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Reliability analysis of latticed steel towers against wind induced displacement

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Abstract. The present study aims at the reliability analysis of steel towers against the limit state of deflection. For this purpose tip deflection of the tower has been obtained after carrying out the dynamic analysis of the tower using modal method. This tip deflection is employed for subsequent reliability analysis. A limit state function based on serviceability criterion of deflection is derived in terms of random variables. A complete procedure of reliability computation is then presented. To study the influence of various random variables on tower reliability, *sensitivity analysis* has been carried out. Design points, important for probabilistic design of towers, are also located on the failure surface. Some parametric studies have also been included to obtain the results of academic and field interest.

Key words: latticed tower; structural reliability; first order reliability method.

1. Introduction

Free standing latticed towers are used extensively in microwave and radio communication systems and for supporting wind energy generators. These towers are vulnerable to failure under oscillatory wind excitations. The failure of these towers may be grouped in two limit states: (1) *limit state of strength or collapse* and (2) *limit state of serviceability*. Failure due to limit state of collapse occurs when tower members reach their ultimate capacity. Serviceability limit state, however, is mainly concerned with excessive tip deflection. Therefore to avoid reaching this limit state, the tip deflection of tower should not exceed the limit specified by standards and codes (Agarawal and Garg 1994, AS: 3995-1994, EIA/TIA-222-E 1991). The present study is concerned with the reliability analysis against the violation of deflection limit state.

A detailed review of the past work shows that though a considerable amount of work has been done on dynamic analysis of latticed steel towers (Agarawal and Garg 1994, Ahmad *et al*. 1984, Davenport and Sparling 1992, Dharaneepathy and Keshavarao 1987, Harikrishna *et al*. 1999, Holmes 1994a, 1994b, Masood *et al*. 1995) and reliability analysis of other structures (Siddiqui and Ahmad 2000, 2001, Siddiqui *et al.* 2002, Choudhury *et al.* 2002, Amanullah *et al*. 2002) but work on reliability analysis of latticed steel towers is not widely reported. Some investigators such as (Menon and Rao 1998, Deoliya and Datta 2000, 1998) studied the reliability of steel and reinforced concrete towers against strength limit state. However, the work on reliability analysis against the serviceability limit

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state of deflection is very scanty. The present study aims at the reliability analysis of steel towers against the limit state of deflection. For this purpose tip deflection of the tower has been obtained after carrying out the dynamic analysis of the tower using modal method. This tip deflection is employed for subsequent reliability analysis. A limit state function based on serviceability criterion of deflection is derived in terms of random variables. A complete procedure of reliability computation is then presented. To study the influence of various random variables on tower reliability, *sensitivity analysis* has been carried out. Design points, important for probabilistic design of towers, are also located on the failure surface. Some parametric studies have also been included to obtain the results of academic and field interest.

2. Analysis for wind load

For the analysis of tower against wind load, the tower has been idealized as multi-degree lumped mass system by assuming masses and projected areas to be concentrated at various nodes along the height (Fig. 1). Only horizontal motions have been considered, and these are assumed to be

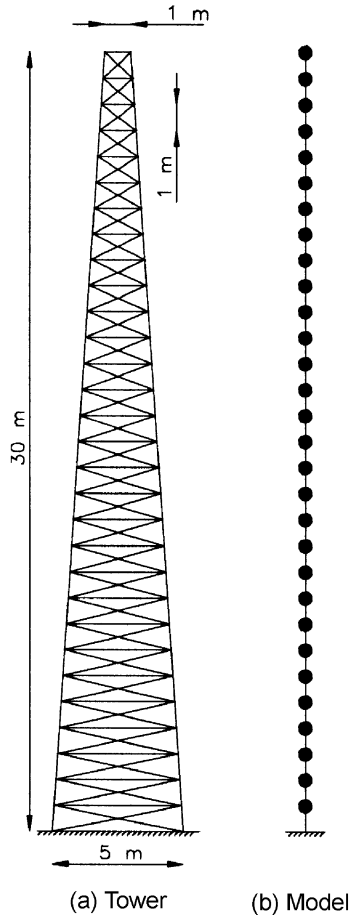

Fig. 1 Latticed steel tower

independent of vertical as well as rotational displacements, which are neglected because of their relatively small magnitudes. The dynamic analysis is thus restricted to that of a “lumped-mass” planar system for which a single degree of freedom (horizontal motion) is assigned to each mass.

