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The optimisation method of the elastic-plastic spatial grid structures

Jan Karczewski†

The Building Structures Institute, Warsaw University of Technology
Ul. Armii Ludowej 16, 00-673 Warsaw, Poland

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Abstract. The low boundary of load carrying capacity of the elastic-plastic spatial grid structures depend on numerous values and their variability assumed in designing process. Analysed influence all this values in searching for optimal variant of the structure lead to too great problem even taking into consideration actual computational power we have in disposal. Therefore one can take only a few values which have greatest influence on the optimal choice. In optimal analysis of the elastic-plastic spatial grid structures the previously proposed method with subsequent modification (Karczewski 1980), (Karczewski, Barszcz and Donten 1996), (Karczewski and Donten 2001) as well as computer program which was worked out by Donten K. to make possible practical utilisation this method was employed. The paper deal with evaluation of influence dimensions of particular values for choice of optimal variant of the structure. One among this values is distribution of the struts in the structure.

Key words: spatial grid structures; plastic analysis; optimisation; decisive variable; objective function; distribution of the strut.

1. Introduction

Plastic analysis method employing in designing process must give exact modelling of real behaviour of the structure as well as full safety during their further exploitation. After analysis used incremental method work out by Author, (Karczewski 1980) and presented in Proceeding of Asia-Pacific Conference on Shell and Spatial Structures which was held in Bejing in 1996 with subsequent modifications (Karczewski Barszcz and Donten 1996), (Karczewski 1997), (Karczewski and Donten 2001).

An axially loaded strut with one end free to move collapses immediately at its ultimate load due to yielding or buckling. Preservation of its equilibrium is impossible without immediate unloading. Struts which are a members of the spatial grid structures exhibit different behaviour. They interact with the whole structure even if buckled in compression or yielded in tension and, therefore their deformations are constrained through the displacements of their ends nodes. Although their stiffness vanishes or even become negative they still are able to carry axial forces of magnitudes compatible with their deformations. The structure collapses if and only if a sufficient number elements has buckled or yielded. There must be so many of them that the remaining elastic substructure becomes a mechanism.

In above mentioned method analysis is performed by “step by step” way and their main assumption

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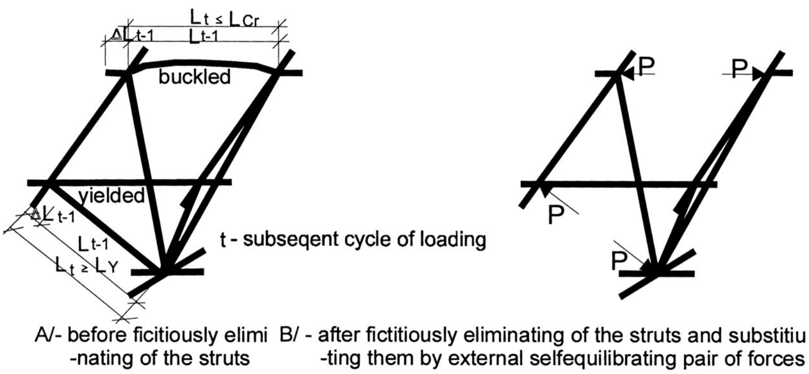

Fig. 1 Main assumption of the method used

consist in fictitious eliminating struts which start, during increasing of loading, in non-linear behaviour and substituting their by pair selfequlibrating external forces, (Karczewski 1980), see Fig. 1. In general relations at an arbitrary instant of the loading and unloading process can be shown as below, Eq. (1):

$$\begin{gathered} \underline{u} = \underline{C}\ \underline{\delta}: \\ \underline{F} + \underline{F_F} = \underline{C}^T\ \underline{P}; \\ \underline{u} = \underline{E}^{-1}\ \underline{P} + \underline{u}^R; \end{gathered} \tag{1}$$

