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A hybrid DQ-TLBO technique for maximizing first frequency of laminated composite skew plates

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Abstract. The differential quadrature (DQ) and teaching-learning based optimization (TLBO) methods are coupled to introduce a hybrid numerical method for maximizing fundamental natural frequency of laminated composite skew plates. The fiber(s) orientations are selected as design variable(s). The first-order shear deformation theory (FSDT) is used to obtain the governing equations of the plate. The equations of motion and the related boundary conditions are discretized in space domain by employing the DQ method. The discretized equations are transferred from the time domain into the frequency domain to obtain the fundamental natural frequency. Then, the DQ solution is coupled with the TLBO method to find the maximum frequency of the plate and its related optimum stacking sequences of the laminate. Convergence and applicability of the proposed method are shown and the optimum fundamental frequency parameter of the plates with different skew angle, boundary conditions, number of layers and aspect ratio are obtained. The obtained results can be used as a benchmark for further studies.

Keywords: a hybrid numerical method; differential quadrature method; teaching-learning based optimization method; laminated composite skew plates

1. Introduction

According to proper characteristics of laminated composites, laminated composite plates of different shapes are widely used in modern industries such as aerospace and nuclear engineering (Hirwani *et al*. 2017, Sahoo *et al*. 2016, 2018). Fibers orientation and stacking sequence of these elements play an important role to obtain optimum design (Honda *et al*. 2009, Topal 2012, Malekzade *et al*. 2014, Vosoughi and Nikoo 2015, Shafei and Shirzad 2017, Vosoughi *et al*. 2017, 2018a, b). On the other hand, the fundamental natural frequency of a structural element is an important characteristic affecting its dynamic response. So, maximizing the fundamental frequency of a structural element, in addition to help to prevent the resonance phenomena, can improve its dynamic behavior.

Some optimization problems of laminated rectangular plates have been investigated in the past. Kam and Lai (1995) used the finite element and constrained multi-start global optimization methods to obtain optimal dynamic characteristics of moderately thick laminated composite square plates. Natural frequencies of thin laminated composite square plates and panels were maximized using the Ritz method and layerwise optimization approach by Narita and his co-workers (Narita 2003, Narita and Hodgkinson 2005, Narita and Robinson 2006). Also, Narita (2006) maximized fundamental natural frequency of thin laminated composite square plate by using a mixed layerwise optimization approach and the finite element method. Topal and Uzman (2008) employed the mixed modified feasible direction and finite element method to maximize the fundamental frequency of moderately thick laminated composite plates with a circular hole and skew composite laminated plates (Topal and Uzman 2008, 2009). Apalak *et al*. (2008) solved the problem of maximizing the fundamental frequency of thin laminated composite square plates with employing the genetic algorithms and the finite element method. Karakaya and Soykasap (2011) used genetic algorithm and simulating annealing methods to maximize natural frequency of simply supported laminated composite plates. They found frequency of the plate analytically. A combination of the Elitist-Genetic algorithm and the finite strip method was employed by Sadr and Ghashochi Bargh (2012) to find the maximum fundamental frequency of thin laminated composite square plates. Honda *et al*. (2013) extended the layerwise optimization procedure to find maximum fundamental frequency of thin and moderately thick laminated composite square plates by combining the Ritz refined zigzag theory. A combination of the finite element method and the artificial bee colony algorithm was employed by Apalak *et al*. (2014) to obtain the optimum fundamental natural frequency of thin

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laminated composite square plates. Vosoughi *et al*. (2016) presented a hybrid method to maximize the fundamental natural frequency of thick laminated composite square plates. They used a mixed finite element, genetic algorithms and particle swarm optimization methods to solve the problem.

