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# A simplified method to predict sway of rigid multi-storey steel frames

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**Abstract.** The lateral sway of a multi-storey steel frame should be limited so as to ensure the comfort of the occupants and for the protection of mechanical and architectural systems. This paper investigates the applicability of Schueller's equation for predicting sway of rigid steel frames, and proposes a number of modifications to the equation so that it can produce results that are almost identical to those given by accurate Finite Element (FE) analysis. The behaviour of irregular steel frames has also been studied and proposals are made so that Schueller's Equation can also be used to predict sway of such frames.

**Key words:** FE analysis; steel frames; sway; rigid connection.

## 1. Introduction

Limiting the lateral sway of multi-storey steel frames is an important design consideration, particularly in those cases for which so-called frame action is relied upon to provide the necessary lateral stiffness. Anderson and Benterkia (1991) observed that for multi-storey building frames the serviceability limit of acceptable lateral sway is likely to control the design rather than ultimate strength. Accurate prediction of lateral sway is therefore an important design consideration.

The allowable sway limit for the service condition depends on a number of factors e.g., occupancy comfort, weathertightness, cladding, fixings, etc. The commonly accepted range varies from 0.0016 to 0.0035 times the height of the building (Fisher andd West 1990). For example, the Task Committee on Drift Control of Steel Building Structures of the American Society of Civil Engineers has recommended that the maximum sway be less than 0.002 times the height of the building for normal wind pressure (Task Committee on Drift Control of Steel Building Structures 1988).

The sway deflection of a rigid frame is caused by shear racking resulting in bending of columns and beams and by the cantilever behaviour of the frame causing axial deformation of the columns. Both effects need to be adequately represented in any prediction method. This paper shows how a very

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simple technique, originally proposed by Schueller (1977), may be adopted directly so as to provide very good and very easily obtained estimates for both regular and irregular frames.

## 2. Schueller's Equation

Schueller (1977) proposed an equation for the prediction of the sway of multi-storey building frames based on the Portal Method, a technique that is widely recognised as a very useful simplification when analysing frames subjected to lateral loads. This section briefly describes the basis of Schueller's Equation.

### 2.1. Lateral sway due to bending of beams and columns

Fig. 1 shows the deformed states of an individual beam and column. The relationship between beam rotation and deformation as obtained from Fig. 1(a) is

$$\frac{\Delta_g/2}{L/2} = \tan\theta \approx \theta \quad \text{or,} \ \theta = \frac{\Delta_g}{L} \tag{1}$$

while the relationship between column rotation and deformation according to Fig. 1(b) is

$$\frac{\Delta_c/2}{h/2} = \tan\varphi \approx \varphi \quad \text{or,} \ \varphi = \frac{\Delta_c}{h} \tag{2}$$

in which, $\theta$ = angle of rotation due to bending of beams
$\Delta_g$ = deflection due to bending of beams
$\phi$ = angle of rotation due to bending of columns
$\Delta_c$ = deflection due to bending of columns

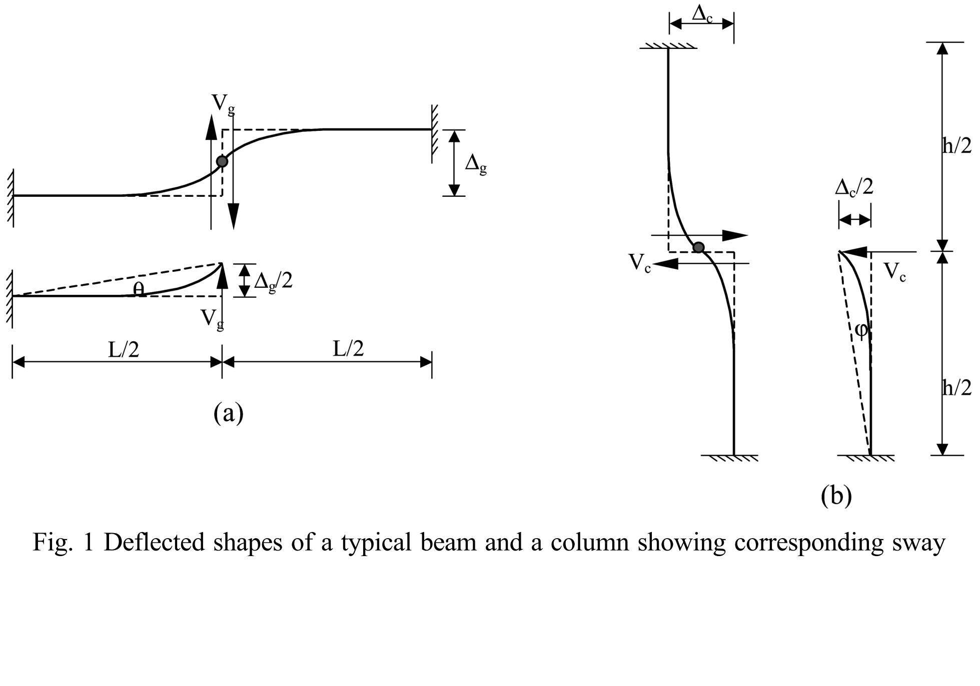



Fig. 1 Deflected shapes of a typical beam and a column showing corresponding sway

