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A size-dependent quasi-3D model for wave dispersion analysis of FG nanoplates

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Abstract. In this paper, a new size-dependent quasi-3D plate theory is presented for wave dispersion analysis of functionally graded nanoplates while resting on an elastic foundation and under the hygrothermaal environment. This quasi-3D plate theory considers both thickness stretching influences and shear deformation with the variations of displacements in the thickness direction as a parabolic function. Moreover, the stress-free boundary conditions on both sides of the plate are satisfied without using a shear correction factor. This theory includes five independent unknowns with results in only five governing equations. Size effects are obtained via a higher-order nonlocal strain gradient theory of elasticity. A variational approach is adopted to owning the governing equations employing Hamilton's principle. Solving analytically via Fourier series, these equations gives wave frequencies and phase velocities as a function of wave numbers. The validity of the present results is examined by comparing them with those of the known data in the literature. Parametric studies are conducted for material composition, size dependency, two parametric elastic foundation, temperature and moisture differences, and wave number. Some conclusions are drawn from the parametric studies with respect to the wave characteristics.

Keywords: functionally graded materials; wave propagation; quasi-3D plate theory; higher-order nonlocal strain gradient elasticity theory; hygrothermal environment

1. Introduction

There are various benefits over the composite materials such as withstanding very high temperature gradients, less stress concentrations, further corrosive resistance, higher toughness and higher fracture resistance, therefore many researchers have focused on mechanical behavior of structures made of functionally graded materials (FGMs) (Xiong and Tian 2017, Ghadiri *et al*. 2017, Moradi-Dastjerdi and Momeni-Khabisi 2016, Rajanna *et al*. 2016, Sharma *et al*. 2017, Ehyaei *et al*. 2017, Ebrahimi and Jafari 2016, Arefi 2015, She *et al*. 2017a, b, c). These materials have continuous changes in thermo-mechanical properties according to a change in the microstructure or atomic order with a specific gradient. These types of materials are for achieving and designing specific properties for potential applications such as biological implants (Suresh and Mortensen 1998), biosystems, transport systems, energy conversion systems, semiconductors, cutting tools, wear resistant structures, thermal barrier coatings used in gas turbines and rocket nozzles (Movchan and Yakovchuk 2004) ballistic impact resistance, multi-functional structures (Vecchio 2005), optics (Miyamoto *et al*. 2013), biomedical engineering, electrical devices, diodes, computer circuit boards, sensors, turbine blades, car engine cylinders, inner wall of nuclear reactors, optical fibers, etc. (El-Wazery and El-Desouky 2015).

Research on nanostructures has become an important issue with the development of nanotechnology. Given the fact that classical theories do not have the ability to consider this effect, the use of size dependent continuum theories such as nonlocal elasticity theory (Eringen and Edelen 1972), strain gradient theory (Papargyri-Beskou and Beskos 2008), and nonlocal strain gradient theory (Lim *et al*. 2015) are developed to consider the small scale effects. The nonlocal and strain gradient elasticity theories is used to study the mechanical behaviors of nanostructures by many investigators (Bouafia *et al*. 2017, Besseghier *et al*. 2017, Mouffoki *et al*. 2017, Shahsavari *et al*. 2017, Hanifi Hachemi Amar *et al*. 2017, Karami *et al*. 2017a, Shahsavari and Janghorban 2017, Karami *et al*. 2018f, Karami and Janghorban 2016). Investigation of size effects on thermal buckling analysis of embedded FG nanoplates resting on an elastic medium is performed by Khetir *et al*. (2017). The analysis of size effects on wave propagation behavior of nanoplates resting on an elastic medium is studied by Wang *et al*. (2010a). Wang *et al*. (2010b) have also studied the size effects on axial wave propagation of nanoplates via nonlocal elasticity. Small size effect on the wave propagation of a piezoelectric nanoplate is investigated by Zhang *et al*. (2014) based on nonlocal elasticity theory. However, there are researchers who have discussed the limitations and inabilities of nonlocal theory of elasticity: the efficiency of differential form of nonlocal elastic law is predicting mechanical responses of nanobeams, especially those with clamped-free boundary conditions was examined

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by Romano *et al*. (2017). They discussed on the inability of nonlocal differential elasticity in analysis of nano-cantilevers and proposed a solution for such problems. Recently, it has been shown that nonlocal differential elasticity based model maybe ill-posed. Of course, according to the simplification of the nonlocal differential elasticity, some works have been focused on the size-dependent behaviors based upon the nonlocal differential models. More recently, it is shown that the nonlocal differential and integral elasticity based models may be not equivalent to each other. A nonlocal integral model to analyze the twisting static response of through-radius FG nanotubes was presented by Zhu and Li (2017d) using Eringen's nonlocal integral elasticity. The authors have shown that in comparison to the widely-used nonlocal differential model in the literature, the nonlocal integral model developed there was self-consistent and well-posed. Longitudinal and torsional dynamic problems for small-scaled rods were modeled by utilizing an integral formula of two-phase nonlocal theory by Zhu and Li (2017b).

Koutsoumaris *et al*. (2015) examined the application of bi-Helmholtz nonlocal elasticity model incorporating two nonlocal parameters in vibration analysis of carbon nanotubes. By comparing obtained results with those of molecular dynamics simulation, they concluded that present bi-Helmholtz nonlocal elasticity in more appropriate than one parameter nonlocal elasticity in predicting the mechanical behavior of nanostructures. It was seen by Shaat and Abdelkefi (2017) that the wave dispersion curves of nanobeams cannot be matched with experimental results by using nonlocality with one length scale parameter. So, they adopted a general nonlocal elasticity theory containing two length scale parameters to study the wave propagation in nanobeams with more accuracy. Therefore, the nonlocal strain gradient elasticity theory is used to investigate the mechanical response of different nanostructures (Karami *et al*. 2018a, b, c, 2017c, Shahsavari *et al*. 2018a, She *et al*. 2017d, 2018). Li *et al*. (2015) studied the wave dispersion of FG nanobeams using the nonlocal strain gradient theory, in which the stress accounts for not only the nonlocal elastic stress field but also the strain gradients stress field. A size-dependent Timoshenko beam model, which accounts for through-thickness power-law variation of a two-constituent functionally graded (FG) material, was derived in the framework of the nonlocal strain gradient theory by Li *et al*. (2016b). The longitudinal dynamic problem of a size-dependent elasticity rod was formulated by utilizing an integral form of nonlocal strain gradient theory by Zhu and Li (2017c). In another study, a size-dependent integral elasticity model was developed for a small-scaled rod in tension based on the nonlocal strain gradient theory by Zhu and Li (2017a). Based upon the nonlocal strain gradient theory wave propagation of fluid-conveying double-walled carbon nanotube was investigated by Zeighampour *et al*. (2017). Also, the effect of van der Waals force between the two intended walls and the DWCNT surroundings was modeled as Winkler foundation. Karami *et al*. (2018d) studied the wave propagation of single layer graphene sheets under the magnetic field effects using bi-Helmholtz nonlocal strain gradient theory. A size-dependent Euler–Bernoulli beam model was formulated and devoted to investigating the scaling effect on the post-buckling responses of FG nano-size beams with the von Kármán geometric nonlinearity using nonlocal strain gradient theory by Li and Hu (2017). Li *et al*. (2018) present a nonlocal strain gradient beam model incorporating the thickness effect for the size-dependent buckling analysis of nanobeams for the first time, and closed-form solutions were derived for post-buckling configuration and critical buckling force.