The tip deflection can be obtained by superimposing the static and dynamic response of the tower. For the purpose of static response, the wind force in the different panels is estimated using the formula:

$$F = \frac{1}{2}\rho C_D A V^2 \tag{1}$$

where,
ρ = mass density of air;
A = projected area of the panel at the reference height;
C_D = coefficient of drag; and
V = velocity of wind at the panel height.

The dynamic response of latticed tower has been determined by solving the equation of motion for along wind response of the damped multi-degree of freedom system, thus

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{C}\dot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{F}(t) \tag{2}$$

where, $\mathbf{x}$, $\dot{\mathbf{x}}$, $\ddot{\mathbf{x}}$= displacement, velocity and acceleration vector;
$\mathbf{M}$ = mass matrix of tower;
$\mathbf{C}$ = damping matrix;
$\mathbf{K}$ = stiffness matrix; and
$\mathbf{F}(t)$ = forcing function due to mean / fluctuating wind load.

The above equation has been solved using *Modal method*. For this purpose Eq. (2) has been normalized. Thus normalized equation of motion in the r^{th} mode becomes

$$\ddot{x}_r(t) + 2\beta_r\dot{x}_r(t) + \omega_r^2 x_r(t) = \frac{P_r(t)}{m_r} \tag{3}$$

where, m_r, $P_r(t)$ are the generalized mass and force and $\beta_r = \delta_r\omega_r / 2\pi$. Here, δ_r and ω_r represents logarithmic decrement and frequency in r^{th} mode. The solution of Eq. (3) will give the r^{th} mode displacement of any point z at time t along the tower height using the relation.

$$x_r(z, t) = a_r(t)\phi_r(z) \tag{4}$$

where, $a_r(t)$ is a constant quantity (at time t) in the r^{th} mode and $\phi_r(z)$ is the modal displacement of point z. The total dynamic response is obtained by superposition of the response in different modes. The along-wind mean response (displacement, bending moment, shear force) of towers is calculated using the force and flexibility matrices. The mean drag force $P(z, t)$ can be taken as the sum of a static part $\overline{P}(z)$ and fluctuating component $p(z, t)$, thus

$$\begin{aligned} P(z, t) &= \overline{P}(z) + p(z, t) \\ &= \frac{1}{2}\rho_a C_D(z)A(z)\overline{V}^2(z) + \rho_a C_D(z)A(z)\overline{V}(z)v(z, t) \end{aligned} \tag{5}$$

where, $C_D(z)$ = Coefficient of drag at height z;
= $(4.38-5.11\phi)\,(1+0.64\phi)$ for latticed tower with $\phi < 0.4$;
ϕ = solidity ratio;
$A(z)$ = projected area per unit length at z;
ρ_a = mass density of air = 1.208 kg/cm^3;
$\overline{V}(z)$ = mean wind velocity at height z;

$$= \overline{V}_{10}\left(\frac{z}{10}\right)^{\alpha};$$

$\overline{V}_{10}$ = the mean velocity at reference height of 10 m;
α = the power law index = 1/7.5 for exposed and windy area; and
$v(z, t)$ = time varying component of wind velocity centered on zero mean.

2.1. Non resonant response

The rms value of the non-resonant response of tower due to background effect of gustiness are separately estimated in r^{th} mode using equation (Ahmad *et al*. 1984):

$$\bar{x}_{r1}^2(z) = \frac{3C_T B\overline{V}_{10}^2}{n_r^2 m_r^2 \omega_r^4}\left[\sum_0^h \rho_a C_D(z) A(z) \overline{V}(z) \phi_r(z)\right]\phi_r(z) \tag{6}$$

where,
B = Back ground turbulence factor and a function of width and height of the structure;
= 0.06 for α = 1/7.5;
C_T = Terrain coefficient;
= 0.005 for α = 1/7.5;
$\bar{x}_{r1}^2(z)$ = Non-resonant deflection at z in the r^{th} mode;
ω_r = natural frequency in r^{th} mode = $2\pi n_r$;
h = height of tower.

