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Forced vibration response in nanocomposite cylindrical shells - Based on strain gradient beam theory

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Abstract. In this paper, forced vibration of micro cylindrical shell reinforced by functionally graded carbon nanotubes (FG-CNTs) is presented. The structure is subjected to transverse harmonic load and modeled by beam model. The size effects are considered based on strain gradient theory containing three small scale parameters. The mixture rule is used for obtaining the effective material properties of the structure. Based on sinusoidal shear deformation theory of beam, energy method and Hamilton's principle, the motion equations are derived. Applying differential quadrature method (DQM) and Newmark method, the frequency curves of the structure are plotted. The effect of different parameters including, CNTs volume percent and distribution type, boundary conditions, size effect and length to thickness ratio on the frequency curves of the structure is studied. Numerical results indicate that the dynamic deflection of the FGX-CNT-reinforced cylindrical is lower with respect to other type of CNT distribution.

Keywords: forced vibration; sinusoidal shear deformation theory of beam; micro cylindrical shell; FG-CNTs; strain gradient theory

1. Introduction

CNTs have superior properties such as high tensile strengths, high aspect ratio, high stiffness and low density and however, can be used as the reinforce phase for the composite materials. However, in this paper, the effect of CNTs on the forced vibration of micro cylindrical shell is presented.

In nano and micro scales, considering size effect is essential. Mechanical analysis of nanostructures has been reported by many researchers (Zemri *et al*. 2015, Larbi Chaht *et al*. 2015, Belkorissat *et al*. 2015, Ahouel *et al*. 2016, Bounouara *et al*. 2016, Bouafia *et al*. 2017, Besseghier *et al*. 2017, Bellifa *et al*. 2017, Mouffoki *et al*. 2017, Li *et al*. 2017).

In the field of forced vibration and nanocomposite structures, Virgin and Plaut (1993) presented effect of axial load on forced vibrations of beams. Orhan (2007) investigated analysis of free and forced vibration of a cracked cantilever beam Repetto *et al*. (2012) studied forced vibrations of a cantilever beam. Dynamic analysis of an embedded single-walled carbon nanotube (SWCNT) traversed by a moving nanoparticle, which is modeled as a moving load, was investigated by Simsek (2012). A forced vibration analysis of functionally graded (FG) nanobeams was considered by Uymaz (2013) based on the nonlocal elasticity theory. Bhushan *et al*. (2014) investigated the hardening–softening nature of primary resonance curves of a doubly clamped cylindrical beam oscillator which represents a silicon nanowire. Forced vibrations of orthotropic shells when there is viscous resistance were considered by Ghulghazaryan (2015), when two versions of the spatial boundary conditions are given on the upper face of the shell, and the displacement vector is given on the lower face. A numerical approach was presented by Chen (2015) for the analysis of the forced vibration of a rigid surface foundation with arbitrary shape. The nonlinear forced vibration behavior of a cantilevered nanobeam was investigated by Dai *et al*. (2016), essentially considering the effect due to the surface elastic layer. Vibration analysis of embedded functionally graded (FG)-carbon nanotubes (CNT)-reinforced piezoelectric cylindrical shell subjected to uniform and non-uniform temperature distributions were presented by Madani *et al*. (2016). Chen *et al*. (2016) investigated the free and forced vibration characteristics of functionally graded (FG) porous beams with non-uniform porosity distribution whose elastic moduli and mass density are nonlinearly graded along the thickness direction. Akbarov and Mehdiyev (2017) studied forced vibration of the elastic system consisting of the hollow cylinder and surrounding elastic medium under perfect and imperfect contact. Su *et al*. (2018) studied the free and forced transverse vibrations of a nanowire on elastic substrate, in a systematic way. Nonlinear amplitude-frequency response, unstable boundary and dynamic responses of an axially moving viscoelastic sandwich beam under low- and high-frequency principle resonances were discussed and compared by Li *et al*. (2018). Akbaş (2018) investigated forced vibration analysis of functionally graded porous deep beams under dynamically load. Mohamed *et al*. (2018) presented a novel numerical procedure to predict nonlinear

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free and steady state forced vibrations of clamped–clamped curved beam in the vicinity of postbuckling configuration.