Up to now, one can find confusing results from studying length scale parameters including nonlocal and strain gradient theories in open literature. Researches on the basis of nonlocality have been reported a stiffness-softening behaviors in contrast with stiffness-hardening behaviors seen in conventional strain gradient theory. Besides, on the basis of nonlocal-strain gradient elasticity by Lim *et al*. (2015) with considering experimental data, it was shown that wave dispersion of nanobeams could not have accurate results with just using nonlocal elasticity or strain gradient theories. Most recently, few studies have been carried out to examine combined effects of nonlocal and strain gradient elasticity in wave propagation, vibration and buckling analysis of nanostructures (Karami *et al*. 2017b, Ebrahimi and Barati 2017, Li *et al*. 2016a, 2017, Barati 2017a, b).

Up to now, several different theories have been derived to study the static, vibration, buckling and wave dispersion of different structures like plates. The simplest and the most user friendly theory is called classical plate theory (CPT) which ignores the shear deformation influences. The assumptions of this theory cause inaccurate results in some cases. Next, we have first order shear deformation theory (FSDT) which has some limitations such as shear correction factor which is unknown. The limitations of above theories were solved by introducing higher order shear deformation theories (HSDT). Some of these higher order theories were considered the thickness stretching effect in their suggested displacement fields. A large application of the thickness stretching effect in FG plates has been proved in the study of Carrera *et al*. (2011). Buckling, bending and vibration of functionally graded sandwich beams were studied by Bennai *et al*. (2015) based on used a new higher-order shear and normal deformation theory; Atmane *et al*. (2015) investigated the effect of thickness stretching and porosity on mechanical behaviors of a FG beams resting on elastic foundations. This effect has an important role in moderately thick FG plates and beams; it was taken into account Shahsavari *et al*. (2018b), Hamidi *et al*. (2015), Chaht *et al*. (2015), Atmane *et al*. (2017).

It is obvious that based on the size-dependent quasi 3D theory, vibration and buckling analysis of FG structures are studied several times, however, there are few articles that investigate the wave propagation of such structures. Wave propagation analysis of FG nanobeams in thermal environment were studied by Ebrahimi and Barati (2016) based on nonlocal strain gradient theory and quasi-3D theory. Therefore, there isn't any research on wave propagation of an FG nanoplate subjected to hygro-thermal loading via higher-order nonlocal strain gradient theory and quasi-3D plate theory.

In this work, a new size-dependent five-variable plate theory is developed to investigate the wave propagation of FG nanoplates by introducing the thickness stretching effect of FGM plates. In order, to consider the small size effects a higher-order nonlocal strain gradient elasticity theory incorporating three scale factor is used. The governing equations are obtained based on Hamilton's principle and an analytical solution is applied to find the wave frequency and phase velocities of FG nanoplate. Influences of different parameters such as temperature and moisture rise, nonlocality, length scale parameter, material composition, elastic foundation parameters and wave number on wave characteristics of rectangular FG nanoplate are investigated.

2. Formulation of the problem

The configuration of rectangular functionally graded plate with the length a, the width b and the thickness h and resting on Winkler-Pasternak elastic foundation is illustrated in Fig. 1. The plate referred to a system of rectangular coordinate system *xyz*.

2.1 Generalized higher-order nonlocal strain gradient theory of elasticity

Due to the generalized higher-order nonlocal strain gradient elasticity theory that reported by Lim *et al*. (2015), the nonlocal stress at a reference point *x* depends not only on the strain at that point, but also on the strains at all other points within the volume *V*. Hence, the stress can be expressed by

$$\sigma_{ij} = \sigma_{ij}^{(0)} - \nabla\sigma_{ij}^{(1)} \tag{1}$$

in which σ_{ij} and σ_{ij}^{1} denote to strain ε_{ij} and strain gradient $\nabla\varepsilon_{ij}$, respectivly and the stress can be writen as follows

$$\sigma_{ij}^{(0)} = \int_V Q_{ijkl}\,\alpha_0\left(x, x', e_0 a\right)\varepsilon'_{kl}(x')dx' \tag{2}$$

$$\sigma_{ij}^{(1)} = \lambda^2 \int_V Q_{ijkl}\,\alpha_1\left(x, x', e_1 a\right)\nabla\varepsilon'_{kl}(x')dx' \tag{3}$$

where Q_{ijkl} are the elastic coefficients, λ is the strain gradient length scale, which is introduced to consider the significance of strain gradient stress field, e_0a and e_1a are lower and higher order nonlocal parameters, which are introduced to consider the significance of nonlocal elastic stress fields. α_0 (x, x', e_0a) and α_1 (x, x', e_1a) are the nonlocal functions for the classical stress tensor and the strain gradient stress tensor, respectively (Eringen 1983). The linear nonlocal differential operator $\mathbf{Ll}_i$, which can be written as the following form, is applied to the both sides of Eq. (1).

$$\mathrm{Ll}_i = 1 - \left(e_i a\right)^2 \nabla^2 \qquad \text{for } i = 0,1 \tag{4}$$

in which ∇^2 is the Laplacian operator. Due to this fact that solving differential equations are easier than integral equations, Lim *et al*. (2015) reported a general and extended constitutive equation for the higher-order nonlocal strain gradient theory as

$$\begin{aligned} &\left[1-\mu_1^2\nabla^2\right]\left[1-\mu_0^2\nabla^2\right]\sigma_{ij} = Q_{ijkl}\left[1-\mu_1^2\nabla^2\right]\varepsilon_{kl} \\ &-Q_{ijkl}\,\lambda^2\left[1-\mu_0^2\nabla^2\right]\nabla^2\varepsilon_{kl} \end{aligned} \tag{5}$$

where

$$\mu_0 = e_0 a, \mu_1 = e_1 a \tag{6}$$

The equivalent format of Eq. (5) is presented as

$$\mathrm{L}_\mu \sigma_{ij} = \mathrm{L}_l Q_{ijkl}\,\varepsilon_{kl} \tag{7}$$

where the linear operators are defined as

$$\begin{aligned} &\mathrm{L}_\mu = (1-\mu_1\nabla^2)(1-\mu_0\nabla^2), \mathrm{L}_l = (1-\mu_1\nabla^2) \\ &-\lambda^2(1-\mu_0\nabla^2)\nabla^2 \end{aligned} \tag{8}$$