Here, $\underline{u}$, $\underline{\delta}$, $\underline{F}$, $\underline{F_F}$, $\underline{P}$, $\underline{u}^R$ denote the following vectors: strut deformations, nodal displacements, applied loads, self-equilibrating pair of external point loads, strut forces and predeformations due to assembling, respectively. $\underline{C}$, $\underline{E}$ are matrices of compatibility (equilibrium) and strut stiffness accordingly. In such a way, the actual spatial grid structure is modelled by a modified elastic structure, resulting from the original one through the removal of the buckled or yielded members, acted upon by the actual load and the reactions of the eliminated members. In such a situation one can easily notice that in the case of the proposed method of analysis, properly simulated strut behaviour is of great importance for the accuracy and safety of the estimation of subsequent values of the function showing relation between strut forces and displacements of the ends of the strut, i.e. the values of the eliminated strut reactions for a given deformation.

In used method, magnitude of the external forces substituting interaction previously eliminated struts on is taken from the model of strut behaviour, (Boutros 1991), (Nonaka 1977), (Karczewski 1980, 1997), (Karczewski Barszc and Donten 1996), (Karczewski and Donten 2001), see Fig. 2. The model of strut behaviour is described by the function $P(u)$ i.e. relationship between axial force $\boldsymbol{P}$ and deformation u expressed by the displacements of strut ends in relation to each other. In used methods u is composed of five integral components, (Karczewski and Barszcz 1995), Eq. (2).

$$u = u^P + u^M + u^Y + u^E + u^R \tag{2}$$

where: u^P - deformation resulting from the elastic deformation produced by axial force, u^M - deformation resulting from variability of the strut geometry with regard formulas proposed by Boutros (Boutros 1991), u^Y - deformation resulting from the plastic axial deformation produced during plastic rotation in plastic hinge, u^E - residual deformation resulting from the plastic axial deformation during plastic rotation in compression, u^R - deformation resulting from the plastic elongation produced during extension when P equal yielding force. The model of strut behaviour (Karczewski and Barszcz 1995), in used

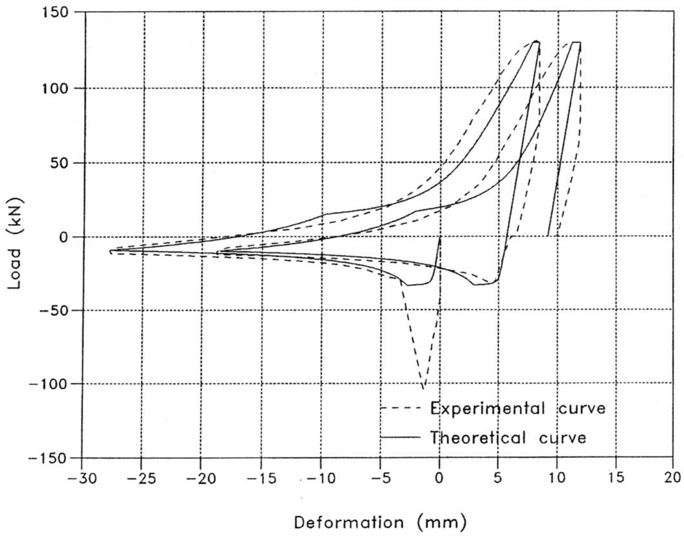

Fig. 2 Model of the strut behaviour

method is shown in Fig. 2.

The computer program TRS was worked out by Krzysztof Donten, (Karczewski and Donten 2001), to make possible practical utilisation above mentioned plastic analysis method. The general flow chart of the program and its plastic procedure are shown at Figs. 3 and 4. The results of analysis can be presented numerically or graphically on the computer monitor. The results can be printed in both cases. The computer program TRS was worked out for environment Windows 9x/NT. In original version program was written and tested on computer P333 with graphic cart Matrox-Millenium II/16MB and operational memory 128MB. For example computer program TRS was employed in analysis of the structure composed with 4802 nodes and 23425 struts. In version used in investigation the computer program TRS was limited to 1000 nodes and 4000 struts.