On the other hand, searching for new combination of different numerical techniques and optimization algorithms to introduce efficient numerical solvers for optimization problems of composite structural elements are of interest for the researchers in the field (Vosoughi and Gerist 2014, Vosoughi 2015, Darabi and Vosoughi 2016, Vosoughi and Darabi 2017, Vosoughi and Anjabin 2017, Khalili and Vosoughi 2018, Vosoughi *et al*. 2018b). In the recent years, the computational efficiency together with the high accuracy of the differential quadrature method (DQM) for solving the different structural problems has been demonstrated; for example see Refs. (Malekzadeh and Vosoughi 2008, Malekzadeh and Vosoughi 2009, Vosoughi 2014, Vosoughi *et al*., 2018a). In these studies, it has been shown that only with a few number of grid points, in spite of the other conventional approximate method such as the finite element method and the Ritz method, the accurate results can be achieved for complicated structural problems. So, here, as a first endeavor, a coupled form of the DQM and teaching-learning based optimization methods is introduced for maximizing fundamental natural frequency of moderately thick laminated composite skew plates. It should be mentioned that as reveals from the literature survey, most of the previously solved optimization problems of composite plates concerned with rectangular plates ones. But in this study, a class of important non-rectangular composite plates, i.e. skew plates, is considered. To solve the problem, the first-order shear deformation theory is adopted to obtain governing equations of the plates, which includes both the transverse shear deformation and the rotary inertia effects. Then, the differential quadrature method is employed to discretize the equations of motion and the related boundary conditions to obtain the natural frequency of the plate. For maximizing the fundamental natural frequency, the plies' fiber orientations are considered as the design variables and the teaching-learning based optimization method is used to find the optimum solution. After validating the present approach, different examples of skew laminated composite plates are solved and the corresponding results are tabulated, which can be used as benchmark solution by other researchers in the field.

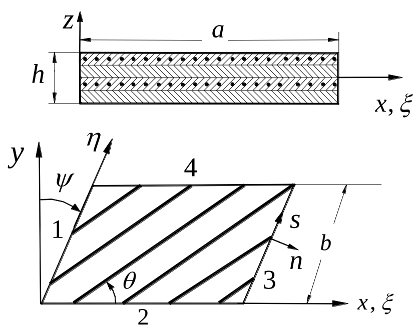

Fig. 1 The geometry of the laminated composite plate

2. The governing equations

A symmetric laminated composite plate with N_L perfectly bonded orthotropic layers of length a, width b and total thickness h is considered (Fig. 1).

Based on the first-order shear deformation theory (FSDT), the constitutive relations of the resultant stresses at an arbitrary material point on the mid-plane of the plates can be obtained as follows (Reddy 1997)

$$\begin{Bmatrix} M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \begin{Bmatrix} \dfrac{\partial \varphi^x}{\partial x} \\ \dfrac{\partial \varphi^y}{\partial y} \\ \dfrac{\partial \varphi^x}{\partial y} + \dfrac{\partial \varphi^y}{\partial x} \end{Bmatrix} \tag{1a}$$

$$\begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \begin{bmatrix} A_{55} & A_{45} \\ A_{45} & A_{44} \end{bmatrix} \begin{Bmatrix} \varphi^x + \dfrac{\partial w}{\partial x} \\ \varphi^y + \dfrac{\partial w}{\partial y} \end{Bmatrix} \tag{1b}$$

where $D_{ij} = \sum_{k=1}^{N_L} \int_{z_{k-1}}^{z_k} \bar{Q}_{ij}^{(k)} z^2 dz$, ($i$, j = 1, 2, 6); $A_{ij} = \kappa_{ij} \sum_{k=1}^{N_L} \int_{z_{k-1}}^{z_k} \bar{Q}_{ij}^{(k)} dz$ (i, j = 4, 5); vis the shear correction factor; z_{k-1} and z_k (k = 1,..., N_L) are the lower and upper surface thickness coordinate of the k^{th} layer; $\bar{Q}_{ij}^{(k)}$ (i, j = 1, 2, , 4, 5, 6) are the transformed reduced stiffness coefficients of the k^{th} layer (Reddy 1997); φ^x and φ^y are the rotations of the transverse normal to the mid-plane of the laminated composite plate about the y- and x-axis, respectively. Also, w is the vertical displacement component of an arbitrary point on the mid-plane of the plate along the z-direction. The governing equations of the laminated composite plate can be stated as

$$\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} = I_{oo} \frac{\partial^2 w}{\partial t^2} \tag{2}$$

$$\frac{\partial M_{xx}}{\partial x} + \frac{\partial M_{xy}}{\partial y} - Q_x = I_{22} \frac{\partial^2 \varphi^x}{\partial t^2} \tag{3}$$

$$\frac{\partial M_{xy}}{\partial y} + \frac{\partial M_{yy}}{\partial y} - Q_y = I_{22} \frac{\partial^2 \varphi^y}{\partial t^2} \tag{4}$$

where $I_{kk} = \int_0^h \rho z^2 dz$.