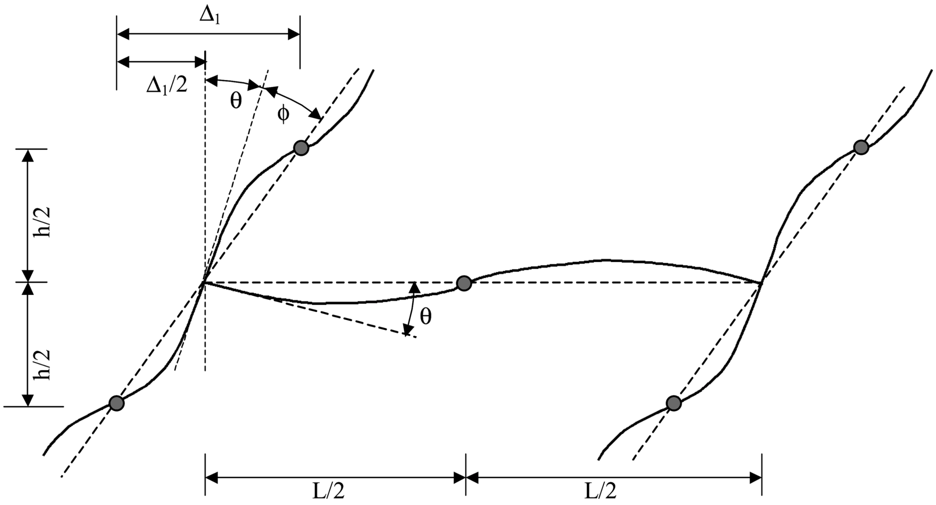



Fig. 2 Deformed shape of a typical part of a rigid frame

Fig. 2 shows the deformed state of the part of a typical rigid frame with the assumed hinge locations at mid-span of beams and mid-height of columns.

The total deformation of a single rigid framework as shown in Fig. 2 is

$$\frac{\Delta_1}{h} = \tan(\theta + \varphi) \approx \theta + \varphi \qquad \Rightarrow \frac{\Delta_1}{h} = \frac{\Delta_g}{L} + \frac{\Delta_c}{h} \tag{3}$$

Considering the beams to be infinitely rigid the deformation caused by the column shear acting at the assumed hinge, as based on a simple cantilever is

$$\Delta_c = 2\frac{V_c(h/2)^3}{3EI_c} = \frac{V_c h^3}{12EI_c} \tag{4}$$

Similarly if the columns are considered to be infinitely rigid, the deflection of a simple cantilever caused by the beam shear at mid-span of the bay yields

$$\Delta_g = 2\frac{V_g(L/2)^3}{3EI_g} = \frac{V_g L^3}{12EI_g} \tag{5}$$

Substituting the values of $\Delta_g$ and $\Delta_c$ in Eq. (3), the sway of a single bent becomes

$$\frac{\Delta_1}{h} = \frac{V_g L^3}{12EI_g L} + \frac{V_c h^3}{12EI_c h} \tag{6}$$

Schueller (1977) assumed that the ratio $\Delta_1/h$ for each floor is constant and may be taken as equal to the ratio of the overall sway of the frame caused by the bending of beams and columns $\Delta_{1\max}$ to the total height of the frame $H$ (see Section 5 for explanation). So, the total sway of the building due to the bending of beams and columns can be obtained using the following expressions

$$\frac{\Delta_1}{h} = \frac{\Delta_{1\max}}{H} = \frac{V_g L^2}{12EI_g} + \frac{V_c h^2}{12EI_c} \quad \Rightarrow \Delta_{1\max} = \frac{V_g L^2 H}{12EI_g} + \frac{V_c h^2 H}{12EI_c} \tag{7}$$

### *2.2. Lateral sway due to axial deformation of columns*

Schueller (1977) considered that the resultant wind pressure acts at the top of the building, thereby causing a linear increase in axial stress in the columns as shown in Fig. 3. This assumption permits the calculation of the lateral sway due to the axial deformation of the columns in a simple way.

If the total width and total moment of inertia of the building is taken as $B$ and $I_B$ respectively then the maximum deformation due to the cantilever action is given by

$$\Delta_c' = \frac{(wH)H^3}{3EI_B} = \frac{wH^4}{3EI_B} = \frac{M_{\max}H^2}{3EI_B} \quad \text{where, } M_{\max} = (wH)H = wH^2 \tag{8}$$

The maximum wind stresses at the exterior columns for a symmetrical frame are

$$f_{\max} = \frac{M_{\max}(B/2)}{I_B} \quad \text{or, } M_{\max} = \frac{2f_{\max}I_B}{B} \tag{9}$$

So from Eq. (8), $\Delta'_c$ can be expressed as $\Delta_c' = \dfrac{2f_{\max}H^2}{3EB}$

Now, if $A_c$ is the cross-sectional area of the base column, $f_{\max}$ can be substituted by $\dfrac{N_c}{A_c}$. Thus the lateral sway caused by column force action can be expressed as