2.2 Kinematics

To owning the governing equations, the displacement field of present theory is chosen on the basis of following assumptions: (1) the transverse displacement is partitioned into bending, shear and stretching components; (2) the in-plane displacement is partitioned into extension, bending and shear components; (3) the bending parts of the in-plane displacements are similar to those given by CPT; and (4) the shear parts of the in-plane displacements give rise to the hyperbolic variations of shear strains and hence to shear stresses through the thickness of the plate in such a way that the shear stresses vanish on the top and bottom surfaces of the plate. According to these assumptions, the following displacement field relations can be obtained

$$\begin{aligned} u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\ v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\ w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) + g(z)\varphi(x,y,t) \end{aligned} \tag{9}$$

in which u_0 and v_0 are the displacements along the *x* and *y* coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse displacement, respectively; and the additional

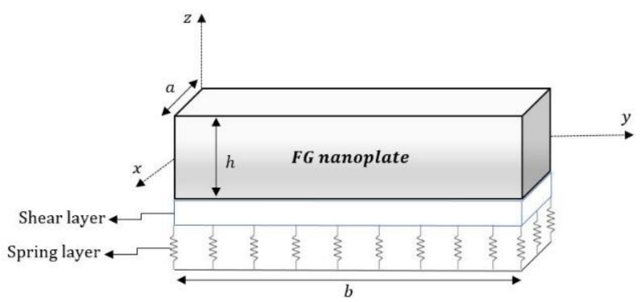


Fig. 1 Geometry of functionally graded nanoplates

displacement φ accounts for the effect of normal stress (stretching effect). In this study, the shape functions $f(z)$ and $g(z)$ are chosen based on the hyperbolic function presented by Shahsavari *et al*. (2018b) as

$$f(z) = -\left[r_1\left(\frac{z}{h}\right) + r_2 \sinh\left(\frac{z}{h}\right)\right]h \tag{10}$$

in which

$$r_1 = \frac{\cosh(\theta)}{24\sinh(\theta) - 11\cosh(\theta)} - 1, \quad r_2 = \frac{-1}{24\sinh(\theta) - 11\cosh(\theta)}, \ \theta = \frac{1}{2} \tag{11}$$

And

$$g(z) = 1 - f'(z) \tag{12}$$

The shape function that may be chosen to satisfy the following conditions.

$$f(0) = 0, \quad f\left(\pm\frac{h}{2}\right) = 0, \quad \int_{-\frac{h}{2}}^{\frac{h}{2}} f(z)dz = 0 \tag{13}$$

The common two-dimensional theories (Reddy 1984, Mahi and Tounsi 2015) satisfy the first conditions. The second condition show that the transverse shear stress is vanished at the top and bottom surfaces plate. While, the third condition means that the shape function should be odd function.

One can find that with the divide of displacement in thickness direction w into bending, shearing and stretching parts (i.e., $w = w_b = w_s = g(z)\varphi$) and by considering following assumptions defined as $\theta_1 = \frac{\partial w_s}{\partial x}$ and $\theta_2 = \frac{\partial w_s}{\partial y}$ where θ_1 and θ_2 are rotations of the yz and xz planes, the displacements introduced in present paper becomes simpler.

According to strain-displacement relations and with considering the displacement field in Eq. (9), the strains defined as follow

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xx}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xx}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \varepsilon_z = g'(z)\varepsilon_z^0 \tag{14}$$

where

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial y} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xx}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \tag{15}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xx}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \begin{bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi_z}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi_z}{\partial x} \end{Bmatrix}, \ \varepsilon_z^0 = \varphi_z \tag{15}$$

and

$$g'(z) = \frac{\partial g(z)}{\partial z} \tag{16}$$

2.3 Equation of motion

Based on the Hamilton's principle theory we try to find the Euler-Lagrange equations of FG nanoplate in thermal environment

$$\int_0^t \delta(U - K + V)dt = 0 \tag{17}$$

where δU is the variation of strain energy; δK is the variation of kinetic energy and δV is the variation of work done by external (applied) forces. The variation of strain energy is

$$\begin{aligned} \delta U &= \int_{-h/2}^{h/2}\int_A \begin{bmatrix} \sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \sigma_z \delta\varepsilon_z + \tau_{yz}\varepsilon_{yz} \\ +\tau_{xz}\varepsilon_{xz} + \tau_{xy}\varepsilon_{xy} \end{bmatrix} dA\,dz \\ &= \int_A \begin{bmatrix} N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_z \delta\varepsilon_z^0 + N_{xy}\delta\gamma_{xy}^0 + M_x^b \delta k_x^b \\ + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b + M_x^s \delta k_x^s + M_y^s \delta k_y^s \\ + M_{xy}^s \delta k_{xy}^s + Q_{yz}^s \gamma_{yz}^0 + Q_{xz}^s \gamma_{xz}^0 \end{bmatrix} dA = 0 \end{aligned} \tag{18}$$

where the stress resultants N, M, and Q are defined by

$$\begin{Bmatrix} N_x & N_y & N_z \\ M_x^b & M_y^b & M_{xy}^b \\ M_x^s & M_y^s & M_{xy}^s \end{Bmatrix} = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \tau_{xy}) \begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \quad N_z = \int_{-h/2}^{h/2} \sigma_z g'(z)dz, \tag{19}$$

and

$$(Q_{xz}^s, Q_{yz}^s)\int_{-h/2}^{h/2} (\tau_{xz}, \tau_{yz}) g(z)dz. \tag{20}$$

The first variation of work done by applied forces can be stated as

$$\delta V = \begin{bmatrix} \int_{-h/2}^{h/2}\int_A (N^T + N^H)\left(\dfrac{\partial^2\delta(w_b + w_s)}{\partial x^2} + \dfrac{\partial^2\delta(w_b + w_s)}{\partial y^2}\right) \\ -k_W\,\delta(w_b + w_s) + k_P\left(\dfrac{\partial^2\delta(w_b + w_s)}{\partial x^2} + \dfrac{\partial^2\delta(w_b + w_s)}{\partial y^2}\right) \end{bmatrix} dA\,dz \tag{21}$$

in which k_w and k_P are linear and shear coefficient of elastic foundation parameters; and the external forced N^T and N^H according to changes of temperature and moisture for FGMs are

$$N^T = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\alpha(z)(\Delta T)\mathrm{d}z \tag{22}$$

$$N^H = \int_{-\frac{h}{2}}^{\frac{h}{2}} E(z)\beta(z)(\Delta H)\mathrm{dz} \tag{23}$$

where $\Delta T = T - T_0$ and $\Delta H = H - H_0$ where T_0 and H_0 can introduce as the reference temperature and moisture, respectively.

