Steel and Composite Structures, Vol. 4, No. 6 (2004) 453-469
DOI: http://dx.doi.org/10.12989/scs.2004.4.6.453

Genetic algorithm based optimum design of non-linear steel frames with semi-rigid connections

M. S. Hayalioglu† and S. O. Degertekin‡

Civil Engineering Department, Dicle University, Mühendislik-Mimarlık Fakültesi, 21280 Diyarbakır, Turkey

(Received December 2, 2002, Accepted July 29, 2003)

Abstract. In this article, a genetic algorithm based optimum design method is presented for non-linear steel frames with semi-rigid connections. The design algorithm obtains the minimum weight frame by selecting suitable sections from a standard set of steel sections such as European wide flange beams (i.e., HE sections). A genetic algorithm is employed as optimization method which utilizes reproduction, crossover and mutation operators. Displacement and stress constraints of Turkish Building Code for Steel Structures (TS 648, 1980) are imposed on the frame. The algorithm requires a large number of non-linear analyses of frames. The analyses cover both the non-linear behaviour of beam-to-column connection and P-Δ effects of beam-column members. The Frye and Morris polynomial model is used for modelling of semi-rigid connections. Two design examples with various type of connections are presented to demonstrate the application of the algorithm. The semi-rigid connection modelling results in more economical solutions than rigid connection modelling, but it increases frame drift.

Key words: genetic algorithm; structural optimization; semi-rigid connections; steel design; unbraced frame.

1. Introduction

Beam-to-column connections are assumed either perfectly pinned or fully rigid in most design of steel frames. This simplification leads to an incorrect estimation of frame behaviour. In fact, the connections are between the two extreme assumptions and possess some rotational stiffness. Full scale testing requires so as to explain the real behaviour of these connections. Bolted and welded beam-to-column connections rotates at an angle due to applied bending moment. This connection deformation has negative effect on frame stability, as it increases drift of the frame and causes a decrease in effective stiffness of the member which is connected to the joint. An increase in frame drift will multiply the second-order (P-Δ) effects of beam-column members and thus will affect the overall stability of the frame. Hence, the non-linear features of beam-to-column connections have important function in structural steel design. As a result of experimental works done by several researchers, various semi-rigid connection modelling and their moment-rotation relationships are proposed. The main of these are linear, polynomial, cubic B spline, power and exponential models (Abdalla and Chen 1995). Some important research works have been reported for the analysis and design of semi-rigid frames (Abdalla and Chen 1995, Dhillon and O'Malley 1999, Kim and Chen 1998, Goto and Miyashita 1998).

†Professor, Corresponding Author, E-mail: hsedat@dicle.edu.tr
‡Research Assistant

American Institute of Steel Construction (AISC) Load and Resistance Factor Design (LRFD) specification (AISC 1995) describes two types of steel construction: fully restrained (FR type) and partially restrained (PR type). This specification requires that the connections of the PR type constructions be considered flexible (semi-rigid) and, this flexibility be evaluated by a reasonable analysis or experimental works. On the other hand, Eurocode 3 (1992) proposes three type connections: rigid; semi-rigid and normally pinned or flexible. Giving clear demarcation lines with exact values among these types of connections is the difference of Eurocode 3 from AISC-LRFD. There has not been any information on semi-rigid connections in Turkish Steel Design specifications (TS 648, 1980 and TS 4561, 1985).

In recent years, some researchers have attended to the design of steel frames with semi-rigid connections (Xu and Grierson 1993, Almusallam 1995, Simoes 1996, Kameshki and Saka 2001). In all these works except the last one, mathematical programming techniques are used to obtain the optimum design solution. Because of discrete character of the optimization problem, the solution techniques of mathematical programming are complex and not very efficient for large scale structures. In the last work, the optimum design of steel frames are obtained using a genetic algorithm. The stress and serviceability constraints of British standard (BS 5950) are imposed on the frames and design examples are given only for a connection type.

Genetic algorithms, which are applications of biological principles into computational algorithms, have been used to obtain the optimum structural design solutions in recent years. They are able to deal with discrete optimum design problems and do not need derivatives of functions, unlike mathematical programming methods.

The aim of the present study is also to consider semi-rigid connections in the optimum design of steel frames according to the specifications of TS 648 and thus to account the non-linear behaviour due to connection characteristics and P-Δ effects of beam-column members. A polynomial model proposed by Frye and Morris (1975) is adopted as semi-rigid connection model.

In the present study, a genetic algorithm is presented for the optimum design of non-linear steel frames with semi-rigid connections subjected to displacement, and stress constraints of TS 648 specifications. A set of available steel sections, European wide flange beams (most sorts of these sections are also available in Turkish standards), are used as discrete design variables. Optimum designs of two frames with various type of semi-rigid connections are performed under the applied loads. The effect of the connection stiffness on the optimum designs is also investigated.

2. Genetic algorithms

Genetic algorithms are search techniques based on the mechanism of natural genetics and natural selection. They make use of the artificial survival of the fittest concept with genetic operators taken from nature to constitute a strong search mechanism. There are various genetic operators used in genetic algorithms. The present work employs a genetic algorithm with reproduction, crossover and mutation operators. A detailed explanation of these operators can be found in the book by Goldberg (1989). Genetic algorithms are used as an optimization method so as to minimize or maximize an objective function. They can be used in the optimum design of steel structures (Hayalioglu 2000, 2001). In the present work, a genetic algorithm given by Rajeev and Krishnamoorthy (1992) is used but improved by employing a uniform crossover operator instead of two-point crossover and adding a mutation operator. Fitness scaling as explained by Goldberg (1989) has also been added to the algorithm in order to prevent significant divergence from the optimum solution and provide fast convergence.