Without loss of generality, in this paper, the skew plates with clamped and simply supported edges and also some combination of these boundary conditions are considered. The conditions of these edge restraints can be stated as

<u>*Simple boundary condition*</u>:

$$w = 0, \tag{5a}$$

$$\varphi^s = 0, \tag{5b}$$

$$M_{nn} = 0 \tag{5c}$$

Clamp boundary condition:

$$w=0, \tag{6a}$$

$$\varphi^s=0, \tag{6b}$$

$$\varphi^n=0 \tag{6c}$$

where $\varphi^n = n_x\varphi^x + n_y\varphi^y$, $\varphi s = -n_y\varphi^x + n_x\varphi^y$, $M_{nn} = M_{xx}n_x^2 + M_{yy}n_y^2 + 2M_{xy}n_x n_y$; also, n_x and n_y are the x and y-components of unit normal vector to an arbitrary edge of the plate, respectively; M_{xx} , M_{yy} and M_{xy} are the bending moments about y and x-axis and twisting moment, respectively (Reddy 1997).

Before utilizing the DQM, the governing equations and the boundary conditions should be transformed into the computational domain of the DQM, which is a rectangular domain one. For this purpose, the following linear transformation rules can be used

$$x=\xi+(\sin\psi)\eta, \qquad y=(\cos\psi)\eta \tag{7}$$

where ξ and η are the natural coordinate variables of the computational domain of the problem; ψ is the skew angle of the plate.

Using the differential quadrature rules (Malekzadeh and Vosoughi 2008, Vosoughi 2016), the discretized form of the equations of motion can be obtained as follows

Eq. (2)

$$\begin{aligned}&(A_{55}-A_{45}\tan\psi)\sum_{m=1}^{N_\xi}A_{im}^{\xi}\phi_{im}^{x}+A_{45}\sec\psi\sum_{n=1}^{N_\eta}A_{jn}^{\eta}\phi_{in}^{x}+\\&(A_{45}-A_{44}\tan\psi)\sum_{m=1}^{N_\xi}A_{im}^{\xi}\phi_{im}^{y}+(A_{55}-2A_{45}\tan\psi+\\&A_{44}\tan^2\psi)\sum_{m=2}^{N'_\xi}B_{im}^{\xi}w_{mj}+2\sec\psi(A_{45}-A_{44}\tan\psi)\times\\&\sum_{m=2}^{N'_\xi}\sum_{n=2}^{N'_\eta}A_{im}^{\xi}A_{jn}^{\eta}w_{mn}+A_{44}\sec^2\psi\sum_{n=2}^{N'_\eta}B_{jn}^{\eta}w_{in}=\\&-I_{00}\frac{d^2w_{ij}}{dt^2}\end{aligned} \tag{8}$$

Eq. (3)

$$\begin{aligned}&(A_{45}\tan\psi-A_{55})\sum_{m=2}^{N'_\xi}A_{im}^{\xi}w_{mj}-A_{45}\sec\psi\sum_{n=2}^{N'_\eta}A_{jn}^{\eta}w_{in}+\\&\left(D_{11}-2D_{16}\tan\psi+D_{66}\tan^2\psi\right)\sum_{m=1}^{N_\xi}B_{im}^{\xi}\phi_{mj}^{x}+2\sec\psi\times\\&(D_{16}-D_{66}\tan\psi)\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}\phi_{mn}^{x}+D_{66}\sec^2\psi\times\\&\sum_{n=1}^{N_\eta}B_{jn}^{\eta}\phi_{in}^{x}+[D_{16}-\tan\psi(D_{12}+D_{66})+D_{26}\tan^2\psi]\times\\&\sum_{m=1}^{N_\xi}B_{im}^{\xi}\phi_{mj}^{y}+\sec\psi(D_{12}+D_{66}-2D_{26}\tan\psi)\times\\&\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}\varphi_{mn}^{y}+D_{26}\sec^2\psi\sum_{n=1}^{N_\eta}B_{jn}^{\eta}\varphi_{in}^{y}-A_{55}\varphi_{ij}^{x}-\\&A_{45}\varphi_{ij}^{y}=-I_{22}\frac{d^2\varphi_{ij}^{y}}{dt^2}\end{aligned} \tag{9}$$