In the field of nanocomposite structures, an investigation on the nonlinear dynamic response and vibration of the imperfect laminated three-phase polymer nanocomposite panel resting on elastic foundations was presented by Duc *et al*. (2015). Van Thu and Duc (2016) presented an analytical approach to investigate the non-linear dynamic response and vibration of an imperfect three-phase laminated nanocomposite cylindrical panel resting on elastic foundations in thermal environments. Duc *et al*. (2017a, b, c) studied thermal and mechanical stability of a functionally graded composite truncated conical shell, plates and double curved shallow shells reinforced by carbon nanotube fibers. Based on Reddy's third-order shear deformation plate theory, the nonlinear dynamic response and vibration of imperfect functionally graded carbon nanotube-reinforced composite plates was analyzed by Thanh *et al*. (2017). Duc *et al*. (2018) presented the first analytical approach to investigate the nonlinear dynamic response and vibration of imperfect rectangular nanocompsite multilayer organic solar cell subjected to mechanical loads using the classical plate theory.

In this paper, the nonlinear forced vibration of micro cylindrical shell reinforced by FG-CNTs is studied based on sinusoidal beam model. The size-dependent are considered based on strain gradient theory. Based on energy method and Hamilton's principal, the motion equations are derived. Applying DQM and Newmark method, the frequency response of the structure is obtained. The effects of CNTs volume percent and distribution type, boundary conditions, size effect and length to thickness ratio are discussed in detail.

2. Sinusoidal beam model

Fig. 1 shows the geometry of the embedded micro cylindrical shell with radius, *R*, length, *L*, and thickness *h*. The structure is reinforced by FG-CNTs and is subjected to harmonic load.

There are many new theories for modeling of different structures. Some of the new theories have been used by

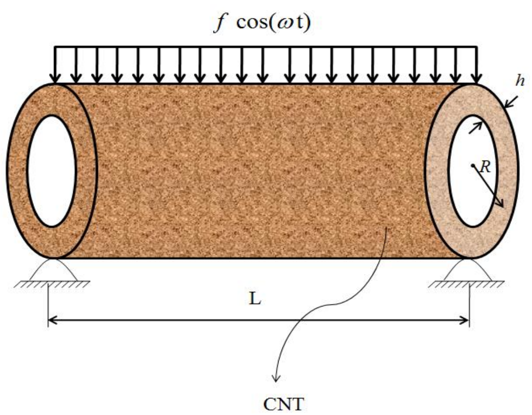

Fig. 1 The schematic of FG-CNT-reinforced cylindrical shell

Tounsi and co-authors (Bessaim *et al*. 2013, Bouderba *et al*. 2013, Belabed *et al*. 2014, Meziane *et al*. 2014, Zidi *et al*. 2014, Bourada *et al*. 2015, Bousahla *et al*. 2016, Beldjelili *et al*. 2016, Boukhari *et al*. 2016, Draiche *et al*. 2016, Bellifa *et al*. 2015, Attia *et al*. 2015, Mahi *et al*. 2015, Bennoun *et al*. 2016, El-Haina *et al*. 2017, Menasria *et al*. 2017, Chikh *et al*. 2017). Based on sinusoidal beam model, we have (Şimşek and Reddy 2013)

$$u_x(x,z,t)=u(x,t)-z\frac{\partial w(x,t)}{\partial x}+\Phi(z)\varphi(x,t)\ , \tag{1}$$

$$u_y(x,z,t)=0\ , \tag{2}$$

$$u_z(x,z,t)=w(x,t)\ , \tag{3}$$

where u and w are the axial and the transverse displacement; φ and ϕ are the transverse shear strain of and the total bending rotation of the cross-sections, respectively; $\Phi(z)$ can be defined as

$$\varphi(x,t)=\frac{\partial w(x,t)}{\partial x}-\phi(x,t)\ , \tag{4}$$

$$\Phi(z\ )=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right) \tag{5}$$

The strain relations of the structure can be expressed as

$$\varepsilon_x=\frac{\partial u}{\partial x}-z\ \frac{\partial^2 w}{\partial x^2}+\frac{h}{\pi}\frac{\partial^2 w}{\partial x^2}-\frac{h}{\pi}\frac{\partial\phi}{\partial x}f^{\ (\sin)}\ , \tag{6}$$

$$\varepsilon_{xz}=\frac{1}{2}\frac{\partial w}{\partial x}f^{\ (\cos)}-\frac{1}{2}\phi f^{\ (\cos)}\ , \tag{7}$$

where $f^{(\sin)}=sin\left(\frac{\pi z}{h}\right)$ and $f^{(\cos)}=\cos\left(\frac{\pi z}{h}\right)$.