$$\Delta_c' = \frac{2N_cH^2}{3EA_cB} \tag{10}$$

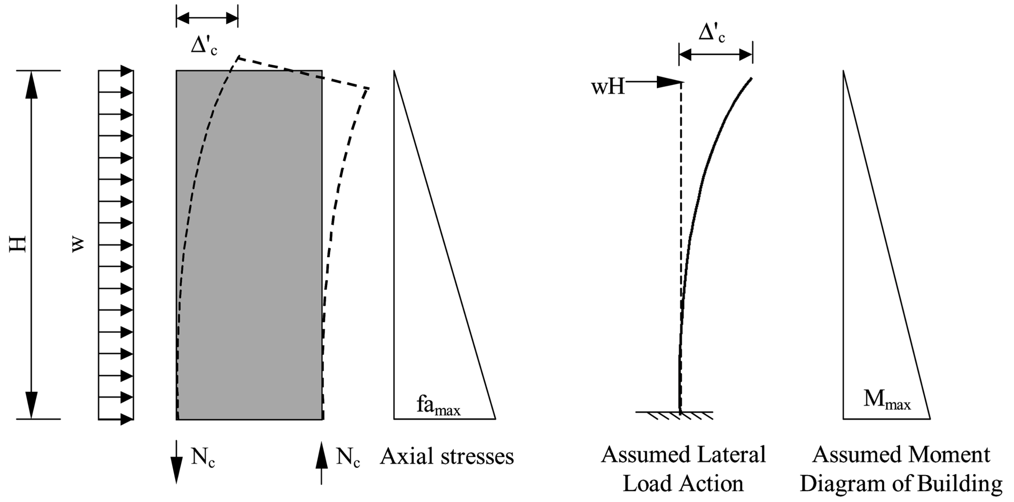



Fig. 3 Cantilever action of a building due to axial stresses in columns

### 2.3. Total sway of a rigid frame

The approximate total sway of a rigid frame consists of the summation of the deformations due to shear racking and those produced by cantilever action. Thus

$$\Delta_{max} = \Delta_c + \Delta_g + \Delta_c'$$

$$\Delta_{max} = \frac{HV_c h^2}{12EI_c} + \frac{HV_g L^2}{12EI_g} + \frac{2N_c H^2}{3EA_c B} \tag{11}$$

in which,

$N_c$ = Axial force in exterior column at the base due to wind
$V_c$ = Shear force in exterior column due to wind above third level (see Section 5 for explanation)
$I_c$ = Moment of inertia of column at the same level as $V_c$ about axis of bending
$A_c$ = Area of the exterior column at the base
$V_g$ = Shear force in girders due to wind at the same level as $V_c$
$I_g$ = Moment of inertia of girder about x-axis at the same level as $V_c$
$H$ = Total height of the frame
$B$ = Total base width of the frame
$h$ = Typical story height
$L$ = Girder span
$E$ = Modulus of elasticity

## 3. Numerical models

### 3.1. FE modelling of frame behaviour

The success of FE method of analysis in solving numerous physical problems has resulted in its widespread use for analyzing structural problems. Although the analysis of structures by finite element method has become a fairly common practice due to the abundance of computers as well as softwares, the major drawback lies in the simplifications that are to be made for the analysis, i.e., the faithful simulation of the actual problem. In course of idealization, three-dimensional geometry of the structure is generally simplified into a skeletal one depending on the orientation of beams and columns. In the case of building frames where this sort of simplification is possible, beams and columns are simplified in a two-dimensional coordinate system to save computational time (Ahmed 1996, Kishi *et al.* 1996).

In the present study, the general purpose FE program ANSYS V5.4 (1998) was used for the modelling of frame response. A uniaxial beam element, BEAM3 - 2D Elastic Beam, with tension, compression, and bending capabilities was used to model both the beams and columns. This element has three degrees of freedom at each node: translations in the nodal $x$ and $y$ directions and rotation about the nodal $z$-axis.

The adopted numerical technique was verified using some analytical results and some previous research. The frames shown in Figs. 4 and 5 were modelled and some selected parameters are compared in Tables 1 and 2. The results show a good agreement between the FE results and those obtained by using analytical techniques (Wang 1983).

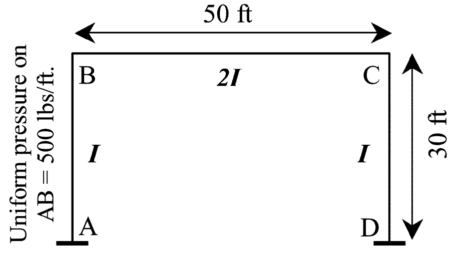



Fig. 4 Simple portal frame used for verification

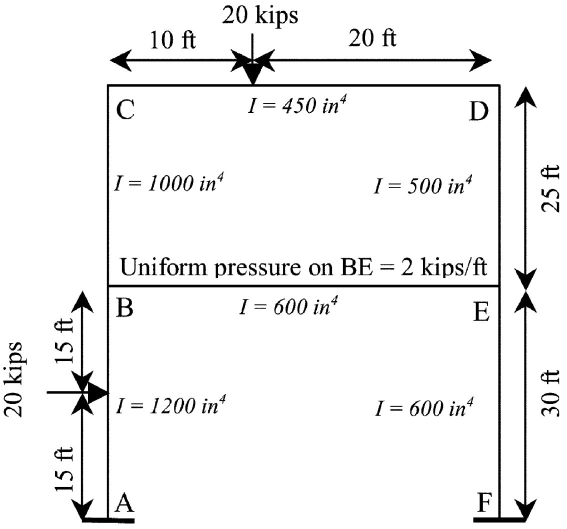



Fig. 5 Two storey portal frame used for verification

Table 1 FE analysis and theoretical results for frame shown in Fig. 4

| Considered parameter | FE results | Theoretical results | % variation |
|---|---|---|---|
| $M_{AB}$ (kip-inch) | 104.2 | 104.2 | 0 |
| $M_{CD}$ (kip-inch) | 39.9 | 40 | -0.25 |
| Sway (inch) | 0.0864 | 0.0864 | 0 |

Table 2 FE analysis and theoretical results for frame shown in Fig. 5

| Moment (kip-ft) | FE results | Theoretical results | % variation |
|---|---|---|---|
| $M_{AB}$ | 34.4 | 34.4 | 0 |
| $M_{BC}$ | 125.7 | 125.7 | 0 |
| $M_{BE}$ | 215.7 | 215.7 | 0 |
| $M_{EB}$ | 56.3 | 55.9 | 0.72 |
| $M_{EF}$ | 48.2 | 48.2 | 0 |

Lui and Chen (1988) analysed a two storey single bay frame considering both rigid and semi-rigid beam-to-column connections to verify their proposed method for the analysis of sway frames. The beams and columns were W14×48 and W12×96 sections respectively. Axial compressive loads $P$ were applied to the top of each of the columns and small lateral forces, $0.001P$ and $0.002P$ to the top and bottom storeys respectively, were applied to induce sway. The results are compared in Fig. 6.

