (ASCE 2005) even when advanced analysis is used. However, the fundamental period computed based on advanced analysis is often on the order of 1.6 to 2.0 times that of an equivalent rigid frame. The result is that the code-based base shear and story drifts for these structures are often substantially larger than those calculated from more rigorous techniques, as the period for these structures corresponds to the range of the elastic design response spectrum where the spectral acceleration is reduced (Faella *et al*. 2000).

Aside from the overall effect of connection and support flexibility, the modeling of the member stiffness is also important. The axial load effect can be included in the analysis via the use of either a geometric stiffness matrix or stability functions (ASCE 1997). The changes in connection stiffness at both the column base and the beam-to-column connections can also be incorporated by the use of suitably modified stiffness matrices (ASCE 1997, Tram *et al*. 2001). Finally, the effects of foundation type (spread footings, grade beams or mat foundations) and the soil stiffness can also be accounted for by treating these elements as beams or plates on elastic foundations (Selvadurai 1979). In order to simulate the elastic soil under the footings, either (a) a number of rigid beam elements, supported elastically at their nodes by discrete Winkler springs or (b) an absolutely rigid beam supported at its center by a translational and a rotational elastic spring are commonly used. In the case of a Winkler-type soil, these simple modeling techniques yield acceptable results (Morfidis and Avramidis 2002). In general, the most important aspect when using a Winkler-type subgrade model is to determine the application-specific constant value of the coefficient of subgrade reaction, $\alpha_n$. These techniques do not incorporate either geometrical or material non-linearities in the soil behavior since it is assumed that period calculations are essentially elastic ones as the rotations tend to be relatively small (Hayalioglu and Degertekin 2005).

The basic model used in this study is shown in Fig. 1, where these effects are lumped into equivalent linear and rotational springs and where the possibility of having individual footings or foundation beams is included. The paper describes first the analytical modifications required to include the foundation effects directly, and then briefly treats the calibration and verification procedures used. A simple one-bay, one-story model was initially used to carry out a parametric study to determine the effect of PR (semi-rigid) connections at both the column base and beam to column connections (including and the effects of rigid zones) on the natural vibration characteristics of steel frames on an elastic foundation. The paper then discusses the relative magnitude of these effects with the aid of a one-bay, two-story structure. Finally, the potential impact in design is assessed with the aid of the three-story PR frames designed as part of the SAC project (Maison and Kasai 2000).

## 2. Analysis Considerations

In this research, the finite element (FE) program KCW (Anh and Hien 2001) was modified to study the effects of foundation stiffness and type on the fundamental frequencies of slender, low-rise steel frames. The KCW program was originally written to perform linear and nonlinear analyses of two and three dimensional frames and includes functionalities such as stability analysis, second-order elastic

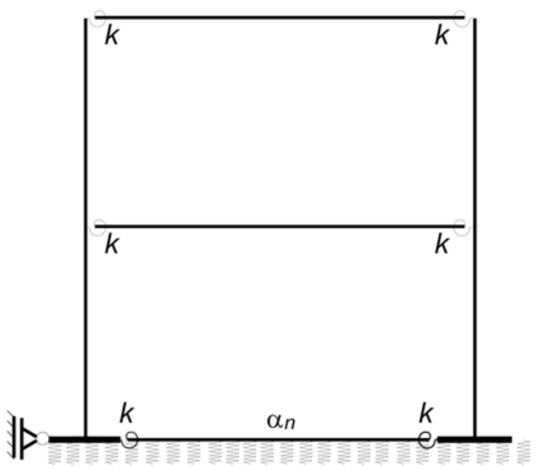


Fig. 1 General model

and inelastic analysis, and non-linear dynamic analysis of semi-rigid steel frames.

To incorporate the connection flexibility into the member stiffness, it is common practice to model the semi-rigid connection as a massless and zero length rotational spring, although far more sophisticated techniques, including distributed plasticity, are also available (Chen *et al.* 1996, Chan and Chiu 2000). Although PR connections exhibit nonlinear response characteristics throughout their whole loading history, it is generally accepted that under service loading conditions the connections can be assumed to behave linearly after shakedown has occurred (Leon 1994). This connection stiffness can be used as the basis for the calculation of dynamic properties in design codes that permit the use of advanced analysis.

Fig. 2 shows one beam with semi-rigid connections with and without rigid-zones (Fig. 2(a) and (b)). Fig. 2(g) shows a similar model for the foundation beam incorporating the subgrade reaction. In Fig. 2, $k_1$ and $k_2$ are the rotational stiffness of connections and $e_1$ and $e_2$ are the lengths of any connection rigid offsets. The nodal displacements are {X} and {X}* and the nodal forces are $\{F\}$ and $\{F\}^*$, with the asterisk indicating the model with rigid offsets. Figs. 2(c) through 2(f) show the displacement and force transformations for the case without rigid offsets for the four degrees of freedom of interest in the connection, while Fig. 2(h) shows the transformations for one case with rigid offsets.

