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Inelastic distortional buckling of hot-rolled I-section beam-columns

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Abstract. The inelastic lateral-distortional buckling of doubly-symmetric hot-rolled I-section beam-columns subjected to a concentric axial force and uniform bending with elastic restraint which produce single curvature is investigated in this paper. The numerical model adopted in this paper is an energy-based method which leads to the incremental and iterative solution of a fourth-order eigenproblem, with very rapid solutions being obtained. The elastic restraint considered in this paper is full restraint against translation, but torsional restraint is permitted at the tension flange. Hitherto, a numerical method to analyse the elastic and inelastic lateral-distortional buckling of restrained or unrestrained beam-columns is unavailable. The prediction of the inelastic lateral-distortional buckling load obtained in this study is compared with the inelastic lateral-distortional buckling of restrained beams and the inelastic lateral-torsional buckling solution, by suppressing the out-of-plane web distortion, is published elsewhere and they agree reasonable well. The method is then extended to the lateral-distortional buckling of continuously restrained doubly symmetric I-sections to illustrate the effect of web distortion.

Key words: beam-column; continuous restraint; distortion; residual stress; yielding.

1. Introduction

The aim of this study is to investigate the inelastic lateral-distortional buckling behaviour of continuously restrained beam-columns subjected to uniform bending and concentric axial compression, which produces single curvature deformations. For an unrestrained doubly-symmetric I-section member, the effect of web distortion in the lateral-torsional buckling behaviour of a beam does not normally arise. However, the lateral-torsional buckling behaviour of the beam is questionable when a beam is continuously restrained along the tension flange, particularly against torsion (Bradford 1988). Typical examples are the hogging region of continuous concrete-steel composite beams and roof sheeting of portal frames where the cladding combined with purlins or girts that forms a diaphragm restraint and continuous restraint along the beam may increase the buckling resistance of the I-section. The in-plane analysis of a beam-column is somewhat different to a beam subjected to uniform bending because additional moment is caused by the axial force (N - δ effect). The major axis flexural rigidity is not constant due to yielding of the cross-section caused by the combination of residual stresses and applied load. The inelastic in-plane analysis of a beam-column to determine the stress resultants is therefore more complicated than an elastic in-plane analysis. Newmark (1943) has presented an integration

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technique to determine the end moment, and this study adopts such a method.

Research on the elastic lateral-torsional buckling of beam-columns is also plentiful. Over the years, a number of researchers have investigated the inelastic lateral-torsional buckling behaviour of beam-columns subjected to a moment gradient and axial force. Miranda and Ojalvo (1965) presented differential equations which included the prebuckling displacement to analyse the unrestrained lateral-torsional buckling of beam-columns without residual stress. The inelastic buckling of beam-columns subjected to an axial force and moment applied at one end of member was considered by Fukumoto and Galambos (1966) with simplified residual stresses and used the finite difference method to solve the differential equations for bending and torsion. Abdel-Sayed and Aglan (1973) studied the inelastic buckling of beam-columns using a finite difference approximation with simplified residual stresses. The monosymmetric effect due to residual stress and applied load was included in their buckling analysis with the tangent modulus theory based on the assumption that E_t was equal to zero and used for yielded regions, and furthermore the Saint Venant torsional rigidity was used for the elastic and yielded regions of the cross-section based on an assumption that the torsional rigidity was not affected by the yielding of the cross-section.

The inelastic lateral-torsional buckling of unrestrained beam-columns was considered by Bradford *et al.* (1984) who developed a finite element method based on the tangent modulus theory. The finite element method developed by Bradford *et al.* was extended by Bradford and Trahair (1985) to investigate the buckling behaviour of beam-columns with a parabolic distribution of the residual stress in the flange and a quartic distribution in the web. From the numerical results they proposed a simple method to predict buckling loads of beams and beam-columns. Bradford and Trahair (1986) modified the finite element method developed by Bradford *et al.* (1984) which was augmented to include elastic translational, rotational, torsional and warping restraints to compare with the experimental results obtained by Cuk *et al.* (1986) for restrained three-span continuous beam-columns, and they found that they were in good agreement with experimental results.

