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System and member reliability of steel frames

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Abstract. The safety level of a structural system designed per code specifications can not be inferred directly from the reliability of members due to the load redistribution and nonlinear inelastic structural behavior. Comparison of the system and member reliability, which is scarce in the literature, is likely to indicate any possible inconsistency of design codes in providing safe and economical designs. Such a comparative study is presented in this study for moment resisting two-dimensional steel frames designed per AISC LRFD Specifications. The member reliability is evaluated using the resistance of the beam-column element and the elastic load effects that indirectly accounts for the second-order effects. The system reliability analysis is evaluated based on the collapse load factor obtained from a second-order inelastic analysis. Comparison of the system and member reliability is presented for several steel frames. Results suggest that the failure probability of the system is about one order of magnitude lower than that of the most critically loaded structural member, and that the difference between the system and member reliability depends on the structural configuration, degree of redundancy, and dead to live load ratio. Results also suggest that the system reliability is less sensitive to initial imperfections of the structure than the member reliability. Therefore, the system aspect should be incorporated in future design codes in order to achieve more reliability consistent designs.

Key words: probability of failure; reliability index; system; frame; nonlinear; inelastic.

1. Introduction

The load and resistance factors in design codes are calibrated such that structural members designed according to codes meet, on average, a set of pre-selected target reliability levels (Ellingwood *et al.* 1980). Therefore, it is expected that a well designed structural system is at least as safe as the most critically loaded structural element since the system reliability is always larger than or equal to the element reliability. The difference between the structural system reliability and the reliability of the most critically loaded structural element depends to a large extent on the structure redundancy and degree of force redistribution after the member failure. Knowing this difference has important implications for improving codified designs since the objective of a design is to ensure the safety of the structural system as well as the safety of the structural element alone. As the consequence of a structural system

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failure is much higher than that of a member failure, a structural system is therefore expected to be much safer than its most critically loaded member. In other words, knowing that a structure is at least as safe as its more critical member may not be adequate. In order to incorporate system reliability consideration in our codified designs we need to first answer questions such as what is the relation between the member and system reliability of structures designed per current design code and what is the effect of system redundancy and force redistribution on the system reliability.

Reliability analyses of single members in steel structures have been carried out extensively in the past twenty years due to the development of the limit states design or load and resistance factor design (LRFD) criteria for steel building structures (Ravindra and Galambos 1978, Bjorhovde *et al*. 1978, Yura *et al*. 1978, Ellingwood and Reinhold 1980, Ellingwood *et al*. 1980). For the purpose of code calibration, the steel members are treated as if they are not linked to the structural system. In other words, the analysis of the member behavior does not incorporate the overall structural system behavior or load redistribution. Since there is no load redistribution, the assessment of member reliability becomes relatively simple. It involves in providing the probabilistic characterization of all the random variables to be considered, and evaluating the probability of failure, i.e., the probability that the total load effect exceeds the overall load carrying capacity of the member. A set of load and resistance factors may be selected such that their use will lead to a relatively consistent reliability level for the designed members. It should be emphasized that the member reliability discussed in here does not directly relate the member reliability for the member in the structural system because of the load redistribution. Further, note that a structural element will not fail (or collapse) and will sustain at most a maximum load effect equal to its load carrying capacity if the structural system is not in collapse state and the elastic-perfectly plastic behavior is assumed. Therefore, the assessment of the reliability of structural element in a structural system by considering the entanglement between the individual elements and the structural system may not be fruitful. On the other hand, a comparison of system reliability and reliability of structural elements which is obtained using the procedure similar to the procedure used for code calibration will provide indications to the importance of redundancy, the load redistribution, and the implied safety levels of structural systems designed according to current design codes.

Many approaches for evaluating the system reliability of steel frames have been proposed in the literature. One type of these approaches including the β-unzipping approach (Thoft-Christensen and Murotsu 1986), the stable configuration approach (Bennett and Ang 1987), and the mathematical programming techniques (Zimmerman *et al*. 1992), requires identifying all the dominant failure modes of the structure. However, efficiently and robustly identifying all the dominant failure modes generally presents considerable difficulty. Moreover, the computation of overall failure probability contributed from the dominant failure modes is also a formidable task mainly due to the correlation among the failure modes. Another type of approaches is to use a limit state function of the system established directly from the so-called collapse load factor of the system (Kam *et al*. 1983, Haldar and Zhou 1992, Zhao and Ono 1998). For a structure subjected to proportional loading, the collapse load factor can be calculated from a first- or second-order inelastic analysis, and there is no need for identifying failure modes. The structure is safe if the collapse load factor is larger than unity whereas it is unsafe if the collapse load factor is less than or equal to unity. In other words, the limit state function of the structure can be defined as the collapse load factor minus one. It is noted that the limit state function for structures subjected to loading conditions other than proportional loading can also be established by using the collapse load factor (Zhou and Hong 2000). Based on such a limit state function, the probability of failure or of collapse of a structural system may be estimated by reliability methods such as the first- and second-order reliability methods (FORM/SORM) and the simulation techniques. Kam *et al*. (1983) and Zhao and Ono (1998) investigated the system reliability of framed structures using the collapse load factor-based limit state function. In both of these studies, the collapse load was calculated by using a first-order analysis without considering the interaction between the axial load and bending moment, and the distributed plasticity in structural members. Furthermore, the response surface approach was used to approximate the limit state function, which was expressed in terms of basic random variables such as external loads and yielding moments of the members. Direct numerical integration, which can be very time-consuming as the number of random variables increases, was carried out by Kam *et al*. (1983) to evaluate the failure probability of the structure, whereas Zhao and Ono (1998) used the FORM and SORM (Madsen *et al.* 1986) to calculate the probability of failure. Haldar and Zhou (1992) established the limit state function for the collapse of a framed structure based on the virtual work principle, which is similar to the collapse load factor-based limit state function. The collapse load was calculated by considering the second-order effect and the interaction between the axial load and bending moment. However, the distributed plasticity was ignored.

