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Crack identification in Timoshenko beam under moving mass using RELM

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Abstract. In this paper, a new method has been proposed to detect crack in beam structures under moving mass using regularized extreme learning machine. For this purpose, frequencies of beam under moving mass used as input to train machine. This data is acquired by the analysis of cracked structure applying the finite element method (FEM). Also, a validation study used for verification of the FEM. To evaluate performance of the presented method, a fixed simply supported beam and two span continuous beam are considered containing single or multi cracks. The obtained results indicated that this method can provide a reliable tool to accurately identify cracks in beam structures under moving mass.

Keywords: crack detection; moving mass; frequencies; regularized extreme learning machine

1. Introduction

In this paper a novel approach is proposed to crack identification in Timoshenko beam under a moving mass. In recent years, response of beam structures excited by a moving mass was a major research field in the realm of the structural engineering and studied by many researchers (Azam *et al*. 2013, Kim *et al*. 2017, Roshandel *et al*. 2015). Ariaei *et al*. (2009) proposed an analytical approach, as well as a calculation method for determining the dynamic response of the undamped Euler–Bernoulli beams with breathing cracks under a point moving mass using the so-called discrete element technique (DET) and the finite element method (FEM). In other work, Cavadas *et al*. (2013) discussed the application of data-driven methods on moving-load responses in order to detect the occurrence and the location of damage. This work focused on two data-driven methods consisting moving principal component analysis (MPCA) and robust regression analysis (RRA). Also, Lee and Eun (2015) considered damage detection of the beam structure subject to a moving load including the inertia effect based on the only measurement data from strain gages and accelerometers without any baseline data. It is shown in the beam tests that the measured strain data can be more explicitly utilized in detecting damage than the acceleration data, and the mass magnitude and its velocity affect the feasibility of damage detection. Mirzaee *et al*. (2015) presented bridge damage detection procedure based on vibration measurements collected from a set of accelerometers. In other work, Xu *et al*. (2015) proposed a new simple and computationally efficient optimization algorithm to damage detection by using finite element model updating.

Pala and Reis (2012) studied the effects of inertial, centripetal, and coriolis forces on the dynamic response of a simply supported beam with a single crack under moving mass load. Nguyen and Tran (2010), presented a method for detecting a multi-cracked beam-like structure subjected to a moving vehicle. This method presented an idea for measuring the vibration directly from the vehicle for crack detection problem in practice. Also, Ariaei *et al*. (2010) presented an analytical method for the application of piezoelectric patches for the repair of cracked Timoshenko beams subjected to a moving mass. The criterion used for the repair is altering the first natural frequency of the cracked beam towards that of the healthy beam using a piezoelectric patch. Also, Yin *et al*. (2017) presented dynamic behavior of damaged bridge with multi-cracks under moving vehicular loads. They developed a new method for considering the effect of cracks and road surface roughness on the bridge response. Recently, Nobahari *et al*. (2017) presented a two-step damage detection method in truss structures using genetic algorithm. In the first step, using the residual force vector concept, the suspected damaged members are detected. In the second step, the precise location and severity of damage in the members are identified using the genetic algorithm and the results of the first step.

Various learning machines is used to detect damage and crack in structures due to their excellent pattern recognition capability. However, the focus of the majority of these studies was centered in damage detection. Recently, Kourehli (2017) used extreme learning machine to damage detection of plate-like structures. Also, Djemana *et al*. (2017) used electromechanical impedance and Extreme Learning Machine to Detect and Locate damage in structures. In this paper, an extreme learning machine (ELM) based algorithm is developed for estimating the damage location by using piezoelectric sensors data. Results showed that ELM can be used as a tool to predict of a single damage in structures. Comparatively few researches have

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been reported in the literatures aimed at studying crack identification in beam like structures under moving mass using artificial intelligence methods. Gökdağ (2013) presented a crack detection method for Beam-Like Structures under moving vehicle using particle swarm optimization. In this study, an objective function defined based on the difference of damaged beam dynamic response and the response calculated by the mathematical model of the beam. The optimization problem solved by the particle swarm optimization. Recently, Ghadimi and Kourehli (2017) proposed a method to multiple crack identification in Euler beams using extreme learning machine. In this study, the extreme learning machine used the modal strain energy and natural frequencies of cracked beam as input and crack states in beam elements as output. In other work, modified extreme learning machine (MELM) presented to detect crack of beam and frame structured by Ghadimi and Kourehli (2017).

In this paper a new application of regularized extreme learning machine (RELM) proposed to identify crack in beam structures under moving mass. Frequencies of beam under moving mass with 40 m/s velocity used as input and crack depth and position in beam elements as output to train regularized extreme learning machine. The proposed method has been applied to two numerical examples, namely a simply supported beam and two-span continuous beam. The obtained results reveal that the presented method is effective to detect crack in beam structures under moving mass.

2. The finite element formulation of a beam under a moving mass

In this section, the finite element formulation of a beam under a moving mass have been presented. Fig. 1 shows mesh discretion of a beam under a moving mass and the *j*th beam element on which the moving mass *mp* applies, at time *t*. The *j*th beam element has two equivalent nodal forces and displacements at each nodal point. The time dependent global position of the moving mass in the span is *xp*(*t*). The beam has (*n*-1) elements and *n* nodes.

When the beam is in vibration, the transverse (*y*) force component, between the moving mass and the beam, induced by the vibration and curvature of the deflected beam is (Cifuentes 1989)

$$f_y(x,t) = m_p\left(g - \frac{d^2 y(x_p,t)}{dt^2}\right)\delta(x - x_p) \tag{1}$$

With

$$x_p = x_0 + v_0 t + \frac{a_m}{2}t^2, \frac{dx_p}{dt} = v_0 + a_m t$$
$$\frac{d^2 x_p}{dt^2} = a_m \tag{2}$$

Where *fy* (*x*,*t*) is the applied force by the accelerating mass at point *x*, and time *t*. δ ($x - x_p$) and *g* are respectively the Dirac-delta function and the gravitational acceleration. Besides, x_0 and v_0 are, respectively, the initial position and

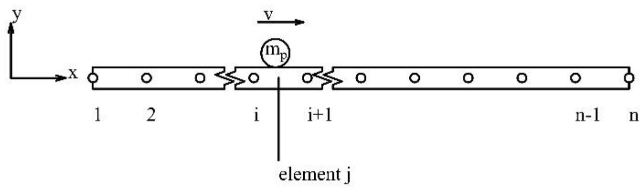

Fig. 1 Finite element of a beam subjected to a moving mass

initial speed of the mass at the time is zero; and a_m is the constant acceleration of the moving-mass. In this study, acceleration of the moving-mass is zero because of constant velocity of moving mass

$$x_p = x_0 + v_0 t, \frac{dx_p}{dt} = v_0, \frac{d^2 x_p}{dt^2} = 0 \tag{3}$$

The velocity and acceleration is computed from the total differential of the second order of the function *y*(*x*,*t*) with respect to time *t*, with variable contact point *xp* (Frýba 2013)

$$\frac{dy(x_p,t)}{dt} = \frac{\partial y}{\partial t} + \frac{\partial y}{\partial x}\frac{dx_p}{dt}$$
$$\frac{d^2 y(x_p,t)}{dt^2} = \frac{\partial^2 y(x,t)}{\partial t^2} + 2\frac{\partial^2 y(x,t)}{\partial t \partial x}\frac{dx_p}{dt} + \tag{4}$$
$$\frac{\partial^2 y(x,t)}{\partial x^2}\left(\frac{dx_p}{dt}\right)^2 + \frac{\partial y(x,t)}{\partial x}\frac{d^2 x_p}{dt^2}$$