Despite extensive research work on the elastic lateral-distortional buckling behaviour of the I-section member, the inelastic range of structural response is rather limited. The energy-based method has been employed to study the inelastic lateral-distortional buckling of I-section beams that incorporate residual stress models is extended here to include the N–δ effect in non-sway members. This study validates the accuracy of this method with an independent study, and considers the effect of restraining the tension flange fully against translation and lateral rotation, but elastically against twist rotation, which models the diaphragm restraint provided by cladding combined with purlins or girts in an industrial frame building.

2. Theory

2.1. General

The concept of an arbitrary axis located at mid-height of the web is adopted in this study to investigate the inelastic buckling behaviour of the beam-column as was done by Broadford and Cuk (1988) as shown in Fig. 1(a). Fig. 1(b) shows a simply supported beam doubly-symmetric beam-column subjected to equal end moments M and axial compressive force N. The stress-strain curve model used in this study is a trilinear idealisation with a constant strain hardening modulus $E_{st} = E / h'$ is shown in Fig. 2. The simplified pattern of residual stress, which has been used by a number of researchers assumes the distribution of residual stress in the flange to be bilinear and a constant tensile stress in the

Fig. 1 Beam-column axes and loading

Fig. 2 The trilinear idealisation of stress-strain curve

web as shown in Fig. 3.

$$\sigma_{rc} = 0.3\sigma_y \tag{1}$$

and

$$\sigma_{rt} = \left[\frac{BT}{BT + t_w(D - T)}\right]\sigma_y \tag{2}$$

where B and T are the flange width and thickness respectively, D is the overall depth of the I-section and t_w is the thickness of the web. The residual stresses should be satistied with equilibrium conditions as

$$\int_A \sigma_r dA = \int_A (x^2 + y^2)\sigma_r dA = 0 \tag{3}$$

The energy-based method is employed in this study to analyse I-section beam-columns with equal and opposite end moments and a constant concentric axial compressive force. A full description of the out-of-plane buckling analysis using the energy method is presented by Lee and Bradford (2002) and Lee (2001) but the in-plane analysis is different to that of uniform bending due to the N–δ effect as given in Lee and Bradford (2002) and Lee (2001).

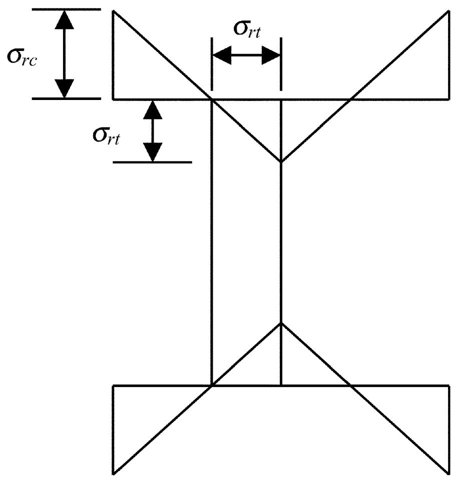

Fig. 3 Simplified residual stresses

2.2. In-plane analysis

When the beam-column of length L is subjected to end moment M and an axial compressive force N, the applied strain $\varepsilon_a(x, y)$ at any point of the cross section can be expressed as

$$\varepsilon_a(x, y) = \varepsilon_o + (y + \bar{y})\rho + \varepsilon_r(x, y) \tag{4}$$

where $\bar{y}$ is assumed neutral axes, ε_0 is strain due to axial force, ρ is curvature, and the residual strain is given by $\varepsilon_r(x, y) = \sigma_r(x, y) / E$.