3. Motion equations

Based on the SGT, the potential strain energy of the structure can be expressed as (Lei *et al*. 2013)

$$U=\frac{1}{2}\int_V\left(\sigma_{ij}\varepsilon_{ij}+P_i\gamma_i+\tau_{ijk}^{(1)}\eta_{ijk}^{(1)}+m_{ij}\ \chi_{ij}\ \right)dV\ , \tag{8}$$

where ε_{ij}, γ_i, $\eta_{ijk}^{(1)}$, χ_{ij} represent the strain, the dilatation gradient, the deviatoric stretch gradient and the symmetric rotation gradient tensors, respectively, which are defined by

$$\varepsilon_{ij}=\frac{1}{2}(\frac{\partial u_j}{\partial x_i}+\frac{\partial u_i}{\partial x_j})\ , \tag{9}$$

$$\gamma_i=\frac{\partial\varepsilon_{mm}}{\partial x_i}\ , \tag{10}$$

$$\eta_{ijk}^{(1)}=\frac{1}{3}\left(\frac{\partial\varepsilon_{jk}}{\partial x_i}+\frac{\partial\varepsilon_{ki}}{\partial x_j}+\frac{\partial\varepsilon_{ij}}{\partial x_k}\right)-\frac{1}{15}\left[\delta_{ij}\left(\frac{\partial\varepsilon_{mm}}{\partial x_k}+2\frac{\partial\varepsilon_{mk}}{\partial x_m}\right)+\delta_{jk}\left(\frac{\partial\varepsilon_{mm}}{\partial x_i}+2\frac{\partial\varepsilon_{mi}}{\partial x_m}\right)+\delta_{ki}\left(\frac{\partial\varepsilon_{mm}}{\partial x_j}+2\frac{\partial\varepsilon_{mj}}{\partial x_m}\right)\right], \tag{11}$$

$$\chi_{ij}=\frac{1}{2}\left(e_{ipq}\frac{\partial\varepsilon_{qj}}{\partial x_p}+e_{jpq}\frac{\partial\varepsilon_{qi}}{\partial x_p}\right), \tag{12}$$

where u_i, δ_{ij} and e_{ijk} are the displacement vector, the knocker delta and the alternate tensor, respectively. The classical stress tensor, σ_{ij}, the higher-order stresses, p_i, $\tau_{ijk}^{(1)}$ and m_{ij} are given by

$$\sigma_{ij}=E\delta_{ij}\varepsilon_{mm}+2G\left(\varepsilon_{ij}-\frac{1}{3}\varepsilon_{mm}\delta_{ij}\right), \tag{13}$$

$$\mathrm{p}_i=2l_0^2G\gamma_i\ , \tag{14}$$

$$\tau_{ijk}^{(1)}=2l_1^2G\eta_{ijk}^{(1)}\ , \tag{15}$$

$$m_{ij}=2l_2^2G\chi_{ij}\ , \tag{16}$$

where E and G are the bulk modulus and the shear modulus, respectively, (l_0, l_1, l_2) are independent material length scale parameters. The kinetic energy of the structure can be expressed as

$$K=\frac{1}{2}\int_{-\frac{h}{2}}^{\frac{h}{2}}\int_0^{2\pi}\int_0^{l}\rho\left[\left(\frac{\partial u_x}{\partial t}\right)^2+\left(\frac{\partial u_z}{\partial t}\right)^2\right]dx\,d\theta dz \tag{17}$$

where ρ denotes the density of structure. The external work due to the harmonic transverse load can be given as

$$W_t=-\int\left(f\ \cos(\omega\mathrm{t})\right)w\,dA, \tag{18}$$

where f and ω are amplitude and excitation frequency, respectively. However, based on Hamilton's principle, the motion equations can be written as