Most previous studies on the system reliability of steel frames employed more or less simplified models to calculate the load carrying capacity of the structure, where one or more of several important issues such as the interaction between the axial load and bending moment, initial imperfections, the geometric nonlinearity (second-order effects), and the distributed plasticity was ignored. Therefore, the validity of the estimated system reliability based on simplified structural analysis methods should be questioned. To obtain more realistic assessments, a second-order inelastic frame analysis that can accurately predict the load carrying capacity of the structure is desired. Many models for such type of analysis have been developed. A plastic-zone analysis that includes distributed yielding effects, residual stresses, initial geometric imperfections, and many other significant behavioral effects will certainly be the most refined and accurate one (Chen *et al.* 1996). However, this model is generally too computationally intensive to be used in the reliability analysis. A second-order elastic-plastic hinge analysis employs concentrated plastic hinges to approximate the inelastic behavior of the members in a frame. This type of analysis is simplified compared with a plastic-zone analysis and is sufficiently accurate for many frame problems. However, the study of Liew (1992) suggested that it can lead to significantly unconservative errors for a number of benchmark frame problems. Liew (1992, see also Chen *et al.* 1996) proposed a second-order refined plastic-hinge model as an improvement over the elastic-plastic hinge model. Although this model also uses concentrated plastic hinges to represent the inelastic behavior of a member, the distributed yielding effects are accounted for. The model uses a column tangent-modulus E_t to represent the distributed yielding due to axial-force effects and a plastic-hinge stiffness-degradation model to represent the distributed yielding due to flexure. Therefore, it provides an excellent compromise between the computational efficiency associated with the elastic-plastic hinge model and the accuracy associated with the plastic-zone model. Benchmark studies (Liew 1992, Chen *et al*. 1996) suggested that this model is able to predict the load carrying capacities of a wide range of structural elements and systems with sufficient accuracy. The efficiency and accuracy of this model make it suitable for the present study.

The objective of this study is directed at assessing the member and system reliability of steel frames designed according to AISC LRFD (1986). For the member reliability evaluation, first-order elastic analyses of the frames are carried out to calculate the load effects of each member, then the FORM is employed to evaluate the probability of failure. For the system reliability evaluation, the collapse load factor-based limit state function is adopted (Zhao and Ono 1998), in which the load factor is calculated by using the second-order refined plastic-hinge model. The probability of failure is estimated by using simulation and the FORM (whenever possible). The difference between the member and system reliability is examined for framed structures with different configurations, the dead to live load ratio, and the initial out-of-plumb of the structure. The study is limited to rigid-jointed planar steel frames subjected to static loads only. Since all the members in the frames have compact sections, local buckling effects are not considered.

2. Approaches for member reliability evaluation

2.1. Beam-column analysis and design

All the beams and columns in a regular moment-resisting steel frame can be generalized as beam-columns. Most limit states design methods for beam-columns make use of the interaction equation. The members are proportioned by ensuring the member forces, which are obtained from an elastic analysis that directly or indirectly includes second-order effects, to satisfy the interaction equation. The AISC LRFD Specifications (*Load* 1986) provide the following interaction equations for designing beam-columns in a planar steel structure:

$$\frac{P_u}{\phi P_n} + \frac{8}{9}\frac{M_u}{\phi_b M_n} \le 1.0, \qquad \frac{P_u}{\phi P_n} \ge 0.2 \tag{1a}$$

$$\frac{P_u}{2\phi P_n} + \frac{M_u}{\phi_b M_n} \le 1.0, \qquad \frac{P_u}{\phi P_n} < 0.2 \tag{1b}$$

where P_u=factored axial force; P_n=nominal tensile or compressive strength; M_u=factored bending moment; M_n=nominal bending moment capacity in the plane of bending; ϕ_b=resistance factor for flexure (=0.9), and ϕ=0.9 if P_u is a tensile force or ϕ=0.85 if P_u is a compressive force. The nominal tensile strength of a beam-column can be written as $f_y A_g$, where f_y is the steel yield strength, and A_g is the gross cross-sectional area of the member whereas its nominal compressive strength is calculated from

$$P_n = \begin{cases} f_y A_g(0.658^{\lambda_c^2}), & \lambda_c \le 1.5 \\ f_y A_g\left(\dfrac{0.877}{\lambda_c^2}\right), & \lambda_c > 1.5 \end{cases} \tag{2}$$

where λ_c is the normalized column slenderness ration, $\lambda_c=(KL/\pi r)\sqrt{f_y/E}$; K is the effective length factor of the column; L is the column length; r is the radius of gyration about axis of buckling, and E is the modulus of elasticity. Note that effects of initial geometric imperfections (initial out of straightness at midlength equal to L/1500) and residual stresses on P_n are implicitly taken into account in Eq. (2) (Chen *et al.* 1996).

Ideally, the factored moment M_u should be derived from a second-order elastic analysis. If such an analysis is carried out, the obtained load effects cannot be considered linearly proportional to the applied load. This makes the assessment of the member reliability difficult if it is not impossible. In lieu of such an analysis, the AISC LRFD Specifications (*Load* 1986) suggest using

$$M_u = B_1 M_{nt} + B_2 M_{lt} \tag{3}$$

to calculate M_u. In Eq. (3), M_{nt} and M_{lt} are obtained based on a first-order elastic analysis. M_{nt} is the maximum moment in the member assuming that lateral translation is restrained in the frame (NT) while M_{lt} is the maximum moment in the member by releasing the lateral restraints (LT). The P-δ moment amplification factor B_1 and the P-Δ moment amplification factor B_2 can be calculated as follows

$$B_1 = \frac{C_m}{1 - P_u/P_{ek}} \geq 1.0 \tag{4}$$

$$B_2 = \frac{1}{1 - \sum P_u/P_{ek}} \tag{5}$$

where $P_{ek} = \pi^2 EI/(KL)^2$, K is the effective length factor, C_m is an equivalent moment factor, and the summations in Eq. (5) are carried through all columns in a story. The calculation of C_m can be found in Chen and Lui (1985). Note that K used for Eq. (4) is calculated based on the assumption of no lateral translation whereas K used for Eq. (5) is calculated by considering the lateral translation.