The stress distribution of the cross-section is

$$\sigma(x, y) = \int_{\varepsilon_r}^{\varepsilon_a} E_t d\varepsilon_a + E\varepsilon_r \tag{5}$$

For a given compressive force N and an assumed curvature ρ, Eqs. (4) and (5) are solved iteratively for the position of the neutral axis by satisfying the equilibrium condition of axial force and is given as

$$N = \int_A \sigma(x, y) dA \tag{6}$$

The maximum moment $M_{\max}$ at midspan can be determined using the assumed curvature and the new neutral axis as determined from Eq. (6) and $M_{\max}$ is given as

$$M_x^{\max} = \int_A \sigma(x, y) y dA \tag{7}$$

2.3. Determination of end moment

The end-moment M is obtained from the midspan moment M_{max} iteratively by employing the Newmark (1943) integration technique. The flexural rigidity about the major axis is

$$\rho = \frac{M_x^{max}}{EI_x} \tag{8}$$

where $(EI)_s$ is the secant flexural modulus of rigidity.

The procedure to determine the end-moment is

1. Assume an end moment M
2. Assume the defected shape, and the elastic deflection by Trahair and Bradford (1998) is used for a first approximation.
3. Compute values of curvature at four equally spaced stations using $\rho = \dfrac{M + Nv}{(EI)_s}$, where v is deflection.
4. Correct the assumed deflections based on the curvature calculated in step 3.
5. Repeat steps 1 to 4 until the assumed deflection at the mid span is equal to the deflection determined in step 4.
6. Calculated the end moment, $M = M_x^{max} - Nv_{midspan}$

2.4. Out-of-plane analysis

Fig. 4 shows the out-of-plane buckling deformation of the cross-section. The buckling deformation of the flange is assumed to be half sine curves with n harmonics while a cubic polynomial is used for the web. The flanges are treated as rigid bars and the well-known beam theory (Timoshenko and Gere 1961) is used conjunction with tangent modulus theory that assumes the elastic modulus for the elastic regions and the strain hardening modulus for the yielded and strain hardened region. Isotropic (Timoshenko and Woinowsky-Krieger 1959) and orthotropic plate theory (Dawe and Kulak 1984, and Haaijer 1957) is used for elastic and inelastic regions of the web respectively. The stiffness matrices $[k_f]$ for the flange and $[k_w]$ for the web can be developed from the tangent modulus theory for the flanges and the isotropic and orthotropic plate theory for the web respectively as was done by Bradford (1986), and Lee and Bradford (2002), and Lee (2001). The pattern of residual stresses used in this study is simplified model. The simplified residual stress does not satisfy the torsional equilibrium condition and therefore the torsional rigidity should be changed to $((GJ)_t - \int_A \sigma_r(x^2 + y^2)dA)$ for the flange and the web as was done by Trahair (1993).

The strain energy due to the continuous elastic restraint, as was done by Lee and Bradford (2002) and Lee (2001), is also included in this study and the elastic restraint matrix can be expressed as $[k_r]$. Fig. 5 shows the types of restraint considered in this study with the restraining action. The out-of-plane analysis assumes that the member is sensibly “prismatic”, in that the extent of yielding along the member is uniform. Owning to the N–δ effect, this is not true, and the member is both sensibly monosymmetric and tapered. The effects of the tapering are handled by use of the theory described in Lee and Bradford (2002) and Lee (2001), but with the moment assumed conservatively to equal M_{max} throughout. While the N–δ effect has a fairly profound effect on the in-plane response, there appears to be some evidence (Bradford *et al.* 1984) that the effect on the buckling is less significant. Tapering caused by the moment gradient is, of course, a major consideration in inelastic buckling.

Fig. 4 Buckling deformations in the plane of the cross-section

Fig. 5 Beam restraints

2.5. Stiffness and stability matrices

The stiffness matrices of the flange $[k_f]$, the web $[k_w]$ and restraint $[k_r]$ can be assembled into global stiffness matrix $[K]$, while the stability matrix may be assembled into the global matrix $[G]$. The total change in potential may be expressed as

$$\Pi = \frac{1}{2}\{q\}^T\{[K]-\lambda[G]\}\{q\} \tag{9}$$

The buckling solution of the beam-column can be obtained from minimising Eq. (9) with respect to $\{q\}$ and may be written as

$$\{[K] - \lambda[G]\} = 0.0 \tag{10}$$

This is an iterative procedure, the global stiffness and stability matrices are adjusted at the value of applied curvature ρ until the eigenvalue (λ) equals 1.

