Steel and Composite Structures, *Vol. 29, No. 2 (2018) 243-256*
DOI: https://doi.org/10.12989/scs.2018.29.2.243

Computational Lagrangian Multiplier Method using optimization and sensitivity analysis of rectangular reinforced concrete beams

Mehran Shariat [1], Mahdi Shariati [*2], Amirhossein Madadi [**1] and Karzan Wakil [3,4]

[1] *Department of Civil Engineering, Hakim Sabzevari University, Sabzevar, Iran*
[2] *Faculty of Civil Engineering, University of Tabriz, Tabriz, Iran*
[3] *Information Technology Department, Technical College of Informatics, Sulaimani Polytechnic University, Iraq*
[4] *University of Human Development, Iraq*

(*Received June 12, 2018, Revised August 15, 2018, Accepted September 4, 2018*)

Abstract. This study conducts an optimization and sensitivity analysis on rectangular reinforced concrete (RC) beam using computational Lagrangian Multiplier Method (LMM) as programming optimization computer software. The analysis is conducted to obtain the minimum design cost for both singly and doubly RC beams according to the specifications of three regulations of American concrete institute (ACI), British regulation (BS), and Iranian concrete regulation (ICS). Moreover, a sensitivity analysis on cost is performed with respect to the effective parameters such as length, width, and depth of beam, and area of reinforcement. Accordingly, various curves are developed to be feasibly utilized in design of RC beams. Numerical examples are also represented to better illustrate the design steps. The results indicate that instead of complex optimization relationships, the LMM can be used to minimize the cost of singly and doubly reinforced beams with different boundary conditions. The results of the sensitivity analysis on LMM indicate that each regulation can provide the most optimal values at specific situations. Therefore, using the graphs proposed for different design conditions can effectively help the designer (without necessity of primary optimization knowledge) choose the best regulation and values of design parameters.

Keywords: optimum design; Lagrangian Multiplier Method; sensitivity analysis; reinforced concrete beam

1. Introduction

Optimization is widely used in civil and mechanical engineering, particularly in reinforced concrete (RC) structures (Aghaee *et al*. 2014, Fanaie *et al*. 2016, Madadi *et al*. 2018, Nasrollahi *et al*. 2018, Paknahad *et al*. 2018). RC optimization is influenced by structural criteria to estimate the structural capability against external forces with the goal of minimizing manufacturing and construction costs. To date, optimization process in the field of reinforced concrete beams has been widely performed using various types of methods such as metaheuristic optimizers (Shariati *et al*. 2010, Fanaie *et al*. 2012, Toghroli *et al*. 2014, Awal *et al*. 2015, Kaveh and Shokohi 2015, Safa *et al*. 2016b, Shah *et al*. 2016, Korouzhdeh *et al*. 2017, Heydari and Shariati 2018), principal stress lines (Li and Chen 2010), strut and tie modelling (Chakrabarty 1992, Chetchotisak *et al*. 2014, Chae and Yun 2015, Long and Lee 2015, Ardalan *et al*. 2017, Farzampour 2017, Joshaghani *et al*. 2017, Bahadori and Ghassemieh 2016), geometric design optimization (Sharafi *et al*. 2012), and artificial neural networks (ANNs) (Toghroli *et al*. 2014, Mohammadhassani *et al*. 2015, Mansouri *et al*. 2016, Safa *et al*. 2016a, b, Toghroli *et al*. 2016, Karamshahi *et al*. 2017, Khorramian *et al*. 2017, Mansouri *et al*. 2017, Zandi and Toghroli 2018).

Furthermore, empirical (El Debs *et al*. 2005) and analytical approaches (Balaguru 1980, Barros *et al*. 2005, Nigdeli and Bekdas 2013, Bazzaz *et al*. 2014, Eskandari and Madadi 2015, Khorami *et al*. 2017a, b, Khorramian *et al*. 2017, Andalib *et al*. 2018, Shariat *et al*. 2018a, Wei *et al*. 2018) has been used to obtain the optimal resistance, mass and cost of the beam with simple supports and cantilever beams. For instance, Balaguru (1980) developed an algorithm to calculate the optimum dimensions and the amount of reinforcement for a doubly reinforced rectangular beam based on the ultimate strength design using rectangular stress blocks for concrete to determine whether the use of a doubly reinforced section is more economical than a singly reinforced section for the same ultimate load capacity (Ozturk *et al*. 2012, Rahmanian *et al*. 2014). However, it can be noticed that the optimization of RC beams should be considered as a nonlinear problem, where the presence of discrete and integer variables along with continuous variables increases the complexity of the optimization problem (Huedo *et al*. 2005). Therefore, efficient methods are better to be considered to solve this type of problems so that more accurate results can be anticipated. Lagrangian multiplier method (LMM) is revealed to be a capable approach in such engineering optimization problems (Fortin and Glowinski 2000, Bertsekas 2014). For instance, Barros *et al*. (2005) presented a model for the optimal design of RC beams by

∗Corresponding author, Ph.D.,
E-mail: shariati@tabrizu.ac.ir

∗∗ M.Sc., E-mail: madadhsu@yahoo.com

http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

considering the stress–strain diagrams using Lagrangian multiplier method (LMM) and compared the results with other optimization models. They also developed the economic bending moment, the optimal area of steel and the optimal steel ratio between upper and lower steel for four different classes of concrete.

Although there are different studies conducted on application of LMM in evaluation of engineering problems, there is still a significant need for effectively utilization of this method in investigation of RC beams in order to optimize various parameters affecting the design and cost of these structures. Sensitivity analysis is also established as an essential work to facilitate the decision making process in engineering problems (Ceranic and Fryer 2000, Shariat *et al*. 2018b), especially where different design regulations are available. Therefore, this study aims to perform an optimization and sensitivity analysis on the rectangular RC beams. Specifically, the minimum cost of design for both singly and doubly RC beams is obtained using LMM regarding to the specifications of three applicable regulations as ACI 318-14 (ACI 2014), British standard 8110 (Standard 1985), and Iranian concrete regulation (ICS) (Tahouni 2005). In addition, a sensitivity analysis is conducted on the effective parameters such as length, width, and depth of RC beam, and area of reinforcement (Panjehpour *et al*. 2014). Further, various curves are developed that can be feasibly utilized in design of RC beams. Finally, numerical examples are represented according to the three regulations to better clarify the design steps.

2. LMM principles based on ACI, BS, and ICS regulations

The LLM is a numerical method which optimizes a multivariate formula under defined constraints. The area of LMM for constrained optimization confronted with a substantial transformation when the augmented Lagrangian functions and methods of multipliers were introduced by Hestenes and Powell (Bertsekas 2014). Later, various efforts were done for better understanding and improving the LMM properties. These efforts were further accompanied by the aids of fresh ideas based on exact penalty functions, resulting in a variety of fascinating methods utilizing Lagrange multiplier iterations to solve different optimization problems.

In the LMM optimization, a multivariate objective function expressed as

$$y = f(x_1, x_2, \dots, x_n) \quad (1)$$

Subjected to equality constraints of the form

$$g_i(x_1, x_2, \dots., x_m) = 0 \;\; i = 1,2, \dots, m \quad (2)$$

Where n is the number of independent variables and m is the number of constraints; m must be less than n based on the problem. The following procedure can be applied to construct the unconstrained Lagrangian function L of the form

$$L(x_1, x_2, \dots, x_n, \lambda_1, \lambda_2, \dots, \lambda_m) = f(x_1, x_2, \dots, x_n) + \sum_{i=1}^{m} \lambda_i g_i(x_1, x_2, \dots, x_n) \quad (3)$$