2.2. Resonant response

For the estimation of resonant response to a given wind speed, the spectrum of horizontal gustiness at frequency n at a reference height of 10 m, as expressed by Eq. (7) given below is first calculated (Ahmad *et al*. 1984).

$$S_{v10}(n) = \frac{4C_T}{n}\overline{V}_{10}^2 \frac{y^2}{(1+y^2)^{4/3}} \tag{7}$$

where, $y = 1200\, n / \overline{V}_{10}$. Next, the rms value of the gust resonant response has been computed separately in r^{th} mode using following equation for the resonant deflection at point z in r^{th} mode (Ahmad *et al*. 1984):

$$\bar{x}_{r2}^2(z) = \frac{1}{n_r^2}\left(\frac{\pi^2 n_r}{2\delta_r}\right)\frac{2S_{V10}(n_r)}{C_1 m_r^2 \omega_r^4}\left[\sum_0^h \rho_a C_D(z) A(z) \overline{V}(z) \phi_r(z)\right]^2 \phi_r^2(z) \tag{8}$$

where, n_r is the natural frequency of the system in the r^{th} mode; $C_1 = 7n_r h / \overline{V}_{10}$; δ_r = Logarithmic decrement in r^{th} mode. The total material damping is first assumed and then aerodynamic damping is estimated with the help of the expression for the logarithmic decrement in the r^{th} mode due to aerodynamic drag

$$\delta_a(r) = \frac{\sum_0^h \rho_a C_D(z)\bar{v}(z)A(z)\phi_r^2(z)}{2n_r \sum_0^h m(z)\phi_r^2(z)} \tag{9}$$

Introducing the value of total damping (which is the sum of material and aerodynamic damping) in Eq. (8), response of tower is computed by iteration. The rms value of the total dynamic response due to gustiness of wind has been calculated by adding non-resonant and resonant responses. The maximum fluctuating response in the r^{th} mode has been obtained by multiplying the rms value of response by a factor g_r, known as the peak factor given by Davenport and Sparling (1992):

$$g_r = \sqrt{2\ln(n_r \bar{t})} + 0.57/\sqrt{2\ln(n_r \bar{t})} \tag{10}$$

where, $\bar{t}$ is the average period taken as 1 hr. This maximum fluctuating response is the largest response expected to be experienced by a structure in its lifetime. The total peak response is obtained by superposition of the mean and maximum fluctuating component. The tip deflection of the tower has thus been obtained for its subsequent reliability analysis.

3. Reliability analysis

In the present study the reliability analysis has been carried out using First Order Reliability Method (FORM (Madsen *et al*. 1986)). In brief, in this approach of reliability estimation, the reliability is measured in terms of a *reliability index*, β and it is related to the probability of failure or probability of limit state violation for any limit state as

$$\beta = -\Phi^{-1}(P_f) \tag{11}$$

where, P_f is the probability of failure and $\Phi(\ )$ is standard normal distribution function. The reliability index β is found from the solution of the constrained optimization problem:

$$\text{Minimize } \beta(y) = (\mathbf{y}^T\mathbf{y})^{1/2} \quad \text{subject to } G(\mathbf{y}) = 0 \tag{12}$$

where, $\mathbf{y}$ is the vector of basic random variables in the standard normal space and $G(\mathbf{y})$ is the limit state function. A limit state function is a mathematical representation of a particular limit state of failure. Depending on the problem under consideration, different formulations for the limit state function can be employed; this may include as variety of strength and serviceability limit states. In the present paper, serviceability limit state of tower deflection is considered for the derivation of limit state function.

The reliability index and the corresponding vector $\mathbf{y}^*$, usually referred to as a design point, obtained from the solution of Eq. (12) can also be used to estimate the influence of individual random variables on the tower reliability in terms of the so-called *sensitivity factors*. For the j^{th} random variable, the sensitivity factor, α_j, is defined as

$$\alpha_j = \left.\frac{\partial\beta}{\partial y_j}\right|_{y^*} = \frac{y_j^*}{\beta} \tag{13}$$

where, y_j^* is the value of this variable at the design point.