3. Verification

Since tabulated or finite element solutions for inelastic lateral-torsional and lateral-distortional buckling of restrained beam-columns is not generally available, the accuracy of the current method is compared with inelastic lateral-distortional buckling of a restrained beam under uniform bending. Such results has been obtained by Lee and Bradford (2002), and inelastic lateral-torsional buckling results obtained by Abdel-Sayed and Aglan (1973) for a doubly-symmetric unrestrained beam-column subjected to axial compressive force *N* and equal end moment *M*. The cross-section considered in Lee and Bradford's (2002) study was 610UB101 (BHP Hot-Rolled Products 1998) and the material properties can be found in their paper. The beam-column considered in Abdel-Sayed and Aglan's (1973) study was the North American 8WF31 and assumed pattern of residual stress was simplified pattern. Details of the material properties used can be found in their paper and is also shown in Fig. 6. In order to compare the current method with the results of Abdel-Sayed and Aglan (1973) inelastic lateral-torsional buckling solution, the out-of-plane flexure of the web has been suppressed as was done by (Bradford and Trahair 1982)

$$U_{wp} = \frac{1}{2}\gamma_r \int_0^L \int_{-\frac{h}{2}}^{\frac{h}{2}} D_w \left(\frac{\partial^2 U_w}{\partial y^2}\right) dy dz \tag{11}$$

Fig. 6 Inelastic lateral-torsional buckling of beam-column with simplified residual stress

Table 1 Inelastic lateral-distortional l buckling of beam under uniform bending with $\alpha_z = 1000$

Length (m)	Lee and Bradford (2002) (KNm)	This study (KNm)
25	622.654	622.653
20	625.173	625.172
15	626.700	626.700
10	627.848	627.847
5	628.563	628.562

where U_{wp} is the strain energy due to out-of-plane plate flexure of the web, and D_w is the plate rigidity factor and γ_r is set to a large value (say 10^8)

Table 1 shows a comparison between this study and the inelastic buckling results of Lee and Bradford (2002) with the normalised torsional restraint value of α_z equal to 1000. α_z is given as

$$\alpha_z = \frac{k_z}{\pi^2 GJ/L^2} \tag{12}$$

where G is the torsional rigidity, L is length of the beam and k_z is the torsional restraint.

The inelastic lateral-distortional buckling results of a restrained beam under uniform bending using the current method is obtained with a very small value of strain due to the axial force (ε_o). It can be seen in Table 1 that the buckling results obtained from the current method and Lee and Bradford (2002) are in good agreement. Fig. 6 shows the comparison between this study and the results of Abdel-Sayed and Aglan (1973), where the end buckling moment is normalised with respect to the yield moment M_y and the axial force is constant at 0.6 N_s, where N_s is the squash load. The slenderness ratio for which the inelastic buckling moments are obtained is the major axis slenderness ratio L / r_x, where r_x is the radius of gyration about the x-axis. Abdel-Sayed and Aglan (1973) used the tangent modulus theory with E_t in the yielded region of the cross-section equal to zero. This study has adopted the same approach as Abdel-Sayed and Aglan for the tangent modulus in the yielded region and they are agreed reasonably well. Fig. 6 also shows the buckling results obtained with tangent modulus theory that is equal to E_{st} in the yielded and strain hardened region. The inelastic buckling results obtained with $E_t = E_{st}$ for the yielded and strain hardened region shows that the onset of strain hardening buckling occurs at a higher slenderness ratio than that using the more conservative assumption of $E_t = 0$ in the yielded regions. Abdel-Sayed and Aglan's solution is somewhat unconservative compared to the present model for higher values of L/r_x. Using the approximate magnifier of $1/(1-N/N_{ox})$ (Trahair and Bradford 1998) at L/r_x =50, where N_{ox} is the elastic major axis buckling load, indicates that the independent solution is about 20% unconservative, and incorporation of this effect renders the independent solution very close to that of the present method.