The variation of kinetic energy of the plate can be written in the form

$$\begin{aligned}
\delta K &= \int_{-h/2}^{h/2}\int_A \left[\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + \dot{w}\,\delta\dot{w}\right] dA\,dz \\
&= \int_A \Big\{ I_0\left(\dot{u}_0\delta\dot{u}_0 + \dot{v}_0\delta\dot{v}_0 + (\dot{w}_b + \dot{w}_s)(\delta\dot{w}_b + \delta\dot{w}_s)\right) \\
&- I_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\delta\dot{v}_0\right) \\
&- J_1\left(\dot{u}_0\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\delta\dot{u}_0 + \dot{v}_0\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\delta\dot{v}_0\right) \\
&+ I_2\left(\frac{\partial\delta\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\delta\dot{w}_b}{\partial y}\frac{\partial\delta\dot{w}_b}{\partial y}\right) + K_2\left(\frac{\partial\delta\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\delta\dot{w}_s}{\partial y}\frac{\partial\delta\dot{w}_s}{\partial y}\right) \\
&+ J_2\left(\frac{\partial\dot{w}_b}{\partial x}\frac{\partial\delta\dot{w}_s}{\partial x} + \frac{\partial\dot{w}_s}{\partial x}\frac{\partial\delta\dot{w}_b}{\partial x} + \frac{\partial\dot{w}_b}{\partial y}\frac{\partial\delta\dot{w}_s}{\partial y} + \frac{\partial\dot{w}_s}{\partial y}\frac{\partial\delta\dot{w}_b}{\partial y}\right) \\
&+ J_1^s\left((\dot{w}_b + \dot{w}_s)\delta\dot{\varphi} + \dot{\varphi}\delta(\dot{w}_b + \dot{w}_s)\right) + K_2^s\dot{\varphi}\delta\dot{\varphi}\Big\} dA
\end{aligned} \tag{24}$$

In above relation, the differentiation with respect to the time variable t is defined with the dot-superscript; and (I_0, I_1, J_1, I_2, J_2, K_2) are mass inertias introduced as below

$$\begin{aligned}
&\left\{I_0, I_1, J_1, J_1^s, I_2, J_2, K_2, K_2^s\right\} \\
&= \int_{-h/2}^{h/2} \left\{1, z, f, g, z^2, zf, f^2, g^2(z)\right\}\rho(z)dz
\end{aligned} \tag{25}$$

Now, by substituting the expressions for δU, δV and δK from Eqs. (18), (21), and (24) into Eq. (17) and applying integrating by parts, with collecting the coefficients of δu_0, δv_0, δw_b, δw_s and $\delta\varphi$, following equilibrium equations are achieved

$$\delta u_0 : \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = I_0\ddot{u}_0 - I_1\frac{\partial\ddot{w}_b}{\partial x} - J_1\frac{\partial\ddot{w}_s}{\partial x} \tag{26}$$

$$\delta v_0 : \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = I_0\ddot{v}_0 - I_1\frac{\partial\ddot{w}_b}{\partial y} - J_1\frac{\partial\ddot{w}_s}{\partial y} \tag{27}$$

$$\begin{aligned}
&\delta w_b : \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x\,\partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - k_W w \\
&+ (k_P + N^T + N^H)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) = I_0\left(\ddot{w}_b + \ddot{w}_s\right) \\
&+ I_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - I_2\nabla^2\ddot{w}_b - J_2\nabla^2\ddot{w}_s + J_1^s\ddot{\varphi}
\end{aligned} \tag{28}$$

$$\begin{aligned}
&\delta w_s : \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x\,\partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} \\
&- k_W w + (k_P + N^T + N^H)\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right) \\
&= I_0\left(\ddot{w}_b + \ddot{w}_s\right) + J_1\left(\frac{\partial\ddot{u}_0}{\partial x} + \frac{\partial\ddot{v}_0}{\partial y}\right) - J_2\nabla^2\ddot{w}_b \\
&- K_2\nabla^2\ddot{w}_s + J_1^s\ddot{\varphi}
\end{aligned} \tag{29}$$

$$\delta\varphi : \frac{\partial Q_{xz}^s}{\partial x} + \frac{\partial Q_{yz}^s}{\partial y} - N_z = J_1^s\left(\ddot{w}_b + \ddot{w}_s\right) - K_2^s\ddot{\varphi} \tag{30}$$

2.4 Constitutive equations

The effective material properties of FG plate change continuously in the thickness direction according to the power-law distribution. The effective material properties (P_f) of FGM plate by using the power-law rule of mixture can be expressed by Reddy (2000)

$$P_f = P_c V_c + P_m V_m \tag{31}$$

where P_c and P_m are the material properties of ceramic and metal sides, and V_c and V_m are the volume fraction of ceramic and metal surfaces, respectively, and are related by

$$V_c + V_m = 1 \tag{32}$$

Then the volume fraction of ceramic side is defined as follows

$$V_c = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{33}$$

where ($n \geq 0$) is a non-negative parameter (power-law index or the volume fraction index) which determines the material distribution across the plate thickness.

According to Eqs. (31)-(32), the effective material properties of FG plates are variable across the thickness direction with the following form

$$p(z) = P_m + P_c\left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{34}$$

In the present investigation, the material properties such as Young's modulus E, Poisson's ratio v, thermal expansion α, moisture expansion coefficient β, shear modulus G, and mass density ρ can be determined by Eq. (33).

The linear constitutive relations of a FG plate can be written as

$$\begin{bmatrix}\sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy}\end{bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66}\end{bmatrix} \begin{bmatrix}\varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy}\end{bmatrix} \tag{35}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively.