3.1. Deflection limit state function

The following deflection limit state function is derived for reliability analysis

$$G(\underline{X}) = z_{\max} - z_s d_s - z_d d_d \tag{14}$$

where, z_{max} is the maximum allowable deflection, which depends on the height of tower. d_s, d_d are the magnitudes of static and dynamic deflections obtained from the analysis discussed in section 2 and z_s, z_d are model uncertainty factors for these deflections. These factors incorporate uncertainties involved in the estimation of responses (i.e. d_s and d_d) in a same fashion as it is considered by Siddiqui and Ahmad (Siddiqui and Ahmad 2000, 2001, Amanullah *et al.* 2002) for offshore structures. The $z_{\max}$ is usually obtained from a non-dimensional expression

$$\frac{h}{z_{\max}} = 300 \tag{15}$$

where, h is the height of the tower.
Substituting Eq. (15) into Eq. (14) we have,

$$G(\underline{X}) = \frac{h}{300} - z_s d_s - z_d d_d \tag{16}$$

where, $\underline{X} = (h, z_s, z_d)$ (17)

4. Numerical study

For numerical study, a latticed steel tower of 30 m height with 1 m top width and constant panel height has been chosen for reliability analysis. Details of the tower are shown in Fig. 1 and the necessary data employed for dynamic analysis are given in Table 1. Moreover, for reliability analysis statistical data of various random variables are required which are shown in Table 2. In this table z_s and z_d are considered normal and their mean values are taken as unity in a same fashion as it is considered by Siddiqui and Ahmad (Siddiqui and Ahmad 2000, 2001, Amanullah *et al.* 2002) for offshore structures. The distribution of h is taken as normal because in the absence of appropriate study on probability distribution, geometrical properties may be considered as normal (Madsen *et al.* 1986). The mean value of h is considered same as its nominal height (Fig. 1). COVs in the Table 2, however, are assumed on the basis of degree of uncertainty involved in these parameters.

Table 1 Data for static and dynamic analysis of tower

Parameter	Value
Height of the tower (h)	30 m
Top width of the tower	1.0 m
Base width	5.0 m
Grade of steel	Fe 250
Mean wind velocity	40 m/s
Reference height	10.0 m
Size of column members (2ISA)	
Panel 1-7	11 cm × 11 cm × 0.8 cm
Panel 8-15	10 cm × 10 cm × 0.6 cm
Panel 16-23	6 cm × 6 cm × 0.5 cm
Panel 24-30	5 cm × 5 cm × 0.3 cm
Size of girder members (ISA)	
Panel 1-7	8 cm × 8 cm × 1 cm
Panel 8-15	5.5 cm × 5.5 cm × 0.6 cm
Panel 16-23	4 cm × 4 cm × 0.5 cm
Panel 24-30	2.5 cm × 2.5 cm × 0.3 cm
Size of Lacing members (ISA)	
Panel 1-7	7 cm × 7 cm × 0.8 cm
Panel 8-15	5.5 cm × 5.5 cm × 0.8 cm
Panel 16-23	5 cm × 5 cm × 0.4 cm
Panel 24-30	2.5 cm × 2.5 cm × 0.5 cm
Terrain roughness coefficient (α)	0.133
Background factor	0.6
Gust factor	1.52

Table 2 Statistical data (COV, coefficient of variation)

Random variables	Distribution	Mean	COV
Height of the tower (h)	Normal	30 m	5%
Static response uncertainty factor (z_s)	Normal	1.0	10%
Dynamic response uncertainty factor (z_d)	Normal	1.0	15%

5. Discussion of results

Figs 2 and 3 show the variation of static and dynamic response with wind velocity and base width respectively. These variations have been employed for present reliability analysis.