4. Elastic torsional restraint

The inelastic lateral-distortional buckling behaviour of the cross-sections 610UB101, 180UB18.1 and 310UC158 manufactured in Australian (BHP Hot-Rolled Products 1998) and the North American 8WF31 are investigated herein, where the tension flange is fully restrained against translation and lateral rotation, but where the restraint against twist rotation during buckling is elastic with a stiffness

Table 2 Geometric dimensions of the I-sections

Designation	Flange width (b_f) (mm)	Flange thickness (t_f) (mm)	Web height (h) (mm)	Web thickness (t_w) (mm)	h / t_w	b_f / h
610UB101	228.0	17.3	589.7	11.2	52.652	0.387
180UB18.1	90.0	8.0	167	5.0	33.4	0.539
310UC158	311.0	25.0	302	15.7	19.236	1.029
8WF31	203.2	10.998	192.202	7.315	26.275	1.057

value of k_z. The dimension of these cross-sections is shown in Table 2. The normalised torsional restraint (a_z) is used in this section to investigate the inelastic lateral-distortional buckling behaviour as was the previous sub-section in the comparison study. The relevant material properties are E (elastic modulus) = 200 GPa, σ_y (yield stress) = 250 MPa, ε_{st} (strain hardening) = 10ε_y (yield strain), h' (E/E_{st}) = 33 and v (elastic Poisson's ratio) = 0.3. The torsional and translational restraint is obtained by Eqs. (12) and (13) respectively.

$$k_t = \frac{k_{ry}\pi^2 n^2}{L^2} \tag{13}$$

where k_t = translational restraint, k_{ry} = the minor axis rotational restraints and n is number of harmonics.

The completed restrained translation can be obtained by an infinite value of minor axis rotation (say 100^6). Figs. 7 and 8 show the normalised buckling moments when the 610UB101 beam-columns are subjected to a constant axial force 0.2N_s and 0.6N_s respectively, while Figs. 9 and 10 are for 180UB18.1 beam-columns subjected to a constant axial force 0.2N_s and 0.6N_s respectively. These inelastic lateral-torsional and lateral-distortional results are plotted, and as expected the lateral-torsional buckling

Fig. 7 Inelastic buckling of beam-column 610UB101 with elastic twist rotational restraint

Fig. 8 Inelastic buckling of beam-column 610UB101 with elastic twist rotational restraint

moments overestimate the true (distortional) buckling moments as the degree of torsional restraint α_z increases. In design, the member strength determined in accordance with AS4100 (SA 1988) would be based on the inelastic lateral-torsional buckling curves (which are modified empirically (Trahair and Bradford 1998), but clearly these strengths are unconservative.

The buckling mode of a beam-column is lateral-torsional and buckles in one harmonic (n=1) when

Fig. 9 Inelastic buckling of beam-column 180UB18.1 with elastic twist rotational restraint

Fig. 10 Inelastic buckling of beam-column 180UB18.1 with elastic twist rotational restraint

fully restrained with translation applied at the tension flange only. As the torsional restraint α_z increases, the beam-column buckles in a distortional mode. The results for the 180UB18.1 show that there are up to 6 buckling harmonics (n = 1,….,6) in the buckling mode. The results for the 610UB101 show that for α_z = 100, there are four harmonics represented in the solution (*ie*. the analysis must be performed for increasing numbers of harmonics (n) and minimum solution adopted, *ie*. n=4 in this case). Compared with the elastic solution (Bradford 1997) which is again dependent on the number of harmonics, the transition in the solution between adjoining harmonics is much smoother for inelastic buckling than for elastic buckling, where the curves are of the characteristic local buckling 'garland' form. Finally, it is worth noting that increasing the degree of torsional restraint increases the strength of the beam-column subjected to a constant axial compression. In Figs. 8 and 10, it is clear that the beam-column has no reserve of bending capacity for $L / h > 11$ and $L / h > 22$ respectively in the absence of torsional restraint, but for large values of α_z the beam-column has a considerable reserve of bending capacity for large member lengths, while it buckles into a number of harmonics.