Eq. (4)

$$\begin{aligned}&(A_{44}\tan\psi-A_{45})\sum_{m=2}^{N'_\xi}A_{im}^{\xi}w_{mj}-A_{44}\sec\psi\sum_{n=2}^{N'_\eta}A_{jn}^{\eta}w_{in}+\\&\left[D_{16}-(D_{12}+D_{66})\tan\psi+D_{26}\tan^2\psi\right]\sum_{m=1}^{N_\xi}B_{im}^{\xi}\phi_{mj}^{x}+\\&\sec\psi(D_{12}+D_{66}-2D_{26}\tan\psi)\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}\phi_{mn}^{x}+\\&D_{26}\sec^2\psi\sum_{n=1}^{N_\eta}B_{jn}^{\eta}\phi_{in}^{x}+[D_{66}-\tan\psi(2D_{26}-D_{22}\times\\&\tan^2\psi)]\sum_{m=1}^{N_\xi}B_{im}^{\xi}\phi_{mj}^{y}+2\sec\psi(D_{26}-D_{22}\tan\psi)\times\\&\sum_{m=1}^{N_\xi}\sum_{n=1}^{N_\eta}A_{im}^{\xi}A_{jn}^{\eta}\phi_{mn}^{y}+D_{22}\sec^2\psi\sum_{n=1}^{N_\eta}B_{jn}^{\eta}\phi_{in}^{y}-A_{45}\phi_{ij}^{x}\\&-A_{44}\phi_{ij}^{y}=-I_{22}\frac{d^2\varphi_{ij}^{y}}{dt^2}\end{aligned} \tag{10}$$

where N_α ($\alpha = \xi$, η) is the number of grid points along the α-direction and $N'_\alpha = N_\alpha - 1$; also, A_{ij}^{α} and B_{ij}^{α} ($\alpha = \xi$, η) are the weighting coefficients of the first and second-order derivatives with respect to α variables ($\alpha = \xi$, η), respectively.

In a similar manner, the discretized form of the boundary conditions

Eq. (5)

$$w_{ij}=0, \tag{11a}$$

$$\varphi_{ij}^{s}=-n_y\varphi_{ij}^{x}+n_x\varphi_{ij}^{y}=0, \tag{11b}$$

$$\begin{aligned}&(M_{nn})_{ij}=[n_x^2D_{11}+2n_xn_yD_{16}+n_y^2D_{12}-\\&(n_x^2D_{12}+2n_xn_yD_{26}+n_y^2D_{22})\tan\psi]\times\\&\sum_{m=1}^{N_\xi}A_{im}^{\xi}\phi_{mj}^{x}+[n_x^2D_{16}+2n_xn_yD_{66}+\\&n_y^2D_{26}-\tan\psi(n_x^2D_{12}+2n_xn_yD_{26}+\\&n_y^2D_{22})]\sum_{n=1}^{N_\eta}A_{jn}^{\eta}\phi_{in}^{y}=0\end{aligned} \tag{11c}$$

Eq. (6)

$$w_{ij}=0, \tag{12a}$$

$$\varphi_{ij}^{s}=-n_y\varphi_{ij}^{x}+n_x\varphi_{ij}^{y}=0, \tag{12b}$$

$$\varphi_{ij}^{n}=n_x\varphi_{ij}^{x}+n_y\varphi_{ij}^{y}=0 \tag{12c}$$

where

$$\begin{cases} i=1 & \text{at } \xi=0 \\ i=N_\xi & \text{at } \xi=a \end{cases}; \quad j=1,2,\dots,N_\eta \tag{13a}$$

$$\begin{cases} j=1 & \text{at } \eta=0 \\ j=N_\eta & \text{at } \eta=b \end{cases}; \quad i=1,2,\dots,N_\xi \tag{13b}$$

i = 1 at the edge ξ = 0 and $i = N_\xi$ at $\xi = a$ with j = 1,…, N_η along these edges; also, j = 1 at the edge η = 0, $j= N_\eta$ at $\eta = b$ and i = 1,…, N_ξ along these edges.

Due to harmonic behavior of motion in free vibration of plates, the transverse displacement and the rotation components can be explained as

$$w(x,y,t)=W(x,y)\sin(\omega t), \tag{14a}$$

$$\varphi^x(x,y,t)=\phi^x(x,y)\sin(\omega t), \tag{14b}$$

$$\varphi^y(x,y,t)=\phi^y(x,y)\sin(\omega t) \tag{14c}$$

where W, ϕ^x and ϕ^y are the amplitude of transverse displacement and corresponding rotation components, respectively; also, ω is the circular frequency of the plate.