$$\begin{aligned}\delta u:\ &\left(\frac{4}{5}\frac{l_1^2G\pi P_0}{h}\right)\frac{\partial^3 w}{\partial x^3}-\left(\frac{2hEP_0}{\pi}\right)\frac{\partial^3 w}{\partial x^3}+\left(-\frac{4}{5}\frac{l_1^2G\pi P_0}{h}\right)\frac{\partial^2\phi}{\partial x^2}\\&+\left(\frac{8}{5}\frac{l_1^2hGP_0}{\pi}+4\frac{l_0^2hGP_0}{\pi}\right)\frac{\partial^5 w}{\partial x^5}-\left(-2\frac{hEP_0}{\pi}\right)\frac{\partial^2\phi}{\partial x^2}\\&+\left(-\frac{8}{5}\frac{l_1^2hGP_0}{\pi}-4\frac{l_0^2hGP_0}{\pi}\right)\frac{\partial^4\phi}{\partial x^4}-\left(2EA\right)\frac{\partial^2 u}{\partial x^2}\\&+\left(\frac{8}{5}l_1^2GA+2l_0^2GA\right)\frac{\partial^4 u}{\partial x^4}+\rho A\frac{\partial^2 u}{\partial t^2}=0,\end{aligned} \tag{19}$$

$$\begin{aligned}\delta w:\ &\left(\frac{16}{15}\frac{l_1^2G\pi^2L}{h^2}+\frac{1}{2}\frac{l_2^2GL\pi^2}{h^2}+2GO\right)\frac{\partial\phi}{\partial x}\\&-\begin{pmatrix}\frac{8}{5}l_1^2GI+\frac{4}{5}\frac{l_1^2h^2}{\pi^2}GI-\frac{8}{5}\frac{l_1^2h}{\pi}GP_1+2l_0^2GI\\-4\frac{l_0^2h}{\pi}GP_1+2\frac{l_0^2h^2}{\pi^2}GL\end{pmatrix}\frac{\partial^6 w}{\partial x^6}\\&-\left(-\frac{4}{5}l_1^2GL+\frac{4}{5}\frac{l_1^2G\pi P_1}{h}\right)\frac{\partial^3\phi}{\partial x^3}-\begin{pmatrix}\frac{8}{5}\frac{l_1^2h}{\pi}GP_0\\+4\frac{l_0^2h}{\pi}GP_0\end{pmatrix}\frac{\partial^5 u}{\partial x^5}\\&+\begin{pmatrix}-\frac{64}{15}l_1^2GO+\frac{32}{15}l_1^2GT_0-\frac{1}{2}l_2^2GO+l_2^2GT\\+4l_0^2GT_0-4l_0^2GO-2\frac{h^2}{\pi^2}EL+2\frac{h}{\pi}EP\end{pmatrix}\frac{\partial^3\phi}{\partial x^3}\\&-\left(2\frac{8}{15}\frac{l_1^2G\pi^2L}{h^2}+\frac{1}{4}\frac{l_2^2GL\pi^2}{h^2}+GO\right)\frac{\partial^2 w}{\partial x^2}\\&-\begin{pmatrix}-\frac{8}{5}\frac{l_1^2h^2}{\pi^2}GL+\frac{8}{5}\frac{l_1^2h}{\pi}GP_1\\-4\frac{l_0^2h^2}{\pi^2}GL+4\frac{l_0^2h}{\pi}GP\end{pmatrix}\frac{\partial^5\phi}{\partial x^5}-\left(-\frac{4}{5}l_1^2GL\right)\frac{\partial^3\phi}{\partial x^3}\\&-\left(\frac{4}{5}\frac{l_1^2G\pi P_0}{h}\right)\frac{\partial^3 u}{\partial x^3}+\left(2\frac{h}{\pi}EP\right)\frac{\partial^3 u}{\partial x^3}-\begin{pmatrix}\frac{8}{5}l_1^2GL\\-\frac{4}{5}\frac{l_1^2G\pi P_1}{h}\end{pmatrix}\frac{\partial^4 w}{\partial x^4}\\&+2\begin{pmatrix}-\frac{32}{15}l_1^2GT+\frac{8}{15}l_1^2GA+l_1^2\frac{32}{15}GO-l_2^2GT_0\\+l_2^2\frac{1}{4}GO+l_2^2GA-4l_0^2GT_0+2l_0^2GO\\+2l_0^2GA+EI-2\frac{h}{\pi}EP_1+\frac{h^2}{\pi^2}EL\end{pmatrix}\frac{\partial^4 w}{\partial x^4}\\&-\frac{\rho hP_1}{\pi}\frac{\partial^3\phi}{\partial x\partial t^2}-\frac{\rho h^2L}{\pi^2}\frac{\partial^4 w}{\partial x^2\partial t^2}+\frac{\rho h^2L}{\pi^2}\frac{\partial^3\phi}{\partial x\partial t^2}+\rho A\frac{\partial^2 w}{\partial t^2}=0,\end{aligned} \tag{20}$$