In order to solve the optimization issue, by taking the derivative of the function with respect to x and λ, the optimal values of x and λ are achieved

$$\frac{dL}{dx_k} = \frac{df}{dx_k} + \sum_{i=1}^{m} \lambda_i \frac{dg_i}{dx_k} = 0, \quad (4)$$

$$k = 1,2, \dots, n. \quad (5)$$

And

$$\frac{dL}{dx_k} = g_i = 0\,, \quad (6)$$

$$i = 1,2, \dots, m. \quad (7)$$

Eqs. (8) and (9) are cost functions that need to be minimized:

for a singly reinforced concrete beam

$$L = C_c.b.d(1 + r) + C_s.\rho.b.d \quad (8)$$

for a doubly reinforced concrete beam

$$L = C_c.b.d(1 + r) + C_s.(\rho + \rho').b.d \quad (9)$$

Table 1 Ultimate design moment for all codes

Code	Beam	Ultimate design moment	Formula number
BS		$M = 0.87bd^2 f_y \rho(1 - 0.98\frac{f_y}{f_c}\rho)$ (Standard 1985)	(12)
ACI	Singly	$M = A_s f_y (d - \frac{A_s f_y}{1.7 f_c b})$ (ACI 2014)	(13)
ICS		$M = A_s f_y (d - \frac{A_s f_y}{1.02 f_c b})$ (Tahouni 2005)	(14)
BS		$M = 0.156bd^2 f_c + bd^2 f_c(0.87\frac{f_y}{f_c}\rho - 0.2)(1 - r)$ (Standard 1985)	(15)
ACI	Doubly	$M = (A_{s_{double}} - A'_s) f_y (d - \frac{\rho f_y}{1.7 f_c} d) + A'_s f_y d(1 - r)$ (ACI 2014)	(16)
ICS		$M = (A_{s_{double}} - A'_s) f_y (d - \frac{\rho f_y}{1.02 f_c} d) + A'_s f_y d(1 - r)$(Tahouni 2005)	(17)

where Cs and Cc are the costs of steel and concrete per unit volume, ρ and ρ' are the tension and compression reinforcement ratios, b and d are the width and effective depth of the section, respectively, and r is the ratio of the reinforcement cover to the effective depth. Reinforced concrete beams of a rectangular section are primarily designed to resist the action of flexural bending, and both singly and doubly reinforced beams are classified in BS (Standard 1985), ACI (ACI 2014), and ICS (Tahouni 2005). To satisfy the desired flexural capacity, the functions of Eqs. (11) to (17), which are presented in Table 1, are considered as constraints on the main objective function.

For all codes

$$\rho_{min} \leq \rho \leq \rho_{max} \tag{10}$$

$$q = C_s/C_c \tag{11}$$

Table 2 Optimum reinforcement ratio, depth, and cost for BS code

Code	Beam	Optimum conclusion	Formula number
BS	Singly	$\rho_{opt} = \dfrac{1}{\frac{q}{1+r} + 1.96\frac{f_y}{f_c}}$	(22)
		$d_{opt} = \sqrt{\dfrac{M}{0.87\,\rho_{opt}\,b(1 - 0.98\rho_{opt}\frac{f_y}{f_c})}}$	(23)
		$Cost_{opt} = \left(qbd_{opt}\,\rho_{opt} + bd_{opt}\,(1 + r)\right) C_c L$	(24)
	Doubly	$\rho_{opt} = 0.3445\dfrac{f_c}{f_y} - 0.3585\dfrac{f_c}{f_y}\dfrac{1}{1 - r} + \dfrac{1 + r}{2q}$	(25)
		$d_{opt} = \sqrt{\dfrac{M}{f_c\ b(0.156 + \left(0.87\rho_{opt}\frac{f_y}{f_c} - 0.2\right)(1 - r)}}$	(26)
		$Cost_{opt} = \left(qbd_{opt}\left(2\rho_{opt} - \rho_u\right) + bd_{opt}\,(1 + r)\right) C_c L$	(27)
		$\rho_u = 0.2314\dfrac{f_c}{f_y}$	(28)

Table 3 Optimum reinforcement ratio, depth, and cost for ACI code

Code	Beam	Optimum solution	Formula number
ACI	Singly	$\rho_{opt} = \dfrac{1}{\frac{q}{1+r} + \frac{f_y}{0.85f_c}}$	(29)
		$d_{opt} = \sqrt{\dfrac{M}{\rho_{opt}\,f_y b(1 - \rho_{opt}\frac{f_y}{1.7f_c})}}$	(30)
		$Cost_{opt} = \left(qbd_{opt}\,\rho_{opt} + bd_{opt}\,(1 + r)\right) C_c L$	(31)
	Doubly	$d_{opt} = \sqrt{\dfrac{M}{f_c\ b(\rho_u\left(1 - \rho_{opt}\frac{f_y}{1.7f_c}\right) + \rho'_{opt}\,(1 - r)}}$	(32)
		$\rho'_{opt} = \dfrac{\rho_u q\left(\frac{\rho_u f_y}{0.425f_c} - (3 + r)\right) + (1 - r^2)}{2q(1 - r)}$	(33)
		$Cost_{opt} = \left(qbd_{opt}\left(\rho_u + 2\rho'_{opt}\right) + bd_{opt}\,(1 + r)\right) C_c \times L,$	(34)
		$\rho_u = 0.27\dfrac{f_c}{f_y}$	(35)

3. Brief LMM formula for ACI, BS and ICS regulations

According to the LMM, deriving the objective function represents the optimum reinforcement ratio and the optimum effective depth of RC beam on the basis of BS (Standard 1985), ACI (ACI 2014), and ICS (Tahouni 2005) regulations, which are presented in Tables 2, 3, and 4, respectively. For example, the procedure of calculation of LMM formula according to the ACI regulation for singly and doubly reinforcement beams is given below

$$L(\rho, R, \lambda) = [\rho q + (1+t)]R - \lambda\left[\rho f_y \left(1 - \frac{\rho f_y}{1.7\, f_c}\right) R^2 - 1\right] \tag{18}$$

$$L(\rho, R, \lambda) = [(\rho_u + 2\rho')q + (1+t)]R - \lambda\left[\left[\rho\left(1 - \frac{\rho f_y}{1.7\, f_c}\right) + \rho'(1-t)\right]R^2 - 1/f_y\right] \tag{19}$$

$$R = \sqrt{\frac{b}{M}} \tag{20}$$

$$t = \frac{d'}{d} \tag{21}$$

By deriving from Eqs. (18) and (19) with respect to the three independent variables of ρ, R and λ, the optimal values of reinforcement ratio, depth, and cost are obtained, which are represented as Eqs. (29) to (42).

By comparing the optimized solutions of singly and doubly reinforced beams for different values of the ratio of steel to concrete tension, we can identify the zones for a particular solution that give least-cost offers. By applying the boundary conditions of the reinforcement ratio and the equations obtained for the optimal reinforcement ratio for the three regulations, Eqs. (43) to (48) are defined toachieve a border point of singly and doubly reinforced beams with boundary conditions in accordance with the fixed data presented in Table 5. Over the defined range of the stress ratios (f_y/f_c) in Table 5, three distinct zones of optimum reinforcement ratios for all codes as well as the boundaries between these zones are depended on the values of the ratios q and r.

Fig. 1 shows a graphical representation of these zones with q = 25 and r = 0.15. Zone 1 corresponds to a singly

Table 5 Optimum boundary of beam

Code	Beam	Optimum solution	Formula number
BS	SRO	$\frac{f_y}{f_c} = 0.422\frac{q}{1+r}$	(43)
ACI	SRO	$\frac{f_y}{f_c} = 0.396\frac{q}{1+r}$	(44)
ICS	SRO	$\frac{f_y}{f_c} = 0.57\frac{q}{1+r}$	(45)
BS	DRO	$\frac{f_y}{f_c} = 2\left(\frac{0.3585}{1-r} - 0.1135\right)\frac{q}{1+r}$	(46)
ACI	DRO	$\frac{f_y}{f_c} = 0.27\frac{q(2.37+r)}{1-r^2}$	(47)
ICS	DRO	$\frac{f_y}{f_c} = 0.27\frac{q(1.94+r)}{1-r^2}$	(48)

Table 4 Optimum reinforcement ratio, depth, and cost for ICS code

Code	Beam	Optimum solution	Formula number
ICS	Singly	$\rho_{opt} = \frac{1}{\frac{q}{1+r} + \frac{f_y}{0.51 f_c}}$	(36)
ICS	Singly	$d_{opt} = \sqrt{\frac{M}{\rho_{opt} f_y b(1 - \rho_{opt}\frac{f_y}{1.02 f_c})}}$	(37)
ICS	Singly	$Cost_{opt} = \left(qbd_{opt}\rho_{opt} + bd_{opt}(1+r)\right)C_c L$	(38)
ICS	Doubly	$d_{opt} = \sqrt{\frac{M}{f_c\, b(\rho_u\left(1 - \rho_{opt}\frac{f_y}{1.02 f_c}\right) + \rho'_{opt}(1-r)}}$	(39)
ICS	Doubly	$\rho'_{opt} = \frac{\rho_u q\left(\frac{\rho_u f_y}{0.255 f_c} - (3+r)\right) + (1-r^2)}{2q(1-r)}$	(40)
ICS	Doubly	$Cost_{opt} = \left(qbd_{opt}(\rho_u + 2\rho'_{opt}) + bd_{opt}(1+r)\right)C_c L,$	(41)
ICS	Doubly	$\rho_u = 0.27\frac{f_c}{f_y}$	(42)