The comparisons made in this section forms the basis for using the numerical technique in parametric studies. The FE models are explained and verified thoroughly in (Ashraf 2001).

### 3.2 Description of the frames used in the parametric study

Table 3 lists the beam and column sections used for the frames considered in parametric studies carried out as a part of the present research. All frames were assumed to have 3 m high storeys and 6 m

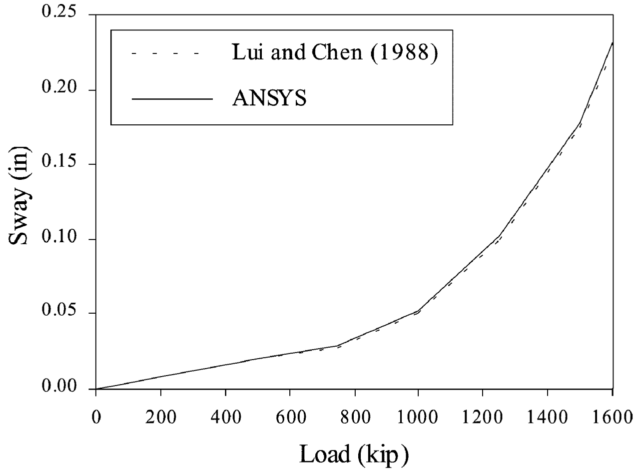



Fig. 6 Comparison for rigid frame analysed by Lui and Chen (1988)

Table 3 Frames considered for parametric study

| No. of storeys | No. of bays | Beam section | Column section |
|---|---|---|---|
| 5 | 1 to 5 | 254×102 UB 25 | 203×203 UC 46 |
| 10 | 1 to 5 | 254×102 UB 25 | 203×203 UC 71 |
| 15 | 1 to 5 | 305×127 UB 37 | 203×203 UC 86 |
| 20 | 1 to 5 | 305×127 UB 37 | 254×254 UC 132 |
| 25 | 1 to 5 | 356×171 UB 45 | 254×254 UC 167 |
| 30 | 1 to 5 | 356×171 UB 45 | 305×305 UC 240 |

Table 4 Wind load intensities considered in the parametric study

| Storey no. | Wind load (kN/m$^2$) | Storey no. | Wind load (kN/m$^2$) | Storey no. | Wind load (kN/m$^2$) |
|---|---|---|---|---|---|
| 5 | 1.84 | 14 | 2.58 | 23 | 3.22 |
| 6 | 1.96 | 15 | 2.65 | 24 | 3.22 |
| 7 | 2.06 | 16 | 2.72 | 25 | 3.22 |
| 8 | 2.15 | 17 | 2.77 | 26 | 3.22 |
| 9 | 2.23 | 18 | 2.82 | 27 | 3.35 |
| 10 | 2.31 | 19 | 2.87 | 28 | 3.35 |
| 11 | 2.39 | 20 | 2.92 | 29 | 3.35 |
| 12 | 2.47 | 21 | 3.08 | 30 | 3.35 |
| 13 | 2.52 | 22 | 3.08 | | |

wide bays. The beam and column sections are selected for the typical load cases as per Bangladesh National Building Code (BNBC 93) (1993). At this point all the frames are considered as regular frames i.e., same beam and column sections are used throughout the whole frame. Section 6 describes the behaviour of irregular frames.

The loading was determined on the basis of BNBC 93. While calculating the wind load intensities, the type of occupancy of the building was considered as general office, representing a standard occupancy structure for which the structure importance coefficient $C_I$ is 1.0. The exposure category was assumed to be urban and sub-urban area which is represented as 'Exposure Category A' in BNBC 93. The basic wind speed was considered as 210 km/h. Table 4 lists the wind load intensities that were applied uniformly on a specific floor.

## 4. Comparison between Schueller's Equation and FE analysis

Table 5 compares results obtained from the FE analysis with the predictions of Schueller's Equation (Eq. 11).

The results of Table 5 show that predictions using Schueller's Equation are too conservative. As the frame becomes taller the discrepancies become too large for the method to be used with confidence. Some form of modification is clearly required for general application.

Table 5 Comparison between Schueller's Equation and FE analysis

| Storeys | Bays | Beam section | Column section | Sway (cm) | | Variation (%) |
|---|---|---|---|---|---|---|
| | | | | Schueller's Equation | FE analysis | |
| 5 | 4 | 254×102 UB 25 | 203×203 UC 46 | 8.71 | 6.44 | +35.2 |
| 10 | 4 | 254×102 UB 25 | 203×203 UC 71 | 42.53 | 30.9 | +37.6 |
| 15 | 4 | 305×127 UB 37 | 203×203 UC 86 | 63.24 | 45.4 | +39.3 |
| 20 | 4 | 305×127 UB 37 | 254×254 UC 132 | 120.3 | 75.8 | +58.7 |
| 25 | 4 | 356×171 UB 45 | 254×254 UC 167 | 129.36 | 82.93 | +56.0 |
| 30 | 4 | 356×171 UB 45 | 305×305 UC 240 | 193.57 | 116 | +66.9 |

## 5. Modifications to the Schueller's Equation

Schueller (1977) proposed to use the column shear $V_c$ just above the 3$^{rd}$ level. It was considered that up to level 3 the sway of a frame may be assumed negligible due to the fixed building base and that a constant slope is achieved after this transition zone which leads to a constant ratio of $\Delta/H$ as explained in Section 2.1. A parametric study was performed to find the justification for using a fixed level for column shear. The level of column shear was taken as a variable and as the level was changed, the value of column shear force $V_c$ was changed as well. So different sway values were obtained for a single frame using different levels for $V_c$ and the sway values were compared with the FE analysis results.

