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Nonlocal free vibration analysis of a doubly curved piezoelectric nano shell

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Abstract. In this paper nonlocal free vibration analysis of a doubly curved piezoelectric nano shell is studied. First order shear deformation theory and nonlocal elasticity theory is employed to derive governing equations of motion based on Hamilton's principle. The doubly curved piezoelectric nano shell is resting on Pasternak's foundation. A parametric study is presented to investigate the influence of significant parameters such as nonlocal parameter, two radii of curvature, and ratio of radius to thickness on the fundamental frequency of doubly curved piezoelectric nano shell.

Keywords: doubly curved piezoelectric nano shells; nonlocal parameter; first-order shear deformation theory; Pasternak's foundation; free vibration

1. Introduction

Advances in production of new materials and structures such as nano materials and curved structures leads to significant progress in theoretical investigations of these structures subjected to various types of loadings. The curved structures such as curved beam and panels and doubly curved shells have been manufactured by engineers and designers for application in bodies of aerospace vehicles such as airplanes and space crafts. One can find various curved structures in these structures that must be analyzed theoretically and experimentally. Due to vast application of single and doubly curved structures in aerospace industries and many other applications, comprehensive studies on the analysis of these structures are required. In addition, these structures can be applied in small scales such as nano and micro. It can be observed that a few works on the bending, vibration and buckling analyses of curved structures in nano or micro scale were developed. Based on our investigations, the number of works in scope of doubly curved structures in nano or micro scales is not adequate for researchers. A comprehensive study on the literature review can show necessity of this work.

Bhimaraddi (1991) studied free vibration analysis of homogeneous and laminated doubly curved shells with very small ratio of thickness to radius of curvature based on three dimensional elasticity theory. The results were validated using comparison with results of two-dimensional shell theories. Qatu and Leissa (1991a) studied free vibration analysis of aminated plates and shallow shells using Ritz method with algebraic polynomial displacement functions. Qatu and Leissa (1991b) studied natural frequencies of cantilevered doubly-curved laminated composite shallow shells using Ritz method. The influence of various parameters such as number of layers, fiber orientation and radii of curvature was studied on the free vibration responses. Orthogonal curvilinear coordinate system was used to derive governing equations of orthotropic doubly curved shells with simply supported boundary condition was studied by Fan and Zhang (1992). Chakravorty *et al*. (1996) employed finite element approach for free vibration analysis of doubly curved laminated composites shell based on first order shear deformation theory. Influence of fibre orientations, lamination schemes and thickness to radius ratio was studied on the responses of structure. Vibration analysis of imperfect single and multilayered composite doubly curved panels was studied by Librescu and Chang (1993) subjected to a system of tangential compressive/ tensile edge loads in the pre- and postbuckling ranges due to influence of transverse shear deformations. Lee and Hsiao (2002) studied free vibration analysis of curved non-uniform beam. Two six-th governing equations of motion with variable coefficients were derived based on Hamilton's principle in terms of longitudinal and transverse displacements. Shi *et al*. (2003) studied free vibration analysis of symmetrically laminated curved panel using a modified Galerkin method based on first-order shear deformation theory. They mentioned that in-plane boundary constraints have significant effects on the vibration behavior of the symmetrically laminated curved panel. In addition, it was concluded that radius of curvature, thickness and lamination schemes have important influences on the vibration responses. Large amplitude nonlinear vibration analysis of doubly curved shell with simply supported boundary condition due to harmonic excitation was studied by Amabili (2005). They used two different non-linear strain–displacement relationships based on the Donnell's and Novozhilov's shell theories. The influence of various dimensionless geometric parameters was studied on the nonlinear responses of system.

Alijani *et al*. (2011) presented nonlinear forced

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vibrations of FGM doubly curved shallow shells based on Donnell's nonlinear shallow-shell theory. Galerkin method was used to convert governing differential governing equations of motion to a system of infinite nonlinear ordinary differential equations with quadratic and cubic nonlinearities. Kiani *et al*. (2012) presented thermo-elastic analysis of a functionally graded doubly curved panel based on first-order shear deformation theory with modified Sanders assumptions. All material properties except Poisson ratio were assumed variable along the thickness direction. Five coupled differential equations were derived based on Hamilton's principle and they were reduced to five ordinary differential equations using Navier's solution. Duc *et al*. (2015) studied nonlinear dynamic analysis and vibration imperfect functionally graded materials thick double curved shallow shells integrated with piezoelectric actuators subjected to electrical, thermal, mechanical and damping loadings. In addition, temperature dependency was assumed for all material properties. Higher order shear deformation theory including thermo-piezoelectric effects was employed for formulation of the problem. Influence of elastic foundation and various types of loadings was studied on the responses of the structure. Tornabene and Ceruti (2013) investigated static and dynamic analysis of laminated doubly-curved shells and panels resting on Winkler-Pasternak elastic foundations using Generalized Differential Quadrature method based on first-order shear deformation theory. The influence of the both shell curvatures was included from the beginning of the theory formulation in the kinematic model. Validation of numerical results was performed through comparison with results of commercial programs. They mentioned that the results are in good agreement with literature. Nonlinear analysis of functionally graded circular and square sandwich plates integrated with piezoelectric layers was studied by Arefi and Allam (2015) and Arefi (2015). In addition, two and three dimensional electro-magneto-elastic analysis of cylindrical shell and thick shell of revolution were studied by Arefi and Rahimi (2014a, b) and Arefi (2014). Nonlinear analyses of functionally graded square and circular plates was studied by Arefi and Rahimi (2011, 2012c). Some useful relation about piezoelectric materials can be observed in literature (Arefi *et al*. 2011, Arefi and Rahimi 2014b, Arefi 2014)