Substituting Eq. (3) in Eq. (4) and rewriting this equation

$$\frac{dy(x_p,t)}{dt} = \frac{\partial y(x,t)}{\partial t} + \frac{\partial y(x,t)}{\partial x}v_0$$
$$\frac{d^2 y(x_p,t)}{dt^2} = \frac{\partial^2 y(x,t)}{\partial t^2} \tag{5}$$
$$+2\frac{\partial^2 y(x,t)}{\partial t \partial x}v_0 + \frac{\partial^2 y(x,t)}{\partial x^2}(v_0)^2$$

Eq. (5) can be written as in a different shape

$$\frac{dy(x_p,t)}{dt} = \dot{y}(x,t) + y'(x,t)v_0$$
$$\frac{d^2 y(x_p,t)}{dt^2} = \ddot{y}(x,t) + \tag{6}$$
$$2\dot{y}'(x,t)v_0 + y''(x,t)(v_0)^2$$

Where " ′ " and " · " are, respectively, spatial and time derivatives of deflection.

Substituting Eq. (4) in Eq. (1) and rewriting this equation

$$f_y(x,t) = (m_p g - m_p\ddot{y}(x,t) - 2m_p\dot{y}'(x,t)v_0$$
$$-m_p y''(x,t)(v_0)^2)\delta(x - x_p) \tag{7}$$

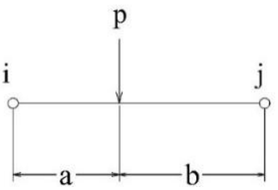

Fig. 2 Concentrated force on beam element

where $m_p\ddot{y}'(x,t)$, $2m_p\dot{y}'(x,t)v_0$ and $m_p y''(x,t)(v_0)^2$ are, respectively, the inertia force, the Coriolis force, the centripetal force of inertial effects of the moving-mass because it moves along the deflected shape of the beam. Besides, the graviton-force of the moving mass is $m_p g$. The jth beam element has two equivalent nodal forces and displacements at each nodal point. The time dependent global position of the moving mass in the span is $xp(t)$, while local position on the length of the element j is a. As can be seen Fig. 2, the equivalent nodal forces of the jth beam element under a concentrated force are

$$F_s = N_s * p, \; s = 1, 2, 3, 4 \tag{8}$$

Where Ns, are shape functions of the beam element given by (Clough and Penzien 2003)

$$\begin{aligned} N_1 &= 1 - 3\xi(t)^2 + 2\xi(t)^3, \\ N_2 &= (\xi(t) - 2\xi(t)^2 + \xi(t)^3)L_e \\ N_3 &= 3\xi(t)^2 - 2\xi(t)^3, \\ N_4 &= (-\xi(t)^2 + \xi(t)^3)L_e \end{aligned} \tag{9}$$

With

$$\xi(t) = \frac{a}{L_e} \tag{10}$$

where L_e is the length of jth beam element and a is the variable distance between the moving mass and the left end of the jth beam element, at time t, as shown in Fig. 2.

$$[F_1, F_2, F_3, F_4]^T = [N_1, N_2, N_3, N_4]^T * p \tag{11}$$

Concentrated force on beam element Fig. 2.

The relation between shape functions and transverse displacements of the jth beam element at position x and time t, is (Clough and Penzien 2003)

$$y = [N_1, N_2, N_3, N_4]*[u_1, u_2, u_3, u_4]^T \tag{12a}$$

$$y = N(x)*U(t) \tag{12b}$$

$$y_p = N(x_p)*U(t) \tag{12c}$$

Partial derivatives of deflection in the desired position on the elements are calculated as follows

$$\begin{aligned} \frac{\partial y}{\partial t}\Big|_{x=x_p} &= N(x_p)*\frac{\partial U}{\partial t}, \\ \frac{\partial U}{\partial t} &= [\dot{u}_1, \dot{u}_2, \dot{u}_3, \dot{u}_4]^T \end{aligned} \tag{13a}$$

$$\frac{\partial y}{\partial x}\Big|_{x=x_p} = \frac{\partial N}{\partial x}\Big|_{x=x_p} * U(t) \tag{13b}$$

$$\begin{aligned} \frac{\partial^2 y}{\partial t^2}\Big|_{x=x_p} &= N(x_p)*\frac{\partial^2 U}{\partial t^2}, \\ \frac{\partial^2 U}{\partial t^2} &= [\ddot{u}_1, \ddot{u}_2, \ddot{u}_3, \ddot{u}_4]^T \end{aligned} \tag{13c}$$

$$\frac{\partial^2 y}{\partial x^2}\Big|_{x=x_p} = \frac{\partial^2 N}{\partial x^2}\Big|_{x=x_p} * U(t) \tag{13d}$$

$$\frac{\partial^2 y}{\partial x\,\partial t}\Big|_{x=x_p} = \frac{\partial N}{\partial x}\Big|_{x=x_p} * \frac{\partial U(t)}{\partial t} \tag{13e}$$

Partial derivatives of shape functions are calculated as follows

$$\begin{aligned} \frac{\partial N_1}{\partial x}\Big|_{x=x_p} &= -6\frac{x_p}{L_e^2} + 6\frac{x_p^2}{L_e^3} = -6\frac{\xi}{L_e} + 6\frac{\xi^2}{L_e}, \; \xi = \frac{x_p}{L_e} \\ \frac{\partial N_2}{\partial x}\Big|_{x=x_p} &= (\frac{1}{L_e} - 4\frac{x_p}{L_e^2} + 3\frac{x_p^2}{L_e^3})L_e = (\frac{1}{L_e} - 4\frac{\xi}{L_e} + 3\frac{\xi^2}{L_e})L_e \\ \frac{\partial N_3}{\partial x}\Big|_{x=x_p} &= 6\frac{x_p}{L_e^2} - 6\frac{x_p^2}{L_e^3} = 6\frac{\xi}{L_e} - 6\frac{\xi^2}{L_e} \\ \frac{\partial N_4}{\partial x}\Big|_{x=x_p} &= (-2\frac{x_p}{L_e^2} + 3\frac{x_p^2}{L_e^3})L_e = (-2\frac{\xi}{L_e} + 3\frac{\xi^2}{L_e})L_e \end{aligned} \tag{14}$$