A design variable has a sequence number in a given discrete set of variables in genetic algorithm. Binary codes are used for these numbers. Individuals of a population are finite length strings formed from either 1 or 0 characters. Individuals and characters are called choromosomes and artificial genes, respectively, in some literature. A string may consist of some substrings so that each of them represents a design variable.

The reproduction operator applies the principle of survival of the fittest in the population. The crossover operator satisfies that individuals from the mating pool recombine genetic information to generate new solutions to the problem. There are several crossover operators existing in the literature. In this work, uniform crossover is employed, which is given in detail by Syswerda (1989). The third operator is mutation which preserves diversification in the search. This operator is applied to each offspring in the population with a predetermined probability. The operator flips the gene of an offspring from 1 to 0 and vice versa at random position.

3. Optimum design problem and its formulation

The optimum design problem of a steel frame with displacement and stress constraints can be stated as follows:

Find the set of design variables, so that the weight of the structure,

$$W(x) = \sum_{k=1}^{ng} A_k \sum_{i=1}^{mk} \rho_i L_i \tag{1}$$

is minimized subject to displacement and stress constraints. In Eq. (1), mk is the total number of members in group k, ρ_i and L_i are density and length of member i, A_k is cross-sectional area of the members belonging to group k, and ng is the total number of member groups in the frame.

The displacement constraints are:

$$\delta_j - \delta_{ju} \le 0 \qquad j = 1, ..., p \tag{2}$$

where δ_j is the displacement of the j-th degree of freedom, δ_{ju} is its upper bound, p is the total number of restricted displacements.

The stress constraints are expressed in terms of the following interaction equations (TS 648, 1980) for members subject to bending moment and axial force:

For members subjected to both axial compression and bending stress,

$$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{C_m \sigma_{bx}}{\left(1 - \frac{\sigma_{eb}}{\sigma'_{ex}}\right)\sigma_{Bx}} \le 1.0 \tag{3}$$

$$\frac{\sigma_{eb}}{0.6\sigma_a} + \frac{\sigma_{bx}}{\sigma_{Bx}} \le 1.0 \tag{4}$$

When $\sigma_{eb} / \sigma_{bem} \le 0.15$, Eq. (5) is permitted in lieu of Eqs. (3) and (4),

$$\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}} \le 1.0 \tag{5}$$

For members subjected to both axial tension and bending stresses,

$$\frac{\sigma_{ec}}{0.6\sigma_a} + \frac{\sigma_{cx}}{\sigma_{cem}} \le 1.0 \tag{6}$$

In Eqs. (3)-(6), the subscript x, combined with subscripts b, B and e indicates the axis of bending about which a particular stress or design property applies, and σ_{bem} is axial compressive stress that would be permitted in the existence of axial force alone, σ_{Bx} is compressive bending stress that would be permitted in the existence of bending moment alone, σ'_{ex} is Euler stress divided by a factor of safety, σ_{eb} is computed axial compressive stress, σ_{bx} is computed compressive bending stress at the point under consideration, C_m is a coefficient whose value is taken as 0.85 for compression members in unbraced frames, σ_a is the yield stress of steel. In Eq. (6), σ_{ec} is the computed axial tensile stress, σ_{cx} is the computed bending tensile stress and σ_{cem} is allowable bending stress which is equal to 0.6 σ_a. Allowable bending stress is increased by 0.15 in accordance with the specification when produced by wind or earthquake acting in combination with the design dead and live loads. Definitions of the permitted and Euler stresses and other details of the specification are given in Appendix II.

The computed stresses are determined from non-linear analysis of steel frames under dead and live loads in combination with wind or earthquake loads.

Effective length factor (K-factor) of columns must be estimated to evaluate the stability of columns in frames with rigid and semi-rigid connections. The factor K is required to determine the permitted compressive stress σ_{bem} and Euler stress σ'_{ex} in the design of frame members. The effective length factor K for the columns in an unbraced frame is determined from the following interaction equation (Kishi *et al*. 1997):

$$\frac{G_A G_B(\pi/K)^2 - 36}{6(G_A + G_B)} = \frac{\pi/K}{\tan(\pi/K)} \tag{7}$$

where G_A and G_B are relative stiffness factors for A-th and B-th ends of columns and given as:

$$G = \frac{\sum I_c/L_c}{\sum I_g/L_g} \tag{8}$$

where the summation is taken over all members connected to the joint, and where I_c is moment of inertia of column section corresponding to plane of buckling, L_c is unbraced length of column, I_g is moment of inertia of beam/girder corresponding to plane of bending, and L_g is unbraced length of beam/girder.

In Eq. (7), it is assumed that the beams and girders are rigidly connected to columns at the joints. The beam/girder stiffness I_g/L_g in Eq. (8) is multiplied by the following factors to consider for different end connections:

The factor is 0.5 for far ends fixed; 0.67 for pinned, and $1/(1+6EI/L.k)$ for flexibly connected, where k is spring stiffness of corresponding end.