The results of the compact section 310UC158 and 8WF31 are shown in Figs. 11 and 12 respectively with a constant axial force of $0.4N_s$. The torsional parameters considered in this study were 0 and 1 because the inelastic buckling moment of compacted I-section (310UC158 and 8WF31) reaches its plastic moment as the torsional parameter is increased. It can be seen that the buckling mode of these members is flexural-torsional rather than lateral-distortional. The number of harmonics for buckling of the 310UC158 member was one, but the 8WF31 shows that there are up to three buckling harmonics (n=1,2,3) in the buckling mode.

The effect of web distortion in the lateral-torsional buckling of I-sections can be influence by the web slenderness (h / t_w) and the flange width to web depth ratio (b_f / h), and top and bottom flange width ratio (B_T / B_B). This study is restricted to doubly-symmetric I-section members and therefore top and bottom flange width ratio (B_T / B_B) is ruled out. Table 2 shows the dimension of the cross-section considered in this study with web slenderness (h / t_w) and flange width to web depth ratio (b_f / h). It has been shown that the cross-section distortion has profound effect on the inelastic distortional buckling of the slender beam-columns (610UB101 and 180UB18.1) which have the web slenderness (h / t_w) of 52

Fig. 11 Inelastic buckling of beam-column 310UC158 with elastic twist rotational restraint

Fig. 12 Inelastic buckling of beam-column 8WF31 with elastic twist rotational restraint

and 33, and the flange width to web depth ratio (b_f / h) is less than 1. The web slenderness of compact I-section (310UC158 and 8WF31) is 19 and 26 and the flange width to web depth ratio (b_f / h) is greater than 1. It can be noted that there is not much difference in web slenderness of the slender I-section 180UB18.1 and compact I-section 8WF31, but the flange width to web depth ratio of 180UB18.1 is much smaller than 8WF31. It is premature to draw conclusion on distortional buckling behaviour of the beam-columns without further study on more realistic loading conditions such as unequal end moment, transverse loading and axial compressive force, but this study found that the effect of web distortion is strongly influenced by the flange width to web depth ratio rather than those of the web slenderness for hot-rolled I-sections.

5. Conclusions

An energy-based method has been employed in this paper to study the inelastic lateral-distortional buckling of doubly-symmetric hot-rolled I-section beam-columns. These beam-columns are subjected to concentric axial compression and equal end moments with continuous elastic restraints that inhibits buckling. The method assumes that the top and bottom flange/web junctions deflect and twist as a sine curve with n harmonics while the web displaces as a cubic curve. Residual stresses are inherent in hot-rolled I-sections during manufacturing and these residual stresses cause a reduction of the buckling strength with their combination of applied load. The buckling load depends on the pattern of residual stress, and therefore the energy method incorporates the so called the simplified pattern of residual stress. The verifications and the accuracy of the method were made with inelastic lateral-distortional buckling results for restrained beams and inelastic lateral-torsional buckling results for unrestrained beam-columns obtained by an independent study and are shown to agree very well.

The effect of the web distortion is important in diaphragm-type restraint of the tension flange in a beam-column, where the flexural rigidity of the diaphragm provides torsional restraint to the beam-column, but in which the shear and translational stiffness of the diaphragm prevent translational and lateral rotation of the tension flange. Four different cross-sections have been used to investigate lateral-distortional buckling behaviour of the beam-column. As would be expected, the beam-column buckles in a lateral-torsional mode when beam-column is completely restrained against translation applied at the tension flange only. The significance of web distortion for compact sections is less important but buckling results for slender I-sections showed that the buckling mode becomes distortional and the web distortion is accentuated as the torsional restraint is increased.

This study considers continuously restrained beam-columns under uniform bending and constant axial force. The pattern of residual stress used in this study is a simplified pattern that suited the North American I-sections but the polynomial pattern of residual stress that suited the British and Australian I-sections has not been considered. To be able to make possible recommendations on the current steel structure standard the further study is required on the inelastic lateral-torsional and lateral-distortional buckling of restrained and unrestrained beam-columns subjected to an unequal end moment, transverse loading and axial compressive force. Thus further study is undertaken for hot-rolled and welded I-section beam-columns subjected to an unequal end moment, transverse loading and axial compressive force using a line type finite element method that incorporates tangent modulus theory and residual stress.

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