$$\begin{aligned}\delta\phi:\ &-\begin{pmatrix}-l_1^2\frac{64}{15}GO+l_1^2\frac{32}{15}GT_0-l_2^2\frac{1}{2}GO+l_2^2GT_0\\+4l_0^2GT_0-4l_0^2GO-2\frac{h^2}{\pi^2}EL+2\frac{h}{\pi}EP_1\end{pmatrix}\frac{\partial^3 w}{\partial x^3}\\&+\begin{pmatrix}-\frac{8}{5}\frac{l_1^2h^2}{\pi^2}GL+\frac{8}{5}\frac{l_1^2h}{\pi}GP\\-4\frac{l_0^2h^2}{\pi^2}GL+4\frac{l_0^2h}{\pi}GP\end{pmatrix}\frac{\partial^5 w}{\partial x^5}+\begin{pmatrix}-\frac{8}{5}\frac{l_1^2h}{\pi}GP\\-4\frac{l_0^2h}{\pi}GP_0\end{pmatrix}\frac{\partial^4 u}{\partial x^4}\\&+2\left(\frac{4}{5}\frac{l_1^2h^2}{\pi^2}GL+2\frac{l_0^2h^2}{\pi^2}GL\right)\frac{\partial^4\phi}{\partial x^4}\\&+\left(-\frac{16}{15}\frac{l_1^2G\pi^2L}{h^2}-\frac{1}{2}\frac{l_2^2GL\pi^2}{h^2}-2GO\right)\frac{\partial w}{\partial x}+\left(-\frac{4}{5}l_1^2GL\right)\frac{\partial^3 w}{\partial x^3}\\&+\left(-\frac{4}{5}\frac{l_1^2G\pi P_0}{h}\right)\frac{\partial^2 u}{\partial x^2}+\left(-\frac{4}{5}l_1^2GL+\frac{4}{5}\frac{l_1^2G\pi P_1}{h}\right)\frac{\partial^3 w}{\partial x^3}\\&+2\left(\frac{8}{15}\frac{l_1^2G\pi^2L}{h^2}+\frac{1}{4}\frac{l_2^2GL\pi^2}{h^2}+GO\right)\phi+2\left(\frac{4}{5}l_1^2GL\right)\frac{\partial^2\phi}{\partial x^2}\\&-2\left(l_1^2\frac{32}{15}GO+l_2^2\frac{1}{4}GO+2l_0^2GO+\frac{h^2}{\pi^2}EL\right)\frac{\partial^2\phi}{\partial x^2}\end{aligned} \tag{21}$$

$$-\left(-2\frac{h}{\pi}EP_0\right)\frac{\partial^2 u}{\partial x^2}-\frac{\rho h P_0}{\pi}\frac{\partial^2 u}{\partial t^2}+\frac{\rho h P_1}{\pi}\frac{\partial^3 w}{\partial x\partial t^2}-\frac{\rho h^2 L}{\pi^2}\frac{\partial^3 w}{\partial x\partial t^2}+\frac{\rho h^2 L}{\pi^2}\frac{\partial^2 \phi}{\partial t^2}=0\,, \tag{21}$$

Where the following integrals are defined

$$\left(A_i, I, P_0, P_1, T_0, L, O\right)=\int_A \beta dA\ , \tag{22}$$

where

$$\beta=\left(1, z^2, f^{(\sin)}, f^{(\cos)}, zf^{(\cos)}, \left(f^{(\sin)}\right)^2, \left(f^{(\cos)}\right)^2\right). \tag{23}$$

In present work, three different types of boundary conditions are considered as following

- Simply supported-Simply supported (SS)

$$@x=0, L\Rightarrow u=w=M_x=0, \tag{24}$$

- Clamped- Clamped (CC)

$$@x=0, L\Rightarrow u=w=\phi=0, \tag{25}$$

- Clamped- Simply supported (CS)

$$\begin{aligned}&@x=0\Rightarrow u=w=\phi=0,\\&@x=L\Rightarrow u=w=M_x=0.\end{aligned} \tag{26}$$