According to the value of ε_z, the elastic constants should have different relations. If we ignore the strain in the z

direction, the elastic constants versus the material properties of FG plate can be written as follow

$$
\begin{aligned}
&C_{11} = C_{22} = \frac{E(z)}{\left(1-\nu^2\right)}, C_{12} = \nu C_{11}, \\
&C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)}
\end{aligned}
\tag{36}
$$

If thickness stretching is considered, $\varepsilon_z \neq 0$, then C_{ij} are the three-dimensional elastic constants, given by

$$
\begin{aligned}
&C_{11} = C_{22} = C_{33} = \frac{(1-\nu)}{\nu}\lambda(z), C_{12} = C_{13} = C_{23} = \lambda(z), \\
&C_{44} = C_{55} = C_{66} = G(z) = \mu(z) = \frac{E(z)}{2(1+\nu)}
\end{aligned}
\tag{37}
$$

where $\lambda(z) = \frac{\upsilon E(z)}{(1-2\upsilon)(1+\upsilon)}$ and $\mu(z) = G(z) = \frac{E(z)}{2(1+\upsilon)}$ are Lamé's coefficients. By substituting Eq. (14) into Eq. (35) and the subsequent results into Eqs. (19)-(20), the stress resultants are readily obtained as

$$
\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix} \begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix} \varepsilon_z^0, \; Q = A^s \gamma,
\tag{38}
$$

$$
N_z = R^a \varphi + L(\varepsilon_x^0 + \varepsilon_y^0) + L^a(k_x^b + k_y^b) + R(k_x^s + k_y^s)
\tag{39}
$$

where

$$
\begin{aligned}
&N = \left\{N_x, N_y, N_{xy}\right\}, \; M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}, \\
&M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}
\end{aligned}
\tag{40}
$$

$$
\begin{aligned}
&\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}, \; k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}, \\
&k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}
\end{aligned}
\tag{41}
$$

$$
\begin{aligned}
&A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \; B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \\
&D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}
\end{aligned}
\tag{42}
$$

$$
\begin{aligned}
&B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \; D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \\
&H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}
\end{aligned}
\tag{43}
$$

$$
\begin{aligned}
&Q = \left\{Q_{xz}^s, Q_{yz}^s\right\}, \; \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}, \; A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \\
&\begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \int_{-h/2}^{h/2} \lambda(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z)\dfrac{1-\nu}{\nu} \end{Bmatrix} g'(z)\, dz
\end{aligned}
\tag{44}
$$

Here the stiffness coefficients are defined as

$$
\begin{aligned}
&\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} \\
&= \int_{-h/2}^{h/2} \lambda(z)(1, z, z^2, f(z), zf(z), f^2(z)) \begin{Bmatrix} \dfrac{1-\nu}{\nu} \\ 1 \\ \dfrac{1-2\nu}{2\nu} \end{Bmatrix} dz
\end{aligned}
\tag{45}
$$

and

$$
(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s) = (A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s),
\tag{46}
$$

$$
A_{44}^s = A_{55}^s = \int_{-h/2}^{h/2} \mu(z)\left[g(z)\right]^2 dz
\tag{47}
$$

2.5 Equations of motion in terms of displacements

Due to generalized nonlocal strain gradient elasticity theory relations and introducing Eqs. (38)-(44) into Eqs. (26)-(30), the size-dependent equations of motion can be expressed in terms of displacements (δu_0, δv_0, δw_b, δw_s, $\delta\varphi$) and the appropriate equations take the form

$$
\begin{aligned}
&\mathcal{L}_l(A_{11}d_{11}u_0 + A_{66}d_{22}u_0 + (A_{12}+A_{66})d_{12}v_0) \\
&+\mathcal{L}_l(-B_{11}d_{111}w_b - (B_{12}+2B_{66})d_{122}w_b) \\
&+\mathcal{L}_l(-B_{11}^s d_{111}w_s (B_{12}^s + 2B_{66}^s)d_{122}w_s - Ld_1\varphi) \\
&= \mathcal{L}_\mu(I_0\ddot{u}_0 - I_1 d_1\ddot{w}_b - J_1 d_1\ddot{w}_s)
\end{aligned}
\tag{48}
$$

$$
\begin{aligned}
&\mathcal{L}_l(A_{22}d_{22}v_9 + A_{66}d_{11}v_0 + (A_{12}+A_{66})d_{12}u_0) \\
&+\mathcal{L}_l(-B_{22}d_{222}w_b - (B_{12}+2B_{66})d_{112}w_b) \\
&-\mathcal{L}_l(B_{22}^s d_{222}w_s + (B_{12}^s + 2B_{66}^s)d_{112}w_s - Ld_2\varphi) \\
&= \mathcal{L}_\mu(I_0\ddot{v}_0 - I_1 d_2\ddot{w}_b - J_1 d_2\ddot{w}_s)
\end{aligned}
\tag{49}
$$

$$
\begin{aligned}
&\mathcal{L}_l(B_{11}d_{111}u_0 + (B_{12}+2B_{66})d_{122}u_0 + (B_{12}+2B_{66})d_{122}v_0) \\
&+\mathcal{L}_l(B_{22}d_{222}v_0 - D_{11}d_{1111}w_b - 2(D_{12}+2D_{66})d_{1122}w_b) \\
&-\mathcal{L}_l(D_{22}d_{2222}w_b + D_{11}^s d_{1111}w_s + 2(D_{12}^s + 2D_{66}^s)d_{1122}w_s) \\
&-\mathcal{L}_l(D_{22}^s d_{2222}w_s - L^a(d_{11}\varphi + d_{22}\varphi)) = \mathcal{L}_\mu(I_0(\ddot{w}_b + \ddot{w}_s)) \\
&+\mathcal{L}_\mu(I_1(d_1\ddot{u}_0 + d_2\ddot{v}_0) - I_2(d_{11}\ddot{w}_b + d_{22}\ddot{w}_b) - J_2(d_{11}\ddot{w}_s + d_{22}\ddot{w}_s)) \\
&+\mathcal{L}_\mu(J_1^s\ddot{\varphi}_z - q)
\end{aligned}
\tag{50}
$$

$$\begin{aligned}
&\mathrm{L}_l(B_{11}^s d_{111}u_0 + (B_{12}^s + 2B_{66}^s)d_{122}u_0 + (B_{12}^s + 2B_{66}^s)d_{122}v_0) \\
&+\mathrm{L}_l B_{22}^s d_{222}v_0 - D_1^s d_{1111}w_b - 2(D_{12}^s + 2D_{66}^s)d_{1122}w_b) \\
&-\mathrm{L}_l(D_{22}^s d_{2222}w_b + H_{11}^s d_{1111}w_s + 2(H_{12}^s + 2H_{66}^s)d_{1122}w_s \\
&-\mathrm{L}_l(H_{22}^s d_{2222}w_s - A_{44}^s d_{11}w_s - A_{55}^s d_{22}w_s) - R(d_{11}\varphi + d_{22}\varphi)) \\
&+\mathrm{L}_l(A_{44}^s d_{11}\varphi + A_{55}^s d_{22}\varphi) = \mathrm{L}_\mu(I_0(\ddot{w}_b + \ddot{w}_s) + J_1(d_1\ddot{u}_0 + d_2\ddot{v}_0)) \\
&-\mathrm{L}_\mu(J_2(d_{11}\ddot{w}_b + d_{22}\ddot{w}_b)K_2(d_{11}\ddot{w}_s + d_{22}\ddot{w}_s) - J_1^s\ddot{\varphi}_z + q)
\end{aligned} \tag{51}$$

$$\begin{aligned}
&\mathrm{L}_l(L(d_1u_0 + d_2v_0) - L^a(d_{11}w_b + d_{22}w_b)(R - A_{44}^s)d_{11}w_s) \\
&+\mathrm{L}_l(+(R - A_{55}^s)d_{22}w_s + R^a\varphi - A_{44}^s d_{11}\varphi - A_{55}^s d_{22}\varphi) \\
&= \mathrm{L}_\mu(J_1^s(\ddot{w}_b + \ddot{w}_s) + K_2^s\ddot{\varphi})
\end{aligned} \tag{52}$$

in which d_i, d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$\begin{aligned}
&d_i = \frac{\partial}{\partial x_i},\ d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j},\ d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \\
&d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad (i, j, l, m = 1, 2)
\end{aligned} \tag{53}$$