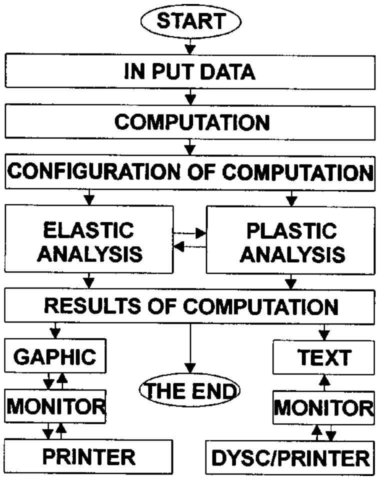

Fig. 3 The general flow chart of the program

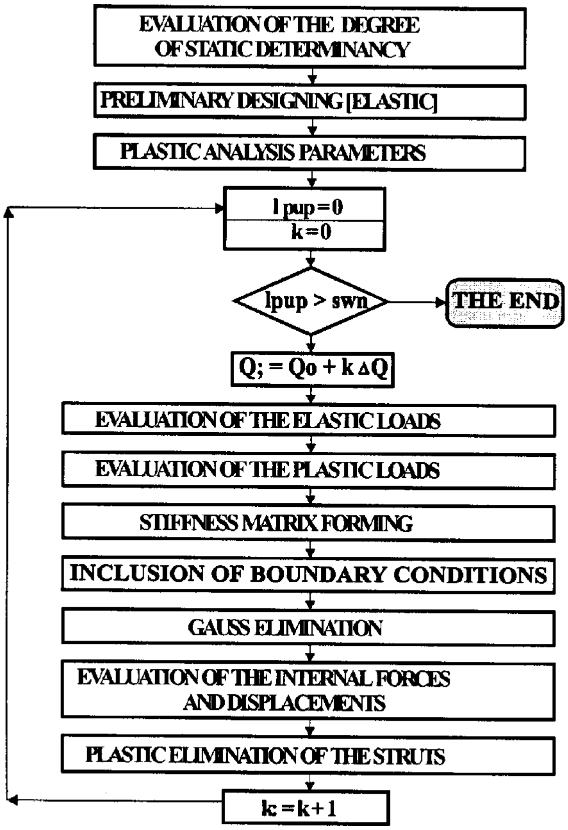

Fig. 4 The flow chart of the plastic procedure implemented in the computer program

2. The basic assumptions of the optimization problem

In optimisation problem was assumed, that the objective functions vector is composed with one element Eq. (3), (Karczewski 1997). The problem can be solved by employing single criterion optimisation, (Karczewski and Paczkowski 1989). The load carrying increment or factor showing relation between increments of load carrying capacity and mass of the all struts in the structure was a criterion of the optimisation, Eq. (3). The optimum of the objective function is univocal with this maximum value.

$$F(\boldsymbol{x}) = [f_1(\boldsymbol{x})]^T \quad \text{or} \quad F(\boldsymbol{x}) = [\, f_1(\beta)]^T \tag{3}$$

where: $f_1(x)$; $f_1\,(\beta)$ - increment of the load carrying capacity or above mentioned factor, adequately.

As a decisive variables was assumed the values qualifying the chosen from fictitiously eliminated struts cross-section increment during their strengthening - **l**, Eq. (4) and values qualifying struts distribution in the structure - $\boldsymbol{p}$, Eq. (5).

$$\mathbf{l} = \boldsymbol{a}, \boldsymbol{b}, \boldsymbol{c}, \boldsymbol{d} \tag{4}$$

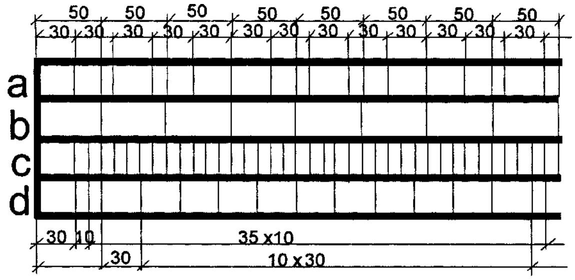

Fig. 5 Decisive variable **l** - increment of the struts cross-sections [%]