$$\begin{aligned} \frac{\partial^2 N_1}{\partial x^2}\Big|_{x=x_p} &= -6\frac{1}{L_e^2} + 6\frac{x_p}{L_e^3} = -6\frac{1}{L_e^2} + 6\frac{\xi}{L_e^2}, \; \xi = \frac{x_p}{L_e} \\ \frac{\partial^2 N_2}{\partial x^2}\Big|_{x=x_p} &= (-4\frac{1}{L_e^2} + 6\frac{x_p}{L_e^3})L_e = (-4\frac{1}{L_e^2} + 6\frac{\xi}{L_e^2})L_e \\ \frac{\partial^2 N_3}{\partial x^2}\Big|_{x=x_p} &= 6\frac{1}{L_e^2} - 12\frac{x_p}{L_e^3} = 6\frac{1}{L_e^2} - 12\frac{\xi}{L_e^2} \\ \frac{\partial^2 N_4}{\partial x^2}\Big|_{x=x_p} &= (-2\frac{1}{L_e^2} + 6\frac{x_p^2}{L_e^3})L_e = (-2\frac{1}{L_e^2} + 6\frac{\xi^2}{L_e^2})L_e \end{aligned} \tag{15}$$

Substituting Eqs. (13), (14), (15) in Eq. (7), interaction force between the beam and mass is calculated as follows

$$\{f\} = \{f_{Inertia}\} + \{f_{Coriolis}\} + \{f_{Centripetal}\} + \{f_{Gravitinal}\} \tag{16}$$

$$\begin{aligned} \{f_{Inertia}\} &= m_p[N]*\left[\frac{\partial^2 y}{\partial t^2}\Big|_{x=x_p}\right] = \\ & m_p\left[N(x_p)\right]^T * \left[N(x_p)\right]*\left[\frac{\partial^2 U}{\partial t^2}\right] \\ \{f_{Inertia}\} &= [m]\{\ddot{u}\} \end{aligned} \tag{17}$$

$$\left\{ f_{Coriolis} \right\} = 2m_p v_0 \left[N \right] * \left[\frac{\partial^2 y}{\partial x \partial t} \Big|_{x=x_p} \right] = m_p \left[N(x_p) \right]^T * \left[\frac{\partial N}{\partial x} \Big|_{x=x_p} \right] * \left[\frac{\partial U(t)}{\partial t} \right] \qquad \left\{ f_{Coriolis} \right\} = \left[c \right] \left\{ \dot{u} \right\} \tag{18}$$

$$\left\{ f_{Centripetal} \right\} = m_p {v_0}^2 \left[N \right]^T * \left[\frac{\partial^2 N}{\partial x^2} \Big|_{x=x_p} \right] * \left[U(t) \right] = m_p {v_0}^2 \left[N(x_p) \right]^T * \left[N''(x_p) \right] * \left[U(t) \right] \qquad \left\{ f_{Centripetal} \right\} = \left[k \right] \left\{ u \right\} \tag{19}$$

$$\left\{ f_{Gravitinal} \right\} = \left[N \right] * m_p g \tag{20}$$

Expanding Eqs. (17), (18), (19), mass, damping and stiffness matrices of two node beam element is calculated as follows

$$\left[m \right] = m_p \begin{bmatrix} {N_1}^2 & N_1 N_2 & N_1 N_3 & N_1 N_4 \\ N_2 N_1 & {N_2}^2 & N_2 N_3 & N_2 N_4 \\ N_3 N_1 & N_3 N_2 & {N_3}^2 & N_3 N_4 \\ N_4 N_1 & N_4 N_2 & N_4 N_3 & {N_4}^2 \end{bmatrix} \tag{21}$$

$$\left[c \right] = 2 m_p v_0 \begin{bmatrix} N_1 N_1' & N_1 N_2' & N_1 N_3' & N_1 N_4' \\ N_2 N_1' & N_2 N_2' & N_2 N_3' & N_2 N_4' \\ N_3 N_1' & N_3 N_2' & N_3 N_3' & N_3 N_4' \\ N_4 N_1' & N_4 N_2' & N_4 N_3' & N_4 N_4' \end{bmatrix} \tag{22}$$

$$\left[k \right] = m_p {v_0}^2 \begin{bmatrix} N_1 N_1'' & N_1 N_2'' & N_1 N_3'' & N_1 N_4'' \\ N_2 N_1'' & N_2 N_2'' & N_2 N_3'' & N_2 N_4'' \\ N_3 N_1'' & N_3 N_2'' & N_3 N_3'' & N_3 N_4'' \\ N_4 N_1'' & N_4 N_2'' & N_4 N_3'' & N_4 N_4'' \end{bmatrix} \tag{23}$$

Where [m], [c] and [k] are the mass, damping and stiffness matrices of the moving finite element; respectively. The values of the mass, damping and stiffness matrices, [m], [c] and [k], of the moving finite element are time-dependent. The dimensions of the mass, damping and stiffness matrices of the moving finite element are equal to the dimensions of the mass, damping, stiffness matrices of two node beam element. Hence, a beam element has two displacements DOF at each end nodal point; the dimensions of the property matrices of the moving finite element will be 4×4.

3. Cracked Timoshenko beam element

In this paper, cracked Timoshenko beam element, which was proposed by Mehrjoo *et al*. (2014) was used to model crack in beam elements. Moreover, it is assumed that no change would occur after crack in the mass matrix. The closed-form of cracked element stiffness matrix may be given as follows (Mehrjoo *et al*. 2014)

$$K_e^c = \frac{1}{B} \begin{bmatrix} 1 & A & -1 & L-A \\ A & CA & -A & A(L-C) \\ -1 & -A & 1 & C \\ L-A & A(L-C) & C & L^2 - 2AL + AC \end{bmatrix} \tag{24}$$

In which

$$K = \frac{Ew\left(h^2\right)}{72\pi P\left(\eta\right)} \tag{25}$$

$$A = \frac{L^2\left(K\right) + 2EIL\alpha}{2L\left(K\right) + 2EI} \tag{26}$$

$$B = \frac{L^3}{3EI} + \frac{L}{GA_S} - \frac{AL^2}{2EI} - \frac{A\alpha L}{K} \tag{27}$$

$$C = \frac{2GA_S KL^2 + 6EIGA_S L\alpha^2 + 6EIK}{3GA_S LK + 6GA_S EI\alpha} \tag{28}$$

$$\eta = \frac{d}{h} \tag{29}$$

$$P(\eta) = 0.638\eta^2 - 1.035\eta^3 + 3.7201\eta^4 - 5.1773\eta^5 + 7.553\eta^6 - 7.332\eta^7 + 2.4909\eta^8 \tag{30}$$

Where d is the crack depth, w represents the beam width, E is the modulus of elasticity, h denotes the beam depth, η represents a non-dimensional crack depth ratio, α represents the position of crack, A_S is the shear cross section, G is the shear modulus of elasticity and L is the length of beam element. Also, K is the equivalent spring stiffness for a single-sided open crack based on the theory of fracture mechanics (Ostachowicz and Krawczuk 1991).