3. Solution procedure

This section is devoted to the solution of the governing equations of a functionally graded (FG) nanoplate. To this end, assuming the displacement fields of the waves propagating in the *x*-*y* plane with the following form of displacement field.

$$\begin{aligned}
&u_0 = A_1 \exp i\left(xk_x + yk_y - \omega t\right), \\
&v_0 = A_2 \exp i\left(xk_x + yk_y - \omega t\right), \\
&w_b = A_3 \exp i\left(xk_x + yk_y - \omega t\right), \\
&w_s = A_4 \exp i\left(xk_x + yk_y - \omega t\right), \\
&w_{st} = A_5 \exp i\left(xk_x + yk_y - \omega t\right)
\end{aligned} \tag{54}$$

where $A_1 - A_4$ are the coefficients of wave amplitude which must be determined; k_x and k_y are the wave numbers of wave propagation along *x*- and *y*- directions, respectively; $i = \sqrt{-1}$; and ω is eigenfrequency. Substituting Eq. (54) into Eqs. (48)-(52) gives

$$\left([K] - \omega^2[M]\right)\{\Delta\} = 0 \tag{55}$$

in which $[K]$ and $[M]$ are the stiffness matrix and the mass matrix, respectively and the eigenvector can be given $\Delta = \{A_1, A_2, A_3, A_4\}^T$.

The dispersion relations of wave propagation in the FG nanoplate can be developed by setting the following determinant to zero

$$\left|[K] - \omega^2[M]\right| = 0 \tag{56}$$

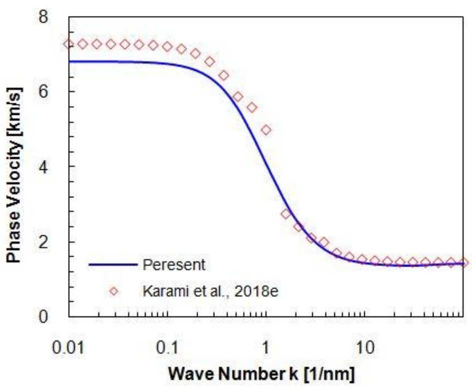


Fig. 2 Comparison of phase velocity in rectangular FG nanoplates versus wave number

By setting $k_x = k_y = k$, the phase velocity can be defined as

$$C = \frac{\omega}{k} \tag{57}$$

The phase velocities of FG nanoplate at $n = 1$, $\lambda = 0.2$, $\mu_0 = \mu_1 = \mu = 1.0$ are compared with those of presented by (Karami *et al*. 2018e) and the results are plotted in Fig. 2. It is revealed that presented model and solution can accurately predict the wave behaviors of FG nanoplates.

4. Numerical results

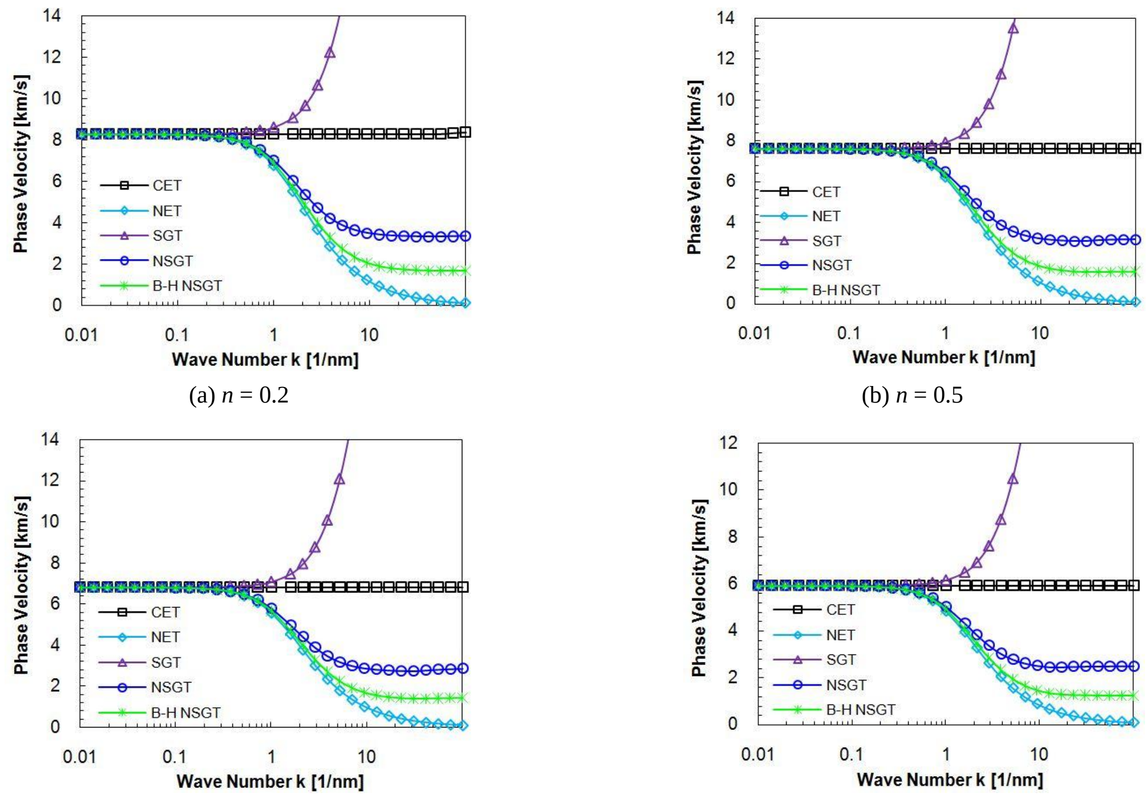
An analytical model is developed to represent a size-dependent wave propagation analysis of FG nanoplate made of Al and Al_2O_3 while is resting on elastic Winkler-Pasternak foundation and under hygrothermal environment. Also, the material properties of Al/Al_2O_3 FG plate are tabulated in Table 1. Governing equations to describe the problem are obtained based on a new size-dependent quasi 3D plate theory. A three parametric nonlocal strain gradient theory of elasticity is applied in order to capture the small-scale effects. In this study, various non-dimensional parameters are used as follows

$$\mathrm{K_W} = \frac{k_W a^4}{D_{11}}, \mathrm{K_P} = \frac{k_P a^2}{D_{11}}, \text{ and } D_{11} = (E_m h^3)/(12(1-\nu^2))$$