The equations that govern the dynamic response of a structure are derived by requiring the work of external forces to be balanced by the work of internal and viscous forces for any small kinematically admissible motion (Cook *et al.* 2001, for example). For a single element with semi-rigid connections and rigid-zones, the work balance with consideration of the geometric nonlinear effects becomes:

$$\begin{aligned}&\int_{Ve}\{\delta\varepsilon\}^T[D]\{\varepsilon\}dV+\{\delta u_k\}^T[k_k]\{u_k\}+\int_{Ve}\{\delta u'\}^T[S]\{u'\}dV\\&+\int_{Ve}\{\delta u\}^T\rho\{\ddot{u}\}dV+\int_{Ve}\{\delta u\}^T k_d\{\dot{u}\}dV=\int_{Ve}\delta u^T q\,dV dV\end{aligned} \tag{1}$$

where $\{\delta u\}$ and $\{\delta\varepsilon\}$ are small arbitrary displacements and their corresponding strains, $[D]$ is the material property matrix, $[k_k]$ is the stiffness matrix of the springs, $\{\delta u_k\}^T$ are the small displacements of the springs, $\{q\}$ are body forces, $\rho$ is the mass density of material, $k_d$ is a material-damping parameter

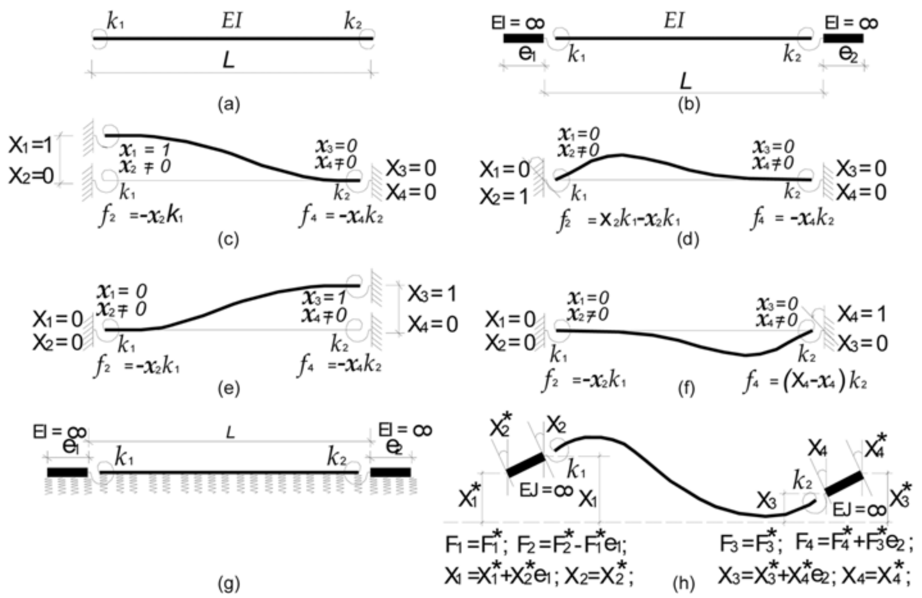


Fig. 2 Beam with semi-rigid connections and rigid zones on an elastic foundation

analogous to viscosity, and the integration is carried out over the element volume $V_e$.

The relationship between the element displacements for the beam with rigid offsets $\{X^*\}$ and the general displacement field {u}, which is a function of both space and time, is given as:

$$\{u\} = [N][T][e]\{X^*\} \tag{2}$$

where,

[N] = shape functions
[T] = transformation matrix
[e] = offset matrix
{X}, $\{X^*\}$ = functions of time only

Eq. (2) represents a local separation of variables. The displacements for the springs become:

$$\{u_k\} = [x_1^k \;\; x_2^k \;\; x_3^k \;\; x_4^k]^T$$

$$\{\delta u_k\}^T[k_k]\{u_k\} = \delta x_2^k . k_1 . x_2^k + \delta x_4^k . k_2 . x_4^k$$

$$\{u_k\} = ([I]-[T])[e]\{X^*\}$$

In general:

$$[k]\sum_{i=1}^{4}\{x\}_i = [k_k]\sum_{i=1}^{4}[\{X\}_i - \{x\}_i] \Rightarrow [k_k] = [k][T]([I]-[T])^{-1} \tag{3}$$

Obtaining a matrix [G] from the shape functions [N] by appropriate differentiation and ordering of terms:

$$\{u'\} = [G]\{x\} = [G][T][e]\{X^*\} \tag{4}$$

and defining the strains {e} as:

$$\{\varepsilon\} = [B]\{x\} \tag{5}$$

where [B] is the strain–displacement matrix, leads to the following work equation from Eqs. (1) and (2):

$$\begin{aligned}\{\delta X^*\}^T[&\int_{Ve}[e]^T[T]^T[B]^T[D][B][T][e]dV\{X^*\} + [e]^T([I]-[T])^T[k_k]([I]-[T])[e]\{X^*\} \\ &+\int_{Ve}[e]^T[T]^T[N]^T\rho[N][T][e]dV\{\ddot{X}^*\} + \int_{Ve}[e]^T[T]^T[N]^T k_d[N][T][e]dV\{\dot{X}^*\} \\ &+\int_{Ve}[e]^T[T]^T[G]^T[S][G][T][e]\{X^*\}dV] = \{\delta X^*\}^T[\int_{Ve}[e]^T[T]^T[N]^T\{q\}dV]\end{aligned} \tag{6}$$

Since $\{\delta X^*\}$ is arbitrary, Eq (6) can be rewritten as:

$$[m^*]\{\ddot{X}^*\} + [c^*]\{\dot{X}^*\} + ([k^*] + [k_\sigma^*])\{X^*\} = \{F^*\} \tag{7}$$

where $[k^*]$, $[k_\sigma^*]$, $[m^*]$, $[c^*]$are the stiffness matrix, geometric stiffness matrix, mass matrix and damping matrix of the element with semi-rigid connections and rigid-zones, and $[k_\sigma^*]$ is the geometric stiffness matrix depending on the load level; $\{F^*\}$ are external load vectors; $\{X^*\}$, $\{\dot{X}^*\}$, $\{\ddot{X}^*\}$ are the displacement vectors, velocity vectors and acceleration vectors of the element with semi-rigid connections and rigid-zones. The stiffness matrix of the beam element with semi-rigid connections and rigid-zones becomes:

$$\begin{aligned}[k^*] &= \int_{Ve}[e]^T[T]^T[B]^T[D][B][T][e]dV + [e]^T([I]-[T])^T[k_k]([I]-[T])[e] \\ &= [e]^T[k][T][e] = [e]^T[T]^T[k_\sigma][T][e]\end{aligned} \tag{8}$$