Pouresmaeeli and Fazelzadeh (2016) studied the influence of carbon nanotube reinforcement on the vibration characteristics of the thick doubly curved functionally graded composite panels. Five different patterns of carbon nanotubes along the thickness direction were used for reinforcements. First order shear deformation theory was used to derive governing equations of motion based on Hamilton's principle. The influences of volume fraction of carbon nanotubes, thickness ratio, aspect ratio and curvature ratio was studied on the responses. Tornabene *et al*. (2016) studied influence of Carbon Nanotube agglomeration on the free vibrations of laminated composite doubly-curved shells and panels. Alankaya and Oktem (2016) employed third-order shear deformation theory for static analysis of cross-ply doubly-curved shells is presented. They presented some numerical results for panel subjected to point load in terms of important parameters of the problem.

The effect of thermal loads on the bending and free vibration results of sandwich functionally graded materials was studied based on new higher order shear deformation theory (Beldjelili *et al*. 2016, Bouderba *et al*. 2013, Attia *et al*. 2015, Hamidi *et al*. 2015, Menasria *et al*. 2017). Free vibration, buckling and bending analyses of functionally graded sandwich plates subjected to mechanical loads was studied based on some new higher order hyperbolic and trigonometric shear deformation theories and new shear and normal deformation theory (Bourada *et al*. 2015, Bennoun *et al*. 2016, Bessaim *et al*. 2013, Bellifa *et al*. 2017, Houari *et al*. 2016). Combination of nonlocal elasticity theory with new higher order shear deformation theory such as quasi 3D theory, refined four variable shear deformation theory and trigonometric shear deformation theory was proposed for size dependent bending, free vibration and buckling analyses of isotropic and functionally graded nanobeams and nanoplates (Bouafia *et al*. 2017, Zemri *et al*. 2015, Karami *et al*. 2017, Larbi Chaht *et al*. 2015, Belkorissat *et al*. 2015, Besseghier *et al*. 2017).

Free vibration analysis of thick doubly curved sandwich panel was studied by Nasihatgozar *et al*. (2017). Static analysis of single and doubly curved panels was studied by Bahadur *et al*. (2017) based on the higher order shear deformation theory and principle of virtual work. The influence of span to thickness ratio, radius of curvature to span ratio and power index was investigated on the results. Thakur *et al*. (2017) studied higher order shear deformation analysis of a doubly curved laminated composite shell. A new displacement function was proposed for static and free vibration analysis of the doubly curved shell. They mentioned that used method in this paper is sufficient for true determination of shear stress distribution along the thickness direction. Free vibration analysis of a size-dependent doubly curved shell in micro scale was studied by Veysi *et al*. (2017) based on a nonlinear analysis. To account size dependency and nonlinearity in the governing equations of motion, modified couple stress theory and nonlinear Von-Karman relations were used. Multiple scales method was used to obtain an approximate analytical solution for nonlinear frequency response. Arefi and Zenkour (2017a) used sinusoidal shear deformation theory for thermo-magneto-electro-elastic analysis of a three layered curved nanobeam. Nonlocal elasticity relations and Hamilton's principle was employed for derivation of the governing equations of motion. They mentioned that applied electric and magnetic potential leads to important changes of responses. Bending analysis sof sandwich curved nanobeam was presented by Arefi and Zenkour (2017b). The sandwich structure was made from a nano core and two piezomagnetic face-sheets. Influence of nonlocal parameter, applied electric and magnetic potentials and two parameters of Pasternak's foundation was studied on the responses of the system. The numerical results indicate that increase of nonlocal parameters leads to decrease of stiffness of structure. Chen *et al*. (2017) studied free vibration analysis of a doubly curved shell made from functionally graded materials. A new shear deformation theory was employed to account stretching effect in governing equations of motion. Static analysis of a single layered curved nano beam was studied based on higher order shear deformation theory by Arefi and Zenkour (2017c). Deformation and stress analysis of the curved nano beam was performed in terms of thermal loads and two parameters of foundation. Magneto-electro-elastic analysis of sandwich curved beam was studied by Arefi and Zenkour (2017d).