Also, mass matrix of Timoshenko beam is given by (Gavin 2014)

$$m_e = \rho AL \begin{bmatrix} \frac{13}{35} + \frac{6}{5}\left(\frac{r}{L}\right)^2 & \frac{11}{210}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & \frac{9}{70} - \frac{6}{5}\left(\frac{r}{L}\right)^2 & -\frac{13}{420}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 \\ \frac{11}{210}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & \frac{1}{105}L^2 + \frac{2}{15}r^2 & \frac{13}{420}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & 0 \\ \frac{9}{70} - \frac{6}{5}\left(\frac{r}{L}\right)^2 & \frac{13}{420}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & \frac{13}{35} + \frac{6}{5}\left(\frac{r}{L}\right)^2 & -\frac{11}{210}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 \\ -\frac{13}{420}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & 0 & -\frac{11}{210}L + \frac{1}{10}\left(\frac{r}{L}\right)^2 & \frac{1}{105}L^2 + \frac{2}{15}r^2 \end{bmatrix} \tag{31}$$

In which

$$r = \sqrt{I/A} \tag{32}$$

Where, ρ is the material mass density; A is cross section of beam, l is the length of beam elements and I is the moment of inertia.

The mass and stiffness of the beam elements under moving mass is obtained as

$$K_e = K_e^c + k \tag{33}$$

$$M_e = m_e + m \tag{34}$$

Where K_e is the stiffness matrix of cracked beam element under moving mass; K_e^c is the cracked element stiffness matrix (Eq. (24)) and k is the stiffness matrices of the moving finite element (Eq. (23)). Also, M_e is the mass matrix of beam element under moving mass; m_e is the mass matrix of Timoshenko beam (Eq. (31)) and m is the mass matrices of the moving finite element (Eq. (21)).

The global stiffness and mass matrices of a cracked structure under moving mass can be made as follow

$$K = \sum_{e=1}^{N_e} K_e \tag{35}$$

$$M = \sum_{e=1}^{N_e} M_e \tag{36}$$

Where N_e is the total number of finite elements. So, the equation of motion of entire system under moving mass can be expressed as

$$M\ddot{X}(t) + C\dot{X}(t) + KX(t) = F(t) \tag{37}$$

Where M, C and K are the overall mass, damping and stiffness matrices, respectively.

4. Solving the equation of motion

In this paper changes in vibration frequencies of beams under moving mass used as a criterion to detect crack in structures. Also, Runge-Kutta 4th Order method have been used to solve the equation of motions of studied beams in MATLAB (2015). Eq. (37) can be written as

$$\ddot{X} = -[M]^{-1}[C]\dot{X} - [M]^{-1}[K]X + [M]^{-1}\{F\} \tag{38}$$

Defining the state vector $Z(t) = [X(t)\ \ \dot{X}(t)]^T$, the Eq. (37) can be written in state space form as

$$\begin{gathered}\{\dot{Z}\} = [A]\{Z\} + [B]\{F\} \\ \dot{Z} = \begin{Bmatrix}\dot{X} \\ \ddot{X}\end{Bmatrix},\ Z = \begin{Bmatrix}X \\ \dot{X}\end{Bmatrix}\end{gathered} \tag{39}$$

Where A is system matrix; and B is a influence vector for the external force. The system matrix and excitation influence vector are given by

$$A = \begin{bmatrix} 0_{n\times n} & I_{n\times n} \\ -M_{n\times n}^{-1}K_{n\times n} & -M_{n\times n}^{-1}C_{n\times n}\end{bmatrix} \tag{40}$$

$$B = \begin{bmatrix} 0_{n\times n} \\ M_{n\times n}^{-1}\end{bmatrix} \tag{41}$$

The most famous of all initial value problem (IVP) methods is the classic Runge-Kutta method of order 4

$$func(t_i, Z) = \dot{Z} \tag{42}$$

$$\begin{gathered}\dot{Z} = \frac{Z_{i+1} - Z_i}{h} \\ \frac{Z_{i+1} - Z_i}{h} = [A]Z_i + [B]\{F\}\end{gathered} \tag{43}$$

Where h is the time steps for solving equations.

$$\begin{aligned} k_1 &= h * func\ (t_i;\ Z_i) \\ k_2 &= h * func\ (t_i + h/2;\ Z_i + k_1/2) \\ k_3 &= h * func\ (t_i + h/2;\ Z_i + k_2/2) \\ k_4 &= h * func\ (t_i + h;\ Z_i + k_3) \\ Z_{i+1} &= Z_i + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4)\end{aligned} \tag{44}$$

Notice that this method uses values of *func* (t_i; Z_i) at 4 different points. In general a method needs n values of *func* to achieve order n. The constants used in this method and other methods are obtained from Taylor's Theorem.

Also, there are many other methods to solve the equation of motions. One of this methods, is the Modified Euler method which is consider f at both the beginning and end of the time step and take the average of the two. Doing this produces the Modified Euler method represented by the following equations

$$\begin{aligned} k_1 &= h * func(t_i, Z_i) \\ k_2 &= h * func(t_i + h, Z_i + k_1) \\ Z_{i+1} &= Z_i + \frac{1}{2}(k_1 + k_2)\end{aligned} \tag{45}$$

Here k_1 captures the information at the beginning of the time step (same as Euler), while k_2 is the information at the end of the time step.

5. Regularized extreme learning machine (RELM)

The ELM algorithm was proposed by Huang *et al*. (2006). The output function of ELM for generalized single-hidden layer feed forward networks is

$$f(x) = \sum_{i=1}^{m} \alpha_i k_i(x) = k(x)\alpha \tag{46}$$

Where $\alpha = [\alpha_1; \dots; \alpha_L]^T$ is the output weight vector between the hidden layer of m nodes to the output nodes, and $k_i(x) = G(a_i . b_i . x)$ is ELM nonlinear feature mapping. Also, a and b are hidden node parameters. The regularized ELM algorithm was proposed by Deng *et al*. (2009). The generalization ability of ELM can be increased by using regularization constant C (Zhang and Luo 2015). This type of ELM is called regularized ELM due to the regularization term. The main difference between ELM and RELM is that the simultaneous minimization of the training error and the norm of the output weights with a regularization parameter (Zhang and Luo 2015). Huang *et al*. (2012) studied the stability and generalization performance of ELM (Huang *et al*. 2015)

$$\min \frac{1}{2}\|\alpha\|^2 + \frac{C}{2}\sum_{i=1}^{N}\|e_i\|^2 \qquad \text{s.t.} \quad k(x_i)\alpha = t_i^T - e_i^T, \quad i = 1, \dots, N \tag{47}$$

By substituting the constraints of Eq. (44) into its objective function, the corresponding Lagrangian is defined by

$$\min L_{ELM} = \frac{1}{2}\|\alpha\|^2 + \frac{C}{2}\|T - K\alpha\|^2 \tag{48}$$