The mass matrix of the beam element with semi-rigid connections and rigid-zones is:

$$[m^*] = \int_{Ve}[e]^T[T]^T[N]^T\rho[N][T][e]dV + [e]^T[T]^T[m][T][e] \tag{9}$$

The damping matrix of the beam element with semi-rigid connections and rigid-zones is:

$$[c^*] = \int_{Ve}[e]^T[T]^T[N]^T\kappa_d[N][T][e]dV = [e]^T[T]^T[c][T][e] \tag{10}$$

where: $[k]$, $[k_\sigma]$, $[m]$ and $[c]$ are the well-known stiffness, geometric stiffness, mass and damping matrices of the beam element.

The element load vector of the beam with semi-rigid connections and rigid-zones is:

$$\begin{aligned}\{F^*\} &= [e]^T[T]^T\{f\} \\ \{f\} &= \int_{Ve}[N]^T\{q\}dV\end{aligned} \tag{11}$$

where $\{f\}$ is the element load vector of beam $\{q\}$ are the body forces.

The formulation of the transformation matrices [T] and [e] are shown in Appendix A (Anh 2002). For a single element, the work balance with consideration of an elastic foundation (see Fig. 2(g)) becomes:

$$\int_{Ve}\{\delta\varepsilon\}^T[D]\{\varepsilon\}dV + \int_{Ve}\{\delta u\}^T[\alpha_N]\{u\}dV + \{\delta u_k\}[k_k]\{u_k\} = \int_{Ve}\{\delta u\}^T\{q\}dV \tag{12}$$

Combining Eqs. (2), (3) and (12) yields:

$$\begin{aligned}&\{\delta X^*\}^T[\int_{Ve}[e]^T[T]^T[B]^T[D][B][T][e]dV\{X^*\} + \\ &\int_{Ve}[e]^T[T]^T[N]^T[\alpha_N][N][T][e]dV\{X^*\} + [e]^T([I]-[T])^T[k_k]([I]-[T])[e]\{X^*\}] \\ &= \{\delta X^*\}^T[\int_{Ve}[e]^T[T]^T[N]^T\{q\}dV]\end{aligned} \tag{13}$$

Since $\{\delta X^*\}$ is arbitrary, Eq (13) can be written as:

$$[k_N^*]\{X^*\} = \{F^*\} \tag{14}$$

The stiffness matrix of the beam element with semi-rigid connections and rigid-zones on an elastic foundation is:

$$\begin{aligned}[k_N^*] &= \int_{Ve}[e]^T[T]^T[B]^T[D][B][T][e]dV + \int_{Ve}[e]^T[T]^T[N]^T[\alpha_N][N][T][e]dV \\ &+ [e]^T([I]-[T])^T[k_k]([I]-[T])[e] = [e]^T\{[k]+[k_n]\}[T][e]\end{aligned} \tag{15}$$

where $[a_N]$ is a flexibility matrix of the soil medium (Selvadurai 1979, Smith and Griffiths 1998). For a single beam element, $[\alpha_N] = \alpha_n$, the stiffness matrix of beam on an elastic foundation, and the stiffness matrix becomes:

$$[k_n] = \int_{Ve}\alpha_n[N]^T[N]dV = \frac{\alpha_n L}{420}\begin{bmatrix}156 & 22L & 54 & -13L \\ 22L & 4L^2 & 13L & -3L^2 \\ 54 & 13L & 156 & -22L \\ -13L & -3L & -22L & 4L^2\end{bmatrix} \tag{16}$$

For the case of a plate on an elastic foundation (the footings in this case), the work (W) done by nodal loads $\{r_e\}$ in moving through nodal displacement $\{d\}$ is equal to work done by distributed loads $\{F\}$ and $\{\Phi\}$ in moving through the displacement field defined by $\{d\}$ and element shape functions. The work done by nodal loads is $W = \{d\}^T\{r_e\}$. Thus, from (Cook et el. 2001):

$$W = \{d\}^T\{r_e\} = \int_{Ve}\{u\}^T\{F\}dV + \int_{Ve}\{u\}^T\{\Phi\}dS \tag{17}$$

The integrals represent the sum of the work of load increments $\{F\}dV$ and $\{\Phi\}dS$ moving through displacement $\{u\}$ produced by nodal DOFs.
From equation (17), the term $\int_{Ve}\{u\}^T\{F\}dV$ can be written as:

$$\int_{Ve}\{u\}^T\{F\}dV = \int_{Ve}\{u\}^T[\alpha_N]\{u\}dV = \int_{Ve}\{d\}^T[N]^T[\alpha_N][N]\{d\}dV \Rightarrow [K]_S = \int_{Ve}[N]^T[\alpha_N][N]dV \tag{18}$$

For bending of a plate on an elastic foundation:

$$[K]_s = t\int_A[N]^T[\alpha_N][N]dA \tag{19}$$

where $[K]_S$ is the stiffness matrix of the plate, A is the area of the plate, t is the plate thickness, [N] are the shape functions and $[\alpha_N]$ = flexibility matrix of soil medium. In the results described in this article, only the geometrical nonlinearities of members are activated. The program, however, is general and can handle non-linear connection behavior connection as well as cyclic and dynamic loads.