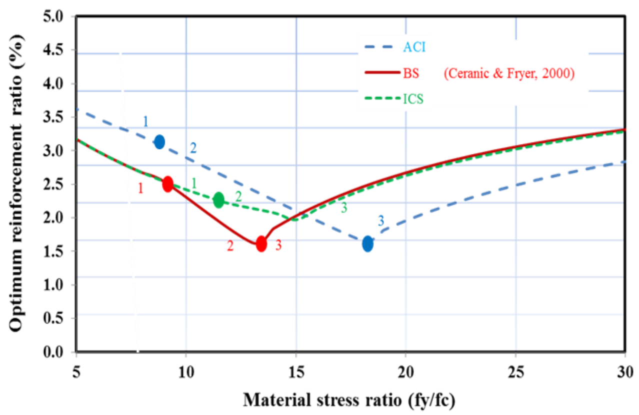

Fig. 1 Optimum reinforcement ratio for q = 25 and r = 0.15

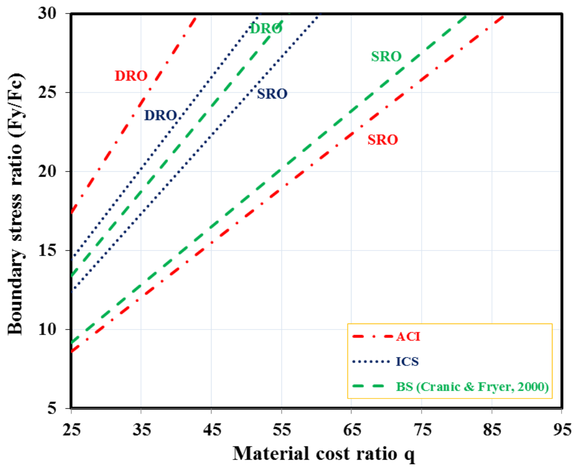

Fig. 2 Optimum solution versus q for r = 0.15

Table 6 Optimum zone points

Code	Beam	Optimum solution
BS		$\frac{f_y}{f_c} = 0.422\frac{25}{1+0.15} = 9.17$
ACI	SRO	$\frac{f_y}{f_c} = 0.396\frac{25}{1+0.15} = 8.6$
ICS		$\frac{f_y}{f_c} = 0.57\frac{25}{1+0.15} = 12.39$
BS		$\frac{f_y}{f_c} = 2\left(\frac{0.3585}{1-r} - 0.1135\right)\frac{25}{1+r} = 13.4$
ACI	DRO	$\frac{f_y}{f_c} = 0.27\frac{25\times(2.37+r)}{1-r^2} = 17.4$
ICS		$\frac{f_y}{f_c} = 0.27\frac{25\times(1.94+r)}{1-r^2} = 14.43$

reinforced section with the ratio of f_y/f_c between its lower bound value of 5 and the point of intersection of the boundary curve at 9.17, 8.6, and 12.39 for the BS, ACI, and ICS regulations, respectively.

Zone 2 corresponds to a singly reinforced section with its optimum reinforcement ratio being set at the boundary value ρ_b for the range of f_y/f_c from 9.17 to 12.39, 8.6 to 13.4, and 12.39 to 26.17 for the BS, ACI, and ICS regulations, respectively. Zone 3 corresponds to a doubly reinforced section with the ratio of f_y/f_c between the point of intersection of the boundary curve at 13.4, 29.07, and 26.17 to the upper bound value of 25, 35, and 45 for the BS, ACI, and ICS regulations, respectively. The calculations of the optimum solutions are presented in Table 6.

According to Fig. 1, by increasing the f_y/f_c ratio in singly reinforced beams (zones 1 and 2), the optimum ratio of reinforcements is decreased, but in doubly reinforced beams (zone 3), the curves have an increasing trend by increasing the f_y/f_c ratio. Similar results are also reported in the study of Ceranic and Fryer (2000) who indicated that in singly reinforced beams an increase in the material cost ratio leads to a corresponding reduction in the reinforcement ratio. This reduction, however, can be compensated by an increase in the effective depth of the section while subjecting to a similar loading. Moreover, it can be observed that at zones 1 and 3, the BS and ICS regulations reveal similar behaviour, while at zone 2, the BS regulation provides lower optimum ratio of reinforcement. On the other hand, the ACI regulation is accompanied by the lowest optimum ratio of reinforcement at zone 3. Therefore, it can be concluded that the BS and ICS regulations can be better for the singly reinforced design than the ACI regulation, while the ACI regulation is better for the doubly reinforced design. Also, for singly reinforced design with boundary conditions, the BS regulation can be the best regulation, resulting in lower cost of the RC beam. The reason of these facts can be associated with a large constant coefficient of the BS and ICS regulations. The problem will be arised when the safety of the BS and ICS regulations at a constant f_y/f_c ratio for singly and doubly reinforced beam design is greater and lower than that of the ACI regulation, respectively. This is in agreement with results previous studies (Ceranic and Fryer 2000, Barros *et al*. 2005).

The effect of varying the ratio of steel cost to concrete cost (q) on the stress ratio for all regulations for the singly and doubly reinforced concrete beams is depicted in Fig. 2, which can suggest an optimal solution and assist the designer. If the designer knows the price ratio of steel to concrete, parameter r, and the stress ratio, the type of beam (singly or doubly reinforced) that should be used can be determined under the minimum cost conditions. In this plot, these limits are shown for the BS, ACI, and ICS regulations. The above zone of these limits is the optimal section of a doubly reinforced section, the zone below this limit is the optimal section of a singly reinforced section, and within these limits, the optimized section of a singly reinforced section or boundary reinforcement can be found.

According to Fig. 2, for material stress ratios beyond 30, which generally happens in Iran, the singly reinforced beam design is preferred, but because of the increases of depth of beam and doubts about the architectural design, it is worthwhile to find the percentage of increase in the cost of using double reinforced beams, which are used to increase the maximum bending capacity of the RC beam.

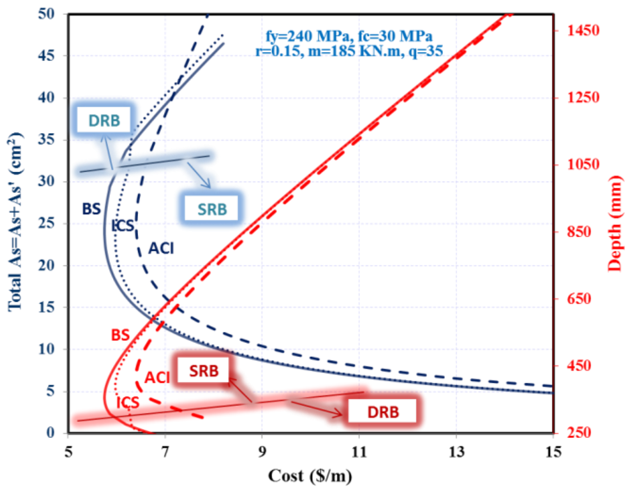

Fig. 3 Analysis of depth and reinforcement area versus cost

4. Verifying LMM and standard methods

To verify the Lagrange solution by the above assumption, which always happens for the singly reinforced beam, it is better to compare regulation designs. An example is analyzed and compared, illustrating situations where the optimum solution is either a singly or doubly reinforced section. A rectangular beam with the width b = 300 mm is subjected to the maximum bending moment of 185 kN·m. The material cost ratio q is set at 35, which depends on the costs of concrete (C_c) and steel (C_s) as 30 \$/m^3 and 1050 \$/m^3; also, the characteristic strengths of steel and concrete are 240 and 30 N/mm^2, respectively. An analysis of depth and reinforcement area versus cost is illustrated in Fig. 3.

It can be observed that the optimum reinforced area as well as the minimum cost of all regulations for the singly reinforced beams and that, by reducing the depth of the beam and increasing the reinforcement area, the cost of the beam is reduced. Then, after entering into the doubly reinforced beam condition, the beam costs increase. In the other words, for any cost, there are two areas and depths as well, which are the optimum conditions. For example, for the cost of 7.03 \$ under the ACI regulation there are 333, 594, and 38 and 16 depths (mm) and total reinforced (cm^2) are respectively. To specify the zone of the doubly and singly reinforced beams, Table 7 shows the zone areas as well as the optimum depths. By this analysis, it is clear that those points are belonging to the singly reinforced beam. This means that, in order to choose the best economical section under a certain moment and initial data of the problem, only the beams with tensile reinforcement should be selected.