Figs. 7 and 8 illustrate the behaviour of a 15 storey 4 bay frame and a 25 storey 1 bay frame respectively. The level of column shear $V_c$ was considered as a variable and the corresponding sway values obtained using the Schueller's Equation are plotted in these figures. To make the predictions of Schueller's Equation close to FE results $V_c$ should be taken near $0.48H$ and $0.62H$ from the base, where $H$ is the total height of the frame, instead of the 3$^{rd}$ level, for the 15 and 25 storey frames respectively.

Once the fact was revealed that $V_c$ can no longer be considered just above the 3$^{rd}$ level for all frames, a total of 30 frames were analysed, varying from 5 to 30 storeys high and having between 1 and 5 bays as given in Table 3. For all these frames the exact level of $V_c$ needed to make the sway values using Equation 11 close to the FE analysis was determined. These results are plotted in Fig. 9. The straight line given in Fig. 10 closely approximates the relationship between $H/B$ and level of $V_c$.

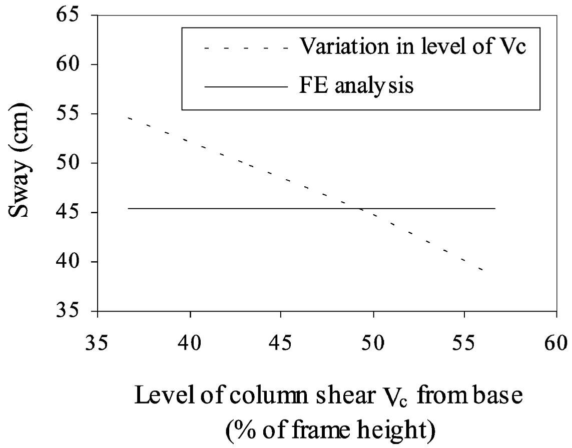



Fig. 7 Variation in sway results due to a change in level of $V_c$ for a 15 storey 4 bay frame

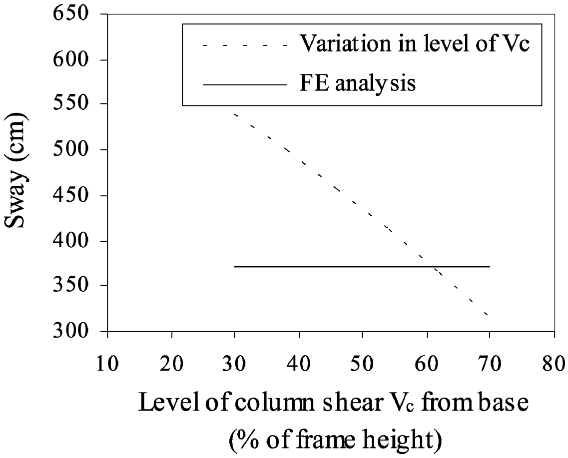



Fig. 8 Variation in sway results due to a change in level of $V_c$ for a 25 storey 1 bay frame

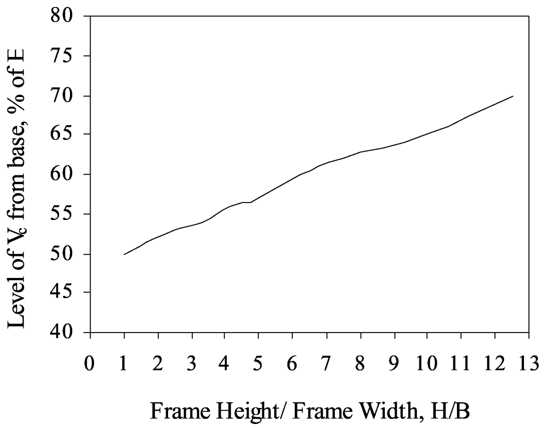



Fig. 9 Actual variation in the level of column shear $V_c$ obtained from parametric study

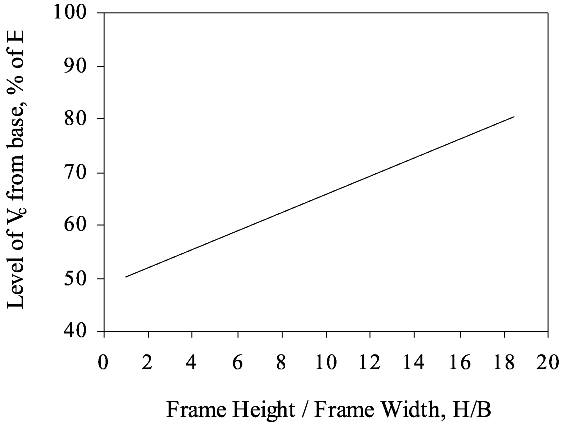



Fig. 10 The level of column shear $V_c$ to be considered in Schueller's Equation

From the results it is proposed that when using Schueller's Equation to predict the sway of a rigid frame the level of column shear $V_c$ should be taken from Fig. 10 knowing the $H/B$ ratio of the frame.