## 3. Numerical Examples

To verify the matrices derived in the preceding section, numerical calculations were conducted on three simple steel frames with pinned – pinned, fixed-fixed and semi-rigid connections. The free vibrations equation for a frame on an elastic foundation can be written as:

$$[m^*][\ddot{X}^*] + ([k_N^*] + [k_\sigma^*])\{X^*\} = \{0\} \tag{20}$$

A general solution of such an equation is:

$$\{X^*\} = \{\bar{X}^*\}\sin(\omega t + \psi) \tag{21}$$

Substituting Eq. (21) into Eq. (20), the natural frequencies w can be determined from:

$$-\omega^2[m^*]\{\bar{X}^*\} + ([k_N^*] + [k_\sigma^*])\{\bar{X}^*\} = \{0\} \tag{22}$$

Table 1 shows a comparison of the natural frequencies for the frame shown in Fig. 3 obtained for two

Table1. Mode 1 - Fundamental frequency without consideration of geometric nonlinearities for the frame in Fig. 3.

| Joint type | Pinned-pinned | Fixed-fixed |
|---|---|---|
| Theory (James et al. 1994) | - | 27.620 |
| Mod. FE (KCW) | 12.745 | 27.579 |
| SAP 2000 | 12.281 | 27.096 |

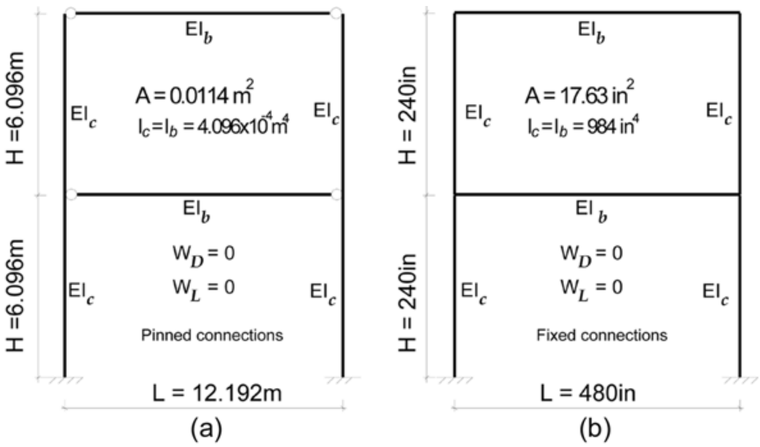


Fig. 3 Portal frame with pinned (a) and fixed (b) beam column connections

limit base conditions for the geometrically linear case. The results indicate very close agreement between theory, a detailed model using an advanced commercial software product (SAP2000) and the program developed herein (KCW). The frame in Fig. 3 is only meant for verification studies and its proportions should not be taken as typical of PR frames. Table 2 presents similar results for the frame in Fig. 4 for various ratios of axial to critical loads ($\beta = P/P_{cr}$, where $P_{cr}$ is the Euler load) and connection stiffness ($k_c$) ($\alpha = k_c/(L/EI)_{beam}$, = 0.001 to 1000) for the case of a rigid foundation. These results indicate that the periods are quite sensitive to the level of axial load as the axial load get large, an issue that will arise if the column members are slender.

Table 3 shows the results for the frame in Fig. 5 but including an assumption of an elastic foundation with a modulus of subgrade reaction ($\alpha_n$) of $6.5 \times 10^3$ KN/m$^3$ (41.4 kcf). This value corresponds to a soft site with loose sand (Bowles 1988). Tables 2 and 3 show that the presence of gravity loads decreases the natural frequencies of the frame primarily because of the P-$\Delta$ effects which tend to reduce frames stiffness. When compressive forces are present, $\omega \rightarrow 0$ as the buckling limit is approached. It can be seen from Tables 2 and 3 that the variation of the natural frequency is considerable when the joint stiffness changes and that the effect of foundation is to decrease the natural frequencies. Consequently, ignoring joint flexibility and foundation stiffness may generally lead to an unacceptable error in the analysis for many types of structures.

## 4. SAC Frames

To assess the impact of foundation flexibility on the design of realistic frames, the 3-story PR frames designed for the SAC project were selected (Maison and Kasai 2000). These frames (See Fig. 6) contained

Table 2. Mode 1 – Fundamental period with consideration of geometric nonlinearities for the frame in Fig. 4

| $\beta = P/P_{cr}$ | Pinned | PR or semi-rigid - Joint Stiffness $\alpha = k_c\ (L/EI)_{beam}$ | | | | | | Fixed |
|---|---|---|---|---|---|---|---|---|
| | | 0.001 | 0.01 | 0.1 | 1 | 10 | 1000 | |
| 0 | 0.493 | 0.493 | 0.490 | 0.468 | 0.363 | 0.256 | 0.228 | 0.228 |
| 0.1 | 0.519 | 0.519 | 0.517 | 0.493 | 0.383 | 0.270 | 0.240 | 0.240 |
| 0.2 | 0.551 | 0.550 | 0.548 | 0.523 | 0.406 | 0.286 | 0.255 | 0.255 |
| 0.3 | 0.588 | 0.588 | 0.585 | 0.559 | 0.434 | 0.306 | 0.273 | 0.272 |
| 0.4 | 0.635 | 0.635 | 0.632 | 0.603 | 0.468 | 0.330 | 0.294 | 0.294 |
| 0.5 | 0.696 | 0.695 | 0.692 | 0.661 | 0.513 | 0.362 | 0.322 | 0.322 |
| 0.6 | 0.778 | 0.777 | 0.773 | 0.738 | 0.573 | 0.404 | 0.360 | 0.360 |
| 0.7 | 0.898 | 0.896 | 0.892 | 0.852 | 0.662 | 0.467 | 0.416 | 0.416 |
| 0.8 | 1.098 | 1.098 | 1.092 | 1.043 | 0.740 | 0.572 | 0.510 | 0.509 |
| 0.9 | 1.551 | 1.551 | 1.544 | 1.475 | 1.145 | 0.809 | 0.720 | 0.720 |
| 0.999 | 48.332 | 49.087 | 48.707 | 46.542 | 36.110 | 25.541 | 22.765 | 22.765 |
| Pcr (KN) | 1180.13 | 1181.52 | 1194.05 | 1315.72 | 2259.74 | 4761.59 | 5997.0 | 6013.82 |