5.1. Design point

Table 3 shows values at design point or the most likely failure point. A point on the failure surface that corresponds to the shortest distance from the origin in the reduced coordinate system is defined as the most likely failure point or design point. These values of different random variables are essential for

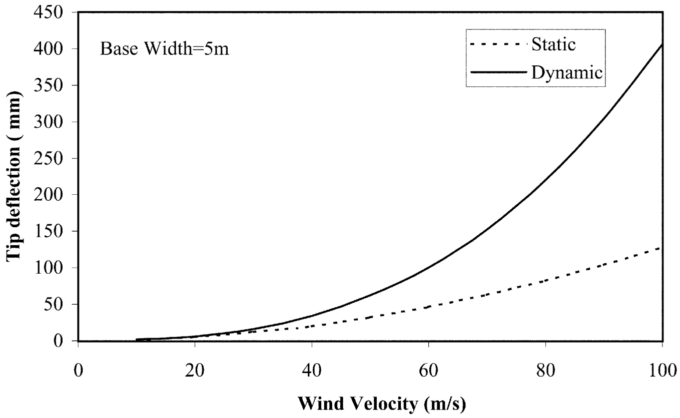

Fig. 2 Static and dynamic response with wind velocity

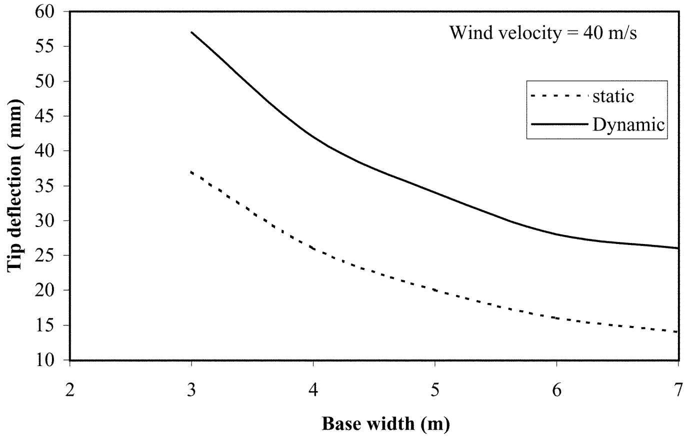

Fig. 3 Static and dynamic response with base width

Table 3 Design values

Random variables	Design values
Height of the tower, h	23.871 m
Static response uncertainty factor, z_s	0.023
Dynamic response uncertainty factor, z_d	0.055

reliability based probabilistic design of towers. In such designs partial safety factors for load and resistance variables are determined for a target reliability (i.e. target reliability index). These safety factors are separately defined for resistance and load variables. For resistance variables it is defined as the nominal, mean or characteristic value divided by the design value and for load variables as the design value divided by the nominal, mean or characteristic values.

5.2. Effect of wind velocity

As expected, Fig. 4 shows that the increase in the wind velocity results in a corresponding decrease in the reliability of tower. The decrease in tower reliability is slow from 10 m/s to 20 m/s velocity

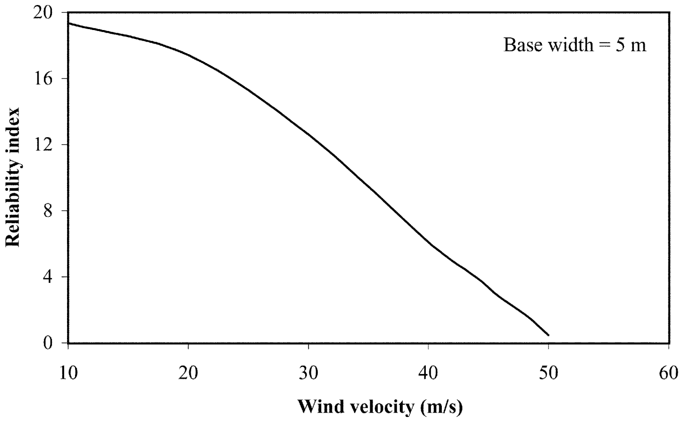

Fig. 4 Tower reliability with wind velocity

excitations. However, from 20 m/s onwards there is a fast decrease in the reliability and it is almost zero near 50 m/s of wind velocity. This shows that a tower safety can be ensured only up to a certain wind velocity. A tower which is safe for lower wind velocity may be completely unsafe and unreliable under a wind velocity of higher magnitude.