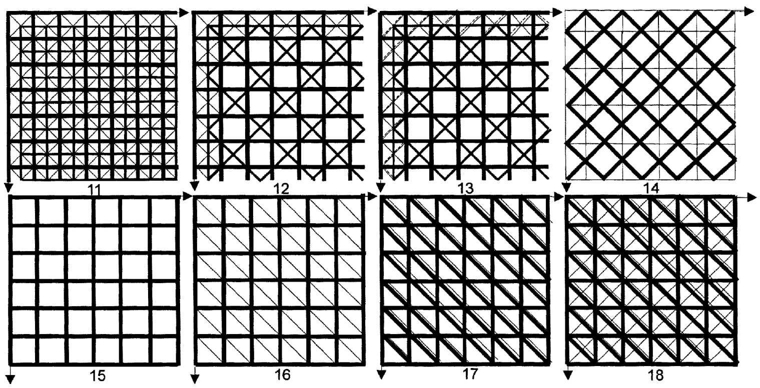

Fig. 6 Decisive variable ***p*** - distribution struts in the structures analysed

The values of the decisive variable **l** - ***a, b, c*** and ***d*** are shown at Fig. 5.

$$\boldsymbol{p} = \mathbf{11, 12, 13, 14, 15, 16, 17, 18} \tag{5}$$

The values of the decisive variable ***p*** - **11, 12, 13, 14, 15, 16, 17** and **18** are shown in Fig. 6. The decisive variables vector was assumed in form

$$\boldsymbol{V} = [\mathbf{l}, \boldsymbol{p}]^T \tag{6}$$

The all variant review method was employed in searching for optimal variant of the structure because the feasible domain was composed only with 32 variants, (Karczewski and Paczkowski 1989).

3. The structures being a subject of optimization

The spatial grid structures used as a roof of sport hall of dimensions 39×39 m in plane is an object of optimisation. 8 kinds of structures were analysed. Exemplary structure No. 11 is shown at Fig. 7. The roof is supported by stanchions on the circumference of the upper layer. The orthogonal spatial grid structure is composed with struts of ring cross-sections made with steel grade 18G2A with mechanical characteristic recommended by (Karczewski and Others 1976), Polish Cod of Practice PN-90/B-03200,

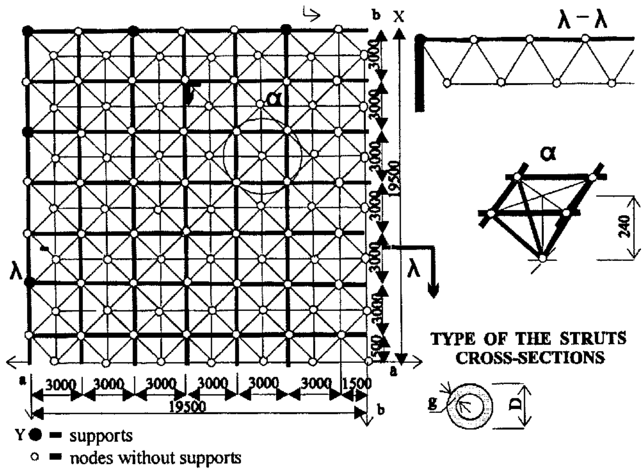

Fig. 7 Exemplary analysed structure No. 11

f_y = 355 MPa (for elements of the thickness up to 16 mm). System strut-node, with nodes type OktaS, was assumed. The mass of the nodes make 10% of the mass of whole structure. The load adopted is typical for the roof structure made of spatial grid and climatic conditions are characteristic for Central Poland. The forces acting on the middle nodes of the upper layer are equal 18 kN and on the middle nodes of the lower layer - 9 kN.