By setting the gradient of L_{ELM} with respect to α to zero, we have

$$L_{ELM} = \alpha^* - CK^T(T - K\alpha^*) = 0 \tag{49}$$

The closed form solution of α can be written as

$$\alpha^* = \left(K^TK + \frac{I}{C}\right)^{-1} K^TT \tag{50}$$

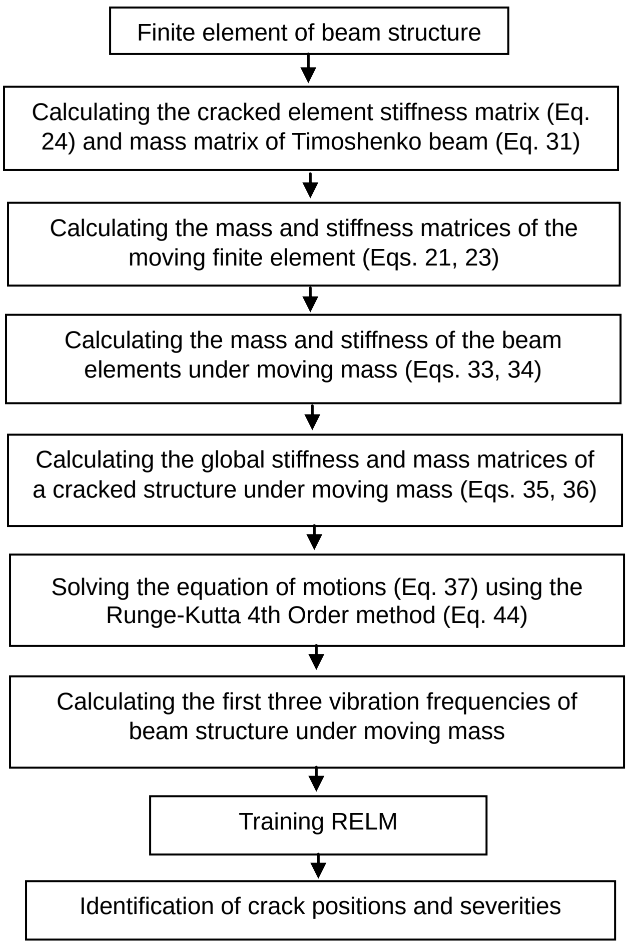

Fig. 3 Flowchart of the crack detection method

where I is an identity matrix.

6. Numerical examples

In this section, the efficiency and effectiveness of the proposed methods is evaluated through some numerically simulated crack identification tests. A simply supported beam and two span continuous beam are chosen with different scenarios of crack for each of them for the purpose. Also, a validation study used for verification of the FEM. The frequencies of the cracked beam under moving mass are used as the input patterns. Also, Fig. 3 shows the flowchart of the proposed method for crack detection and estimation in beam under moving mass.

6.1 A validation study

A simply supported beam subjected to a moving mass with an open crack is located in the middle of the beam proposed by Mahmoud and Zaid (2002). For the considered beam, the material properties include Young's modulus of $E = 2.1 \times 10^{11}$ Nm^{-2} and mass density of ρ = 7860 kg/m^3. The cross-sectional area and the moment of inertia of the beam are: A = 0.5 m^2 and I = 0.0417 m^4. Also, the moving mass is m = 39300 kg (Mahmoud and Zaid 2002, Ariaei *et al*. 2009). Also, beam span is 50 meters. Finite-element model consisting of 4 beam elements and 5 nodes is considered. The time steps are 0.01 to dynamic solution. Also, crack is modeled on second beam element with a = 0.99 (see Fig. 2) and crack depth ratio = 0.5.

In this paper, the classic Runge Kutta method of order 4, which is the most famous of all initial value problem (IVP) methods, have been used to solve the equation of motions of studied beams in MATLAB (2015). To investigate the performance of different methods to solve the equation of motions of simply supported beam, a comparative study has been done. As can be seen in Fig. 4, the obtained results (normalized deflections) are very close. For more study about the different methods to solve the equation of motions see (Shampine *et al*. 1999, Dormand and Prince 1980, Forsythe *et al*. 1977, Shampine and Gordon 1975).

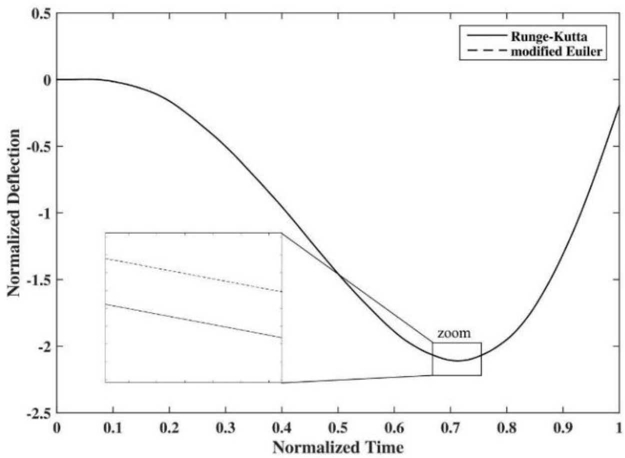

Fig. 4 A comparative study of different methods to solve the equation of motions of simply supported beam

Table 1 A comparative results of maximum normalized deflection of beam subjected to a moving mass

Speed ($m\ s^{-1}$)	Other references		Present study
10	Reference (Mahmoud and Zaid 2002)	1.34	1.321
	Reference (DET) (Ariaei et al. 2009)	1.249	
	Reference (FEM) (Ariaei et al. 2009)	1.288	
	Reference (Ariaei et al. 2010)	1.280	
20	Reference (Mahmoud and Zaid 2002)	1.43	1.418
	Reference (DET) (Ariaei et al. 2009)	1.388	
	Reference (FEM) (Ariaei et al. 2009)	1.411	
	Reference (Ariaei et al. 2010)	1.413	
40	Reference (Mahmoud and Zaid 2002)	2.13	2.097
	Reference (DET) (Ariaei et al. 2009)	2.054	
	Reference (FEM) (Ariaei et al. 2009)	2.046	
	Reference (Ariaei et al. 2010)	2.066	

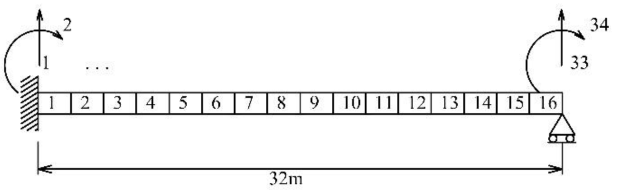

Fig. 5 Finite element model of fixed simply supported beam

Table 1 presents a comparative results of maximum normalized deflection of beam subjected to a moving mass. It should be noted that the deflection-time response is normalized relative to the value $m_p g L^3 / 48EI$, which is the static deflection due to m_p at mid-span. It can be seen that the obtained maximum normalized deflection are so close to deflections reported on other references.