A comprehensive literature review on the various analysis of doubly curved structures especially vibration analysis was performed. This review indicates that although some valuable works on the vibration analysis of doubly curved have been presented, however no vibrational parametric study on the piezoelectric doubly curved nano shell was presented in detail. One can conclude that combination of size-dependent theories with curvilinear coordinate system for a doubly curved piezoelectric nano-shell leads to significant improvement of previous studies. In this work, first order shear deformation theory, Eringen's nonlocal theory and piezoelasticity relations are used to derive governing equations of motion for a doubly curved piezoelectric nano shell. The influence of nonlocal parameter and various geometric parameters are investigated on the free vibration responses of structure.

2. Formulation of doubly curved piezoelectric nano shells

In this section the governing equations of motion for a doubly curved piezoelectric nano shell is presented. The nano shell is subjected to applied electric potential along the thickness direction. Fig. 1 shows the schematic of a α, β, z are indicated coordinates along two plannar and thickness direction. In addition, two principle radii of curvature are depicted with R_1, R_2. First order shear deformation theory is used for description of deformations in doubly curved piezoelectric nano shell as follows (Arefi and Rahimi 2011, 2012a, b, 2014a)

$$\begin{aligned} \bar{u} &= \left(1+\frac{z}{R_1}\right)u + z\phi_1 \\ \bar{v} &= \left(1+\frac{z}{R_2}\right)v + z\phi_2 \\ \bar{w} &= w \end{aligned} \tag{1}$$

in which $\bar{u}$, $\bar{v}.\bar{w}$ are displacement field and $u.v.w$ are displacements of middle surface of doubly curved piezoelectric nano shell. In addition, ϕ_1, ϕ_2 are rotation functions about β, α curves. Based on first order shear deformation theory, strain components are derived as follows

$$\begin{aligned} \varepsilon_1 &= \varepsilon_1^0 + zk_1 \\ \varepsilon_2 &= \varepsilon_2^0 + zk_2 \\ \varepsilon_4 &= \varepsilon_4^0 \\ \varepsilon_5 &= \varepsilon_5^0 \\ \varepsilon_6 &= \varepsilon_6^0 + zk_6 \end{aligned} \tag{2}$$

in which the above variables are expressed as

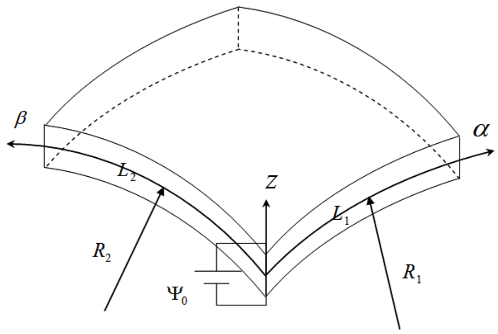

Fig. 1 The schematic of a doubly curved piezoelectric nano shell

$$\begin{aligned}
\varepsilon_1^0 &= \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial u}{\partial \alpha} + \frac{w}{R_1}, \\
\varepsilon_2^0 &= \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial v}{\partial \beta} + \frac{w}{R_2} \\
\varepsilon_6^0 &= \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial v}{\partial \alpha} + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial u}{\partial \beta}, \\
\varepsilon_4^0 &= \phi_2 + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial w}{\partial \beta} - \frac{v}{R_2} \\
\varepsilon_5^0 &= \phi_1 + \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial w}{\partial \alpha} - \frac{u}{R_1} \\
k_1 &= \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial \phi_1}{\partial \alpha}, \qquad k_2 = \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial \phi_2}{\partial \beta} \\
k_6 &= \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial \phi_2}{\partial \alpha} + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial \phi_1}{\partial \beta} \\
&\quad + \frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)\left(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial v}{\partial \alpha} - \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\right)
\end{aligned} \tag{3}$$