4. Mixture rule

According to this theory, the effective Young and shear moduli of structure may be expressed as (Liew *et al*. 2014)

$$E_{11}=\eta_1 V_{CNT}E_{r11}+(1-V_{CNT})E_m, \tag{27}$$

$$\frac{\eta_2}{E_{22}}=\frac{V_{CNT}}{E_{r22}}+\frac{(1-V_{CNT})}{E_m}, \tag{28}$$

$$\frac{\eta_3}{G_{12}}=\frac{V_{CNT}}{G_{r12}}+\frac{(1-V_{CNT})}{G_m}, \tag{29}$$

where E_{r11}, E_{r22} and E_m are Young's moduli of CNTs and matrix, respectively; G_{r11} and G_m are shear modulus of CNTs and matrix, respectively; V_{CNT} and V_m show the volume fractions of the CNTs and matrix, respectively; η_j (j = 1, 2, 3) is CNT efficiency parameter for considering the size-dependent material properties. Noted that this parameter may be calculated using molecular dynamic (MD). However, the CNT distribution for the mentioned patters obeys from the following relations

$$UD:\quad V_{CNT}=V^*_{CNT}, \tag{30}$$

$$FGV:\ V_{CNT}(z)=\left(1+\frac{2z}{h}\right)V^*_{CNT}, \tag{31}$$

$$FGO:\ V_{CNT}(z)=2\left(1-\frac{2|z|}{h}\right)V^*_{CNT}, \tag{32}$$

$$FGX:\ V_{CNT}(z)=2\left(\frac{2|z|}{h}\right)V^*_{CNT}, \tag{33}$$

where

$$V^*_{CNT}=\frac{w_{CNT}}{w_{CNT}+(\rho_{CNT}/\rho_m)-(\rho_{CNT}/\rho_m)w_{CNT}}, \tag{34}$$

where w_{CNT} is the mass fraction of the CNT; ρ_m and ρ_{CNT} present the densities of the matrix and CNT, respectively; v_{r12} and v_m are Poisson's ratios of the CNT and matrix, respectively.

5. Solution method

The main idea of the DQM is that the derivative of a function at a sample point can be approximated as a weighted linear summation of the function value at all of the sample points in the domain. The functions f and their k^{th} derivatives with respect to x can be approximated as (Madani *et al*. 2016)

$$\frac{d^n f(x_i)}{dx^n}=\sum_{j=1}^{N}C_{ij}^{(n)}f(x_j)\qquad n=1,\ldots,N-1, \tag{35}$$

Eventually, the motion equations can be expressed in the matrix form as below

$$\left\{[K][d]+[M][\ddot{d}]\right\}=[f\ \cos(\omega t)], \tag{36}$$

where $[d]$ = $[u\ w\ \phi]^T$ is the displacement vector; $[K]$ the linear stiffness matrices, respectively. Furthermore $[M]$ is the mass matrix. Finally, based on Newmark method, the frequency response of the structure can be calculated (Simsek and Kocatürk 2009).

6. Numerical results

The structure is made from Poly methyl methacrylate (PMMA) with the constant Poisson's ratios of v_m = 0.34, temperature-dependent thermal coefficient of α_m = $(1 + 0.0005\Delta T)\times10^{-6}/K$, and temperature-dependent Young moduli of E_m = $(3.52 - 0.0034T)$ GPa in which $T = T_0 + \Delta T$ and T_0 = 300 K (room temperature) (Madani *et al*. 2016).

The effect of distribution type of CNT on the frequency response of the structure is shown in Fig. 2. The CNT uniform distribution and three types of FG patterns namely as FGV, FGO and FGX are considered. It can be concluded that the FGX pattern is the best choice compared to other cases. It is because, in the FGX mode, the frequency of structure is maximum and the deflection is minimum with respect to other cases. It means the stiffness of system is higher with respect to other three patterns. Meanwhile, the