4. Plastic computation and analysis results obtained

Optimisation was performed for 32 variants of the structure which are in the feasible domain. The computation was realised according to the principles described in the foregoing points. The values of the additional parameters necessary for plastic analysis were assumed as follows: number of the increasing loading-1, increment of the loading-1% maximum load for elastic phase, rigour of elastic line iteration-0,1 [%], primary plastic deflection-0,01 [%L], critical displacement 100 [cm], (Karczewski 1997).

First the elastic designing was performed with regard to the recommendations of the Polish Steelwork Code of Practice. Next the incremental plastic analysis of the structure designed initially in elastic phase was worked out. Of course in each stage of analysis the compliance of the effective maximum displacement (deflection) obtained from the analysis was checked accounting the requirements of the serviceability limit state. The computation starts from the load level corresponding to the yield load for the structure analysed. The analysis was carried on by an incremental "step by step" method, (Karczewski Barszcz and Donten 1996). The load was increased by 1% of the yield load in every subsequent steps. Incremental analysis was continued up to the moment when the ultimate limit state due to a global or a local mechanism in the structure has occurred. After strengthening of the cross-sections of the chosen struts the plastic analysis was performed again for next stage from the initial load level-yield load, but for the structure representing greater mass resulting from changes in cross-section chosen fictitiously eliminated struts. The analysis was ended when structure is failed due to the global mechanism. The exemplary result of the plastic analysis performed for all variants of the structure

Table 1 Correction of fictitiously eliminating struts cross-sections effect (1/4 of the structure) variant 11_a

The stage of analysis	Increment of the cross-sections %	Struts with strengthening cross-sections	Increment of load bearing capacity $v/v_s \cdot 100\%$	Increment of the mass of all struts in the structure $(\delta / \delta_s)100\%$	The factor of the load bearing capacity $\beta = v/\delta$
1	30	20	120	100	1,20
2	30	54	126	102	1,15
3	30	182	129	109,71	The global mechanism

v, δ - actual load bearing capacity and mass of the all struts in the structure, adequately
$\boldsymbol{v_s}$, δ_s - the bearing capacity and mass of all struts in the structure in first stage in the analysis, adequately
β - the factor of the load bearing capacity increment

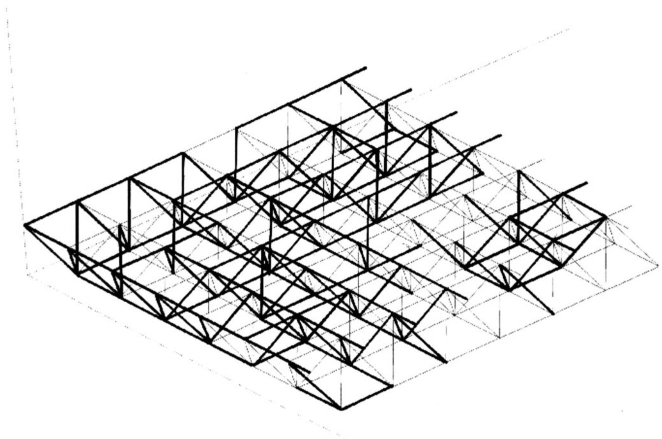

Fig. 8 The plastified or broken struts in the last stage of analysis - variant 11_a

enclosed in the feasible domain defined by decisive variables **l** and ***p*** are shown in Table 1 and Fig. 8, Fig. 9, Fig. 10. After the stiffness adjustment to the structures analysed to the optimal conditions the visible increment of the load bearing capacity, in relation to load bearing capacity of the structure without correction resulting with struts distributions and value of the assumed increment in strengthening of the chosen struts was attained.

The exemplary results of the plastic analysis performed for all variants of the structures with different decisive variables are collected in Table 1 and shown in Fig. 8. In example the decisive variables were equal: **l** = 11 and ***p*** = *a*.