The discretized equations of motion and the related boundary conditions can be expressed in the matrix form as follows

$$\left(\begin{bmatrix} [K_{WW}] & [K_{W\phi}] \\ [K_{\phi W}] & [K_{\phi\phi}] \end{bmatrix} - \omega^2 \begin{bmatrix} [M_{WW}] & [0] \\ [0] & [M_{\phi\phi}] \end{bmatrix}\right) \times \begin{Bmatrix} \{W\} \\ \{\phi\} \end{Bmatrix} = \{0\} \tag{15}$$

where $[K_{ij}]$ (i, j = W, ϕ) and $[M_{ij}]$ (i, j = W, ϕ) are the stiffness and mass matrices, respectively; also, $\{W\}$ and $\{\phi\} = \begin{Bmatrix} \{\phi^x\} \\ \{\phi^y\} \end{Bmatrix}$ are the vectors of vertical displacement and the rotational degrees of degrees of freedom, respectively.

3. TLBO for maximizing the fundamental natural frequency

The objective function of the problem and its related constraints for fiber orientations as design variables are given below

$$\begin{aligned} &\text{Maximize } \Omega \\ &\text{subjected to } -90° < \theta_j < 90°, \quad j = 1, \dots N_L \end{aligned} \tag{16}$$

where $\Omega = \omega\left(\sqrt{\rho/E_2}\right)a^2/h$ is the non-dimensional fundamental natural frequency of the plate and θ_j is the fiber orientation of the j^{th} layer.

Teaching-learning based optimization method is an evolutionary optimization technique which is based on the effects of teachers on learners, and also the interaction between the learners. In this method, design variable(s) are considered as a set of population (learners). The TLBO has two parts. The first one is the teacher phase in which the learners are trained by their teacher and the second part is learners phase in which the learners are trained with interaction between themselves (Rao *et al*. 2011, Baghlani and Makiabadi 2013).

According to TLBO to find the maximum fundamental natural frequency of the plate, the fiber(s) orientations are selected as learners or students. The first population is randomly generated and the related values of frequency of the plate are calculated using the DQM. Then, the fiber(s) orientations related to the maximum frequency is considered as teacher ($X_{teacher}$) and the remained learners would be learned from the teacher according to the following equation

$$X_{new} = X_i + r\left(X_{teacher} - T_F X_{mean}\right) \tag{17}$$

where X_i is the i^{th} learner; r and T_F are two random values between 0 and 1; (X_{mean}) is the mean value of the learners (Rao *et al*. 2011, Baghlani and Makiabadi 2013).

In the second phase, the new learners (X_{new}) can be obtained by comparing fitness values (f) of learners with each other and modifications are made using the following equations (Rao *et al*. 2011, Baghlani and Makiabadi 2013).

$$\text{if} \quad f(X_i) > f(X_i):\ X_{new} = X_i + r\left(X_j - X_i\right) \tag{18}$$

$$\text{if} \quad f(X_j) > f(X_i):\ X_{new} = X_i + r\left(X_i - X_j\right) \tag{19}$$

where X_j is the j^{th} learner.

The above procedure are continued to obtain the

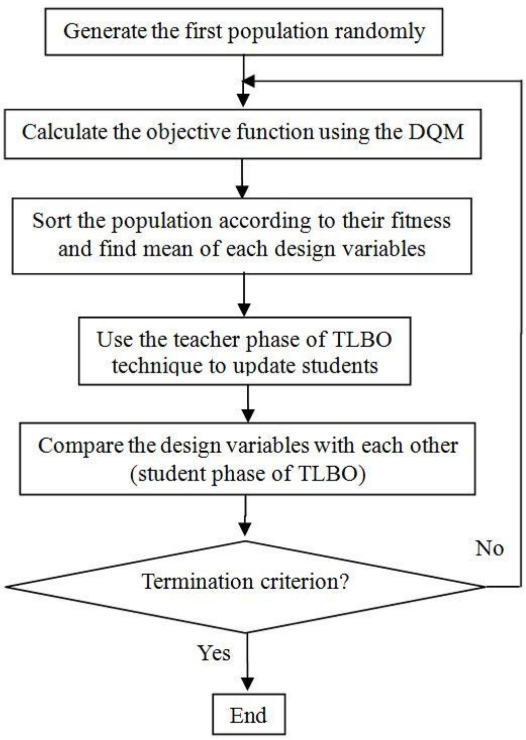

Fig. 2 Flow chart of the hybrid DQ-TLBO solution procedure

maximum fundamental natural frequency and its related optimum fiber(s) orientations with acceptable accuracy. More details of the TLBO method can be found in Refs. (Rao *et al*. 2011, Baghlani and Makiabadi 2013). Also, flow chart of the new hybrid DQM-TLBO solution procedure is shown in Fig. 2.