Relying on various theories of elasticity, the wave dispersion relations between the phase velocity and the wave number are studied and these results are illustrated in Fig. 3 for different values of power-law indices. As expected, the wave dispersion relations for different material distributions seem to be similar, and the power-law

Table 1 Material properties of the used (Al/Al_2O_3) FG nanoplate

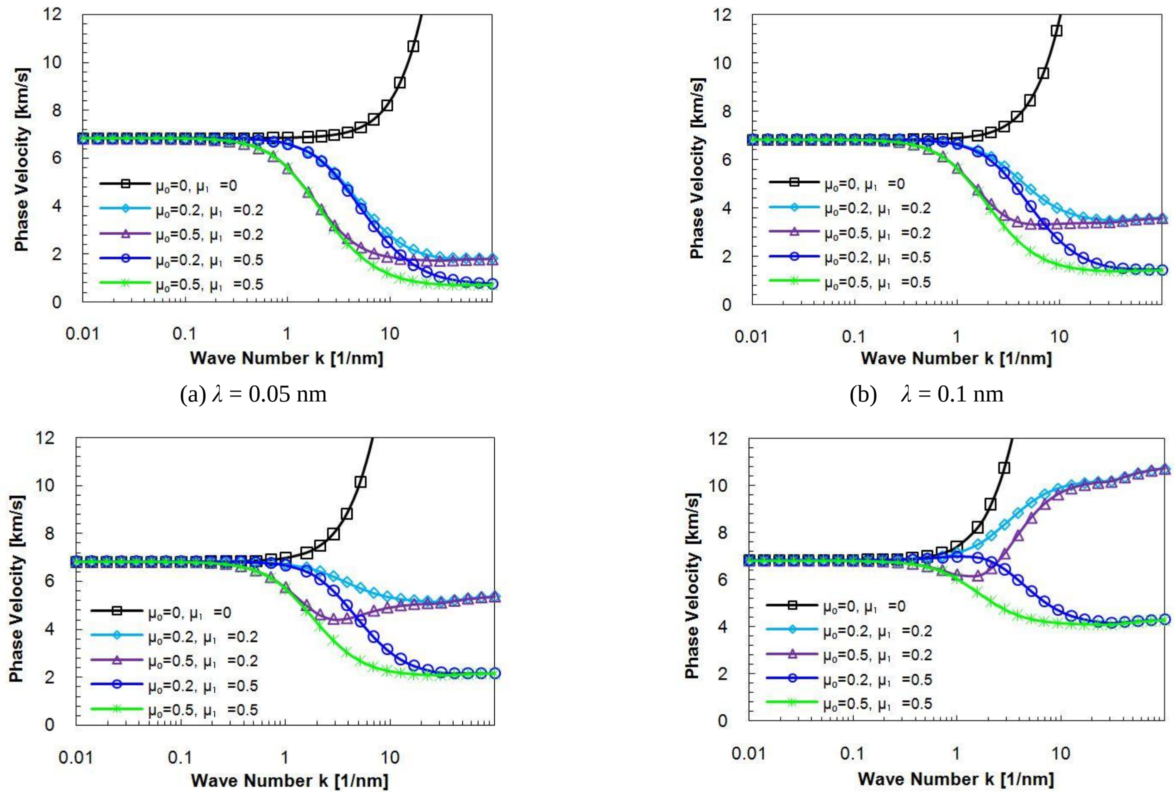
Material	E (GPa)	ρ (kg/m^3)	ν	α (/K)	β (wt.%H^{2}O)$^{-1}$
Aluminum (Al)	70	2702	0.3	23×10^{-6}	0.44
Alumina (Al_2O_3)	380	3800	0.3	7×10^{-6}	0.001



(a) n = 0.2 (b) n = 0.5

(c) n = 1 (d) n = 2

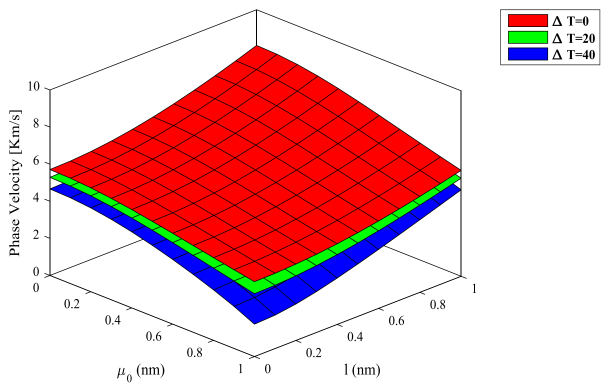
Fig. 3 Wave dispersion curves of FG nanoplate versus wave number for different material distributions and different elasticity theories



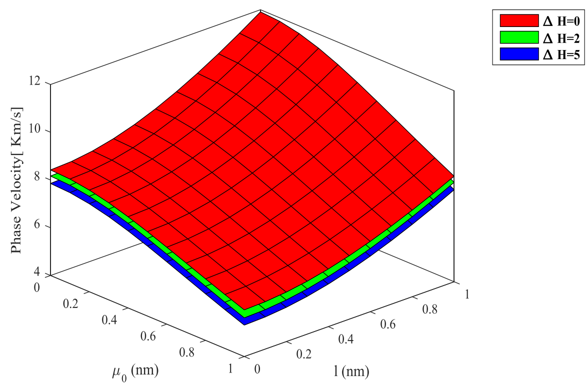
(a) λ = 0.05 nm (b) λ = 0.1 nm

(c) λ = 0.15 nm (d) λ = 0.3 nm

Fig. 4 Wave dispersion curves of FG nanoplate versus wave number for different nonlocal and strain gradient parameters (n = 1)



(a)



(b)

Fig. 5 Phase velocity variation of FG nanoplate under thermal (a) and moisture (b) differences versus lower-order nonlocal and strain gradient parameters

index *n* plays a significant role on the wave dispersion relations between phase velocity and wave number of FG nanoplate. With increasing the power-law index *n* the phase velocities will decrease for all values of wave number. For low wave numbers, the phase velocities of various continuum theories (CET: Classical Elasticity Theory; NET: Nonlocal Elasticity Theory; SGT: Strain Gradient Theory; NSGT: Nonlocal Strain Gradient Theory; B-H NSGT: Bi-Helmholtz Nonlocal Strain Gradient Theory) remain almost unchanged. One of the interesting results is that the phase velocities of different elasticity theories are almost identical when the wave number is smaller than 0.1 1/nm. It means that when $k < 0.1$ 1/nm, all the five continuum theories can product good results. However, when $k > 0.1$ 1/nm, different continuum theories product different trends.