To make better use of optimum depth, the cost and area

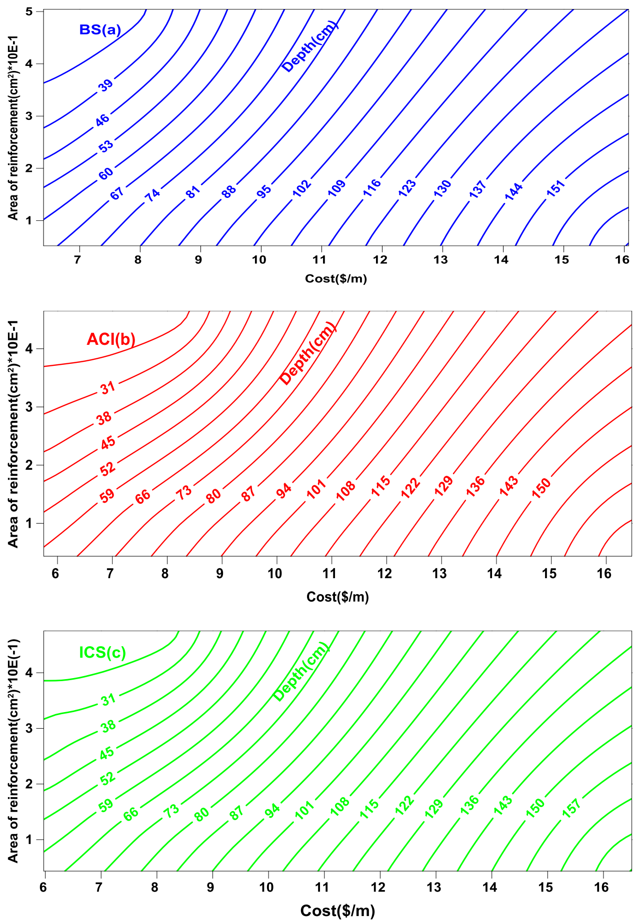

Fig. 4 Contour area of reinforcement versus cost for different depths of beams

Table 7 Solution of boundary reinforced beam

Code	Beam	calculation	Boundary solution
BS	Boundary reinforced beam	$185 \times 10^6 = 0.87 \times 300 \times d^2 \times 240 \times \rho_u (1 - 0.98 \dfrac{240}{25} \rho_u)$ $\rho_u = 0.2314 \dfrac{f_c}{f_y} = 0.0241$	d = 398 mm As = 2878 mm^2
ACI		$185 \times 10^6 = 300 \times \rho_u \times d \times 240 (d - \dfrac{\rho_u \times d \times 240}{1.7 f_c})$ $\rho_u = 0.27 \dfrac{f_c}{f_y} = 0.02813$	d = 347 mm As = 2931 mm^2
ICS		$185 \times 10^6 = 300 \times \rho_u \times d \times 240 (d - \dfrac{\rho_u \times d \times 240}{1.02 f_c})$ $\rho_u = 0.27 \dfrac{f_c}{f_y} = 0.02813$	d = 371 mm As = 3134 mm^2

Table 8 Optimum solution from Lagrange and manual method

Beam	Code	$A_{s_{opt}}$ (mm^2) (LMM)	d_{opt} (mm) (LMM)	$Cost_{opt}$ $\left(\frac{\$}{m}\right)$ (LMM)	$A_{s_{opt}}$ (mm^2)	d_{opt} (mm)	$Cost_{opt}$ $\left(\frac{\$}{m}\right)$
	BS	2635	405	6.41	2622	406	6.08
Single	ACI	2704	359	6.07	2415	357	5.74
	ICS	2448	413	6.29	2235	403	5.97

reinforced for contour of the optimum depth are depicted in Fig. 4. According to this contour, an increase in the cost from 7 \$ to 13 \$ significantly increased the area reinforced. However, at a fixed cost, there are some areas reinforced, which depend on the depth. For example, when the cost is fixed at 8\$ there are five reinforced content, which can be selected by the depths of the beam. In other words, in any design the known depths and total area reinforced can be used to find the optimum cost for different regulations. The LMM and regulation methods are defined above, but for comparison, a typical example is analyzed and compared for different regulations, illustrating situations where the optimum solution is either a singly or doubly reinforced section.

The optimum solution is compared with the regulation design procedure specified in the BS, ACI, and ICS regulations by LMM. The obtained solutions are presented in Table 8. This table shows that the solutions obtained by the LMM are comparable with those of the manual method. This is in agreement with study of Adamu and Karihaloo (1994), who compared the continuum-type optimally criteria (COC) with discretized continuum-type optimally criteria (DCOC) using augmented Lagrangian method to obtain the minimum cost design of RC beams, and the results confirmed the effectiveness of LMM in such optimization problems.

For better understanding the results, numerical examples (based on data released in (Ceranic and Fryer 2000)) are conducted and compared according to specifications of the three regulations. The results have shown that the all-MATLAB code of LMM is correct and can be extended for sensitivity analysis of all regulations as well as various parameters, which have not been done previously.

5. Numerical example

In this section, assuming a constant width (b = 0.26 m), other effective parameters are evaluated and their optimum design values are calculated for a specific bending moment applied to cross section of RC beam. Specifically, significant parameters of structural design of beam like effective depth and area of reinforcement are obtained for singly, boundary and doubly RC beam according to various regulation constraints (Figs. 5-7). Further, in three examples different assumptions were considered to make the findings more obvious, where the optimum solutions for effective depth and area of reinforcement are obtained from the related figures.

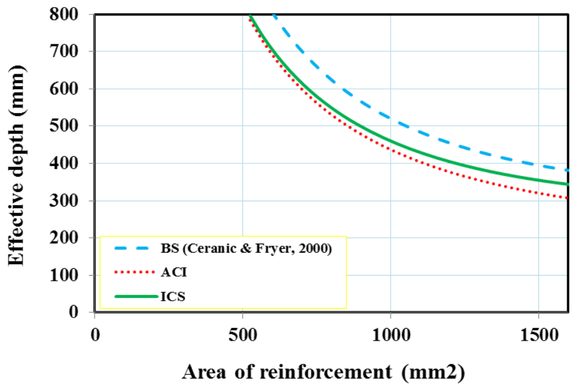

Fig. 5 Singly reinforced optimum solution

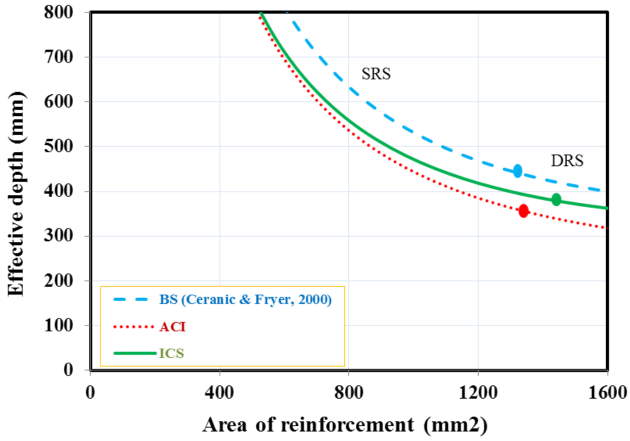

Fig. 6 Boundary reinforced optimum solution

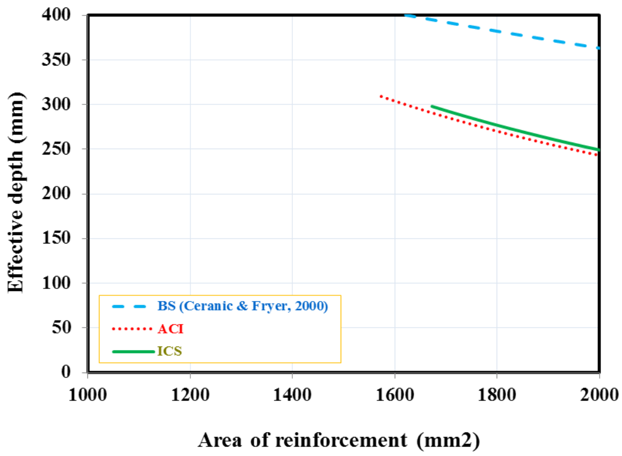

Fig. 7 Doubly reinforced optimum solution

5.1 Design example 1

A beam with a width of b = 260 mm is subjected to the maximum bending moment of 185 kN.m. The ratio r is taken as 0.15, material cost ratio q as 75 and Cc as 50 \$/m^3. Characteristic strength of concrete and steel are 30 and 460 N/mm^2, respectively.

5.1.1 BS solution

From Fig. 2, the optimum solution is shown to be a singly reinforced section (Ceranic and Fryer 2000).