Genetic algorithm is convenient for unconstrained optimization problems. The present problem described by Eqs. (1)-(6) is a constrained one and therefore it is necessary to transform it into an unconstrained problem. This is achieved by using a transformation based on the violations of normalized constraints as suggested by Rajeev and Krishnamoorthy (1992). The normalized form of constraints given in Eqs. (2)-(6) can be expressed as follows:

$$g_j(x) = \frac{\delta_j}{\delta_{ju}} - 1.0 \le 0\,, \qquad j = 1,\dots, p \tag{9}$$

$$g_i(x) = \left[\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{C_m \sigma_{bx}}{\left(1 - \dfrac{\sigma_{eb}}{\sigma'_{ex}}\right)\sigma_{Bx}} \right]_i - 1.0 \le 0\,, \qquad i = 1,\dots, nc \tag{10}$$

$$g_i(x) = \left[\frac{\sigma_{eb}}{0.6\,\sigma_a} + \frac{\sigma_{bx}}{\sigma_{Bx}} \right]_i - 1.0 \le 0\,, \qquad i = 1,\dots, nc \tag{11}$$

or, in case of $\sigma_{eb} / \sigma_{bem} \le 0.15$,

$$g_i(x) = \left[\frac{\sigma_{eb}}{\sigma_{bem}} + \frac{\sigma_{bx}}{\sigma_{Bx}} \right]_i - 1.0 \le 0\,, \qquad i = 1,\dots, nc \tag{12}$$

and,

$$g_i(x) = \left[\frac{\sigma_{ec}}{0.6\,\sigma_a} + \frac{\sigma_{cx}}{\sigma_{cem}} \right]_i - 1.0 \le 0\,, \qquad i = 1,\dots, nb \tag{13}$$

where nc is total number of members subjected to both axial compression and bending stress and nb is total number of members subjected to both axial tension and bending stresses.

The unconstrained objective function $\varphi(x)$ is then written as:

$$\varphi(x) = W(x)\left(1 + C\sum_{i=1}^{m} v_i\right) \tag{14}$$

where C is a constant to be selected depending on the problem. A value of 10 is found suitable for C in all design examples presented in this article. In Eq. (14) v_i is violation coefficient computed as:

$$\begin{aligned} &\text{if } g_i(x) > 0 \quad &&\text{then} \quad &&v_i = g_i(x) \\ &\text{if } g_i(x) \le 0 \quad &&\text{then} \quad &&v_i = 0 \end{aligned} \tag{15}$$

where i varies from 1 to m which is the total number of constraints.

The minimum of the unconstrained function $\varphi(x)$ will be searched by genetic algorithm. The algorithm requires a criteria to carry out selection among the individuals. This is done in such a way that the fittest individual has maximum fitness. Goldberg suggests that $\varphi(x)$ should be subtracted from a large constant for the minimization problem. In the present work, an expression for fitness is selected as:

$$F_i = [\varphi(x)_{\max} + \varphi(x)_{\min}] - \varphi_i(x) \tag{16}$$

where F_i is the fitness of i-th individual, $\varphi(x)_{\max}$ and $\varphi(x)_{\min}$ are the maximum and minimum values of $\varphi(x)$ among the current population, $\varphi_i(x)$ is the value of the same function computed for the i-th individual. The individuals with small fitness die off and the others send copies to the mating pool proportional to their fitness. After the mating pool is created, individuals are coupled

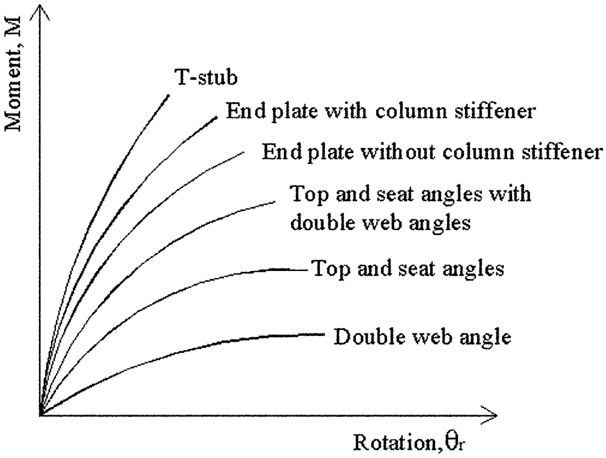

Fig. 1 Moment-rotation curves of semi-rigid connections

randomly and crossover is applied to them.

It is clear that computation of the fitness of an individual requires the values of displacements and stresses in the frame system. This is achieved by carrying out the non-linear analysis of steel frames with semi-rigid connections.

4. Connection modelling and analysis of steel frames with semi-rigid connections

A connection rotates through angle θ_r caused by applied moment M. This is the angle between beam and column from their original position. Several moment-rotation relationships have been derived from experimental studies for modelling semi-rigid connections of steel frames. These relationships vary from linear model to exponential models and are non-linear in nature. Relative moment-rotation curves of extensively used semi-rigid connections are shown in Fig. 1 (Chen *et al.* 1996).

The geometry and size parameters of six types of connections used in the present study are shown in Fig. 2 (Chen *et al.* 1996). In the present work, a polynomial model offered by Frye and Morris (1975) is used because of its easy application. This model is expressed by an odd power polynomial which is in the following form:

$$\theta_r = c_1(\kappa M)^1 + c_2(\kappa M)^3 + c_3(\kappa M)^5 \tag{17}$$

where κ is standardization constant depends upon connection type and geometry; c_1, c_2, c_3 are the curve fitting constants. The values of these constants may be taken from the work by Chen *et al.* (1996) or by Faella *et al.* (2000).

The non-linear analysis of steel frames takes into account both the geometrical non-linearity of beam-column members and non-linearity due to end connection flexibility of beam members. The columns of frames are generally continuous and do not have any internal flexible connections. However, the beams possess semi-rigid end connections, but have small axial forces with a geometric non-linearity of little importance. In the present study, two types of members are adopted for easiness in the design of steel frames with semi-rigid connections:

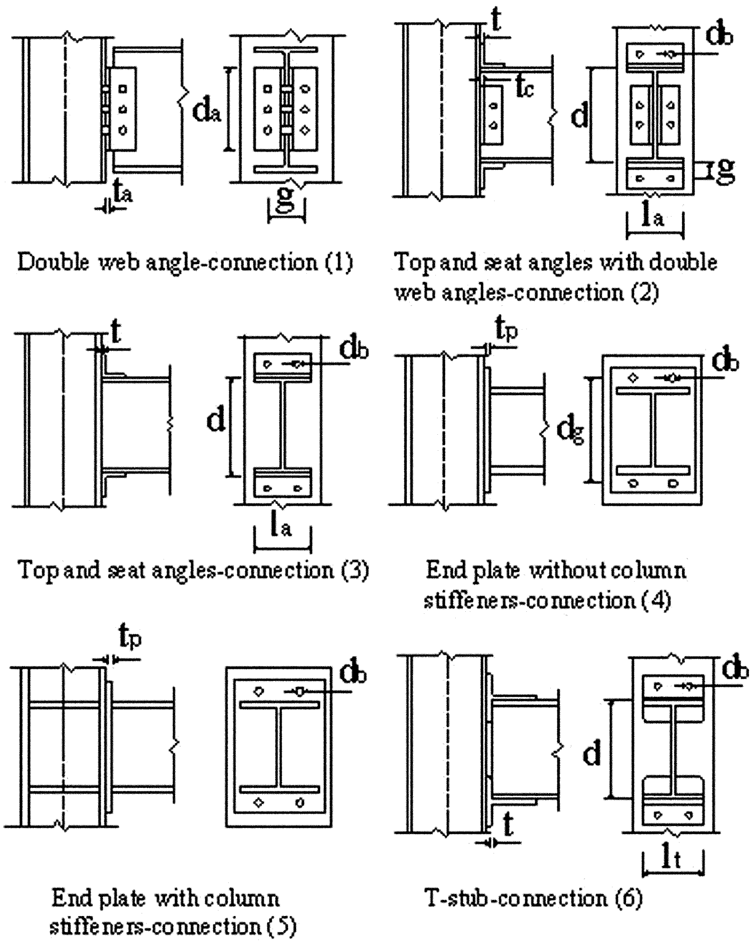

Fig. 2 Semi-rigid connection types and size parameters (type-numbers are given in brackets)

1. Beam-column member: A plane-frame member modified to include geometric non-linearity effect (P-Δ effect).
2. Beam member with semi-rigid end connections: A plane-frame member modified to incorporate end connection flexibility.

4.1. Beam-column member

The stiffness matrix of a beam-column member i with six degree of freedom in local coordinates incorporating P-Δ effect can be expressed as follows:

$$[\bar{k}]_i = [k_E]_i + [k_p]_i \tag{18}$$

where $[k_E]_i$ is the conventional linear-elastic stiffness matrix and $[k_p]_i$ is geometrical stiffness matrix which can be taken from Chajes and Churchill (1987).

4.2. Beam member with semi-rigid end connections

Semi-rigid end connections of a beam can be represented by rotational springs. θ_{rA} and θ_{rB} are the relative spring rotations of both ends (A-end and B-end) and k_A and k_B are the corresponding spring stiffness (connection stiffness) expressed as:

$$k_A = \frac{M_A}{\theta_{rA}} \tag{19}$$

$$k_B = \frac{M_B}{\theta_{rB}} \tag{20}$$

where M_A and M_B are moments of the beam at both ends.

In the present work, a stiffness matrix taken from Chen and Lui (1991), (Chapter 6) is used for a beam member with semi-rigid end connections.

The clear expression for the adopted fixed-end force vector due to in-span gravity loads on the beam can be found in Dhillon and O'Malley (1999).

4.3. Analysis procedure

The structure stiffness matrix is constructed by superimposing the member stiffness matrices contain geometric non-linearity and connection flexibility effects. This matrix is substituted in the structural equilibrium equations which are non-linear and necessitate an iterative solution procedure. The applied loads are divided into a number of small-load increments and structural equilibrium equations are written in the incremental form:

$$[S]\{\Delta D\}=\{\Delta F\} \tag{21}$$

where $[S]$ is structure stiffness matrix, $\{\Delta F\}$ is incremental load vector, and $\{\Delta D\}$ is incremental displacement vector. The incremental Eq. (21) are iteratively solved by a sequence of linear steps. The secant stiffness approach (Dhillon and O'Malley 1999) is utilized for calculating the connection stiffness. The connection secant stiffness, SE, is defined as:

$$SE = \frac{\Delta M}{\Delta \theta_r} \tag{22}$$

where ΔM is the change in end moment during a load increment, $\Delta\theta_r$ is the change in relative spring rotation during the load increment. For each load increment, structure stiffness matrix is formed at the start of each iterative cycle. This requires calculation of the connection secant stiffness at the beginning of each cycle, and changing of the latest geometry and member end forces based on information from previous cycle. The convergent connection secant stiffnesses related to all load increments are shown in Fig. 3. Convergence is obtained when the difference between joint displacements of two consecutive cycles falls below a specified tolerance. As the vertical and lateral loads are assumed to be applied to the frame at the same time starting from zero to its final value with small increments, it is thought that the unloading of connections may not occur and is not taken into account in this study.

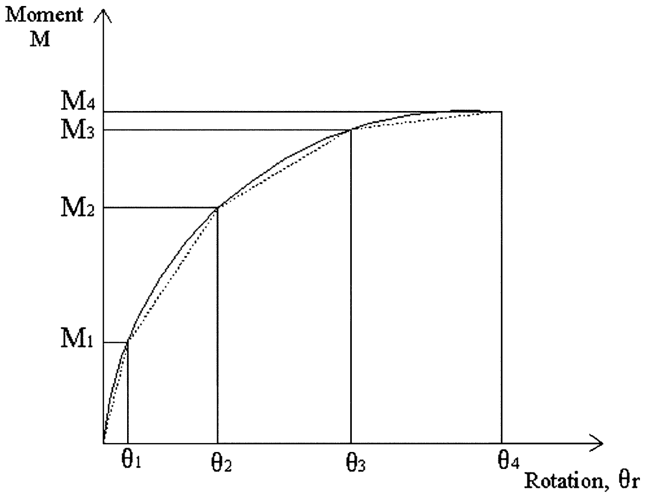

Fig. 3 Connection secant stiffnesses through load increments

A convergent solution of a load increment forms initial values for the next iteration and the iterative procedure goes on until all load increments are taken into account. The solutions for all load increments are added up to acquire a total non-linear response.