6.2 Fixed simply supported beam

Fig. 5 shows a fixed simply supported beam with the same geometrical and physical parameters as those given in validation study. Finite-element model consisting of 16 beam elements and 17 nodes is considered. Also, the mass ratio is 0.2 (m_p (moving mass) / M (beam) = 0.2). The numerical studies are carried out within the MATLAB (2015) environment, which is used for the solution of finite element problems.

In this example, three different hypothetical crack scenarios are simulated as elements with cracks in different positions. The crack depth ratio and position in each element is listed in Table 2.

The first, second and third modes frequencies of cracked and intact beam under moving mass with 40 m/s velocity for the three different crack scenarios are presented in Fig. 6 to show frequency-change of the cracked and intact beams under moving mass. It should be noted that these frequencies are due to the moving mass. In other word, the mass and stiffness of studied system (cracked beam under

Table 2 Three different crack scenarios for fixed simply supported beam

Scenario	Crack element	Crack depth ratio
Scenario 1	7	0.2
Scenario 2	2	0.1
	11	0.3
Scenario 3	14	0.1
	8	0.3
	4	0.2

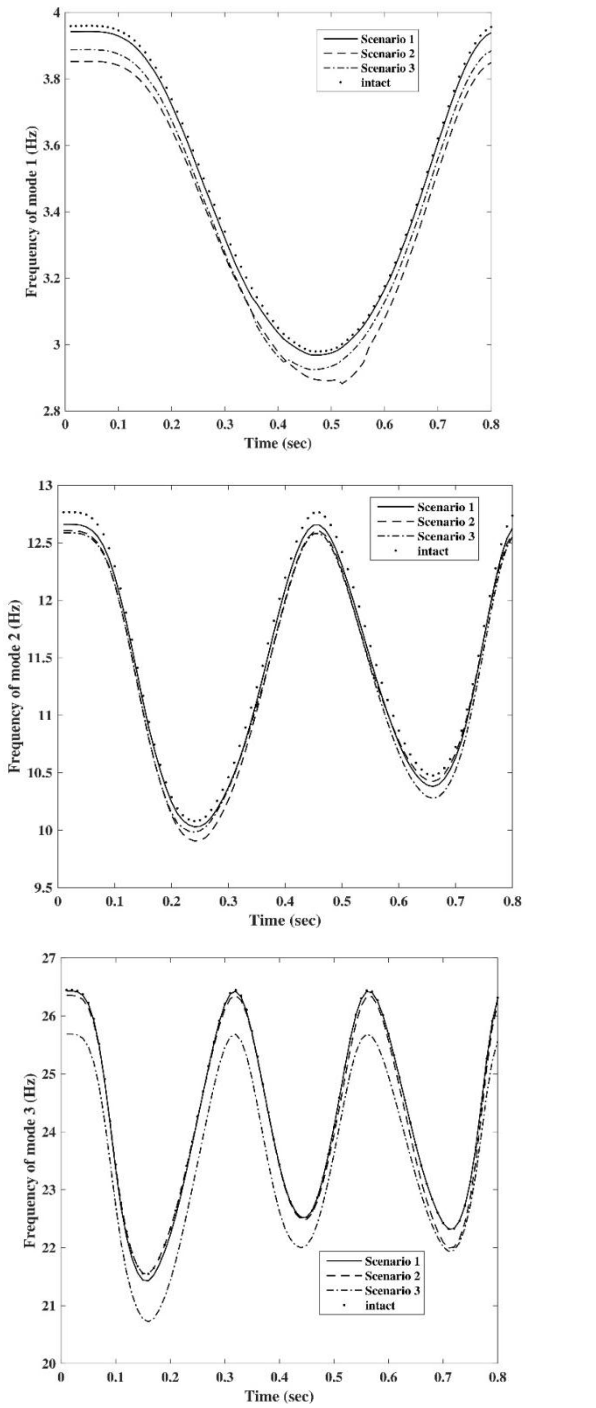

Fig. 6 The first three vibration frequencies of intact and cracked fixed simply supported beam under moving mass

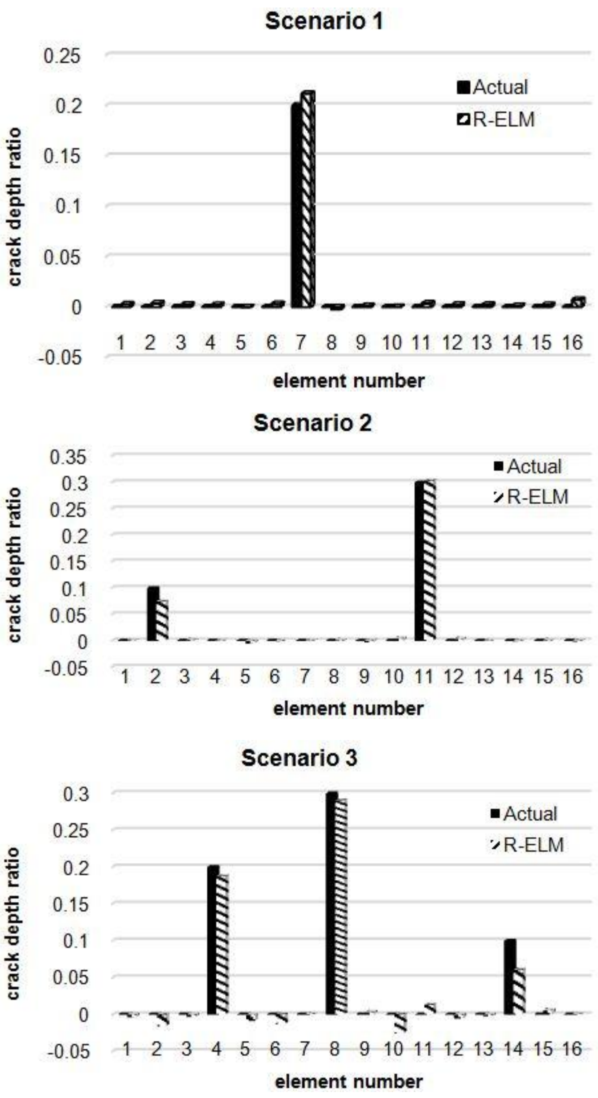

Fig. 7 Results for three crack scenarios of the fixed simply supported beam with RELM

moving mass) are variable and depends on the position of moving mass on the beam.

To generate the training patterns, 750 beam structures under moving mass with different crack depths and positions were considered, and their frequencies were computed using the numerical analyses. Because the time steps are 0.01 seconds to dynamic solution, 80 frequencies amount achieved for each mode. For this case, input is a matrix with 750 rows (number of samples for training) and 240 columns (using first three frequencies) and output is a matrix with 750 rows (number of samples for training) and 16 columns (16 beam elements).