From Eqs. (22), (23), and (24)

$$\rho_{opt} = 0.0105$$

$$d_{opt} = 448 \text{ mm}$$

$$\text{Cost}_{opt} = 0.2256C_c = 11.28 \text{ \$/m}$$

5.1.2 ACI solution

From Fig. 2, the optimum solution is shown to be a singly reinforced section.

From Eqs. (29), (30), and (31)

$$\rho_{opt} = 0.012$$

$$d_{opt} = 380.1 \text{ mm}$$

$$\text{Cost}_{opt} = 0.2025C_c = 10.12 \text{ \$/m}$$

5.1.3 ICS solution

From Fig. 2, the optimum solution is shown to be a singly reinforced section.

From Eqs. (36), (37), and (38)

$$\rho_{opt} = 0.0104$$

$$d_{opt} = 418 \text{ mm}$$

$$\text{Cost}_{opt} = 0.2092C_c = 10.46 \text{ \$/m}$$

5.2 Design example 2

The same design parameter values are used as in the previous example with the following exceptions. The material cost ratio q is 45, fc = 25 N/mm^2. The results are presented in Fig. 6.

Table 9 Optimum concrete and steel cost ratios for BS code

Code	Beam	Material cost ratio	Formula number
BS	Singly	$\frac{C_c}{C_t} = \frac{1}{1 + \frac{q\rho_{opt}}{(1+r)}}$	(51)
		$\frac{C_s}{C_t} = \frac{1}{1 + \frac{1+r}{q\rho_{opt}}}$	(52)
	Doubly	$\frac{C_c}{C_t} = \frac{1}{1 + q\frac{(2\rho_{opt} - 0.231\frac{f_c}{f_y})}{(1+r)}}$	(53)
		$\frac{C_s}{C_t} = \frac{2\rho_{opt} - 0.231\frac{f_c}{f_y}}{2\rho_{opt} - 0.231\frac{f_c}{f_y} + \frac{(1+r)}{q}}$	(54)

Table 10 Optimum concrete and steel cost ratios for ACI code

Code	Beam	Material cost ratio	Formula number
ACI	Singly	$\frac{C_c}{C_t} = \frac{1}{1 + q\rho_{opt}}$	(55)
		$\frac{C_s}{C_t} = \frac{1}{1 + \frac{1}{q\rho_{opt}}}$	(56)
	Doubly	$\frac{C_c}{C_t} = \frac{1}{1 + q\frac{(2\rho_{opt} + 0.27\frac{f_c}{f_y})}{(1+r)}}$	(57)
		$\frac{C_s}{C_t} = \frac{2\rho_{opt} + 0.27\frac{f_c}{f_y}}{2\rho_{opt} + 0.27\frac{f_c}{f_y} + \frac{(1+r)}{q}}$	(58)

Table 11 Optimum concrete and steel cost ratios for ICS code

Code	Beam	Material cost ratio	Formula number
ICS	Singly	$\frac{C_c}{C_t}=\frac{1}{1+q\rho_{opt}}$	(59)
		$\frac{C_s}{C_t}=\frac{1}{1+\frac{1}{q\rho_{opt}}}$	(60)
	Doubly	$\frac{C_c}{C_t}=\frac{1}{1+q\frac{(2\rho_{opt}+0.27\frac{f_c}{f_y})}{(1+r)}}$	(61)
		$\frac{C_s}{C_t}=\frac{2\rho_{opt}+0.27\frac{f_c}{f_y}}{2\rho_{opt}+0.27\frac{f_c}{f_y}+\frac{(1+r)}{q}}$	(62)

5.3 Design example 3

The design parameter values are as those specified in example 1 with the exception that the material ratio q is 25. Fig. 7 indicates that the optimum solution is a doubly reinforced section.
Comparison of the results obtained for these examples with the results of Ceranic and Fryer (2000), it can be stated that the results are close to each other, verifying the model presented in this study for comparison of different regulations.

6. Sensitivity analysis

Sensitivity analysis is a technique which is used to determine how different values of an independent variable will impact a particular dependent variable under a given set of assumptions. This technique is used within specific boundaries that will depend on one or more input variables, such as the effect that changes in interest rates will have on a bond's price. Sensitivity analysis is a way to predict the outcome of a decision if a situation turns out to be different compared to the key prediction(s). The cost factors C_c/C_t and C_s/C_t for optimal solution conditions for unit length are introduced and presented in Tables 9-11 for the three regulations, where C_c, C_s, and C_t are the costs of concrete, reinforcement, and total materials, respectively.

Three distinct zones have to be defined, depending on whether the beam has a singly reinforced, doubly reinforced, or boundary reinforcement ratio as the optimum solution. Because of this, the ratio boundary given by Eq. (28) must be equal to the optimum ratios given by Eqs. (22) and (25) that specify the lower and upper bound values of q for singly and doubly reinforced sections, respectively, for the BS regulation; in a similar way, the results can be obtained for other regulations and are given in Table 12.

The associated curves according to Eqs. (51)-(62) are depicted in Fig. 8. The assumption of this figure is that $f_y = 240$ MPa, $f_c = 30$ MPa, and r = 0.15, but q is varied in ACI and ICS regulations, where a beam is doubly reinforced and the boundary section initial slope of the curve increases in the cost ratio of steel to concrete, the share of the concrete and steel costs increase and decrease, respectively, and in the singly reinforced section, these trends are reversed.

Table 12 Boundary solution for beam section

Code	Beam	Boundary solution (q)
BS		21.8
ACI	SRO	23.23
ICS		16.14
BS		15.09
ACI	DRO	11.5
ICS		13.85

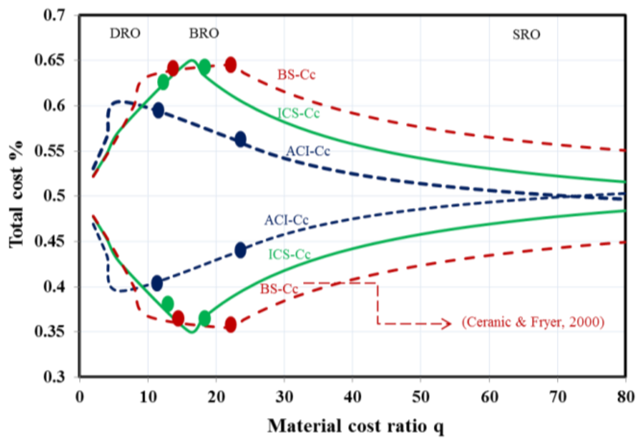

Fig. 8 Percentage of material costs ($f_y = 240$ MPa, $f_c = 30$ MPa, r = 0.15)

The results indicate that the ACI regulation is better than the other regulations because the share of the cost of concrete and steel are closer to each other than in the other regulations. By increases of q, especially where the beam is doubly reinforced, the share of concrete and steel costs increase and then decrease; when the beam is singly reinforced or a boundary section, this share decreased, and ACI performed better for various q than the other regulations.

Fig. 9 depicts the sensitivity analysis of the optimum

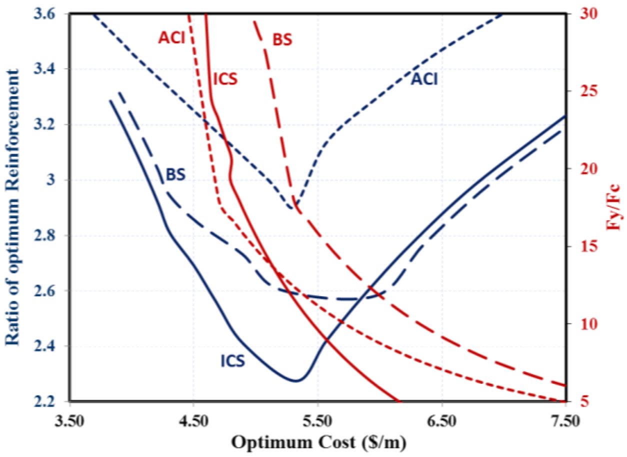

Fig. 9 Sensitivity analysis of optimum reinforcement ratio and stress ratio versus optimum cost

reinforcement ratio and stress ratio versus the optimum cost. As seen, increasing the strength of steels in a beam, leads to the decreased content of consumed steel, and because of the higher cost of steel compared to concrete, the total cost of design is decreased. Under constant conditions and equal stress ratios for the design of the desired beams according to the three regulations, the ICS regulation provides the lowest costs. According to the blue curve of the optimum ratio of reinforcement, initially the reductions in this proportion lead to lower costs and when the beam is closer to the conditions of doubly reinforced, the slope is increased, and the cost reduction growth is reduced further. When the beam is placed in doubly reinforced mode, initially the curve is too steep and increasing the reinforcements will not help much to reduce costs. But at higher reinforcement ratios, the slope decreases. For a constant reinforcement ratio, a comparison of the three regulations shows that the ACI regulation gives the most optimal results for singly reinforced sections, and the BS regulation gives the most optimal results for doubly reinforced sections. Also the intersection of the blue and red