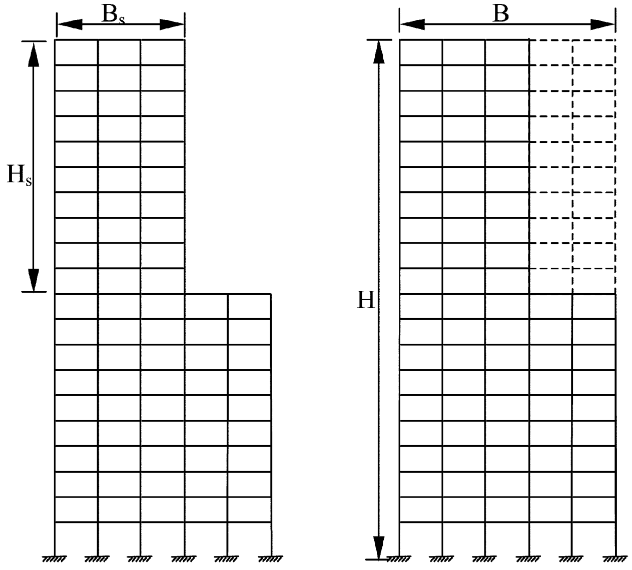
## 6. Modifications for irregular frames

Irregular frames, i.e., frames having unequal storey heights, bay size and column sections along the height or width and having different numbers of storeys in successive bays (this particular type of frame has been termed a 'stepped frame' in this paper) are very common in practice. This section studies the sway behaviour of these frames and devises further modifications to Schueller's Equation.

A total of 41 stepped frames and their corresponding regular frames were analysed using ANSYS V5.4. The geometry of the frames is given in Table 6. Then the results were studied in an attempt to

Table 6 Geometrical description of the stepped frames used to find step factors

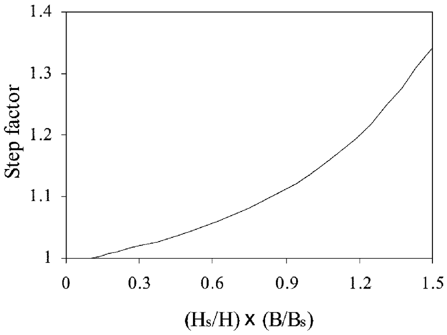
| No. of storeys in step | Regular frame | | Stepped frame | |
|---|---|---|---|---|
| | No. of storeys | No. of bays | No. of storeys taken off from top | No. of bays taken off |
| 10 | 30 | 6 | 10 | 1, 2, 3, 4, 5 |
| | 25 | 5 | 10 | 1, 2, 3, 4 |
| | 20 | 5 | 10 | 1, 2, 3, 4 |
| 5 | 30 | 6 | 5 | 1, 2, 3, 4, 5 |
| | 25 | 5 | 5 | 1, 2, 3, 4 |
| | 20 | 5 | 5 | 1, 2, 3, 4 |
| | 15 | 4 | 5 | 1, 2, 3 |
| 2 | 20 | 5 | 2 | 1, 2, 3, 4 |
| | 15 | 4 | 2 | 1, 2, 3 |
| | 10 | 3 | 2 | 1, 2 |
| | 8 | 4 | 2 | 1, 2, 3 |



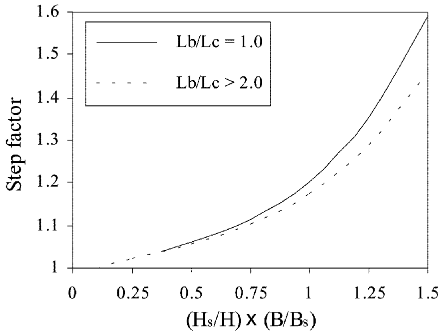

(a) Stepped frame (b) Corresponding regular frame.

Fig. 11 Relation between a stepped frame and its corresponding regular frame

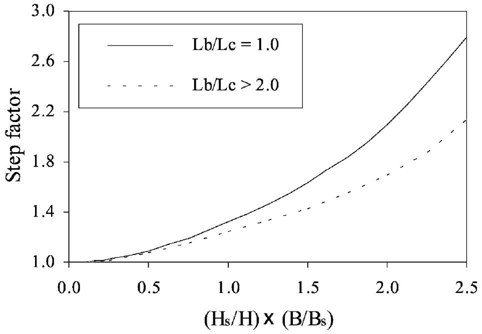
identify a suitable parameter so that the sway of a stepped frame can be predicted by knowing the sway of its corresponding regular frame. The sway of a stepped frame is always higher than the sway of its corresponding regular frame and a multiplying factor step-factor has been introduced to obtain this extra value. Step factor is the ratio of the sway of a stepped frame to the sway of its corresponding regular frame. According to Fig. 11, step-factor for frame (a) is defined as $\Delta_a / \Delta_b$; where $\Delta_a$ and $\Delta_b$ are sway of frames (a) and (b) respectively. Two other new terms $B_s$ and $H_s$, as shown in Fig. 11, refer to the width and height of the step respectively.



(a) Step factors for 2 storey steps



(b) Step factors for 5 storey steps



(c) Step factors for 10 storey steps

Fig. 12 (a) Step factors for 2 storey steps, (b) Step factors for 5 storey steps, (c) Step factors for 10 storey steps

The trends resulting from the parametric study are plotted in Figs 12(a), 12(b) and 12(c). From these figures step factors can be obtained knowing only the geometry of the frame. Thus the sway for a stepped frame can be determined by multiplying the sway of its corresponding regular rigid frame by the step factor for that specific frame.