It is noted that Eq. (3) is a conservative approach to calculate the maximum second-order moment since B_1M_{nt} and B_2M_{lt} may not necessarily be at the same location. Further, it is noted that Eqs. (4) and (5) are only valid for rigid joints (Chen and Lui 1985). The effective length factor K, which mainly depends on the type and degree of end restraints of the member, can be calculated using the alignment chart method, LeMessurier's method (see Appendix I), and the buckling analysis of the overall structural system (Liew 1992).

2.2. Reliability analysis

All the load effects and resistance are modeled as random variables rather than stochastic processes in this study. Let $\boldsymbol{X}_R$, $\boldsymbol{X}_R=[P_R, M_R]^{\mathrm{T}}$, denote the vector of random variables that represent the resistance of a beam-column., where P_R is the axial load capacity, and M_R is the bending moment capacity in the plane of bending. P_R and M_R depend on the material and geometric variables as well as the modeling error. Let $\boldsymbol{X}_S$, $\boldsymbol{X}_S=[P_S, M_S]^T$, denote the vector of random variables that represent the load effects of the member, where P_S is the axial force, and M_S is the bending moment. The load effects are due to the dead load, the live load, and/or the environmental loads. Further let $\boldsymbol{x}_R$ and $\boldsymbol{x}_S$ denote values of $\boldsymbol{X}_R$ and $\boldsymbol{X}_S$, respectively. Based on Eq. (1) and dropping the resistance factors, the limit state function for a beam-column element, $g_m(\boldsymbol{x}_R, \boldsymbol{x}_S)$, can be expressed as

$$g_m(\boldsymbol{x}_R, \boldsymbol{x}_S) = \begin{cases} g_{m1} = 1.0 - \dfrac{p_S}{p_R} - \dfrac{8m_S}{9m_R}, & \dfrac{p_s}{p_R} \geq 0.2 \\ g_{m2} = 1.0 - \dfrac{p_S}{2p_R} - \dfrac{m_S}{m_R}, & \dfrac{p_s}{p_R} < 0.2 \end{cases} \tag{6}$$

The probability of failure P_{fm} can thus be calculated from

$$P_{fm} = \int_{g_m(\boldsymbol{x}_R, \boldsymbol{x}_S) \leq 0} f_{X_R}(\boldsymbol{x}_R) f_{X_S}(\boldsymbol{x}_S) d\boldsymbol{x}_R d\boldsymbol{x}_S \tag{7}$$

where $f_{X_R}(\boldsymbol{x}_R)$ and $f_{X_S}(\boldsymbol{x}_S)$ are the joint probability density functions of $\boldsymbol{X}_R$ and $\boldsymbol{X}_S$, respectively. Since $g_m(\boldsymbol{x}_R, \boldsymbol{x}_S)$ is described by two functions, P_{fm} shown in Eq. (7) can be calculated using the FORM (Madsen *et al.* 1986)

$$P_{fm} = \Phi(-\beta_1, -\beta_2, \rho) \tag{8}$$

where β_1 and β_2 are the reliability indices obtained using the FORM for g_{m1} and g_{m2}, respectively; ρ is the correlation coefficient between the two limit state functions, and $\Phi(\bullet, \bullet, \bullet)$ is the binormal distribution function, which can be easily evaluated using, for example, Mendell and Elston algorithm (see Mendell and Elston 1974, Terada and Takahashi 1988, Hong 1999a).

Based on the above, the procedure for evaluating the reliability of a beam-column designed per AISC LRFD criteria can be summarized as: 1) obtain the nominal second-order load effects associated with each type of load from a first-order elastic frame analysis and by using Eqs. (3)-(5); 2) obtain the nominal resistance (including geometric properties); 3) calculate the mean values of the load effects and the resistance using mean to nominal ratios; 4) assign an appropriate coefficient of variation (cov) and a probability distribution type to each random variable, and 5) calculate P_{fm} by using Eq. (8).

3. Approaches for system reliability evaluation

Let **P** and **R** denote the vectors of random variables that represent the external loads and the resistance of the structure, respectively. **P** may include the dead load, live load, and environmental loads while **R** may include the yield strength and modulus of elasticity of steel, cross-sectional properties of structural members, and the geometry of the structure. Further let **p** and **r** denote values of **P** and **R**, respectively. For a given structure with resistance **r** subjected to loads **p** that are applied proportionally, the load carrying capacity of the structure can be expressed as $\lambda\mathbf{p}$ (Zhao and Ono 1998), where λ that depends on **p** and **r** is known as the load factor or the collapse load factor in the plastic analysis. Therefore, $\lambda>1.0$ indicates that the structure can withstand load **p** while $\lambda\leq 1.0$ indicates that the structure will collapse under **p**. The values of **P** and **R** leading to the collapse can be expressed as $g_s(\mathbf{r},\mathbf{p})\leq 0$, where

$$g_s(\mathbf{r}, \mathbf{p}) = \lambda(\mathbf{r}, \mathbf{p}) - 1 \tag{9}$$

$g_s(\bullet)$ represents the limit state function, and $\lambda=\lambda(\mathbf{r}, \mathbf{p})$ is used to emphasize that λ is a function of **r** and **p**.