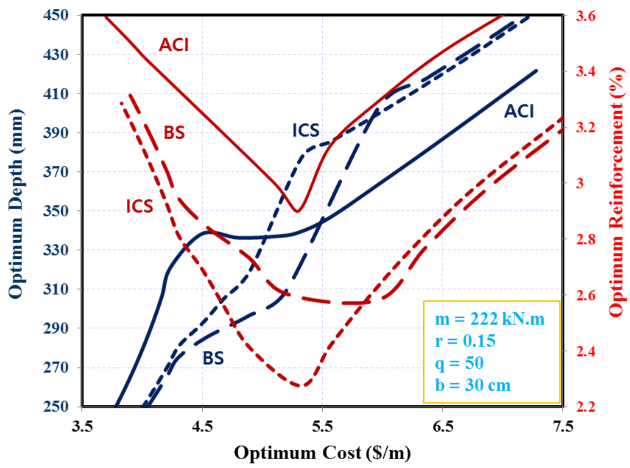

Fig. 10 Sensitivity analysis of ratio of optimum reinforcement and depth versus optimum cost

curves for each regulation shows the minimum costs in the specified section for this stress.

As shown in Fig. 10, the decrease in optimum depth leads to a cost reduction for both singly and doubly reinforced beams, but the slope of this curve in the doubly reinforced section is more than that for the singly reinforced section for a specified range of cost.

In an overview of Fig. 10, we can understand that in the beams with only tensile reinforcements, by reducing the depth of the beam and the optimum reinforcement ratio, the total cost is lowered, and in the boundary between singly and doubly reinforced beams, it is accompanied with a sudden decrease of the optimum reinforcement ratio and increased depth of the beam. In doubly reinforced section, cost reduction corresponds to a decreased depth of the beam, increased slope, and higher optimal reinforcement ratio.

To better use the situations in Figs. 9 and 10, both are depicted in Fig. 11, showing the relationship of the stress ratio, optimum area of reinforcement, and the optimum cost of the beam at constant amounts of m = 222 kN.m, q = 25 and r = 0.15. The crossing of contours shows where the

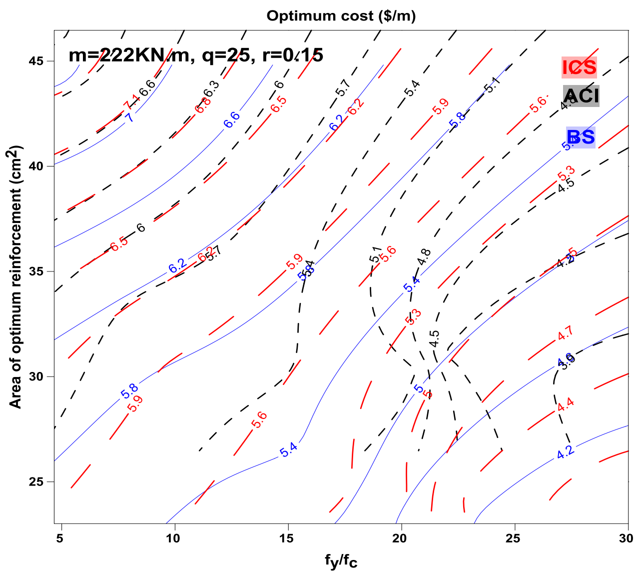

Fig. 11 Contour of optimum cost changes with different optimum areas of reinforcement of the beam and stress ratios

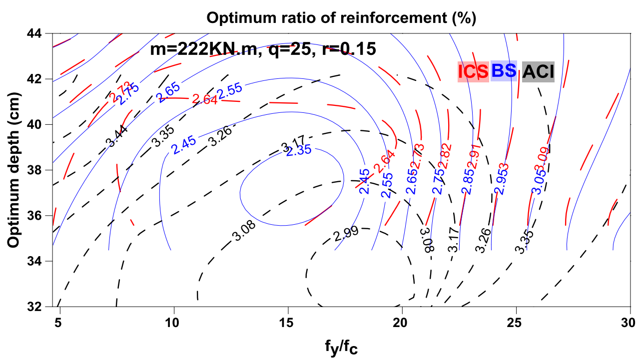

Fig. 12 Optimum depth changes with different optimum area of reinforcement and cost of beam

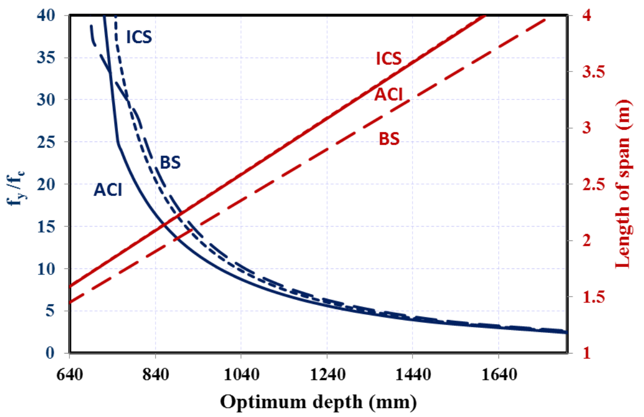

Fig. 13 Sensitivity analysis of length of spans and stress ratio versus optimum depth

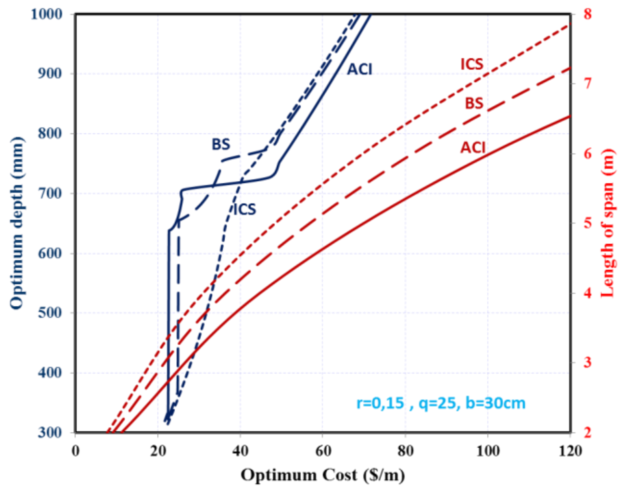

Fig. 14 Sensitivity analysis of length of span and optimum depth versus optimum cost

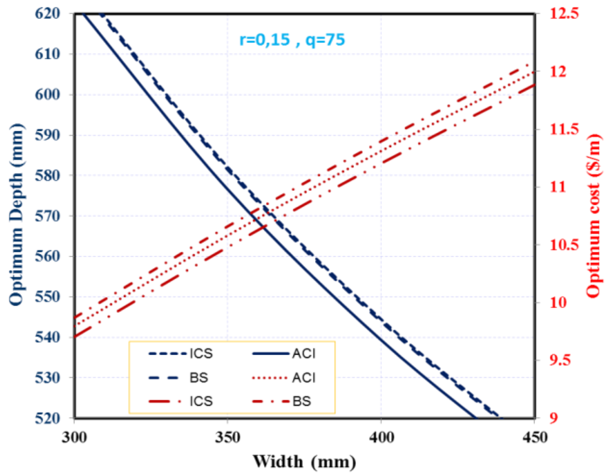

Fig. 15 Sensitivity analysis of optimum cost and depth versus width of beam

optimal reinforcement and depth values are a certain value and which of the regulations has the minimized costs. Comparing each group of three contours with a constant design condition (including one contour each regulation) indicates that the ACI regulation typically provides the lowest optimum cost. For instance, at f_y/f_c = 8 and area of optimum reinforcement = 42 (cm^2), the optimum cost obtained by BS, ACI, and ISC regulations are 7, 7.1, and 6.6 ($/m), respectively, confirming the priority of ACI regulation in order to achieve the minimum design cost.

Fig. 12 shows the relationship of the stress ratio, the optimum depth, and the optimal ratio of reinforcement at constant amounts of m = 222 kN.m, q = 25 and r = 0.15. By considering the f_y/f_c ratio and the optimum depth of beam, the optimum ratio of reinforcement can be obtained according to each regulation. It can be observed that at most cases the BS regulation provides lower ration of reinforcement, resulting in lower cost of the RC beam. For instance, for f_y/f_c = 26 and optimum depth = 39 cm, the optimum ratio of reinforcement is 3.35, 3.05, and 3.09% for ACI, BS, and ICS regulations, respectively. On the other hand, since the practical limitations in some cases may not allow the designer to choose high depths, utilization of the graphs of Fig. 12 can be valuable to obtain the optimum ratio of reinforcement at the desired optimum depth.