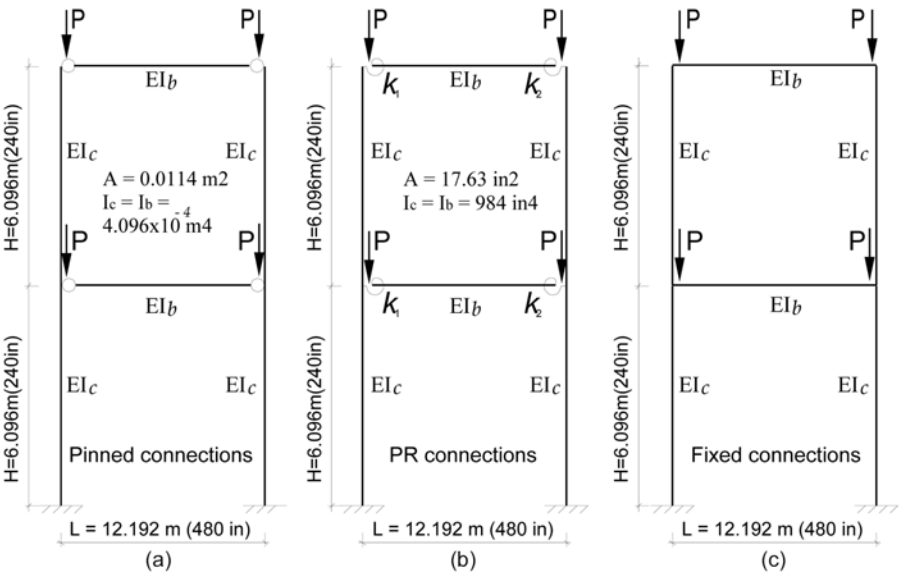


Fig. 4 Portal frame with semi-rigid connections and geometric non-linear

two types of connections, one a composite PR connection with a connection stiffness of about $4.5 \times 10^4$ KN-m/rad ($4 \times 10^5$ kip-in/rad) and one steel PR connection with a stiffness of about $1.7 \times 10^4$ KN-m/rad ($1.5 \text{x} 10^5$ kip-in/rad) (Leon *et al.* 1996). The composite connections were used in the interior connections to the strong side of the column, while the steel connections were used in exterior columns and to the weak side of the columns. The columns sizes used were W14 × 74, W14 × 90 and W14 × 109, while the beams sizes were W18 x 35. The moment-rotation characteristics for the connections are shown as curve C18 in Fig. 7. The foundations were idealized as square footings with dimensions of 1.52 m × 1.52 m × 0.45 m (5 ft. by 5 ft. by 1.5 ft.). Appendix B gives some details of the design of this frame.

Table 3. Mode 1 - Fundamental period with consideration of geometric nonlinearities and an elastic foundation ($\alpha_n$ = 6,500 KN/m$^3$ ) for the frame in Fig. 4

| b = P/P$_{cr}$ | Pinned | PR or semi-rigid - Joint Stiffness $\alpha$ = k$_c$ (L/EI)$_{beam}$ | | | | | | Fixed |
|---|---|---|---|---|---|---|---|---|
| | | 0.001 | 0.01 | 0.1 | 1 | 10 | 1000 | |
| 0 | 1.190 | 1.187 | 1.163 | 0.987 | 0.560 | 0.346 | 0.308 | 0.308 |
| 0.1 | 1.254 | 1.252 | 1.226 | 1.040 | 0.590 | 0.364 | 0.324 | 0.323 |
| 0.2 | 1.330 | 1.328 | 1.300 | 1.103 | 0.625 | 0.385 | 0.342 | 0.342 |
| 0.3 | 1.422 | 1.419 | 1.390 | 1.179 | 0.668 | 0.411 | 0.365 | 0.364 |
| 0.4 | 1.536 | 1.532 | 1.501 | 1.274 | 0.722 | 0.443 | 0.392 | 0.392 |
| 0.5 | 1.682 | 1.679 | 1.644 | 1.396 | 0.790 | 0.485 | 0.429 | 0.428 |
| 0.6 | 1.881 | 1.877 | 1.839 | 1.560 | 0.883 | 0.541 | 0.478 | 0.477 |
| 0.7 | 2.171 | 2.167 | 2.123 | 1.802 | 1.019 | 0.623 | 0.550 | 0.549 |
| 0.8 | 2.658 | 2.654 | 2.600 | 2.207 | 1.248 | 0.762 | 0.671 | 0.670 |
| 0.9 | 3.756 | 3.753 | 3.679 | 3.124 | 1.764 | 1.076 | 0.946 | 0.945 |
| 0.999 | 37.624 | 37.179 | 36.744 | 31.105 | 17.600 | 10.740 | 9.434 | 9.420 |
| Pcr (KN) | 241.08 | 242.36 | 252.77 | 351.74 | 1108.57 | 3061.33 | 4029.10 | 4042.48 |