The variation of phase velocity in FG nanoplate under the nonlocal and strain gradient effects is plotted in Fig. 4 at power law index $n = 1$. Generally, the phase velocities are almost identical for different nonlocal strain gradient parameters before the certain value of wave number. Due to presented model, the variation of phase velocity after that certain wave number depends on the values of nonlocal strain gradient parameters. It is concluded that, the phase velocities become more affected by these scale parameters at higher wave numbers. Especially, by ignoring the nonlocality, the numerical results of conventional strain gradient theory are rendered. In this situation, one can observe that after a certain value of wave number, phase velocity of the system tends to infinity. It is also concluded that when strain gradient parameter is smaller than nonlocal parameters, raising wave number after the peak value leads to decrease in phase velocity of FG nanoplate. But, when strain gradient parameter is larger than a nonlocal parameter, raising wave number leads to larger phase velocities. Expressing in a different way, according to stiffness-hardening mechanism observed for strain gradient elasticity, with the increase of strain gradient parameter, we have larger phase velocities. Another significant note is that higher order nonlocal parameter has more reducing influence on phase velocities in comparison with the lower order nonlocal parameter. Thus, it is very important to have two nonlocal parameters and a strain gradient parameter for wave propagation study in nanostructures.

To study the environmental effect, the variation of phase velocity in FG nanoplate under the temperature and also moisture differences versus lower and higher order nonlocal parameters at $k = 1$/nm, $\mu_1 = 0.2$, $n = 1$ is plotted in Fig. 5. Generally, the FG nanoplate without environmental impact has larger phase velocities than similar nanoplate including that. However, the moisture effect due to the fact that adsorption of water molecules produces swelling and degradation is more efficient than the temperature deference to decrease the phase velocities in FG nanoplate. Also, with an increase in temperature or moisture, the role of size effects on the wave characteristics in FG nanoplate is increased.

Frequency of FG nanoplates versus Winkler-Pasternak parameters for different power-law index is plotted in Fig. 6 at $k = 0.1$/nm, $l = 0.1$, $\mu_0 = 0.2$, $\mu_1 = 0.5$. One can observe that increasing in the stiffness of elastic foundation parameters enhances the rigidity of nanoplates and leads to higher frequencies for all of power-law index. In addition, it is seen that the influence of the Pasternak parameter on the wave frequency is more prominent than that of the Winkler parameter.

As final study on the wave characteristics, the variation of phase velocity of FG nanoplate on elastic foundation with respect to lower order and higher order nonlocal

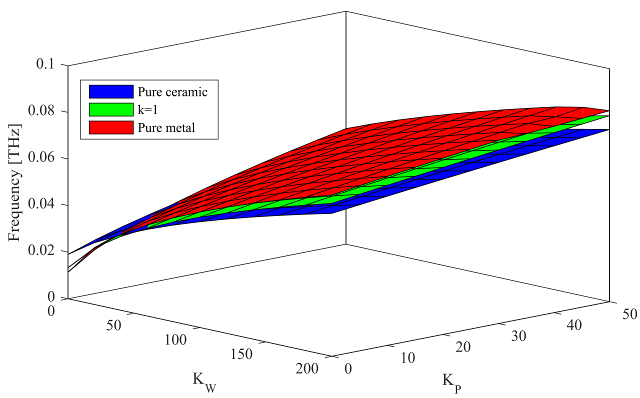
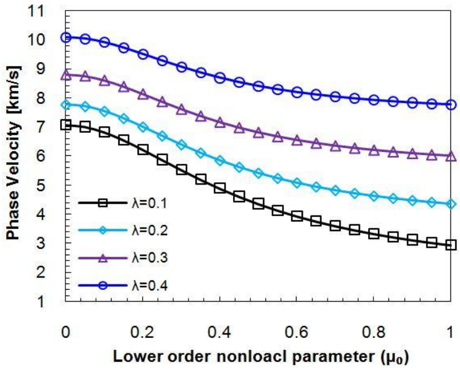
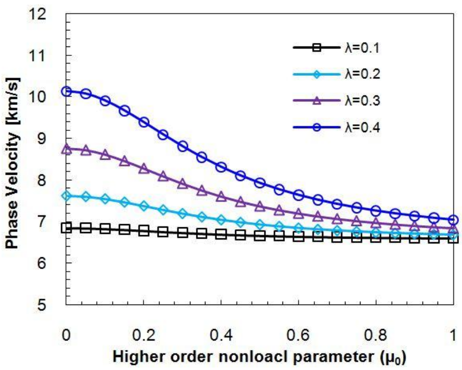


Fig. 6 The effects of material composition on the frequency of FG nanoplates for various Winkler-Pasternak parameters



(a)



(b)

Fig. 7 Phase velocity of FG nanoplate versus (a) lower order nonlocal parameter; and (b) higher order nonlocal parameter for different length scale parameter (μ_0 = 0.1, μ_1 = 0.1)

parameters for different length scale parameters at a fixed wave number k = 1 1/nm is plotted in Fig. 7 when n = 1, K_w = 100, K_P = 20.Generally, the phase velocities decreases for every value of length scale parameter for increasing lower order and higher order nonlocal parameters. Although, with the increase of length scale parameter, one can see enlarges of the phase velocities according to the stiffness-hardening behavior.

5. Conclusions

A new size-dependent quasi-3D plate theory with stretching effect for the wave dispersion analysis of functionally graded plates while resting on elastic foundation and under the hygrothermal environment is presented in this paper. The main advantage of this approach is that, in addition to incorporating the thickness stretching effect. The equations of motion are obtained based on quasi-3D plate theory in conjunction with bi-Helmholtz nonlocal strain-gradient elasticity theory by utilizing the principle of virtual work using Hamilton's principle. The stiffness-softening mechanism in nanostructures is captured via two nonlocal parameters, while the stiffness-hardening mechanism is considered via a strain gradient parameter. It is concluded that nonlocal and strain gradient parameters exert stiffness-softening and stiffness-hardening mechanisms at large wave numbers. Although, influences of nonlocality and strain gradient are negligible at small wave numbers in our study. Also, with an increase in the power-law index, the phase velocity of FG nanoplate will decrease. Moreover, it is shown that hygrothermal environment may play important roles on the variation of phase velocities in some cases.

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