3.1. Evaluation of the load factor

In this study, λ is calculated using a second-order refined elastic-plastic-hinge analysis program, PHINGE (Chen *et al*. 1996). The program can analyze planar steel frames with rigid or semi-rigid connections using two-dimensional frame and truss elements, and the connection element. Key assumptions for the development of the program are: 1) all member cross-sections are fully compact such that local buckling effects are insignificant; 2) all members are adequately braced such that out-of-plane flexural or lateral-torsional buckling does not influence the member response prior to failure; 3) the effects of shear deformation are neglected; 4) plastic hinges are concentrated at the ends of elements; 5) once a plastic hinge has formed, the cross-section forces are assumed to move on the plastic strength surface; 6) large displacement but small strain is assumed throughout the analysis, and 7) the elastic unloading at the locations of plastic hinges is neglected. The distributed plasticity effects due to the axial force as well as member initial imperfections (member out of straightness and residual stresses) are approximated by replacing the elastic modulus E with the column effective stiffness E_t whereas the distributed plasticity effects due to the bending action are approximated by introducing a stiffness-degradation model (Chen *et al*. 1996). Two types of model for E_t are provided in the program. One is derived based on AISC LRFD (1986) column strength equation, and the other is derived based on the Column Research Council (CRC) column strength equations (Chen *et al*. 1996). The former is adopted in this study since it implicitly takes into account the effects of member initial out of straightness and residual stresses, which is consistent with the member reliability analysis. The cross-sectional yield criterion for the frame element is represented by the AISC LRFD cross-sectional plastic strength equations (Chen *et al*. 1996):

$$\frac{P}{P_y} + \frac{8M}{9M_p} = 1.0\,, \qquad \text{for } P/P_y \geq 0.2 \tag{10a}$$

$$\frac{P}{2P_y} + \frac{M}{M_p} = 1.0 \qquad P/P_y < 0.2 \qquad , \qquad \text{for} \tag{10b}$$

where M=bending moment at a cross-section under consideration; M_p=plastic moment capacity for the cross-section; P=axial force at the cross-section, and P_y=squash load.

Note that in PHINGE, λ is obtained from an automatic load-incremental procedure. For detailed theoretical background and solution procedures of the program, readers are referred to Chen *et al*. (1996).

3.2. Reliability analysis

The probability of failure of the structure, P_{fs}, is given by

$$P_{fs} = \int_{g_s < 0} f_{\mathbf{R}}(\mathbf{r}) f_{\mathbf{P}}(\mathbf{p}) d\mathbf{r} d\mathbf{p} \tag{11}$$

where $f_R(\mathbf{r})$=joint probability distribution function of the resistance, $f_{\mathbf{P}}(\mathbf{p})$=joint probability distribution function of the loads, and g_s is the limit state function.

Eq. (11) may be estimated by using reliability methods such as the FORM/SORM, the response surface method, and simulation techniques. The FORM/SORM are very efficient and accurate if there is only one design point (or point of maximum likelihood of failure), and the limit state surface is smooth such that its first order derivatives exist. Since P_{fs} may be contributed from different failure modes, the use of FORM/SORM may lead to considerable underestimate of the probability of failure. The limit state surface is usually approximated by a second-order polynomial of the basic random variables in the response surface method. However, this method may also fail to capture multiple local minimum points on the limit state surface. Although the simulation is less efficient than the FORM/SORM, its accuracy is not affected by multiple failure points on the limit state surface. Furthermore, the simulation is straightforward for implementation, and numerical difficulties are not likely to occur during the analysis. For the purpose of accuracy, the simple Monte Carlo technique is used to estimate P_{fs} in this study although the FORM will be used as well whenever is possible. The intention of using the latter is an attempt to validate the adequacy of the FORM for the system reliability analysis of framed structures.