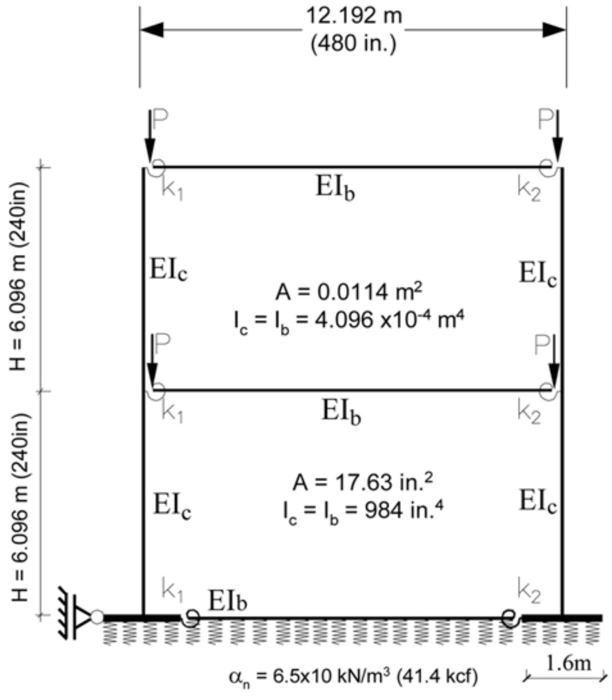


Fig. 5 Portal frame on an elastic foundation with various spring stiffness

The frames were modeled as 3D space frames using beams with semi-rigid connections, with plate elements used for modeling the floor slabs. It was assumed that the foundation has stiffness $\alpha_s$ and reacts in compression as well as tension, i.e., as a Winkler's foundation (Selvadurai 1979).

Table 4 shows the comparison of periods for the case where the foundation is assumed to rest on a

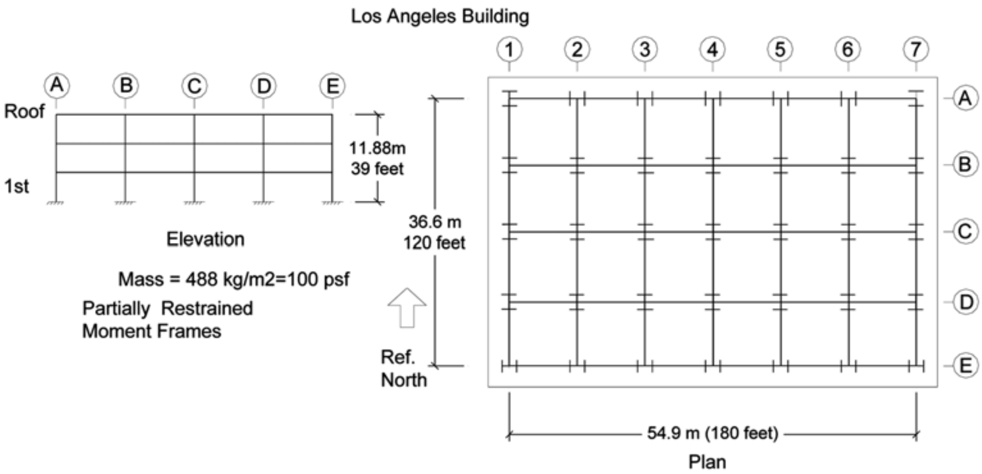


Fig. 6 SAC Frames

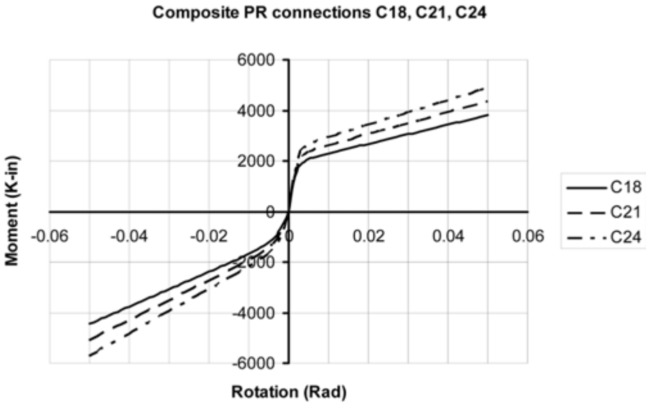


Fig. 7 Typical PR connection moment-rotation curves

rigid surface but both the stiffness of the beam connections and column bases at the first floor are assumed to vary. Interestingly, a comparison between the periods for the case of the beams being pinned-pinned or fixed-fixed with the column bases fixed shows very small differences. This means that much of the stiffness of the frame is related to the column base stiffness and not to that of the connections (i.e., the system is acting more with the columns as cantilevers than as a true frame.) For

Table 4. First three periods for the three-story SAC frame

| Beam end conditions | Fixed - Fixed beams | | Partially restrained beams | | | | Pinned -pinned beams |
|---|---|---|---|---|---|---|---|
| Column base stiffness | Fixed ($k_{base} = \infty$) | Pinned ($k_{base} = 0$) | Fixed ($k_{base} = \infty$) | $k_{base}$= $10(EI/L)_{column}$ | $k_{base}$ = $1(EI/L)_{column}$ | Pinned ($k_{base} = 0$) | Fixed ($k_{base} = \infty$) |
| T1 | 0.867 | 1.350 | 0.875 | 0.925 | 1.134 | 1.358 | 0.882 |
| T2 | 0.444 | 0.681 | 0.448 | 0.472 | 0.575 | 0.687 | 0.453 |
| T3 | 0.298 | 0.354 | 0.299 | 0.311 | 0.341 | 0.356 | 0.301 |

this case, the ratio of beam to connection stiffness is about 10, indicating that the connections will behave mostly as rigid ones with respect to the beams, and that the latter are rather flexible. The large difference in periods given by different assumptions (Fig. 8) for base fixity clearly underscores that not only the foundation-soil interface, but also the base plate connection between the foundation and the column play a significant role

Table 5 compares the first three natural frequencies for the case of a soft site ($\alpha_N = 2.4 \times 10^4$ KN/m$^3$ (150 kcf)) from KCW and SAP 2000. The results are very close for the first two natural periods, which correspond to the first modes in the two perpendicular directions. The third mode, which corresponds to a vertical mode, shows a somewhat larger difference.