5.3. Effect of tower base width

Fig. 5 shows the effect of base width on tower reliability. It is self explanatory, as the base width of tower increases there is corresponding decrease in the tip deflection (Fig. 3). Due to this reason the tower reliability increases as the base width of the tower increases. Fig. 5 also shows that when the base width of tower is 3 m tower reliability index is around 0.5 and when base width is 7 m the reliability improves to around 9.0. It can, therefore, be concluded that for tower reliability the base width plays a very important role and designers must give a due consideration to tower base width for achieving required reliability or safety level (usually 3.5-4.5, Siddiqui and Ahmad 2000, 2001, Siddiqui *et al.* 2002, Choudhury *et al.* 2002, Amanullah *et al.* 2002).

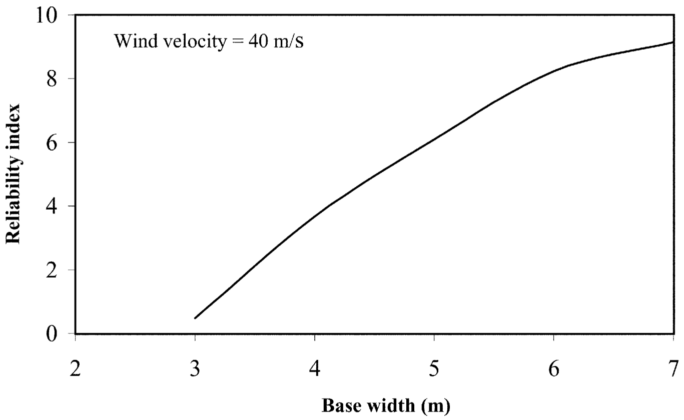

Fig. 5 Tower reliability with base width

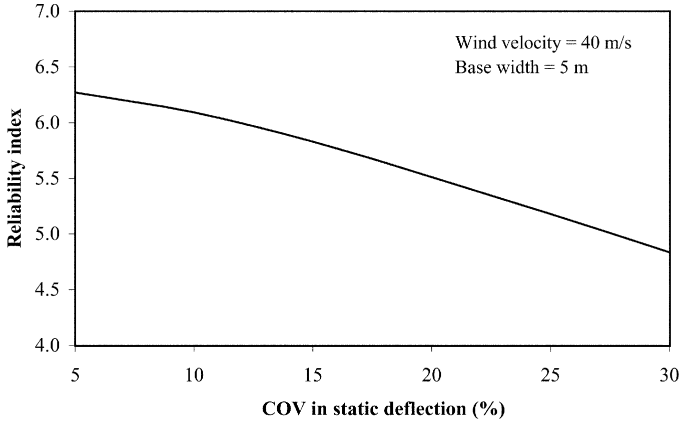

Fig. 6 Tower reliability with uncertainty in static deflection

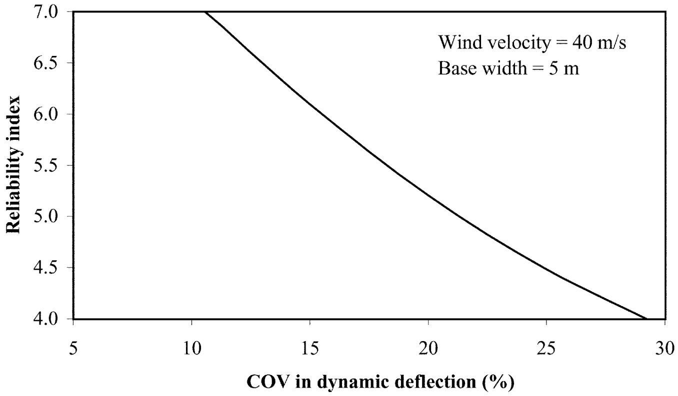

Fig. 7 Tower reliability with uncertainty in dynamic deflection

5.4. Effect of response uncertainty

Fig. 6 shows that as the uncertainty, measured in terms of coefficient of variation (COV), in the estimation of static deflection increases there is corresponding continuous decrease in the reliability index magnitude. This shows that it is not only the mean deflection that controls the reliability or safety of tower but also the COV plays a very significant role in determining the reliability of tower. The figure shows that when COV is 5%, reliability is around 6.2 and it falls to about 4.8 when COV becomes 30%. This indicates that an additional increase of 25% in COV results in about 30% reduction in the reliability index. Similar discussion also applies to Fig. 7 as well.