The sway behaviour of irregular frames with different geometrical and cross-sectional properties has been investigated and reported by Ashraf *et al.* (2004). The parametric studies revealed that if the average values of beam and column properties i.e., $L_b$, $I_b$, $L_c$, $I_c$, $A_c$ are used in Schueller's Equation the sway prediction remains within 5% of the FE results. This fact is explained using worked examples 4 and 5.

## 7. Illustrative examples

**Example 1:** A 12 storey, 3 bay regular frame with typical storey height of 3 m and bay size of 6 m is subjected to a gravity load of 37.5 kN/m. Wind pressure acting on the frame has an intensity of 1.5 times higher than the typical BNBC load as mentioned in Table 4. If the beam and column sections are 305×102 UB 28 and 203×203 UC 86 respectively, calculate the sway of this frame.

For the frame under consideration:

$h = 3$ m $\quad H = 12 \times 3 = 36$ m

$L = 6$ m $\quad B = 6 \times 3 = 18$ m

$I_c = 9462$ cm$^4$ $\quad I_g = 5415$ cm$^4$ $\quad A_c = 110.1$ cm$^2$

For this frame, $H/B = 36\ /18 = 2$

Now, from Fig. 10, it is observed that $V_c$ should be considered at $0.51H$ from the base. Hence, the following values were obtained using the portal method,

$N_c = 700.39$ kN $\qquad V_c = 61.28$ kN $\qquad V_g = 56.63$ kN

So, finally from Eq. (11), $\Delta_{max}$ is obtained as 64.48 cm.

Sway calculated from a FE analysis is 63.94 cm, which differs from the predicted value by less than 1%.

**Example 2:** An 18 storey, 4 bay regular frame with typical storey height of 3 m and bay size of 5 m is subjected to a gravity load of 37.5 kN/m. Wind pressure acting on the frame is as given in Table 4. If the beam and column sections are 356×127 UB 39 and 305×305 UC 97 respectively, calculate the sway of this frame.

For the given frame:

$h = 3$ m $\qquad H = 18 \times 3 = 54$ m

$L = 5$ m $\qquad B = 5 \times 4 = 20$ m

$I_c = 22202\ cm^4$ $\qquad I_g = 10054\ cm^4$ $\qquad A_c = 123.3\ cm^2$

For this frame, $H/B = 54.0/20.0 = 2.70$

Now, from Fig. 10, it is observed that $V_c$ should be considered at $0.55H$ from the base. Hence, the following values were calculated using the portal method,

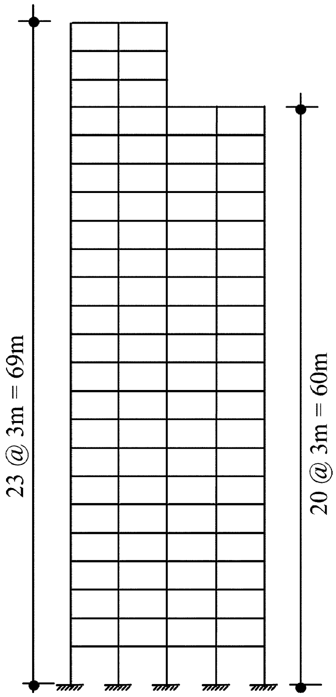



Fig. 13 Stepped frame used for illustration

$N_c$ = 1073.12 kN $V_c$ = 41.79 kN $V_g$ = 59.56 kN

So, finally, from Eq. (11), $\Delta_{\max}$ is obtained as 39.96 cm.

Sway from FE analysis is 40.35 cm, which is again differs from the predicted value by less than 1%.

**Example 3:** The stepped frame shown in Fig. 13 has a typical storey height of 3 m and bay size of 6 m. Gravity load acting over the floors is 37.5 kN/m while wind pressure acting on the frame is taken as the typical BNBC load given in Table 4. If beam and column sections are chosen as 305×127 UB 37 and 254×254 UC 167 respectively, calculate the overall sway of this frame.

For the given frame:

$h$ = 3 m $H$ = 23 × 3 = 69 m

$L$ = 6 m $B$ = 6.0 × 4 = 24 m

$I_c$ = 29914 cm$^4$ $I_g$ = 7143 cm$^4$ $A_c$ = 212.4 cm$^2$

$H/B$ ratio for its corresponding regular frame = 69/24 = 2.875

So, from Fig. 10, it is observed that $V_c$ should be considered at 0.55$H$ from the base. Hence, the following values were calculated using the portal method,

$N_c$ = 1589 kN $V_c$ = 75.49 kN $V_g$ = 67.11 kN

From Eq. (11) sway of its corresponding regular frame is 105.04 cm.

For the determination of step factor, $H_s/H \times B/B_s$ = 9/69×24/12 = 0.26

According to Fig. 9(a), for 2 storey step, 'step factor' = 1.01

According to Fig. 9(b), for 5 storey step, 'step factor' = 1.03

So, for 3 storey step, 'step factor' = 1.017

Thus, $\Delta_{\max}$ = 1.017×105.035 = 107.14 cm.

Sway obtained from FE analysis is 104.4 cm, which is within 2.6% of the predicted value.