5. Optimum design procedure

Genetic algorithm based optimum design algorithm for steel frames with semi-rigid connection consists of the following steps:

1. Construct the initial population randomly which comprises binary digits.
2. Decode the binary codes for the design variables of each individual and find their sequence numbers in the available steel section list. Carry out the non-linear analysis of each steel frame, which represents an individual in the population, under the applied loads and obtain the response of the frame.
3. Calculate the value of unconstrained function $\varphi(x)$ for each individual using Eqs. (9)-(15). Find the maximum and minimum values of this function in the population.
4. Calculate the fitness value for each individual from Eq. (16).
5. Apply linear fitness scaling to the population as explained by Goldberg (1989) to obtain fast convergence on the optimum solution.
6. Apply the reproduction operator. Copy the individuals into the mating pool according to their fitness, and couple them randomly. Generate offspring using uniform crossover and thus obtain the new population.
7. Apply mutation to each offspring in the new population with a specified probability.
8. Replace the initial population by the new population and repeat steps 2 to 8 until the distance between the maximum and average fitness values of current population falls below a specified tolerance. In this case, the individual with the maximum fitness value in current population represents the optimum design.

6. Design examples

A computer program has been developed in the present study, which is implementation of the optimum design procedure. Two design examples are presented to demonstrate the application of the optimum design procedure. The designs of flexibly-connected frames are compared to the designs of rigidly-connected frames under the same design requirements. The design of rigidly-connected frames are performed for both considering and not considering P-Δ effect of beam-column members. The material is steel with a modulus of elasticity of 205940 MPa and yield stress of 235.4 MPa. Material density is 7850 kg/m^3. European wide flange beams (i.e., HE sections) in accordance with Euronorm 53-62 (1993) are used in the optimum design of the frames, due to plenty of sections. The numbers of semi-rigid connection types used in the designs are the same as the ones given in Fig. 2. Relatively larger displacement restrictions are imposed on the flexibly-connected frames than those of rigidly-connected frames, to account geometrical non-linearity due to P-Δ effects and connection flexibility. Therefore the maximum drift is restricted to $H/250$ (H=total height of the frame) for the frames with semi-rigid connections while it is limited to $H/500$ - $H/400$ for the rigidly-connected frames.

6.1. Three-storey, two-bay frame

The configuration, dimensions, loading and numbering of members of three-storey, two- bay frame are shown in Fig. 4. The maximum drifts are restricted to 4.38 cm and 2.74 cm for the frames with semi-rigid and rigid connections respectively. The applied loads shown in Fig. 4 are divided into ten equal parts to carry out the non-linear analysis.

The connection size parameters which remain fixed during the optimum design process are given in Table 1 depending on the connection types. The results of the optimum designs for six types of semi-rigid connections and also rigid connection are presented in Table 2 in the form of frame weight and