After the stiffness adjustment of the structure analysed to the optimal conditions the visible increment of the load bearing capacity of the structure with some struts distribution, in relation to the load bearing capacity of the structure with different struts distribution was attain. This increment, known as plastic reserve of the load bearing capacity, among others depends strongly upon assumed distribution of the struts in the structure has occurred. One can noticed that increasing the load bearing capacity by increasing the cross-sections of chosen fictitiously eliminated struts, simultaneously the mass of the all struts in structure increase too. It lead to phenomena that occur variants no optimal with have lower mass and higher load carrying capacity then optimal ones, Fig. 9, Fig. 10 and Table 2, Table 3. It seems, that is better, for the presentations of the real profits resulting from the optimal analysis as well as influence variability of the chosen values assumed during performance of the calculation, to introduce

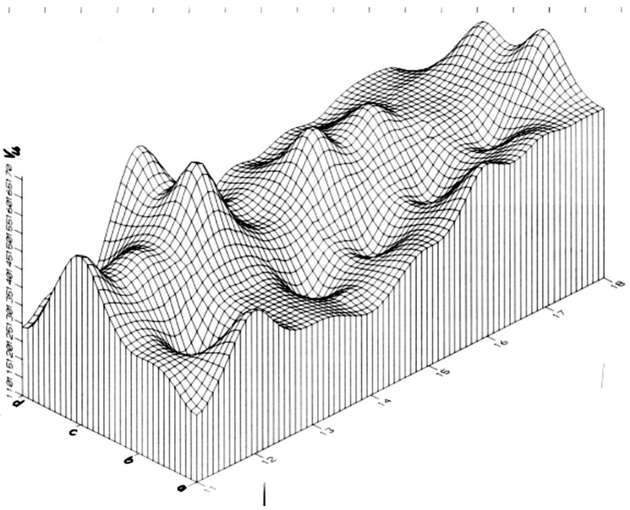

Fig. 9 Low boundary of the load bearing capacity - the spatial view of optimal analysis

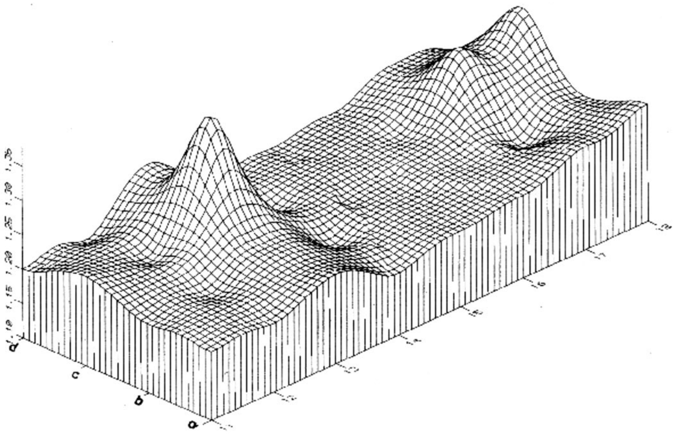

Fig. 10 The factor of load bcaring capacity - the spatial view of optimal analysis

the factor $\beta = \upsilon / \delta$ showing relation between actual load bearing capacity and the actual mass of the all struts in the structure.

For analysed structures with assumed eight different struts distributions and four options of increment of cross-sections during strengthening chosen struts was obtained different factor β, see Table 3. The increment vary from 19% to 38% of the yield load for the above mentioned structures. It does mean, that influence, among others, distribution of the struts in the structure is significant. From output data has obtained during computation result, that load carrying capacity tested variants of the structure achieve maximal value for the variant 18c and is equal 171%, Fig. 11. and Table 2 of the yield load of mentioned variant of the structure. The minimal values of the load bearing capacity was achieved for variant 12b and is equal 123% of the yield load of mentioned variant. This observations are confirmed by data collected at Table 2 and shown in the Fig. 10. The factor of load bearing capacity increment has reached maximal value for the variant 13c of the structure and is equal β=1.38. The minimal value of