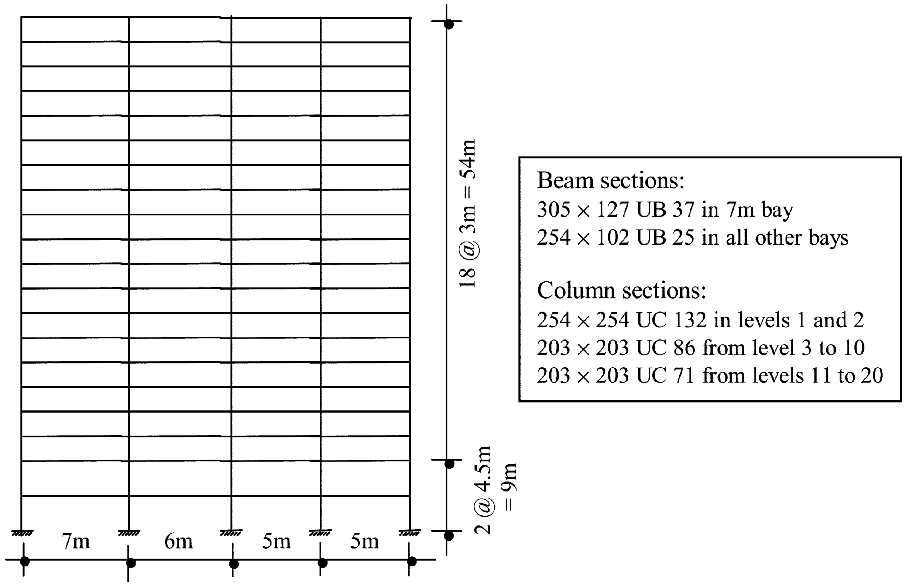



Fig. 14 Twenty storey four bay irregular frame used for illustration

**Example 4:** The 20 storey 4 bay frame shown in Fig. 14 is subjected to a gravity load of 37.5 kN/m while wind pressure acting on the frame is taken as the typical BNBC load as given in Table 4. Calculate the overall sway of this frame.

For the given frame:

$h_{avg}$ = (2×4.5+18×3.0)/20 = 3.15 m $H$ = 9 + 54 = 63 m

$L_{avg}$ = (7.0+6.0+2×5.0)/4 = 5.75 m $B$ = 7 + 6 + 2×5 = 23 m

$I_{c,avg}$ = (2×22416 + 8 ×9462 + 10×7647)/20 = 9760 cm$^4$

$I_{g,avg}$ = (20×7143 + 3×20×3404) = 4339 cm$^4$

$A_{c,avg}$ = (2×167.7 + 8 ×110.1 + 10×91.1)/20 =106.36 cm$^2$

For this frame, $H/B$ = 63/23 = 2.74

Now, from Fig. 10, it is observed that $V_c$ should be considered at 0.54$H$ from the base. Hence, the following values were obtained using the portal method,

$N_c$ = 1250.75 kN $V_c$ = 60.04 kN $V_g$ = 62.84 kN

So, finally from Eq. (11), $\Delta_{\max}$ is obtained as 143.46 cm.

Sway calculated from FE analysis is 145.2 cm, which differs from the predicted value by less than 2%.

**Example 5:** The frame shown in Fig. 15 was analysed by Ahsan (1997). If all necessary details are as given in the figure, calculate the sway of the frame.

For the given frame:

$h$ = 3.5 m $H$ = 21 m

$L$ = 5 m $B$ = 10 m

$A_{c,avg}$ = (3×150.2 + 3×113.3)/6 =131.75 cm$^2$

$I_{c,avg}$ = (3×27670 + 3×14270)/6 = 20970 cm$^4$

$I_g$ = 47540 cm$^4$

For this frame, $H/B$ = 21/10 = 2.1

Now, from Fig. 10, it is observed that $V_c$ should be considered at 0.5$H$ from the base. Hence, the following values were obtained using the portal method,

$N_c$ = 186.81 kN $V_c$ = 30.63 kN $V_g$ = 36.75 kN

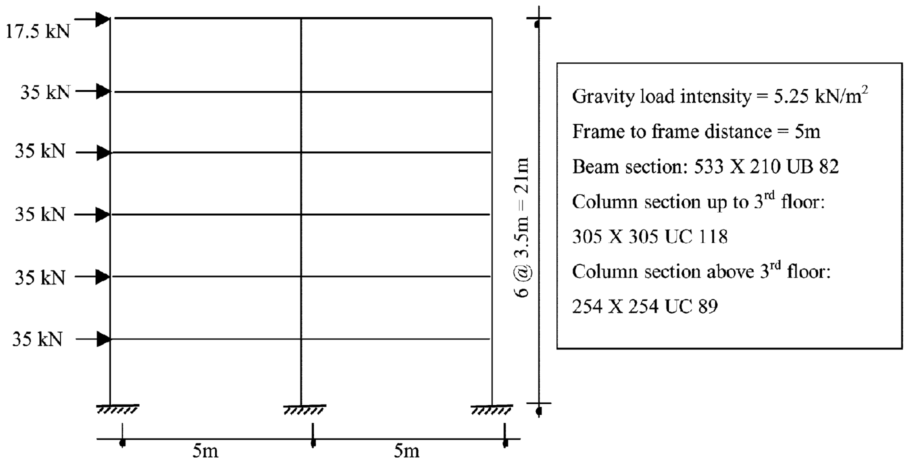



Fig. 15 Six storey two bay frame used by Ahsan (1997)

So, finally from Eq. (11), $\Delta_{max}$ is obtained as 3.35 cm.
Sway calculated from FE analysis is 3.28 cm, which differs from the predicted value by less than 3%.

## 8. Conclusions

A design chart has been proposed to locate the appropriate level of column shear to be considered when using Schueller's Equation. Use of this chart makes the predictions of this equation very close to those given by FE analysis. Further modifications are made to use this equation in the case of irregular frames. Application of this Equation is explained using some worked examples and the results are in good agreement with the FE analysis. At the preliminary stage of design this Equation can serve as a very useful tool for predicting the overall sway of a multi-storey frame without the need to resort to numerical modelling.

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