4. Numerical results

In this section, by using the computer program prepared based on the presented algorithm, the numerical results for the symmetric laminated composite skew plates subjected to different boundary conditions are presented. For this purpose, firstly, the convergence and accuracy of the hybrid DQ-TLBO method for maximizing fundamental natural frequency of laminated composite plates and the related fiber(s) orientations are shown in Table 1. In this table and for all the other solved examples, the following mechanical properties are used for the all layers

$$E_{11}=181\,\text{GPa},\ E_{22}=10.3\,\text{GPa},\ G_{12}=7.17\,\text{GPa},\ G_{13}=G_{12},$$
$$G_{23}=2.39\,\text{GPa},\ \nu_{12}=\nu_{21}=0.28,\ \rho=1500\,\text{kg}/\text{m}^3$$

Table 1 Convergence and accuracy of the presented hybrid method for the non-dimensional frequency of 8-layered symmetric $[\theta_1^\circ/\theta_2^\circ/\theta_3^\circ/\theta_4^\circ]_{\text{Sym}}$. SSSS laminated composite plate (a/b = 1, a = 1, a/h = 100, $N_\xi = N_\eta$ = 17, ψ = 0)

N_p	N_g	Ω	The optimum fibers orientation			
			θ_1°	θ_2°	θ_3°	θ_4°
10	10	56.0587	-42	47	43	79
	20	56.3143	-45	45	45	49
	50	56.3175	-45	45	45	43
	100	56.3175	-45	45	45	43
20	10	56.1179	41	-44	-46	-25
	20	56.2911	46	-47	-46	-46
	50	56.3184	45	-45	-45	-45
	100	56.3184	45	-45	-45	-45
100	10	56.3057	44	-44	-44	-45
	20	56.3067	45	-46	-44	-40
	50	56.3184	45	-45	-45	-45
	100	56.3184	45	-45	-45	-45
Narita (2006)		56.32	45	-45	-45	-45
Sadr and Ghashochi Bargh (2012)		56.370	-45	45	45	48

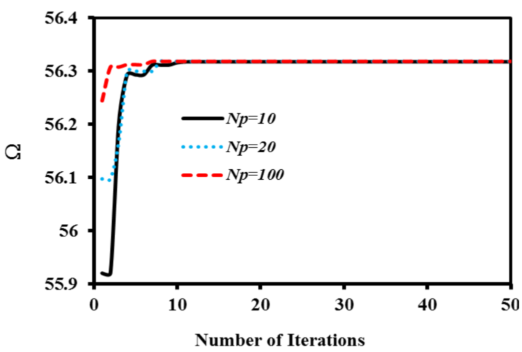

Fig. 3 Convergence of the hybrid DQ-TLBO for maximizing the frequency of 8-layered symmetric $[\theta_1^\circ/\theta_2^\circ/\theta_3^\circ/\theta_4^\circ]_{\text{Sym}}$. SSSS laminated composite plate (a/b = 1, a = 1, a/h = 100, $N_\xi = N_\eta$ = 17)

In Table 1, the results of the presented hybrid DQ-TLBO method for simply supported (SSSS) 8-layered symmetric square plates are compared with those of other approaches (Narita 2006, Sadr and Ghashochi Bargh 2012). As it can be seen from the presented results in Table 1, it is obvious that by increasing the number of population (N_p) and the number of generation (N_g), the results of present study show an excellent agreement with those of others. Also, one can observe that N_p = 20 and N_g = 50 are sufficient to obtain the converged accurate results. So hereafter, these values are used to find the maximum fundamental natural frequency of the laminated composite skew plates and the related fiber(s) orientations. Although the coupled method can give real values for optimum design variables, but with respect to construction limits for the laminated composite plates, the obtained optimum results are rounded and integer optimum fiber(s) orientations are reported in the following solved examples.