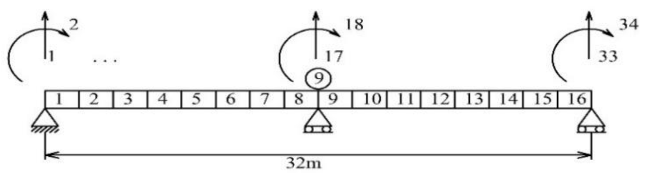

Fig. 8 The two-span continuous beam with the finite element model

Table 3 Two different crack scenarios for three-span continuous beam

Scenario	Crack element	Crack depth ratio
Scenario 1	9	0.3
Scenario 2	3	0.1
	14	0.3
Scenario 3	12	0.1
	6	0.3
	2	0.2

Fig. 7 illustrates the efficiency and effectiveness of the proposed method in detecting and quantifying of three different crack scenarios of the simply supported beam using RELM. It can be seen that the crack severity and locations can be obtained, for three different scenarios using the first three modes frequencies.

6.3 A two span continuous beam

A two-span continuous beam as illustrated in Fig. 8 with the same geometrical and physical parameters as those given for simply supported beam. Finite-element model consists of 16 beam elements and 17 nodes.

In this case, three different hypothetical crack depth ratio and position in each element is listed in Table 3.

The first three vibration frequencies of intact and cracked two span continuous beam under moving mass with 40 m/s velocity for the three different crack scenarios are presented in Fig. 9.

Fig. 10 shows the obtained results to detect crack severity and locations for three different scenarios using the first three modes frequencies and RELM. It can be seen that the crack severity and locations obtained correctly.

7. Conclusions

In this paper a new method proposed to crack identification in beam structure under moving mass. A new learning machine, namely, regularized extreme learning machine has been evaluated to detect crack in beam structures. First three modes frequencies of cracked beams subjected to moving mass used as input in which the crack position and severity in beam elements as output. This data is acquired by the analysis of cracked structure applying the finite element method (FEM). Runge-Kutta 4th Order method has been used to solve the equation of motions of studied beams in MATLAB (2015). A validation study has been done with an example that reported in literature. To evaluate the proposed method, two examples containing a simply supported beam and two span continuous beam has been used. The obtained results show:

- Regularized extreme learning machine is an effective machine to solve crack detection problems.
- Frequencies of dynamic system (beam and moving

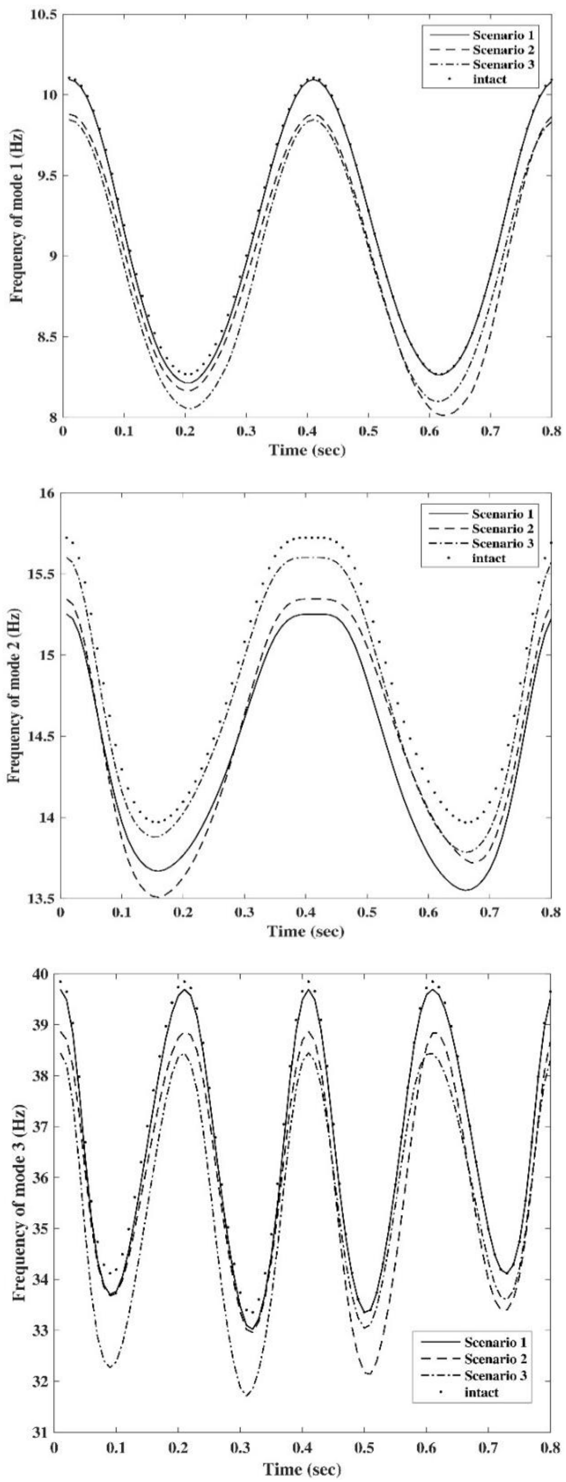

Fig. 9 The first three vibration frequencies of intact and cracked two span continuous beam under moving mass

mass) introduced as a useful characteristics of cracked beam in crack identification procedure.

- An effective method proposed method to detect crack in beam structures under moving mass.

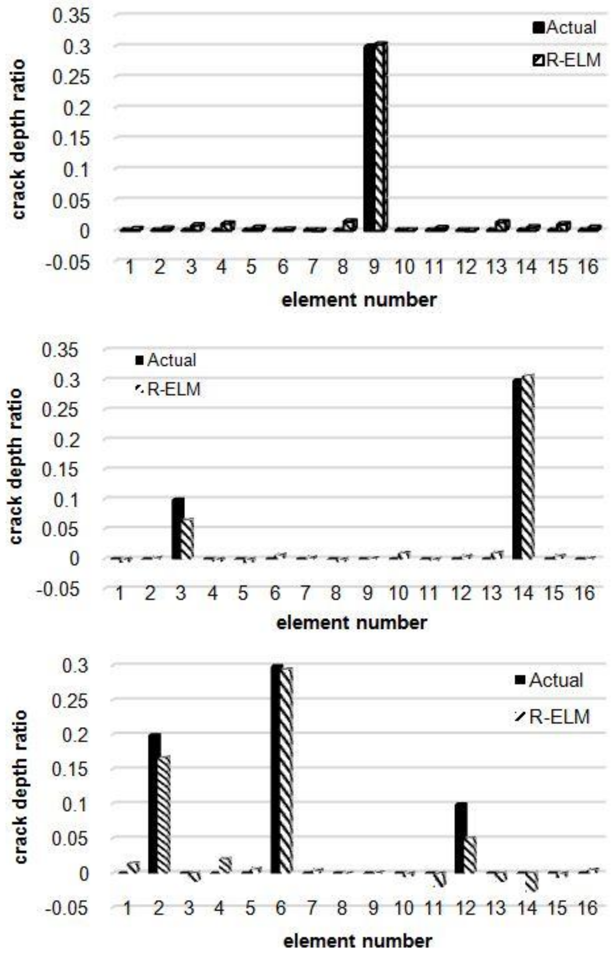

Fig. 10 Results for three crack scenarios of the fixed simply supported beam with RELM

References

Ariaei, A., Ziaei-Rad, S. and Ghayour, M. (2009), "Vibration analysis of beams with open and breathing cracks subjected to moving masses", *J. Sound Vib.*, **326**(3) 709-724.