Table 6 and Fig. 9 show the results for first three periods, base shear design coefficient and elastic displacement for the frame with different soil stiffness. The next to last row of Table 6 indicates the maximum base shear coefficient ($C_{smax}$) needed for design if the periods extracted from this program are used directly. As expected, $C_{smax}$ increased as the stiffness of the foundation increased, but the relationship in non-linear. It is interesting to note that the code-prescribed period for this case is 0.7347 sec. (see Appendix B), with a corresponding base shear coefficient ($C_s$) of 0.0765. The use of this model will decrease the design values of the base shear, lateral forces, and overturning moments but may increase the computed values of the lateral displacements and the secondary forces associated with P-Delta effects (ASCE 7 - 02).

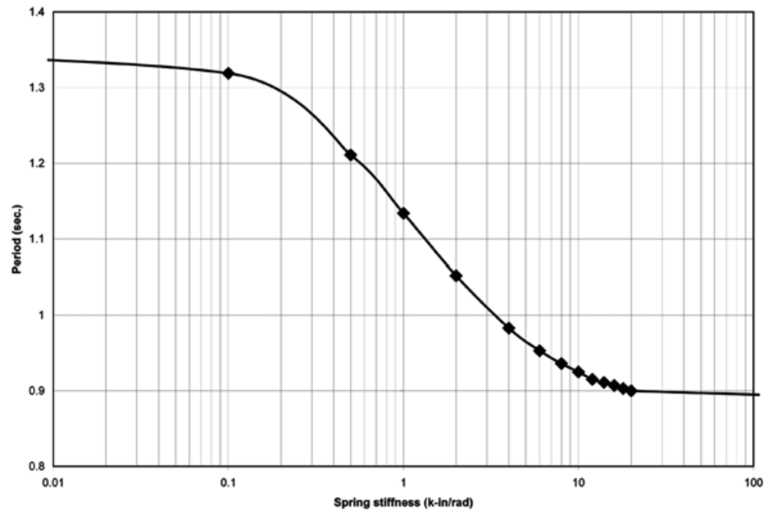


Fig. 8 Effect of column base stiffness on first natural period of SAC frame

Table 5. Compassion the first three periods of three-story SAC frame between KCW and SAP 2000

| PR or semi-rigid connection | Elastic Foundation | |
|---|---|---|
| $\alpha_N = 2.4\text{x}10^4$ kN/m$^3$ (150 kcf) | KCW | SAP |
| T1 | 1.166 | 1.167 |
| T2 | 0.601 | 0.602 |
| T3 | 0.301 | 0.345 |

$\alpha_N$ - Value of modulus of sub-grade reaction

Table 6. The first three periods of three stories SAC frame with consideration of soil stiffness and PR (semi-rigid) connections

| $\alpha_N$ (kcf) | 0 | 30 | 75 | 150 | 200 | 300 | 500 | 700 | 800 | |
|---|---|---|---|---|---|---|---|---|---|---|
| T1 | 1.350 | 1.328 | 1.243 | 1.166 | 1.133 | 1.086 | 1.033 | 1.003 | 0.992 | 0.867 |
| T2 | 0.681 | 0.643 | 0.63 | 0.601 | 0.584 | 0.561 | 0.533 | 0.517 | 0.511 | 0.444 |
| T3 | 0.354 | 0.616 | 0.41 | 0.301 | 0.264 | 0.222 | 0.259 | 0.306 | 0.312 | 0.298 |
| T1 $\rightarrow C_{s,max}$ | 0.041 | 0.042 | 0.045 | 0.048 | 0.049 | 0.052 | 0.054 | 0.056 | 0.057 | 0.064 |
| Displacement (ft) | 0.147 | 0.159 | 0.136 | 0.120 | 0.114 | 0.106 | 0.097 | 0.092 | 0.09 | 0.0723 |

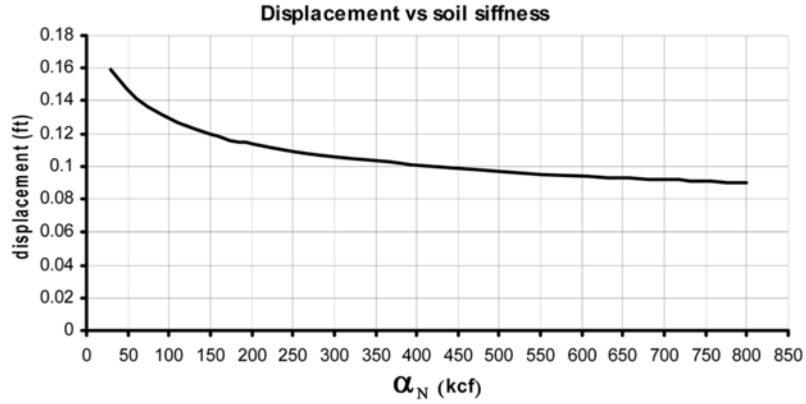


Fig. 9 Displacement of SAC frames vs soil stiffness

## 5. Conclusions

This paper described the development of a program capable of directly incorporating foundation flexibility and foundation type into the analysis of PR frames. The stiffness matrix and geometric stiffness matrix of beam element with semi-rigid connections and rigid zones are derived by using transformation matrices $[T]$ and $[e]$. The derived matrices not only included the effects of axial force (tension or compression) on the flexural stiffness but also effects of flexural semi-rigid connections and rigid zones at the beam's end. Both matrices are necessary in stability and second-order elastic analyses of frames with semi-rigid connections and rigid-zones. The extent of the influence of joint flexibility in steel frames structures resting on an elastic foundation can be gleaned by studying the relationship between various matrices in Eqs. (12) and (13). The validity of the program is substantiated by several verification cases.