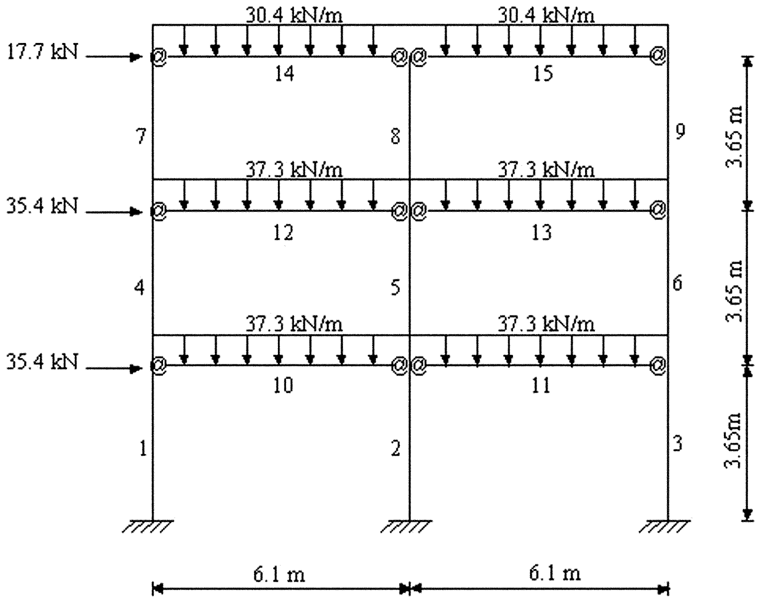

Fig. 4 Three-storey, two-bay frame

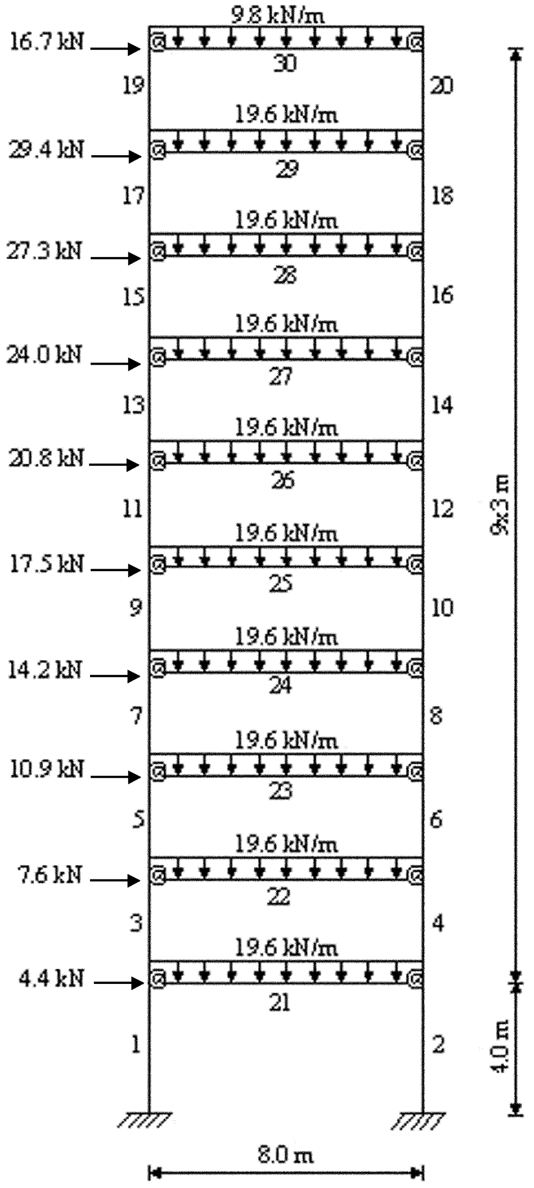

Fig. 5 Ten-storey, single-bay frame

drift. The optimum design sections of the frame with the six semi-rigid connections and rigid connections are given in Table 3.

The member grouping is done by the designer manually. It is not done automatically by the optimization program. A designer can determine member grouping considering savings in the labor cost. Therefore the columns are separated into three distinct groups whose cross-sections remain unchanged along the storeys. The same grouping for the beams of the two storeys is also adopted to reduce the fabrication cost of connections.

Table 1 The fixed connection size parameters for three-storey, two-bay frame

Connection type	Connection size parameters (cm)
1	t_a=2.4 g = 31.0
2	t = 2.0 t_c = 2.0 g = 10.5
3	t = 2.8 d_b = 2.8
4	t_p = 2.8 d_b = 2.8
5	t_p = 2.8
6	t = 2.0 d_b = 2.0

Table 2 Optimum design results of three-storey, two-bay frame (Frame weights and drifts)

Semi-rigid connection type	Weight (kg)			Drift (cm)		
	Semi-rigid connection	Rigid connection P-Δ effect	Rigid connection no P-Δ effect	Semi-rigid connection	Rigid connection P-Δ effect	Rigid connection no P-Δ effect
1	5011	5615	5174	2.04	1.83	0.88
2	4810			2.20		
3	5319			2.11		
4	5251			1.90		
5	5317			2.58		
6	4925			1.86		

Table 3 HE sections of three-storey, two-bay frame at the optimum design

Member No	Semi-rigid connection types (1)	(2)	(3)	(4)	(5)	(6)	Rigid connection P-Δ effect	Rigid connection no P-Δ effect
1,4,7	200AA	180AA	200AA	180AA	180A	180AA	180AA	320A
2,5,8	450AA	500AA	300A	300A	500AA	280A	280A	260A
3,6,9	200AA	200AA	320AA	340AA	200A	340AA	340AA	320AA
10-13	400AA	340AA	400AA	400AA	320A	400AA	400AA	300A
14,15	320AA	240B	320AA	300AA	320AA	320AA	320AA	260A

The results of the optimum designs in Table 2 show that the weights of the flexibly-connected frames decrease by 5-14%, depending on connection types, over the weights of rigidly-connected frames. As regards the drifts, the drifts of flexibly-connected frames increase by 2-41% over the drifts of rigidly-connected frame. The drift values are quite below their upper bounds and this indicates that stress constraints govern the designs for both flexibly and rigidly-connected frames.

To examine the effect of the connection stiffness on the optimum design of frames, the frame with connection type 4 is designed with various connection size parameters and the results are presented in Table 4 in the form of frame weights and drifts. The curve fitting and standardization constants of M-θ_r polynomial relationship given by Eq. (17) for connection type 4 are given by Chen *et al*. (1996) as:

$$c_1 = 1.83\times10^{-3},\ c_2 = 1.04\times10^{-4},\ c_3 = 6.38\times10^{-6}$$

and

Table 4 The effect of connection stiffness on the optimum design of three-storey, two-bay frame

Connection size parameters (cm)		Weight (kg)	Drift (cm)
t_p=2.8	d_b=2.8	5251	1.90
t_p=2.5	d_b=2.5	5378	2.16
t_p=2.1	d_b=2.1	5398	2.48
t_p=1.7	d_b=1.7	5809	2.73

Table 5 The effect of population size on the optimum design of three-storey, two-bay frame

Population size	Weight (kg)	Number of generation	Computing time (min)
60	4810	125	1.83
120	4626	201	5.17

$$\kappa = d_g^{-2.4} t_p^{-0.4} d_b^{-1.5} \tag{23}$$

where the parameters in Eq. (23) are shown in Fig. 2. The values of d_g are calculated depending on the steel section adopted for the beam.

The results of Table 4 indicate that, reducing of connection stiffness causes increase in both frame weight and drift.

A population size of 60 is used in all designs (population size is the number of individuals in a population in genetic algorithm). The frame with the connection type 2 is also designed using a population size of 120 and the results are presented in Table 5. Computing times given in Table 5 belong to a personal computer with an Intel Pentium III 450 MHz microprocessor.

The weight decrease by 3.8% but the number of generations and computing time increase by 61 % and 2.83 times respectively, when doubled the population size.

6.2. Ten-storey, single-bay frame

Fig. 5 shows configuration, dimensions, loading and numbering of members. The maximum drifts are limited to 12.4 cm, 6.89 cm and 6.2 cm for the frames with semi-rigid connections, rigid connections with P-Δ effect and rigid connections with no P-Δ effect.