Ariaei, A., Ziaei-Rad, S. and Ghayour, M. (2010), "Repair of a cracked Timoshenko beam subjected to a moving mass using piezoelectric patches", *Int. J. Mech. Sci.*, **52**(8), 1074-1091.

Azam, S.E., Mofid, M. and Khoraskani, R.A. (2013), "Dynamic response of Timoshenko beam under moving mass", *Scientia Iranica*, **20**(1), 50-56.

Cavadas, F., Smith, I.F. and Figueiras, J. (2013), "Damage detection using data-driven methods applied to moving-load responses", *Mech. Syst. Signal Process.*, **39**(1), 409-425.

Cifuentes, A.O. (1989), "Dynamic response of a beam excited by a moving mass", *Finite Elem. Anal. Des.*, **5**(3), 237-246.

Clough, R.W. and Penzien, J. (2003), *Dynamics of Structures*, (3rd Ed.), Computers and Structures, Inc., Berkeley, CA, USA.

Deng, W., Zheng, Q. and Chen, L. (2009), "Regularized extreme learning machine", *Proceedings of IEEE Symposium on Computational Intelligence and Data Mining, 2009. CIDM'09*, Nashville, TN, USA, March-April, pp. 389-395.

Djemana, M., Hrairi, M. and Al Jeroudi, Y. (2017), "Using electromechanical impedance and extreme learning machine to detect and locate damage in structures", *J. Nondestr. Eval.*, **2**(36), 1-10.

Dormand, J.R. and Prince, P.J. (1980) "A family of embedded Runge-Kutta formulae", *J. Comput. Appl. Math.*, **6**(1), 19-26.

Forsythe, G.E., Moler, C.B. and Malcolm, M.A. (1977), Computer methods for mathematical computations.

Frýba, L. (2013), *Vibration of Solids and Structures under Moving*

Loads, Volume 1, Springer Science & Business Media.
Gavin, H.P. (2014), *Structural Element Stiffness, Mass, and Damping Matrices*, Duke University.
Ghadimi, S. and Kourehli, S.S. (2016), "Multiple crack identification in Euler beams using extreme learning machine", *KSCE J. Civil Eng.*, **1**(21), 389-396.
Ghadimi, S. and Kourehli, S.S. (2017), "Crack detection of structures using modified extreme learning machine (MELM)", *Inverse Probl. Sci. Eng.*, **25**(7), 995-1013.
Gökdağ, H. (2013), "A crack identification approach for beam-like structures under moving vehicle using particle swarm optimization", *Mater. Test.*, **55**(2), 114-120.
Huang, G.B., Zhu, Q.Y. and Siew, C.K. (2006), "Extreme learning machine: theory and applications", *Neurocomputing*, **70**(1), 489-501.
Huang, G.-B., Zhou, H., Ding, X. and Zhang, R. (2012), "Extreme learning machine for regression and multiclass classification", *IEEE Transactions on Systems; Man and Cybernetics, Part B: Cybernetics*, **42**(2), 513-529.
Huang, G., Huang, G.B., Song, S. and You, K. (2015), "Trends in extreme learning machines: A review", *Neural Networks*, **61**, 32-48.
Kim, T., Park, I. and Lee, U. (2017), "Forced vibration of a Timoshenko beam subjected to stationary and moving loads using the modal analysis method", *Shock Vib.*, Article ID 3924921. DOI: 10.1155/2017/3924921
Kourehli, S.S. (2017), "Application of extreme learning machine to damage detection of plate-like structures", *Int. J. Struct. Stabil. Dyn.*, 1750068.
Lee, E.T. and Eun, H.C. (2015), "Damage detection using measurement response data of beam structure subject to a moving mass", *Latin Am. J. Solids Struct.*, **12**(12), 2384-2402.
Mahmoud, M.A. and Zaid, M.A. (2002), "Dynamic response of a beam with a crack subject to a moving mass", *J. Sound Vib.*, **256**(4), 591-603.
MATLAB (2015), Matlab User Manual; Mathwork Inc. Lowell, MA, USA.
Mehrjoo, M., Khaji, N. and Ghafory-Ashtiany, M. (2014), "New Timoshenko-cracked beam element and crack detection in beam-like structures using genetic algorithm", *Inverse Probl. Sci. Eng.*, **22**(3), 359-382.
Mirzaee, A. Shayanfar, M. and Abbasnia, R. (2015), "A novel sensitivity method to structural damage estimation in bridges with moving mass", *Struct. Eng. Mech., Int. J.*, **54**, 1217-1244. DOI: 10.12989/sem.2015.54.6.1217
Nguyen, K.V. and Tran, H.T. (2010), "Multi-cracks detection of a beam-like structure based on the on-vehicle vibration signal and wavelet analysis", *J. Sound Vib.*, **329**(21), 4455-4465.
Nobahari, M., Ghasemi, M.R. and Shabakhty, N. (2017), "Truss structure damage identification using residual force vector and genetic algorithm", *Steel Compos. Struct., Int. J.*, **25**, 485-496. DOI: 10.12989/scs.2017.25.4.485
Ostachowicz, W.M. and Krawczuk, M. (1991), "Analysis of the effect of cracks on the natural frequencies of a cantilever beam", *J. Sound Vib.*, **150**(2), 191-201.
Pala, Y. and Reis, M. (2012), "Dynamic response of a cracked beam under a moving mass load", *J. Eng. Mech.*, **139**(9), 1229-1238.
Roshandel, D., Mofid, M. and Ghannadiasl, A. (2015), "Modal analysis of the dynamic response of Timoshenko beam under moving mass", *Scientia Iranica. Transaction A; Civil Engineering*, **22**(2), 331.
Shampine, L.F. and Gordon, M.K. (1975), Computer solution of ordinary differential equations: The initial value problem.
Shampine, L.F., Reichelt, M.W. and Kierzenka, J.A. (1999), "Solving index-1 DAEs in MATLAB and Simulink", SIAM review, **41**(3), 538-552.
Xu, Y., Qian, Y., Song, G. and Guo, K.M. (2015), "Damage detection using finite element model updating with an improved optimization algorithm", *Steel Compos. Struct., Int. J.*, **19**, 191-208. DOI: 10.12989/scs.2015.19.1.191
Yin, X., Liu, Y., Deng, L. and Kong, X. (2017), "Dynamic Behavior of Damaged Bridge with Multi-Cracks Under Moving Vehicular Loads", *Int. J. Struct. Stabil. Dyn.*, **17**(02), 1750019.
Zhang, K. and Luo, M. (2015), "Outlier-robust extreme learning machine for regression problems", *Neurocomputing*, **151**, 1519-1527.

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