For the solved problem in Table 1, the efficiency and convergence of the present hybrid method are shown in Fig. 3. In this figure, the convergences of the hybrid method for the different number of population are plotted. It is obvious that by increasing the number of population, the method converges rapidly to the optimum value of the frequency. Comparing these results with those achieved using the finite strip element method together with elitist genetic algorithms (E-GAs) solution obtained by Sadr and Ghashochi Bargh (2012) highlights the efficiency of the present hybrid DQ-TLBO solution method. They compared their results with those of the genetic algorithms (GAs) and show that the E-GAs has less computational costs than the GAs (Sadr and Ghashochi Bargh 2012). Also, they found that for the 8 layers laminated composite plates, the GAs gives optimum results with the initial population equal to 35 in 13 iterations. But in this paper it is shown that the present hybrid DQ-TLBO can found the optimum solution with twenty number of population in seven iterations. Reducing the number of iterations together with the less number of population sizes in the optimization problem show the less computational costs of the presented solution technique.

The influence of the number of layer on the maximum non-dimensional frequency of simply supported 8-layered skew plate is studied in Table 2. As it is shown, for a skew plate by increasing the number of layer from 2 to 6, the optimum frequency change considerably; but after that by increasing the number of layer to 12, the maximum frequency has no significant change. Also, by increasing the skew angle of the plate with two layers, the fiber angles are increased clockwise. But by increasing the number of layers the problem become more complex and specific relations

Table 2 Maximum non-dimensional fundamental natural frequency and the related optimum fibers orientations for symmetric $[\theta_1^\circ/\theta_2^\circ/\cdots/\theta_{N_L}^\circ]_{\text{Sym}}$. SSSS laminated composite skew plates with different number of layer and skew angle (a/b = 1, a/h = 100, b = 1, N_p = 20, N_g = 50, $N_\xi = N_\eta$ = 17)

ψ	N_L	Ω	The optimum fibers orientation					
			θ_1°	θ_2°	θ_3°	θ_4°	θ_5°	θ_6°
0°	2	139.1795	0	---	---	---	---	---
	4	160.6730	-45	45	---	---	---	---
	6	174.0579	45	-45	-46	---	---	---
	8	178.0906	45	-45	-45	-45	---	---
	10	178.5160	45	-49	-45	-48	42	---
	12	178.4267	-46	44	43	46	-47	34
15°	2	159.1952	-52	---	---	---	---	---
	4	177.3503	-53	37	---	---	---	---
	6	187.0663	-53	37	38	---	---	---
	8	188.6887	-53	38	37	37	---	---
	10	188.3300	-53	38	-53	41	37	---
	12	188.6650	37	-54	-53	-52	-52	-53
30°	2	204.6379	-60	---	---	---	---	---
	4	219.3670	-60	30	---	---	---	---
	6	224.9180	-60	27	31	---	---	---
	8	225.0531	-60	30	-60	30	---	---
	10	223.6018	-62	-61	29	26	34	---
	12	224.9427	-62	-60	28	30	30	29
45°	2	306.3501	-67	---	---	---	---	---
	4	316.2149	-68	22	---	---	---	---
	6	315.8976	-68	22	-67	---	---	---
	8	316.3431	-73	-58	22	22	---	---
	10	316.8629	-69	-69	22	-54	18	---
	12	316.7214	-65	-77	-59	21	23	26
60°	2	588.8867	-75	---	---	---	---	---
	4	595.4286	-79	-37	---	---	---	---
	6	597.3617	-66	81	80	---	---	---
	8	597.2644	90	-57	-57	90	---	---
	10	597.7465	-60	90	-90	-90	-56	---
	12	597.7379	-60	90	90	-61	-90	-90

between the fibers orientations may not be achieved.

The effect of aspect ratio of laminated composite skew plate on the optimum fundamental natural frequency is shown in Table 3, which includes the results for the 8-layered simply supported skew plates. From the data presented in this table, it is seen that for a given value of the skew angle, by increasing the aspect ratio the optimum non-dimensional frequency increases.

In Table 4, the effect of boundary conditions on the optimum fundamental natural frequency of the skew plate is investigated. To describe a boundary condition, a four letter symbol is used. For example, “CSSS” means that first edge

Table 3 Maximum non-dimensional fundamental natural frequency and the related optimum fibers orientations for 8-layered symmetric $[\theta_1^\circ/\theta_2^\circ/\theta_3^\circ/\theta_4^\circ]_{\text{Sym}}$. SSSS laminated composite skew plates with different number of layer and skew angle (a/b = 1, a/h = 100, b = 1, N_p = 20, N_g = 50, N_L = 8, $N_\xi = N_\eta$ = 17)