Fig. 13 represents the sensitivity of optimum depth to the changes of length of spans and stress ratio. As seen, in beams under a linearly distributed load, the changes along the beam span are quite directly associated with the optimum depth changes in beams, so that the span-to–optimal depth ratio is constant. Also, according to the left vertical axis, by increasing the stress ratios above 15, the slope of the curve increases. This means that the change to the optimal depth is reduced. In other words, the use of high-strength concrete or steel leads to increased and decreased depth changes, respectively.

Fig. 14 represents the sensitivity of optimum cost to the changes of length of spans and optimum depth. As shown, in these beams which are under linearly distributed loads, the final optimum cost of beam has a nonlinear relationship with the beam length. In other words, from static point of view, it can be mentioned that by increasing the length twice, the final optimal cost of the beam increases four times.

Fig. 15 represents the sensitivity of beam width to the changes optimum depth and optimum cost. It can be observed that the width of a beam can have a large impact on the final cost of the beam. Thus, in accordance with Eqs. (34) and (36), the final cost is directly proportional to the square root of the optimal beam width; the optimal depth is inversely proportional to the square root of the beam width; and by reducing the width of the beam, beam depth increases.

Fig. 16 represents the sensitivity of ratio of optimum reinforcement to the variations of q and f_y/f_c. Accordingly, one can see that, for the beams of either only tensile or compressive and tensile reinforcements, with the increase of the cost of steel to concrete, there is a reduction of the reinforcement, and instead, under constant loading conditions, this reduction increases the optimized depth of the beam. On the right vertical axis, the curve consists of two distinct categories of singly and doubly reinforced beams. For the curve associated with the beam with only tensile reinforcement that is below the curve fracture, by

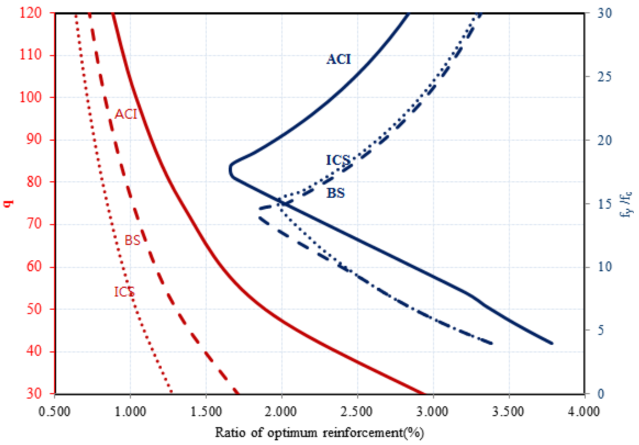

Fig. 16 Sensitivity analysis of cost ratio and stress ratio versus ratio of optimum reinforcement

increasing the stress ratio, we are faced with a decreased optimum ratio of reinforcements. After this amount, the greater the increase in the costs ratio, the greater the increase in the optimum ratio of reinforcements. However, by a comparison of the curves for the three regulations for a specified proportion of the costs, the ICS and BS regulations give more optimal values for the reinforcement ratio. In other words, for singly reinforced sections, for a certain ratio of costs, the BS regulation gives the most optimal results, whereas for higher values in doubly reinforced sections, the ACI and ICS regulations give more optimal results.

7. Conclusions

This study presents an optimization and sensitivity analysis on rectangular reinforced concrete (RC) beam using LMM to obtain the minimum design cost for both singly and doubly RC beams based on the specifications of the ACI, BS, and ICS regulations. The concluding points of this study can be summarized as follows:

- The results indicate that instead of complex optimization relationships, the LMM can be used to optimize and minimize the cost of singly and doubly reinforced beams under flexural bending moment with different boundary conditions.
- The minimum cost of singly RC beams is directly related to the optimum ratio of reinforcements, but it is inversely proportional to the doubly reinforced beams.
- The minimum cost of RC beams, whether singly or doubly reinforced, is directly related to the optimal depth.
- The sensitivity of lowering the depth to cost reduction for doubly reinforced beams is more than that for singly reinforced beams.
- The overall results from the sensitivity analysis on LMM indicate that each regulation can provide the most optimal values at specific situations. Therefore, using the graphs proposed for different design conditions can effectively help the designer (without necessity of primary optimization knowledge) choose the best regulation and values of design parameters, leading to the minimum cost of RC beam.
- These graphs can also be utilized for novel materials and composites to precisely determine the effective parameters on cost function of these materials at each ACI, BS, and ICS regulations.

References

ACI (2014), Building Code Requirements for Reinforced Concrete.

Adamu, A. and Karihaloo, B. (1994), “Minimum cost design of RC beams using DCOC Part II: Beams with uniform cross-sections”, *Struct. Optimiz.*, **7**(4), 252-259.

Aghaee, K., Yazdi, M.A., and Tsavdaridis, K.D. (2014), “Mechanical properties of structural lightweight concrete reinforced with waste steel wires”, *Magaz. Concrete Res.*, **66**(1), 1-9.

Andalib, Z., Kafi, M.A., Bazzaz, M. and Momenzadeh, S. (2018), “Numerical evaluation of ductility and energy absorption of steel rings constructed from plates”, *Eng. Struct.*, **169**, 94-106.

Ardalan, R.B., Joshaghani, A. and Hooton, R.D. (2017), “Workability retention and compressive strength of self-compacting concrete incorporating pumice powder and silica fume”, *Constr. Build. Mater.*, **134**, 116-122.

Awal, A.A., Shehu, I.A. and Ismail, M. (2015), “Effect of cooling regime on the residual performance of high-volume palm oil fuel ash concrete exposed to high temperatures”, *Constr. Build. Mater.*, **98**, 875-883.

Bahadori, A. and Ghassemieh, M. (2016), “Seismic evaluation of I-shaped beam to box-column connections with top and seat plates by the component method”, *Sharif: Civil Eng.*, **322**(21), 129-138.

Balaguru, P.N. (1980), “Cost optimum design of doubly reinforced concrete beams”, *Build. Environ.*, **15**(4), 219-222.

Barros, M.H.F.M., Martins, R.A.F. and Barros, A.F.M. (2005), “Cost optimization of singly and doubly reinforced concrete beams with EC2-2001”, *Struct. Multidiscipl. Optimiz.*, **30**(3), 236-242.

Bazzaz, M., Kafi, M.A., Kheyroddin, A., Andalib, Z. and Esmaeili, H. (2014), “Evaluating the seismic performance of off-centre bracing system with circular element in optimum place”, *Int. J3. Steel Struct.*, **14**(2), 293-304.

Bertsekas, D.P. (2014), *Constrained Optimization and Lagrange Multiplier Methods*, Academic Press.

Ceranic, B. and Fryer, C. (2000), “Sensitivity analysis and optimum design curves for the minimum cost design of singly and doubly reinforced concrete beams”, *Struct. Multidiscipl. Optimiz.*, **20**(4), 260-268.

Chae, H. and Yun, Y. (2015), “Strut-tie model for two-span continuous RC deep beams”, *Comput. Concrete, Int. J.*, **16**(3), 357-380.

Chakrabarty, B. (1992), “Models for optimal design of reinforced concrete beams”, *Comput. Struct.*, **42**(3), 447-451.

Chetchotisak, P., Teerawong, J., Yindeesuk, S. and Song, J. (2014), “New strut-and-tie-models for shear strength prediction and design of RC deep beams”, *Comput. Concrete, Int. J.*, **14**(1), 19-40.

El Debs, A.L.H., Chaves Neto, A.C., Andreczevski Chaves, I., Squarcio, R.M.F. and Araki lira, S. (2005), “Optimization of cross section of reinforced concrete beam using experimental design”, *Proceedings of the 6th World Congresses of Structural and Multidisciplinary Optimization*, Rio de Janeiro, Brazil.

Eskandari, H. and Madadi, A. (2015), "Investigation of ferrocement channels using experimental and finite element analysis", *Eng. Sci. Technol., Int. J.*, **18**(4), 769-775.

Fanaie, N., Aghajani, S. and Shamloo, S. (2012), "Theoretical assessment of wire rope bracing system with soft central cylinder", *Proceedings of the 15th World Conference on Earthquake Engineering*.

Fanaie, N., Aghajani, S. and Dizaj, E.A. (2016), "Theoretical assessment of the behavior of cable bracing system with central steel cylinder", *Adv. Struct. Eng.*, **19**(3), 463-472.