The fixed connection size parameters for six connection types are given in Table 6. The optimum design results for the frames with semi-rigid connections and rigid connections are presented in Table 7. The optimum design sections of the frame are also given in Table 8.

Table 6 The fixed connection size parameters for ten-storey, single-bay frame

Connection type	Connection size parameters (cm)		
1	t_a=2.0	g=22.0	
2	t=1.6	t_c=1.6	g=10.5
3	t=2.4	d_b=2.8	
4	t_p=2.0	d_b=2.0	
5	t_p=2.0		
6	t=2.0	d_b=2.0	

Table 7 Optimum design results of ten-storey, single-bay frame (Frame weights and drifts)

Semi-rigid connection type	Weight (kg)			Drift (cm)		
	Semi-rigid connection	Rigid connection P-Δ effect	Rigid connection no P-Δ effect	Semi-rigid connection	Rigid connection P-Δ effect	Rigid connection no P-Δ effect
1	15862	18818	19520	12.38	6.72	6.18
2	16288			12.39		
3	16712			12.39		
4	17289			12.32		
5	17577			12.31		
6	16475			12.35		

Table 8 HE sections of ten-storey, single-bay frame at the optimum design

Member No	Semi-rigid connection types (1)	(2)	(3)	(4)	(5)	(6)	Rigid connection P-Δ effect	Rigid connection no P-Δ effect
1-6	450B	650A	600A	650A	500B	450B	450×312°	400×299
7-12	550AA	500AA	500AA	360B	500A	550AA	500AA	450AA
13-18	340AA	340AA	340AA	450AA	360A	320AA	340AA	550AA
19,20	450AA	360AA	320AA	320AA	320AA	320AA	400B	320AA
21-23	550AA	500AA	600AA	500AA	600AA	650AA	550AA	550AA
24-26	450AA	650AA	500AA	650AA	450AA	360A	650AA	500AA
27-29	450AA	360AA	600AA	400AA	450AA	340A	400AA	450AA
30	320AA	400AA	320AA	320AA	320AA	320AA	320AA	320AA

The results in Table 7 indicate that the weights of flexibly-connected frames decrease by 7-19% over the weights of rigidly-connected frames. On the other hand, the drifts of flexibly-connected frames increase by 83-100% over the rigidly-connected frames. The drift values are quite close to their upper bounds and this indicates that displacement constraints control the designs of flexibly and rigidly-connected frames.

The weight of rigidly-connected frame with P-Δ effect is supposed to be larger than that of the one without P-Δ effect due to magnified column and beam end moments. However, the results in Table 7 is opposite to this supposition. The reason for this is that the displacement constraints govern the both designs and stress constraints are mostly passive at the optima. In the previous example, the displacement constraints are passive and the stress constraints control design, therefore the weight of frame with P-Δ effect becomes larger than that of the one without P-Δ effect.

A population size of 60 is used in all designs of this example. The frame with connection type 2 is also designed considering a population size of 96 and the results are given in Table 9.

Table 9 The effect of population size on the optimum design of ten-storey, single-bay frame

Population size	Weight (kg)	Number of generation	Computing time (min.)
60	17289	191	6.06
96	17035	276	14.14

The weight decrease 1.5% but the number of generations and computing time increase by 45% and 2.33 times respectively when the population size is increased by 60%.

7. Conclusions

A genetic algorithm based optimum design procedure is presented for steel frames with semi-rigid connections considering non-linear behaviour of frames. Design examples are incorporated to demonstrate the influence of connection flexibility and geometric non-linearity on the design of steel frames.

The following conclusions are drawn from the design examples considered, when using genetic algorithm in the optimum design of non-linear steel frames with semi-rigid connections:

1. The population size plays important role in the values of the optimum weights and in the number of generations produced. An increase in population size results in large increase in the number of generations produced and the computing time, but small decrease in the weight of the frame. A population size between l and $2l$, where l is chromosome length, produces adequate results.
2. Fitness scaling and higher crossover probability increase the speed of convergence. Linear fitness scaling with a value of 2 for the multiplier is included in the algorithm and a value of 0.95 is used for crossover probability. Small mutation probabilities such as 0.001 or 0.002 are found suitable in the examples considered, since greater values of this probability cause significant divergence from the optimum solution.
3. The following terminating criterion is used in the genetic algorithm: $(F_{max}-F_{avg})/F_{max} \leq \varepsilon$, where F_{max} and F_{avg} are the maximum and average fitness values in the current population, and ε is a prescribed small number. Selecting smaller values for ε causes delay in convergence, but larger values for ε yields premature convergence. Values between 0.005 and 0.008 are found appropriate in the design examples presented.
4. It is observed from the results of optimum design examples that semi-rigid connection modelling creates lighter frames when compared to rigid connection modelling. These decreases in the weights are calculated as 5-20% in the examples considered.
5. The semi-rigid connections cause a large increase in the frame drift. These increases in the drifts are calculated up to 100% in the design examples.
6. It is found from the results that reducing of connection stiffness causes increase in both optimum frame weight and drift. The reason for this is that more flexible connections increase the displacements of the frame, but these displacements are adjusted to their restrictions by the optimization process assigning larger sections to the members.
7. More economical optimum frames can be obtained by adjusting the stiffness of the connections.

References

Abdalla, K.M. and Chen, W.F. (1995), "Expanded database of semi-rigid steel connections", *Comput. Struct.*, **56**(4), 553-564.

AISC Manual of Steel Construction (1995), *Load and Resistance Factor Design*, Chicago, ILL.

Almusallam, T.H. (1995), "Effect of connection flexibility on the optimum design of steel frames", *Proc. of Int. Conf. on Developments in Computational Techniques for Civil Engineering*, Edinburgh.

BS5950 (1990), British Standards, Structural use of steelworks in building, British Standard Institution, London, UK.