ψ	a/b	Ω	The optimum fibers orientation			
			θ_1°	θ_2°	θ_3°	θ_4°
0°	0.5	178.3936	0	0	0	0
	1.0	178.0906	45	-45	-45	-45
	1.5	241.8377	-67	62	63	-90
	2.0	356.7873	90	90	90	90
15°	0.5	191.5477	-16	-16	-16	-16
	1.0	188.6887	-53	38	37	37
	1.5	257.3445	-85	-77	57	55
	2.0	383.0366	90	-90	-88	-90
30°	0.5	237.7416	-30	-31	-29	-29
	1.0	225.0531	-60	30	-60	30
	1.5	318.6618	-88	89	-86	-86
	2.0	475.4988	90	90	90	-89
45°	0.5	345.9980	-42	-41	-41	81
	1.0	316.3431	-73	-58	22	22
	1.5	462.5361	-90	-90	83	-33
	2.0	691.8710	86	87	90	-27
60°	0.5	623.4564	-55	-44	67	-90
	1.0	597.2644	90	-57	-57	90
	1.5	870.3157	-90	90	-56	90
	2.0	1247.6547	78	-90	-40	-40

(see Fig. 1) is clamped and second, third and fourth edges are simply supported, respectively. The results show that increasing the redundancy of the plate increases the non-dimensional fundamental frequency. Also, from the results presented in Tables 2-4, one can conclude that by increasing the skew angle, the optimum non-dimensional fundamental frequency increases.

5. Conclusions

As a first attempt, a hybrid differential quadrature (DQ) and teaching-learning based optimization (TLBO) method is introduced to obtain the maximum fundamental natural frequency of laminated composite skew plates. The first-order shear deformation theory (FSDT) is used to obtain the governing equations. Using a linear transformation and the differential quadrature method, the discretized form of equations of motion of the skew plate in computational domain are obtained. Then, the teaching-learning based optimization method is coupled with the DQM to find the maximum fundamental natural frequency of the laminated composite skew plate and the related optimum fiber(s) orientations as design variables. Robustness, accuracy and

Table 4 Influences of the skew angle on the non-dimensional fundamental natural frequency of 8-layered symmetric $[\theta_1^\circ/\theta_2^\circ/\theta_3^\circ/\theta_4^\circ]_{\mathrm{Sym}}$ laminated composite skew plate with different boundary conditions (B.C.) (a/h = 10, a = 1, a/b = 1, N_p = 20, N_g = 50, $N_\xi = N_\eta$ = 17)

ψ	B.C.	Ω	The optimum fibers orientation			
			θ_1°	θ_2°	θ_3°	θ_4°
0°	SSSS	178.0906	45	-45	-45	-45
	SSSC	210.7945	61	-58	-58	-58
	SSCC	227.9043	45	-45	-45	-45
	SCSC	286.8869	-90	-90	90	-87
	CCCS	290.2939	0	0	0	2
	CCCC	295.5449	0	-90	-5	90
15°	SSSS	188.6887	-53	38	37	37
	SSSC	225.4676	-71	47	-71	52
	SSCC	242.1253	-53	37	-52	38
	SCSC	308.4928	-90	-87	-90	-90
	CCCS	312.3703	-17	-18	-16	-17
	CCCC	318.5576	-88	-20	-19	89
30°	SSSS	225.0531	-60	30	-60	30
	SSSC	276.6221	-81	-83	28	36
	SSCC	295.0462	-60	30	-60	-62
	SCSC	382.7721	-90	90	-89	88
	CCCS	389.0818	-32	-34	-31	-32
	CCCC	403.7894	-53	-90	-16	-18
45°	SSSS	316.3431	-73	-58	22	22
	SSSC	401.6987	-90	90	90	-16
	SSCC	422.7459	-60	86	0	-54
	SCSC	559.6993	87	-90	-90	-90
	CCCS	572.9343	-45	-45	-45	-45
	CCCC	620.8962	-66	-64	90	19
60°	SSSS	597.2644	90	-57	-57	90
	SSSC	746.1258	-90	83	82	-39
	SSCC	782.7108	-58	80	86	90
	SCSC	1052.1406	84	-90	-90	-90
	CCCS	1080.1930	-58	-57	-57	-56
	CCCC	1252.8378	-75	-76	-75	-76

applicability of the introduced method for solving the problem under consideration are shown. Then, the effects of different parameters such as skew angle, number of layers and aspect ratio on the maximum frequency of the laminated composite skew plate with different boundary conditions are studied. The presented results can be used as benchmark solution for the future works.

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