Farzampour, A. (2017), "Temperature and humidity effects on behavior of grouts", *Adv. Concrete Constr., Int. J.*, **5**(6), 659-669.

Fortin, M. and Glowinski, R. (2000), *Augmented Lagrangian Methods: Applications to the Numerical Solution of Boundary-Value Problems*, Elsevier.

Heydari, A. and Shariati, M. (2018), "Buckling analysis of tapered BDFGM nano-beam under variable axial compression resting on elastic medium", *Struct. Eng. Mech., Int. J.*, **66**(6), 737-748.

Huedo, J.D., Martínez, J.M. and Montero, P.G. (2005), "Dimensioning of longitudinal reinforcements in concrete beams with a non-rectangular section and variable height", *Spanish J. Agricul. Res.*, **3**(4), 367-376.

Joshaghani, A., Moazenian, A. and Shuaibu, R.A. (2017), "Experimental study on the use of trass as a supplementary cementitious material in pervious concrete", *J. Environ. Sci. Eng. A*, **1**, 39-52.

Karamshahi, A., Alihoseini, Z., Mirzaei, J. and Jafarzadeh, A. (2017), "Interval sampling methods in Zagros forests using GIS", *J. Forestry Res.*, **28**(6), 1261-1266.

Kaveh, A. and Shokohi, F. (2015), "Optimum design of laterally-supported castellated beams using CBO algorithm", *Steel Compos. Struct., Int. J.*, **18**(2), 305-324.

Khorami, M., Alvansazyazdi, M., Shariati, M., Zandi, Y., Jalali, A. and Tahir, M. (2017a), "Seismic performance evaluation of buckling restrained braced frames (BRBF) using incremental nonlinear dynamic analysis method (IDA)", *Eartq. Struct., Int. J.*, **13**(6), 531-538.

Khorami, M., Khorami, M., Motahar, H., Alvansazyazdi, M., Shariati, M., Jalali, A. and Tahir, M.M. (2017b), "Evaluation of the seismic performance of special moment frames using incremental nonlinear dynamic analysis", *Struct. Eng. Mech., Int. J.*, **63**(2), 259-268.

Khorramian, K., Maleki, S., Shariati, M., Jalali, A. and Tahir, M.M. (2017), "Numerical analysis of tilted angle shear connectors in steel-concrete composite systems", *Steel Compos. Struct., Int. J.*, **23**(1), 67-85.

Korouzhdeh, T., Eskandari-Naddaf, H. and Gharouni-Nik, M. (2017), "An improved ant colony model for cost optimization of composite beams", *Appl. Artif. Intell., Int. J.*, **31**(1), 44-63.

Li, Y. and Chen, Y. (2010), "Beam structure optimization for additive manufacturing based on principal stress lines", *Solid Freeform Fabrication Proceedings*.

Long, X. and Lee, C.K. (2015), "Improved strut-and-tie method for 2D RC beam-column joints under monotonic loading", *Comput. Concrete, Int. J.*, **15**(5), 807-831.

Madadi, A., Eskandari-Naddaf, H., Shadnia, R. and Zhang, L. (2018), "Characterization of ferrocement slab panels containing lightweight expanded clay aggregate using digital image correlation technique", *Constr. Build. Mater.*, **180**, 464-476.

Mansouri, I., Safa, M., Ibrahim, Z., Kisi, O., Tahir, M.M., Baharom, S. and Azimi, M. (2016), "Strength prediction of rotary brace damper using MLR and MARS", *Struct. Eng. Mech., Int. J.*, **60**(3), 471-488.

Mansouri, I., Shariati, M., Safa, M., Ibrahim, Z., Tahir, M.M. and Petković, D. (2017), "Analysis of influential factors for predicting the shear strength of a V-shaped angle shear connector in composite beams using an adaptive neuro-fuzzy technique", *J. Intell. Manuf.*, 1-11.

Mohammadhassani, M., Saleh, A., Suhatril, M. and Safa, M. (2015), "Fuzzy modelling approach for shear strength prediction of RC deep beams", *Smart Struct. Syst., Int. J.*, **16**(3), 497-519.

Nasrollahi, S., Maleki, S., Shariati, M., Marto, A. and Khorami, M. (2018), "Investigation of pipe shear connectors using push out test", *Steel Compos. Struct., Int. J.*, **27**(5), 537-543.

Nigdeli, S.M. and Bekdas, G. (2013), "Optimization of RC beams for various cost ratios of steel/concrete", Proceedings of the 4th European conference of civil engineering ECCIE.

Ozturk, H.T., Durmus, A. and Durmus, A. (2012), "Optimum design of a reinforced concrete beam using artificial bee colony algorithm", *Comput. Concrete, Int. J.*, **10**(3), 295-306.

Paknahad, M., Bazzaz, M. and Khorami, M. (2018), "Shear capacity equation for channel shear connectors in steel-concrete composite beams", *Steel Compos. Struct., Int. J.*, **28**(4), 483-494.

Panjehpour, M., Ali, A.A.A., Voo, Y.L. and Aznieta, F.N. (2014), "Effective compressive strength of strut in CFRP-strengthened reinforced concrete deep beams following ACI 318-11", Computers and Concrete, **13**(1), 135-147.

Rahmanian, I., Lucet, Y. and Tesfamariam, S. (2014), "Optimal design of reinforced concrete beams: A review", *Comput. Concrete, Int. J.*, **13**(4), 457-482.

Safa, A., Rashidinejad, H.R., Khalili, M., Dabiri, S., Nemati, M., Mohammadi, M.M. and Jafarzadeh, A. (2016a), "Higher circulating levels of chemokines CXCL10, CCL20 and CCL22 in patients with ischemic heart disease", *Cytokine*, **83**, 147-157.

Safa, M., Shariati, M., Ibrahim, Z., Toghroli, A., Baharom, S.B., Nor, N.M. and Petkovic, D. (2016b), "Potential of adaptive neuro fuzzy inference system for evaluating the factors affecting steel-concrete composite beam's shear strength", *Steel Compos. Struct., Int. J.*, **21**(3), 679-688.

Shah, S.N.R., Sulong, N.R., Khan, R., Jumaat, M.Z. and Shariati, M. (2016), "Behavior of industrial steel rack connections", *Mech. Syst. Signal Processing*, **70-71**: 725-740.

Sharafi, P., Hadi, M.N. and Teh, L.H. (2012), "Geometric design optimization for dynamic response problems of continuous reinforced concrete beams", *J. Comput. Civil Eng.*, **28**(2), 202-209.

Shariat, M., Eskandari-Naddaf, H., Tayyebinia, M. and Sadeghian, M. (2018a), "Finite Element Modeling of Shear Strength for Concrete Deep Beams (Part II)", *Materials Today: Proceedings*, **5**(2), 5521-5528.

Shariat, M., Eskandari-Naddaf, H., Tayyebinia, M. and Sadeghian, M. (2018b), "Sensitivity Analysis of Reinforced Concrete Deep Beam by STM and FEM (Part III)", *Materials Today: Proceedings*, **5**(2), 5529-5535.

Shariati, M., Ramli Sulong, N.H., Maleki, S. and Arabnejad Kh, M.M. (2010), "Experimental and analytical study on channel shear connectors in light weight aggregate concrete", *Proceedings of the 4th International Conference on Steel & Composite Structures*, Sydney, Australia, July.

Standard, B. (1985), "8110: Part 1, Structural use of concrete–code of practice for design and construction", British Standards Institute, London UK, pp. 3-8.

Tahouni, S. (2005), Designing Concrete Structures Based on Iranian Concrete Code.

Toghroli, A., Mohammadhassani, M., Suhatril, M., Shariati, M. and Ibrahim, Z. (2014), "Prediction of shear capacity of channel shear connectors using the ANFIS model", *Steel Compos. Struct., Int. J.*, **17**(5), 623-639.

Toghroli, A., Suhatril, M., Ibrahim, Z., Safa, M., Shariati, M. and Shamshirband, S. (2016), "Potential of soft computing approach for evaluating the factors affecting the capacity of steel–concrete composite beam", *J. Intell. Manuf.*, 1-9.

Wei, X., Shariati, M., Zandi, Y., Pei, S., Jin, Z., Gharachurlu, S., Abdullahi, M.M., Tahir, M.M. and Khorami, M. (2018), "Distribution of shear force in perforated shear connectors", *Steel Compos. Struct., Int. J.*, **27**(3), 389-399.

Zandi, Y. and Toghroli, A. (2018), "Computational investigation of the comparative analysis of cylindrical barns subjected to earthquake", *Steel Compos. Struct., Int. J.*, **28**(4), 439-447.

CC