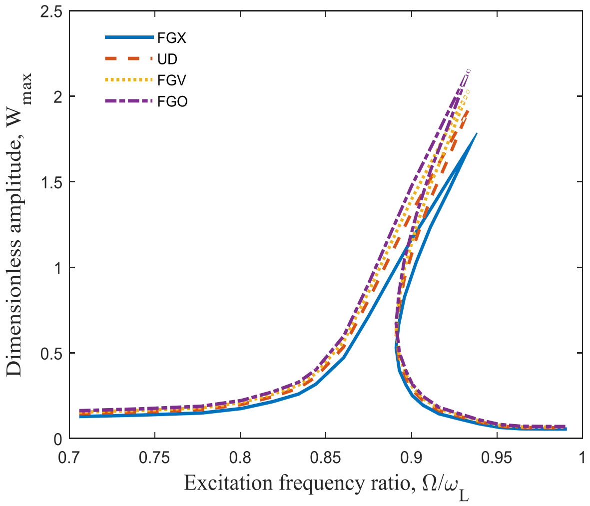

Fig. 2 The effect of CNT distribution on the frequency response of the structure

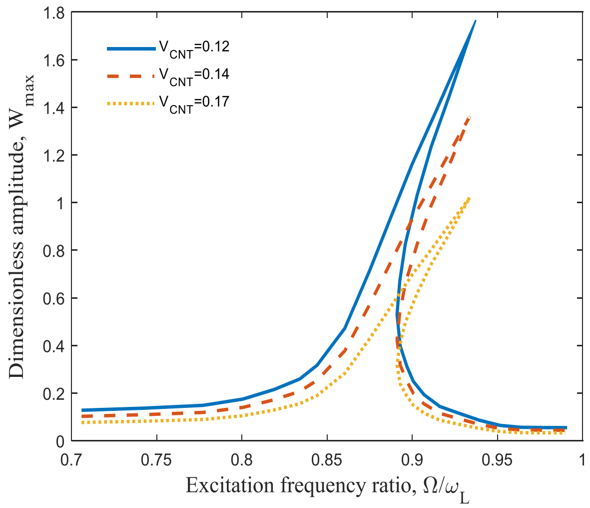

Fig. 3 The effect of CNT volume percent on the frequency response of the structure

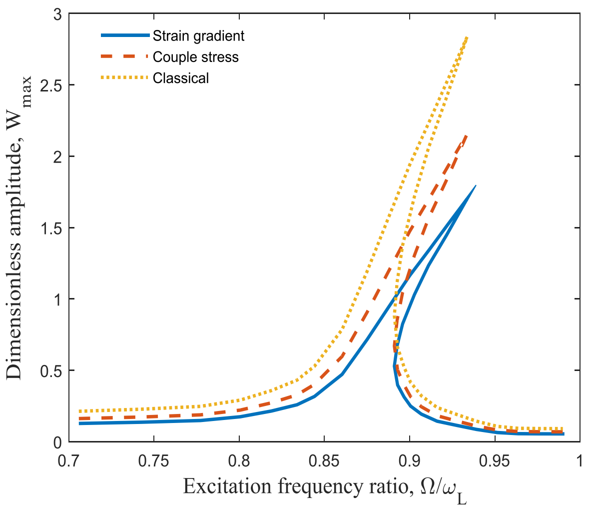

Fig. 4 The effect of different theories on the frequency response of the structure

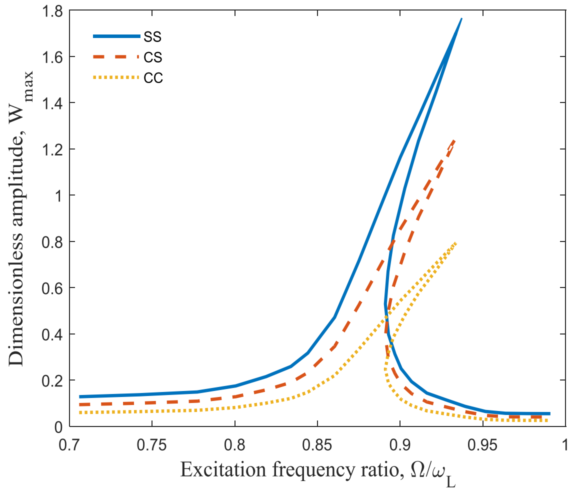

Fig. 5 The effect of different boundary conditions on the frequency response of the structure

frequency of structure with CNT uniform distribution is higher than FGV and FGO models. However, it can be concluded that the CNT distribution close to top and bottom are more efficient than those distributed nearby the mid-plane.