Table 2 Low Boundary of load carrying capacity considered variants of structure

Stage of the analysis	The strut distribution in the structure	Low boundary of load carrying capacity variants analysed			
		The chosen fictitiously eliminated struts cross-sections increment			
		a	b	c	d
1	11	129	137	157	129
2	12	150	123	150	127
3	13	138	138	167	163
4	14	133	127	142	136
5	15	143	132	160	143
6	16	158	139	158	145
7	17	158	146	152	150
8	18	157	171	165	140

Table 3 The factor of load bearing capacity increment in analysed variants - β

Stage of the analysis	The struts distribution in the structure	The factor of load bearing capacity increment -β, for variants analysed			
		The chosen fictitiously eliminated struts cross-sections increment			
		a	b	c	d
1	11	1,20	1,20	1,23	1,20
2	12	1,20	1,20	1,21	1,20
3	13	1,23	1,23	1,38	1,27
4	14	1,19	1,19	1,20	1,19
5	15	1,21	1,21	1,22	1,21
6	16	1,22	1,22	1,22	1,22
7	17	1,25	1,21	1,31	1,25
8	18	1,27	1,24	1,33	1,23

$\beta = \vartheta/\delta$ where: ϑ - low boundary of load bearing capacity, δ - the mass of all struts in the structure
- maximal value of the factor of load bearing capacity increament
- minimal value of the factor of load bearing capacity increament

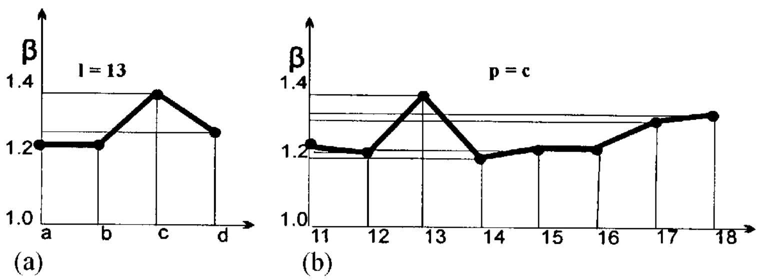

Fig. 11 The factor of load bearing capacity increment for all variants and stable value l = *b* and for all variants and stable value *p* = 11

this factor has reached, in this case, for variants 14a, 14b, 14d and is equal β =1.19. From pattern shown in Fig. 10 results difference between minimal and maximal values equal) $\Delta\beta$ = 0.19.

5. Conclusions

The proposed in paper optimal analysis of elastic-plastic spatial grid structures seem to be enough safety and accuracy for practical designing. The used method of analysis and computer program work out for practical its utilising occurred easy in solving difficult problems.

The results of optimal analysis performed for 32 variants of exemplary structures with assumed different options: of the struts distributions in the structure and size of the cross-sections increment being indicate the appreciable dependence of the maximum load bearing capacity of the structure on above mentioned values. In former investigations the greatest differences was observed if cross-sections of all plasticised struts in subsequent stages of analysis are strengthened. Therefore in presented investigation was assumed that in all stages of analysis all plasticised struts were strengthened.

From optimal analysis result that greatest values of factor β were for the size of cross-section increment equal 30% in first stage and 10% in every subsequent - see Fig. 10. Approximately one can assume the value of struts distribution in the structure influence on the load bearing capacity is significant. For optimal variant of the structure the factor β increase by 38%, see Fig. 11(b). If one take into consideration influence additional value, option of increment of the cross-sections in subsequent stages the factor β increase by 19%.

The results obtained by employing the proposed incremental analysis method confirm observable influence of the assumed values, especially struts distribution in the structure, on finally obtain load bearing capacity. It does mean that during searching for optimal solution, in case of the spatial grid structure, one has taken into consideration struts distribution in the structure. Also it make proper further investigations of the considered problem. First of all, research anticipated should pay attention to find additional values have significant influence on finally load bearing capacity. This values which must be taken into consideration during designing the optimal plastic spatial grid structures.

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