In general, the effects of semi-rigid connections, geometrical nonlinearity and foundation are to reduce the natural frequency of vibration of the frame. The effect of axial load on the natural frequency of a structure depends on the magnitude of the change of the member stiffness by the axial load. For slender structures subjected to high axial load, this effect is significant and vice versa. In general, if column of medium slenderness are designed without the consideration of axial load effect, the frequency of structure will be over-estimated.

The studies on the PR SAC frame indicate that the assumption of rigid connections at beam-column and column-base interfaces, as well as the assumption of a rigid foundation, can lead to significant errors if simplified procedures are used. These errors in an equivalent static analyses are expected to lead to even more serious problems when considering the effect of higher modes under a non-linear dynamic analysis.

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## Appendix A – Transformation Matrices

*Transformation matrix* $[T]$

$$\begin{bmatrix} t_{11} \\ t_{21} \\ t_{31} \\ t_{41} \end{bmatrix} = \begin{bmatrix} 1 \\ \dfrac{-6EJ(2EJ+k_2L)}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)L} \\ 0 \\ \dfrac{-6EJ(2EJ+k_1L)}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)L} \end{bmatrix}$$

$$\begin{bmatrix} t_{12} \\ t_{22} \\ t_{32} \\ t_{42} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{(4EJ+k_2L)Lk_1}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)} \\ 0 \\ \dfrac{-2EJLk_1}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)} \end{bmatrix}$$

$$\begin{bmatrix} t_{13} \\ t_{23} \\ t_{33} \\ t_{43} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{6EJ(2EJ+k_2L)}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)L} \\ 1 \\ \dfrac{6EJ(2EJ+k_1L)}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)L} \end{bmatrix}$$

$$\begin{bmatrix} t_{14} \\ t_{24} \\ t_{34} \\ t_{44} \end{bmatrix} = \begin{bmatrix} 0 \\ \dfrac{-2EJLk_2}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)} \\ 0 \\ \dfrac{(4EJ+k_1L)Lk_2}{(12E^2J^2+4EJk_2L+4Lk_1EJ+L^2k_1k_2)L} \end{bmatrix}$$

*Transformation matrix* $[e]$

$$[e] = \begin{bmatrix} 1 & e_1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -e_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

### *Appendix B – 3-Story SAC Building Calculations*

3 – Story Building designed for Los Angeles, California, LA 90013
Foundation dimensions: Individual footings 5ft × 5ft × 1.5ft.
Stiffness of spring k1 = 145,454 k-in/rad; k2 = 400,000 k-in/rad

- Seismic Design Category (ASCE 7 – 02):
- Seismic Use group I
- Site classification B
- Adjust for site class:

$F_a = 1$ (Table 9.4.1.2.4a )
$F_v = 1$ (Table 9.4.1.2.4 b )
$S_{MS} = (F_a).(S_s) = (1).(1.7753) = 1.7753$
$S_{M1} = (F_v).(S_1) = (1).(0.6745) = 0.6745$
Design spectral response acceleration parameters
$S_{DS} = (2/3)\ S_{MS} = (2/3).(1.7753) = 1.1835$
$S_{D1} = (2/3).\ S_{M1} = (2/3)\ .0.6745 = 0.44966$
Seismic Design Category R = 8
$T_a = C_t.\ h_n^x = (0.028).(39^{0.8}) = 0.52481$ (Table 9.5.5.3.2)
$S_{D1} = 0.44966 > 0.4 \Rightarrow T = C_u.T_a = (1.4).(0.52481) = 0.7347$ (Table 9.5.5.3.1)

$$C_s = \frac{S_{DS}}{R/I} = \frac{1.1835}{8/1} = 0.1479$$

$$C_{SMin} = 0.044.I.S_{DS} = 0.05207$$

$$C_{Smax} = \frac{S_{DS1}}{T(R/I)} = \frac{0.44966}{(0.7347).(8/1)} = 0.0765 \Rightarrow C_s = 0.0765(Eq.9.5.5.2.1\text{-}2)$$

$W = (126\ \text{psf}).(180).(120).(3) = 8164800$ lb
$V = C_s\ .\ W = (0.0765).(8164800) = 624607.2$ lb

$$F_x = C_{vx}\ V;\ C_{vx} = \frac{w_x h_x^k}{\sum_{i=1}^{n} w_i h_i^k}$$

$$k = 1 + \frac{0.7347 - 0.5}{2} = 1.1173$$

| Floor | Height (ft) | Weight (lb) | Wx*hx^k | Cvx | Fx | Displacement (ft) |
|---|---|---|---|---|---|---|
| First | 13 | 2721600 | 47806714.8 | 0.152 | 94892 | 0.02767 |
| Second | 26 | 2721600 | 103715763 | 0.329 | 205866 | 0.05490 |
| Third | 39 | 2721600 | 163154980 | 0.518 | 323848 | 0.07169 |
| | | 8164800 | 314677458 | 1.000 | 624607 | |

- From Table 7, the seismic design coefficient $C_s$ computed with consideration of soil structure interaction has to be less than $C_{smax} = 0.064$

Comparison $C_{smax}$ between ASCE 7 – 02 and proposed method:

$$\Delta C_{Smax} = \frac{0.0765 - 0.064}{0.0765} 100\% = 16.33\%$$