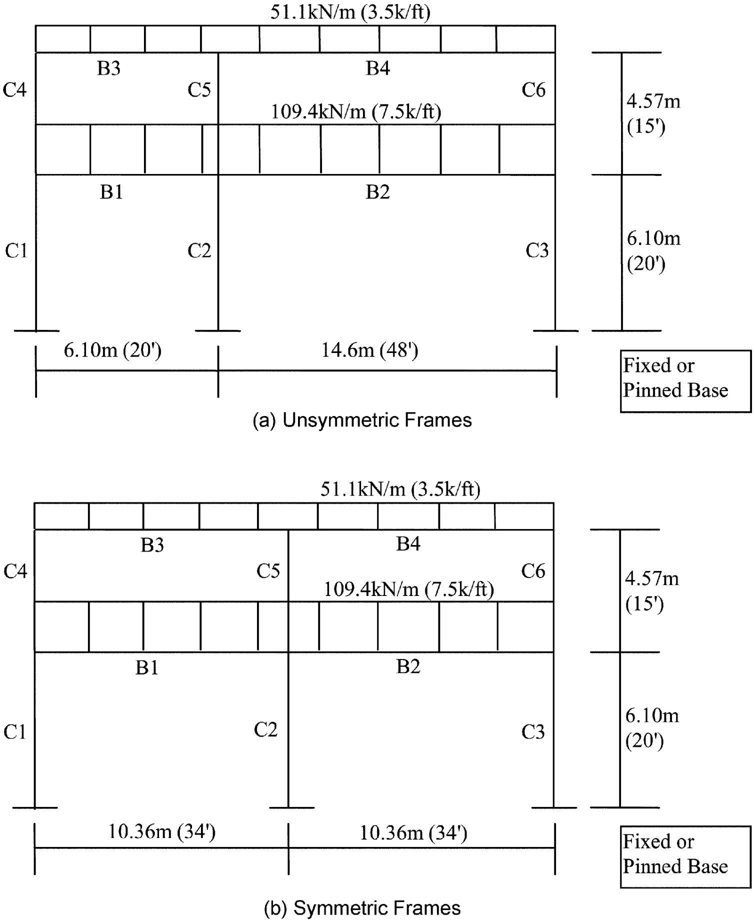

Fig. 1 Dimensions and nominal dead and live loads of the frames

4. Reliability assessments of frames designed per AISC LRFD

4.1. Descriptions of frames and design assumptions

The reliability of each member and the overall structural system of four planar steel frames designed per AISC LRFD Specifications (*Load* 1986) are investigated in the following sections. These frames are taken from Ziemian (1990) and considered to be representative of bents found in typical low-rise industrial buildings. The frames have two types of geometrical configuration, one symmetric and the other unsymmetric. The columns in each frame are either fixed or pinned at the base. The nominal steel yield strength is 248 MPa (36 ksi). Frames are subjected to the dead load D, the live load L, and the wind load W. Several assumptions are made for the design of these frames, which are: 1) all members are bent about major axes; 2) all members are fully restrained to prevent out-of-plane actions; 3) the nominal dead load is equal to the nominal live load; 4) gravity loads are applied as point loads at the beam quarter points; 5) wind loads are applied as point loads at the exterior joints; 6) frame behavior and no composite action are assumed; 7) all the member sections meet compact section classification

Table 1 Member sizes of the frames

Member	U-P	U-F	S-P	S-F
C1	W12×19	W12×14	W14×53	W14×53
C2	W14×159	W14×145	W14×99	W14×74
C3	W14×145	W14×145	W14×53	W14×53
C4	W6×9	W6×9	W14×43	W14×53
C5	W14×145	W14×145	W14×26	W12×22
C6	W14×145	W14×145	W14×43	W14×53
B1	W30×116	W33×118	W36×135	W33×130
B2	W36×182	W36×182	W36×135	W33×130
B3	W24×55	W24×55	W27×84	W24×76
B4	W30×116	W30×108	W27×84	W24×76

provided in the AISC LRFD Specifications; 8) the normalized column slenderness ratio, λ_c, is less than 1.5, and 9) the beam-column interaction equations, Eqs. (1), are satisfied for all the members. The frame design was given by Ziemian (1990) for ultimate strength requirements, serviceability requirements, and some additional requirements that are generally satisfied in typical design practice and are described in Ziemian (1990). For all the frames, the design is controlled by the ultimate strength requirements, and more specifically, by the factored gravity load combination, 1.4(D+L). This load combination is the same as 1.2D+1.6L that is specified in the AISC LRFD Specifications for the dead to live load ratio is unity. The dimensions as well as the nominal dead and live loads of the frames are shown in Fig. 1. Each frame is assigned a two-letter identifier, in which the first letter indicates the geometrical configuration of the frame (Unsymmetric or Symmetric), and the second letter indicates the base condition of the frame (Pinned or Fixed). The selected member sizes based on the design are shown in Table 1 (Ziemian 1990). Since the wind load does not govern the design, it is not considered further in the following analysis.

Since detailed design calculations are not reported in Ziemian (1990), the value of the interaction equation corresponding to 1.4(D+L) for each member of the frames is calculated in this study and shown in Table 2. The second-order bending moment M_u is obtained according to Eq. (3), where M_{nt} and M_{lt} are obtained from frame analyses using SAP2000. The P-δ amplification factor B_1 was calculated using Eq. (4) whereas the P-Δ amplification factor B_2 and the effective length factor K for members in unbraced frames were calculated using LeMessurier's Method (Eqs. (I-3) and (I-4)). Table 2 suggests that the frames were reasonably well designed since the interaction equations are satisfied for most of the members (within 5% unconservativeness). However, values of the interaction equation for four beams exceed 1.0 with maximum of 8%. This may be attributed to the fact that gravity loads were applied as concentrated loads at the beam quarter points in Ziemian (1990) while they were applied as uniformly distributed loads in this study.

4.2. Assumptions for the reliability analysis

Since the design is controlled by the combination of factored dead and live loads, the member and system reliability of the frames under dead and live loads only is investigated in this study. Probabilistic descriptions of the random variables that are commonly suggested in the literature (Galambos and Ravindra 1978, Ellingwood *et al*. 1980) and shown in Table 3 are adopted. All random variables are

Table 2 Member strength check of the frames

Member	Value of the interaction equation			
	U-P	U-F	S-P	S-F
C1	0.744	0.768	0.942	0.916
C2	0.945	0.913	0.886	0.954
C3	0.790	0.778	0.949	0.917
C4	0.573	0.558	0.957	1.032
C5	0.951	0.921	0.836	0.962
C6	0.937	0.896	0.957	1.032
B1	1.037	0.989	0.998	1.049
B2	1.037	1.052	0.998	1.049
B3	1.013	1.032	0.930	1.084
B4	0.969	1.067	0.930	1.084

Table 3 Probabilistic descriptions of the random variables

Random variable	Mean/Nominal	Coefficient of variation	Probability distribution type
Dead load	1.0	0.08	Normal
Live load	1.0	0.25	Gumbel
Steel yield strength	1.05	0.10	Lognormal

assumed to be independent to each other, and geometrical uncertainties are ignored in the analysis. It is also assumed that loads that are the same type but on different floors are fully correlated with each other. The member strength check indicates that B_1 associated with all columns and most of the beams in the four frames equal 1.0. Values of B_1 associated with a small number of beams exceed 1.0 by no more than 1%. These indicate that P-δ effects are negligible in the frames. Therefore, they are neglected in the member reliability assessment, and B_1 is assumed to be equal to 1.0 for all the members. LeMessurier's Method is used in the member reliability analysis to calculate B_2 and the effective length factor K for members in unbraced frames. In using PHINGE for the system reliability analysis, all columns were modeled by one discrete element since the plastic hinge will form only at the column ends due to the absence of transverse loads on columns and due to the neglect of member P-δ effects. All beams were modeled by four elements in order to detect possible formation of plastic hinges between the beam ends. Gravity loads are modeled as concentrated loads at beam quarter points and applied proportionally. The system reliability is estimated by using 50,000 simulation runs.