The effect of the CNT volume fraction on the frequency response of the cylindrical shell is illustrated in Fig. 3. It is observed that increasing the CNT volume fraction increases the frequency and decreases the deflection of the structure. This is due to the fact that the increase of CNT volume fraction leads to a harder structure.

Fig. 4 is plotted to study the effect of different theories of strain gradient, couple stress and classical. As can be seen the deflection of the strain gradient theory is lower than couple stress and the deflection of the couple stress is lower than classical one. This since the strain gradient theory has the three additional expression consisting of dilatation gradient tensor, the deviatoric stretch gradient tensor and the rotation gradient tensor.

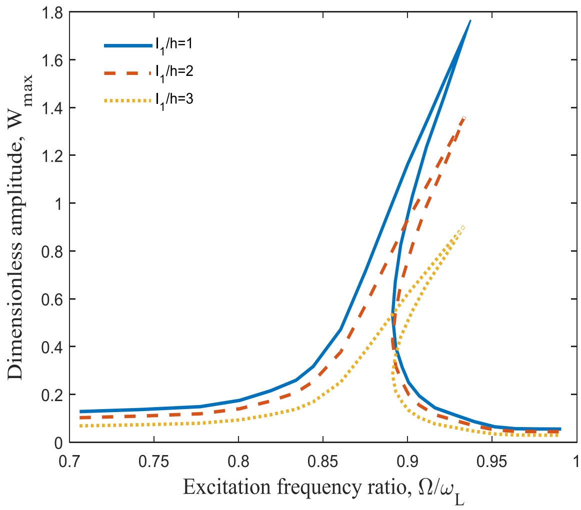

Fig. 6 The effect of material length scale parameter on the frequency response of the structure

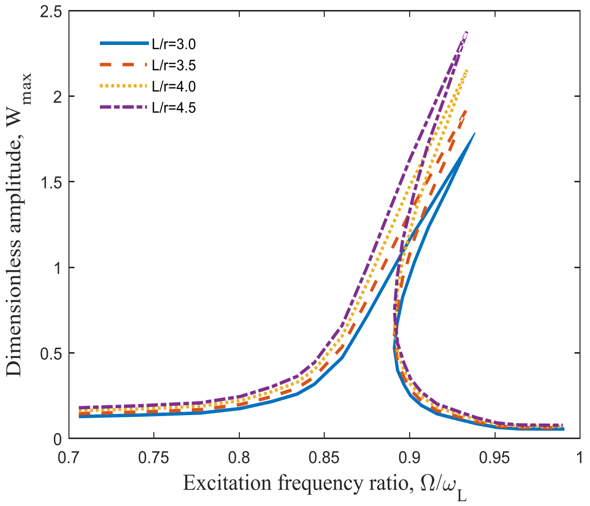

Fig. 7 The effect of length to radius ratio on the frequency response of the structure

Fig. 5 represents the effect of boundary conditions on the frequency response of the system. It can be seen that by considering CC boundary condition, the maximum amplitude decreases and the frequency is increased.

It is since the CC boundary condition leads to more bending rigidity.

The effect of material length scale parameter on the frequency response of the structure is shown in Fig. 6. As can be seen, by increasing the material length scale parameter, the amplitude of the system will be reduced.

The effects of length to radius ratio of the cylindrical shell on the frequency response of the structure are presented in Fig. 7. It is obvious that by increasing the length to radius ratio of the cylindrical shell, the structure becomes more softer, thus the deflection is increased.

7. Conclusions

Forced vibration of micro cylindrical shell reinforced by FG-CNTs was presented in this paper. The mixture rule was used for obtaining the effective material properties of the structure. Based on sinusoidal beam model, the motion equations were derived based on energy method. DQ and Newmark methods were used for obtaining the frequency response of the structure. The effect of different parameters including CNTs volume percent and distribution type, boundary conditions, size effect and length to thickness ratio on the frequency response of the of the system was studied. It can be concluded that the FGX pattern was the best choice compared to other cases. It was observed that increasing the CNT volume fraction increases the frequency and decreases the deflection of the structure. As can be seen the deflection of the strain gradient theory was lower than couple stress and the deflection of the couple stress was lower than classical one. In addition, by increasing the material length scale parameter, the amplitude of the system will be reduced. Furthermore, by considering CC boundary condition, the maximum amplitude decreases and the frequency is increased.

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