5.5. Sensitivity analysis

This analysis has been carried out to study the influence of various random variables on tower reliability. The influence of various random variables on tower reliability is measured in terms of *sensitivity factor* (α_j) which for the j^{th} random variable is defined as

$$\alpha_j = -\frac{\left(\frac{\partial Z_l}{\partial y_j}\right)_*}{\left[\sum_{j=1}^{n}\left(\frac{\partial Z_l}{\partial y_j}\right)^2 *\right]^{1/2}} \tag{18}$$

where Z_1 and y_j indicate the limit state function and j^{th} random variable in reduced coordinate system; and * indicate the most probable or design point on the failure surface.

The above defined sensitivity factors have following characteristics:

1. The lower the magnitude of α_j, less is the influence of j^{th} random variable on the reliability.
2. α_j is positive for load variables and negative for resistance variables.
3. If α_1, α_2, α_3……….. α_n are the sensitivity factors for n random variables appearing in the limit state function then $\sum_{j=1}^{n}\alpha_j^2 = 1$.

In the present study, using above expression, sensitivity factors for each random variable have been determined and shown in graphical form in Fig. 8. This figure shows that the random variable h (tower height) is negative and z_s (model uncertainty in static deflection), z_d (model uncertainty in dynamic deflection) are positive. This indicates that h will contribute to the resistance part and z_s, z_d will contribute to the load part of the limit state function. In other words, if all the parameters are kept same any increase or decrease in the magnitude of h will correspondingly increase or decrease the tower reliability. This is due to the fact that the maximum allowable deflection directly depends on h (Eq. 16). The effect of model uncertainties z_s and z_d, however, will be opposite. A comparison between the magnitudes of sensitivity factors of z_s and z_d show that sensitivity factor magnitude for z_s is less than z_d. This shows that the influence of z_d on tower reliability is comparatively more than the effect of z_s.

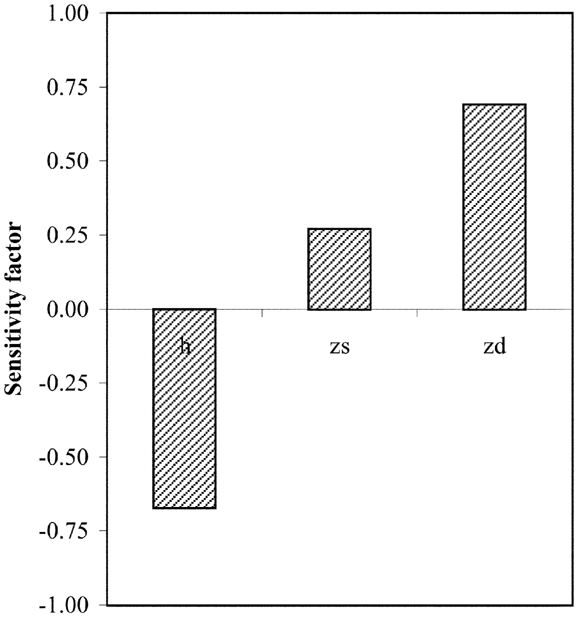

Fig. 8 Results of sensitivity analysis

6. Conclusions

The present paper presents a methodology for the reliability analysis of latticed steel towers against serviceability limit state of deflection. The results of the analysis showed that a tower safety or reliability can be ensured only up to a certain wind velocity. A tower which is safe for lower wind velocity may be completely unsafe and unreliable under a velocity of high magnitude. Effect of base width was also studied on tower reliability and observed that as the base width of the tower increases the tower reliability improves significantly. A study on effect of uncertainty, measured in terms of COV, in various random variables showed that on tower reliability uncertainties involved in the estimation of static and dynamic deflection affects the tower reliability considerably. Further, it is also observed that the model uncertainty in dynamic response affects the tower reliability more than the model uncertainty in static response. To study the influence of various random variables on tower reliability sensitivity analysis has been carried out. The results of this analysis are found to be very important design tool.

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