4.3. Comparison between the member and system reliability

Results of the reliability analysis are shown in Table 4. The reliability index, β, shown in this table is obtained from $\Phi^{-1}(P_f)$, where $\Phi^{-1}(\bullet)$ denotes the inverse normal distribution function and P_f represents P_{fm} or P_{fs}. Table 4 suggests that the β values associated with the individual members range from 2.4 to about 5.5. However, a majority of them falls between 2.5 and 3.5, which is in reasonable agreement with the target reliability index suggested in the code calibration (Ellingwood *et al.* 1980). Comparison between Tables 2 and 4 suggests that the estimated member reliability is consistent with the design. In other words, a member that has a small value of the interaction equation in design is associated with a

Table 4 Reliability analysis results of the frames

Member	U-P		U-F		S-P		S-F	
	β	P_f	β	P_f	β	P_f	β	P_f
C1	4.28	9.4×10^{-6}	4.15	1.7×10^{-5}	3.31	4.6×10^{-4}	3.33	4.3×10^{-4}
C2	3.21	6.6×10^{-4}	3.33	4.4×10^{-4}	3.67	1.2×10^{-4}	3.26	5.6×10^{-4}
C3	4.09	2.1×10^{-5}	4.00	3.2×10^{-5}	3.28	5.2×10^{-4}	3.33	4.4×10^{-4}
C4	5.36	4.1×10^{-8}	5.48	2.2×10^{-8}	2.97	1.5×10^{-3}	2.63	4.3×10^{-3}
C5	3.00	1.4×10^{-3}	3.14	8.4×10^{-4}	3.83	6.5×10^{-5}	3.20	6.8×10^{-4}
C6	3.07	1.1×10^{-3}	3.28	5.3×10^{-4}	2.98	1.5×10^{-3}	2.63	4.3×10^{-3}
B1	2.59	4.8×10^{-3}	2.81	2.5×10^{-3}	2.77	2.9×10^{-3}	2.53	5.8×10^{-3}
B2	2.60	4.7×10^{-3}	2.53	5.7×10^{-3}	2.77	2.9×10^{-3}	2.53	5.8×10^{-3}
B3	2.71	3.4×10^{-3}	2.62	4.4×10^{-3}	3.11	9.3×10^{-4}	2.41	8.0×10^{-3}
B4	2.96	1.5×10^{-3}	2.51	6.0×10^{-3}	3.11	9.3×10^{-4}	2.41	8.0×10^{-3}
System	3.24	6.0×10^{-4}	3.43	3.0×10^{-4}	3.49	2.4×10^{-4}	3.30	4.8×10^{-4}

Table 5 Comparison between the member and system reliability

	U-P	U-F	S-P	S-F
Most critical member(s)	B1	B4	B1&B2	B3&B4
P_{fm}	4.8×10^{-3}	6.0×10^{-3}	2.9×10^{-3}	8.0×10^{-3}
P_{fs}	6.0×10^{-4}	3.0×10^{-4}	2.4×10^{-4}	4.8×10^{-4}
P_{fm}/P_{fs}	8	20	12	17

high reliability index, and vice versa. Table 4 also suggests that the reliability of the overall system is always higher than the reliability of the member that controls the design of the structure (or the most critically loaded member). This is expected since all the frames investigated are redundant structures, and the beneficial effect of force redistribution is accounted for in the system reliability analysis but not in the member reliability analysis. In Table 5, the system reliability is compared with the reliability of the most critically loaded member(s) for each frame. Table 5 suggests that the difference between the system and member reliability depends on the structural configuration and the degree of redundancy of the structure. That values of P_{fm}/P_{fs} are larger for the two fixed-base frames (U-F and S-F) than for those of the pinned-base frames (U-P and S-P) may be explained by the fact that a more redundant structure generally benefits more from the force redistribution. Further, Table 5 suggests that the probability of failure of the overall system is roughly one order of magnitude lower than the probability of failure of the most critically loaded member in the structure.

4.4. Effects of dead to live load ratio

The results shown in Table 4 are based on the nominal dead to live load ratio equal to unity. Two other dead to live load ratios, 0.75 and 1.25, are used for investigating the difference between the member and system reliability of Frame U-P. In both cases, the nominal values of the dead and live loads are calculated such that the total factored gravity loads remain unchanged. Hence, the design of the frame will not be affected by the change of the dead to live load ratio. The member and system reliability of Frame U-P for the dead to live load ratio equal to 0.75, 1.0, and 1.25 is shown in Table 6. It can be seen

Table 6 Member and system reliability of Frame U-P for different dead to live ratios

Nominal dead/live	0.75		1.0		1.25	
Member	β	P_f	β	P_f	β	P_f
C1	4.08	2.2×10^{-5}	4.28	9.4×10^{-6}	4.46	4.1×10^{-6}
C2	3.08	1.0×10^{-3}	3.21	6.6×10^{-4}	3.33	4.4×10^{-4}
C3	3.92	4.5×10^{-5}	4.09	2.1×10^{-5}	4.26	1.0×10^{-5}
C4	5.13	1.5×10^{-7}	5.36	4.1×10^{-8}	5.58	1.2×10^{-8}
C5	2.89	1.9×10^{-3}	3.00	1.4×10^{-3}	3.09	1.0×10^{-3}
C6	2.96	1.5×10^{-3}	3.07	1.1×10^{-3}	3.17	7.8×10^{-4}
B1	2.52	5.6×10^{-3}	2.59	4.8×10^{-3}	2.66	4.0×10^{-3}
B2	2.53	5.8×10^{-3}	2.60	4.7×10^{-3}	2.67	3.8×10^{-3}
B3	2.63	4.3×10^{-3}	2.71	3.4×10^{-3}	2.79	2.7×10^{-3}
B4	2.86	2.1×10^{-3}	2.96	1.5×10^{-3}	3.05	1.2×10^{-3}
System	3.10	9.6×10^{-4}	3.24	6.0×10^{-4}	3.37	3.8×10^{-4}