Chajes, A. and Churcill, J.E. (1987), "Nonlinear frame analysis by finite element methods", *J. Struct. Engrg.*,

ASCE, **113**(6), 1221-1235.
Chen, W.F. and Lui, E.M. (1991), *Stability Design of Steel Frames*, CRC Press, Inc.
Chen, W.F., Goto, Y. and Liew J.Y.R. (1996), *Stability Design of Semi-rigid Frames*, John Wiley & Sons, Inc.
Dhillon, B.S. and O'Malley, J.W. (1999), "Interactive design of semirigid steel frames", *J. Struct. Engrg.,* ASCE, **125**(5), 556-564.
Eurocode 3 (1992), Design of Steel Structures, Part I: General rules and rules for buildings, Comite European de Normalisation (CEN), Brussels, Belgium.
Euronorm, 53-62, (1993), European Wide Flange Beams, CEN, Brussels, Belgium.
Faella, C., Piluso, V. and Rizzano, G. (2000), *Structural Steel Semirigid Connections*, CRC Press, LLC.
Frye, M.J. and Morris, G.A. (1975), "Analysis of flexibly connected steel frames", *Can. J. Civ. Engrg*., **2**, 280-291.
Goldberg, D.E. (1989), *Genetic Algorithms in Search, Optimization and Machine Learning*, Addisson-Wesley, Reading, MA.
Goto, Y. and Miyashita, S. (1998), "Classification system for rigid and semirigid connection", *J. Struct. Engrg*., ASCE, **124**(7), 750-757.
Hayalioglu, M.S. (2000), "Optimum design of geometrically non-linear elastic-plastic steel frames via genetic algorithm", *Comput. Struct*., **77**(5), 527-538.
Hayalioglu, M.S. (2001), "Optimum load and resistance factor design of steel space frames using genetic algorithm", *Struct. Multidisc. Optim*., **21**(4), 292-299.
Kameshki, E.S. and Saka, M.P. (2001), "Optimum design of nonlinear steel frames with semi-rigid connections using a genetic algorithm", *Comput. Struct*., **79**, 1593-1604.
Kim, Y. and Chen, W.F. (1998), "Practical analysis for partially restrained frame design", *J. Struct. Engrg*., ASCE, **124**(7), 736-749.
Kishi, N., Chen, W.F. and Goto, Y. (1997), "Effective length factor of columns in semirigid and unbraced frames", *J. Struct. Engrg*., ASCE, **123**(3), 313-320.
Rajeev, S. and Krishnamoorthy, C.S. (1992), "Discrete optimization of structures using genetic algorithms", *J. Struct. Engrg*., ASCE, **118**(5), 1233-1250.
Simoes, L.M.C. (1996) "Optimization of frames with semi-rigid connections", *Comput. Struct*., **60**(4), 531-539.
Syswerda, G. (1989), "Uniform crossover in genetic algorithms", *Proc. of 3-rd Int. Conf. on Genetic Algorithms*, Los Altos, CA.
TS 648 (1980), Building Code for Steel Structures, Turkish Institute of Standards, Ankara.
TS 4561 (1985), Rules for Plastic Design of Steel Structures, Turkish Institute of Standards, Ankara.
Xu, L. and Grierson, D.E. (1993), "Computer automated design of semi-rigid steel frameworks", *J. Struct. Engrg*., ASCE, **119**(6), 1740-1760.

Appendix I. Permitted and Euler stresses in TS 648

Permitted axial compressive stress:

When λ, the largest effective slenderness ratio of a compressive member, is less than λ_p:

$$\sigma_{bem} = \frac{\left[1 - \frac{1}{2}\left(\frac{\lambda}{\lambda_p}\right)^2\right]\sigma_a}{n} \tag{24}$$

where

$$\lambda_p = \sqrt{\frac{2\pi^2 E}{\sigma_a}} \tag{25}$$

When λ exceeds λ_p:

$$\sigma_{bem} = \frac{2}{5}\frac{\pi^2 E}{\lambda^2} \tag{26}$$

In Eq. (24), n is a factor of safety which is defined as:

$$\text{if } \lambda < 20 \qquad \text{then} \quad n = 1.67$$

$$\text{if } 20 \leq \lambda < \lambda_p \qquad \text{then} \quad n = 1.5 + 1.2\left(\frac{\lambda}{\lambda_p}\right) - 0.2\left(\frac{\lambda}{\lambda_p}\right)^3 \tag{27}$$

Permitted compressive bending stress:

When the compression flange is solid and approximately rectangular in cross section and its area is not less than that of the tension flange:

$$\sigma_{Bx} = \frac{82500 C_b}{ld/F_b} \tag{28}$$

In the foregoing,

l = distance between cross sections braced against twist or lateral displacement of the compression flange (cm).

d = depth of column, beam or girder (cm).

F_b = area of the compression flange (cm^2).

σ_{Bx} = permitted compression bending stress which is not more than 0.6 σ_a (MPa).

C_b= $1.75+1.05(M_1/M_2)+0.3(M_1/M_2)^2$, but not more than 2.3, where M_1 is smaller and M_2 is larger bending moment at the ends of unbraced length, taken about the strong axis of the member, and where M_1/M_2 is positive when M_1 and M_2 have the same sign and negative when they are of opposite signs. When the bending moment at any point within an unbraced length is larger than that at both ends of this length, the value of C_b shall be taken as unity.

Euler stress divided by a factor of safety:

$$\sigma'_{ex} = \frac{\pi^2 E}{(Kl_b/i_b)^2}\frac{1}{2.5} \tag{29}$$

where K is the effective length factor in the plane of bending, l_b is the actual unbraced length in the plane of bending and i_b is the corresponding radius of gyration.

DN