Table 7 Comparison between the member and system reliability for Frame U-P

Nominal dead/live	0.75	1.0	1.25
Most critical member	B1	B1	B1
P_{fm}	6.0×10^{-3}	4.8×10^{-3}	4.0×10^{-3}
P_{fs}	9.6×10^{-4}	6.0×10^{-4}	3.8×10^{-4}
P_{fm}/P_{fs}	6	8	10

that both the member and system reliability decreases as the dead to live load ratio decreases because the variability of the live load is higher than that of the dead load. The comparison between the system reliability and the reliability of the most critically loaded member (B_1) shown in Table 7 suggests that the difference between the system and member reliability depends on the nominal dead to live load ratio, and the system reliability seems to be more sensitive to the variation of such a ratio.

4.5. Effects of column initial out of plumbness

According to Ziemian (1990), effects of column initial out of plumbness on the design are significant for the symmetric frames subjected to factored gravity load combination. Hence, the impacts of these effects on the member and system reliability of the two symmetric frames are investigated in this study. All three columns lines in Frames S-P and S-F were assumed to have a deterministic initial out of plumbness of $\Delta=H/500$ in the same direction, where H (=10.67 m or 35') is the total height of the frame (see Fig. 2). This imperfection, which corresponds to the AISC erection tolerance (Ziemian 1990), is then modeled explicitly in the member and system reliability evaluations. The member and system reliability of Frames S-P and S-F with and without the initial out of plumbness is shown in Table 8. Two trends can be observed from Table 8. First, the sensitivities of the member and system reliability to the initial out of plumbness depend on the degree of redundancy of the structure. The effects of initial out of plumbness on the member reliability of Frame S-F are less significant compared with the corresponding effects on Frame S-P. The system reliability of Frame S-F increases while the system reliability of Frame S-P decreases if column initial out of plumbness is accounted for in the analysis. Second, the member reliability is more sensitive to the initial out of plumbness than the system

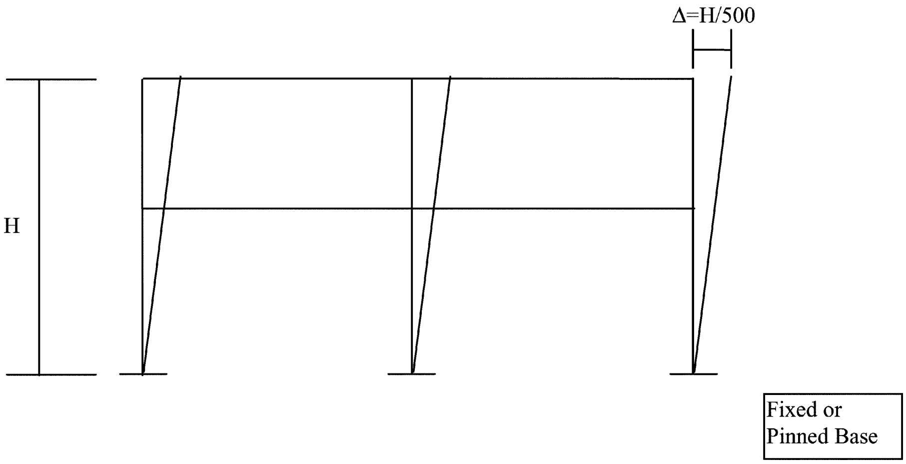

Fig. 2 Column initial out of plumbness of symmetric frames

Table 8 Effects of initial out of plumbness on the reliability of Frames S-P and S-F

Member	S-P				S-F			
	without initial out of plumbness		with initial out of plumbness		without initial out of plumbness		with initial out of plumbness	
	β	P_f	β	P_f	β	P_f	β	P_f
C1	3.31	4.6×10^{-4}	3.69	1.1×10^{-4}	3.33	4.3×10^{-4}	3.46	2.7×10^{-4}
C2	3.67	1.2×10^{-4}	3.30	4.8×10^{-4}	3.26	5.6×10^{-4}	3.12	9.0×10^{-4}
C3	3.28	5.2×10^{-4}	2.97	1.5×10^{-3}	3.33	4.4×10^{-4}	3.20	6.8×10^{-4}
C4	2.97	1.5×10^{-3}	3.02	1.3×10^{-3}	2.63	4.3×10^{-3}	2.67	3.9×10^{-3}
C5	3.83	6.5×10^{-5}	3.75	8.7×10^{-5}	3.20	6.8×10^{-4}	3.14	8.4×10^{-4}
C6	2.98	1.5×10^{-3}	2.93	1.7×10^{-3}	2.63	4.3×10^{-3}	2.59	4.8×10^{-3}
B1	2.77	2.9×10^{-3}	2.73	3.2×10^{-3}	2.53	5.7×10^{-3}	2.51	6.0×10^{-3}
B2	2.77	2.9×10^{-3}	2.80	2.5×10^{-3}	2.53	5.7×10^{-3}	2.52	5.9×10^{-3}
B3	3.11	9.3×10^{-4}	3.11	9.5×10^{-4}	2.41	8.0×10^{-3}	2.40	8.1×10^{-3}
B4	3.11	9.3×10^{-4}	3.12	9.1×10^{-4}	2.41	8.0×10^{-3}	2.41	7.9×10^{-3}
System	3.49	2.4×10^{-4}	3.451	2.8×10^{-4}	3.30	4.8×10^{-4}	3.40	3.4×10^{-4}

reliability. This may be due to that the elastic frame analysis on which the member reliability evaluation is based is affected more by the initial out of plumbness than the inelastic frame analysis on which the system reliability evaluation is based. That the system reliability of Frame S-F increases very slightly by considering column initial out of plumbness seems to be against the intuition. However, causes for such an increase are not clear to the authors at this stage.

4.6. System reliability analyses using the FORM

The system reliability of the frames shown previously was evaluated using a simple Monte Carlo technique in this study. Although this approach is straightforward to implement and numerically stable, it may become very time-consuming for more complicated framed structures. Therefore, the possibility

Table 9 System reliability obtained from the simulation and the FORM

Frame	β	
	FORM	Simulation
U-P	3.22	3.24
U-F	3.39	3.43
S-P	Failed	3.492
S-F	Failed	3.302

of directly using the FORM to estimate P_{fs} was explored. The obtained P_{fs} is shown in Table 9. The results suggest that the FORM worked very well on the two unsymmetric frames (U-P and U-F) but failed on the symmetric ones (S-P and S-F). The adequacy of the FORM for Frames U-P and U-F suggests that the limit state surfaces for these two frames are relatively linear at the design points in the normal space. The failure of the FORM on S-P and S-F is due to the difficulty in evaluating the derivatives of the limit state function with respect to the basic random variables. Such a difficulty did not occur for the unsymmetric frames. However, how to solve this difficulty is left for future studies. Further, the use of response surface in conjunction with the FORM/SORM is not explored since the obtained results can be affected by the selected points to be used for the fitting of the response surface (Hong 1999b).

5. Conclusions

System and member reliability evaluation are carried out for four planar steel frames designed according to AISC LRFD Specifications (*Load* 1986). The frames are considered to be typical of low-rise industrial buildings. The member forces used in the member reliability evaluation are obtained from a first-order elastic frame analysis and equations that indirectly take into account the second-order effects. The probability of failure of each member in the frames is evaluated by using the first-order reliability method (FORM). The system reliability is assessed using the simple simulation technique and the FORM whenever possible. The limit state function for the system reliability analysis is based on the collapse load factor that is obtained from the so-called refined second-order elastic-plastic hinge analysis approach given by Chen *et al*. (1996).

Analysis results for frames under the dead and live loads suggest that the system reliability is much higher than the reliability of the most critically loaded structural member due to the beneficial effects of force redistribution. The ratio between the probability of failure of the most critical member and of the system depends on the structural configuration, the degree of redundancy, and the nominal dead to live load ratio. Its value ranges from 6 to 20 for the frames investigated. Results also suggest that the sensitivities of the member and system reliability to column initial out of plumbness also depends on the degree of redundancy of the structure and that the system reliability is less sensitive to the column initial out of plumbness than the member reliability. Therefore, the reliability of structures designed according to design codes that do not incorporate the system aspect will not be consistent. To achieve more safety consistent designs, this aspect should be incorporated in future design codes.

This study is primary concerned with structures whose designs are controlled by the ultimate strength requirements, and should be extended to structures whose designs are controlled by the serviceability requirements. Also, the effects of partial correlation or dependency between the loads and resistance deserve a further investigation.

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Appendix I

LeMessurier (1977) proposed a method for calculating the effective length factor and the P-Δ moment amplification factor for unbraced frames. A modified form of his formula for the effective length factor K can be expressed as (Liew 1992).

$$K = \sqrt{\frac{\pi^2 EI_i}{P_{ui}L^2}\left[\frac{\sum P_{ui} + \sum C_{Li}P_{ui}}{\sum P_{Li}}\right]} = \sqrt{\frac{\pi^2 I_i}{P_{ui}}\left[\frac{\sum P_{ui} + \sum C_{Li}P_{ui}}{\sum \eta_i I_i}\right]} \tag{I-1}$$

where the subscript i refers to the ith column in the story; the summations are with respect to all the columns in the story, and the corresponding elastic second-order moment amplification factor B_2 is

$$B_2 = \frac{1}{1 - \dfrac{\sum P_{ui}}{\sum P_{Li} - \sum C_{Li}P_{ui}}} \tag{I-2}$$

The parameters in Eqs. (I-1) and (I-2) are defined as follows:
ΣP_{ui}= sum of vertical forces acting on the story at factored load level
P_{ui} = axial forces on column i calculated based on a first-order analysis
I_i = moment of inertia of column i
L = the column length in the story
P_L = the force that produces a unit rotational displacement of the member, P_L is defined as $\frac{\eta EI}{L}$
C_L = stiffness correction factor for a column, which accounts for P-δ effects and is defined as $\frac{\eta K_n^2}{\pi^2} - 1$.
K_n is determined from the alignment charts.

η = column end restraint coefficient defined as $\frac{6(G_A + G_B) + 36}{2(G_A + G_B) + G_A G_B + 3}$, where G_A and G_B are the column to beam stiffness ratios at the column ends and defined as $G = \frac{\sum\left(\frac{EI}{L}\right)_{column}}{\sum\left(\frac{EI}{L}\right)_{beam}}$. If the column is hinged at one end, $G_B = \infty$. The corresponding value of η can be calculated as $\frac{3}{(1 + G_A/2)}$, and the value of C_L can be approximated as $\frac{0.22}{(1 + G_A/2)^2}$.

By considering that C_L is usually small for columns in unbraced frames and can be neglected in many designs, K and B_2 shown in Eqs. (I-1) and (I-2) are simplified to (Liew 1992)

$$K = \sqrt{\frac{\pi^2 I_i}{P_{ui}}\left[\frac{\sum P_{ui}}{\sum \eta_i I_i}\right]} \tag{I-3}$$

$$B_2 = \frac{1}{1 - \frac{\sum P_{ui}}{\sum P_{Li}}} \tag{I-4}$$

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