The nonlocal stress-strain relations based on nonlocal piezo-elasticity relations for doubly curved piezoelectric nano shell are expressed as (Arefi 2016a, b, Arefi and Zenkour 2016a, b, c, 2017e, f, g, h, i)

$$(1-\mu^2\nabla^2)\begin{Bmatrix}\sigma_\alpha \\ \sigma_\beta \\ \sigma_{\beta z} \\ \sigma_{\alpha z} \\ \sigma_{\alpha\beta}\end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{44} & 0 & 0 \\ 0 & 0 & 0 & Q_{55} & 0 \\ 0 & 0 & 0 & 0 & Q_{66}\end{bmatrix} \begin{Bmatrix}\varepsilon_\alpha \\ \varepsilon_\beta \\ \gamma_{\beta z} \\ \gamma_{\alpha z} \\ \gamma_{\alpha\beta}\end{Bmatrix} - \begin{bmatrix} 0 & 0 & e_{13} \\ 0 & 0 & e_{23} \\ 0 & e_{42} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0\end{bmatrix}\begin{Bmatrix}E_\alpha \\ E_\beta \\ E_z\end{Bmatrix} \tag{4}$$

in which $\mu = e_0 a$ is nonlocal parameter. In addition, e_0 is nonlocal material constant and determined from experimental or by dynamics simulations results. a defines internal characteristic scale and external characteristic scale correspondingly. This parameter show the size-dependency associated with Eringen's nonlocal elasticity theory. To predict the behavior of structures in nano scale, this theory is employed with change of nonlocal parameter. The electric displacement relations are expressed as

$$\begin{Bmatrix} D_\alpha \\ D_\beta \\ D_z \end{Bmatrix} = \begin{bmatrix} 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & e_{24} & 0 & 0 \\ e_{31} & e_{32} & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} \varepsilon_\alpha \\ \varepsilon_\beta \\ \gamma_{\beta z} \\ \gamma_{\alpha z} \\ \gamma_{\alpha\beta} \end{Bmatrix} - \begin{bmatrix} k_{11} & 0 & 0 \\ 0 & k_{22} & 0 \\ 0 & 0 & k_{33} \end{bmatrix} \begin{Bmatrix} E_\alpha \\ E_\beta \\ E_z \end{Bmatrix} \quad (5)$$

Electric potential distribution is assumed a combination of linear function that reflects applied voltage and a cosine function that satisfies homogeneous boundary conditions. Based on these expressions, the electric potential is expressed as follows (Arefi and Zenkour 2017j, k, l, m)

$$\breve{\Psi} = \frac{2z}{h}\Psi_0 - \Psi(\alpha,\beta)\cos\frac{\pi z}{h} \quad (6)$$

Electric field components are derived using electric potential distribution as follows

$$\begin{aligned} E_\alpha &= -\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\breve{\Psi}}{\partial\alpha} = \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\Psi}{\partial\alpha}\cos\frac{\pi z}{h} \\ E_\beta &= -\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\breve{\Psi}}{\partial\beta} = \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\Psi}{\partial\beta}\cos\frac{\pi z}{h} \\ E_z &= -\frac{2}{h}\Psi_0 - \frac{\pi}{h}\Psi(\alpha.\beta)\sin\frac{\pi z}{h} \end{aligned} \quad (7)$$

In this stage and with substitution of strain and electric filed components into stress and electric displacement relations, the stress and electric displacement components can be updated as follows

$$\begin{aligned} &(1-\mu^2\nabla^2)\sigma_\alpha \\ &= Q_{11}\left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial u}{\partial\alpha} + \frac{w}{R_1} + z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\phi_1}{\partial\alpha}\right\} \\ &\quad +Q_{12}\left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial v}{\partial\beta} + \frac{w}{R_2} + z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\phi_2}{\partial\beta}\right\} \\ &\quad -e_{13}\{-\frac{2}{h}\Psi_0 - \frac{\pi}{h}\Psi(\alpha.\beta)\sin\frac{\pi z}{h}\} \end{aligned} \quad (8)$$

$$\begin{aligned} &(1-\mu^2\nabla^2)\sigma_\beta \\ &= Q_{12}\left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial u}{\partial\alpha} + \frac{w}{R_1} + z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\phi_1}{\partial\alpha}\right\} \\ &\quad +Q_{22}\left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial v}{\partial\beta} + \frac{w}{R_2} + z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\phi_2}{\partial\beta}\right\} \\ &\quad -e_{13}\{-\frac{2}{h}\Psi_0 - \frac{\pi}{h}\Psi(\alpha,\beta)\sin\frac{\pi z}{h}\} \\ &(1-\mu^2\nabla^2)\sigma_{\beta z} \\ &= Q_{44}\left\{\phi_2 + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial w}{\partial\beta} - \frac{v}{R_2}\right\} \\ &\quad -e_{42}\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\Psi}{\partial\beta}\cos\frac{\pi z}{h}\} \\ &(1-\mu^2\nabla^2)\sigma_{\alpha z} \\ &= Q_{55}\left\{\phi_1 + \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial w}{\partial\alpha} - \frac{u}{R_1}\right\} \\ &\quad -e_{51}\left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\Psi}{\partial\alpha}\cos\frac{\pi z}{h}\right\} \\ &(1-\mu^2\nabla^2)\sigma_{\alpha\beta} \\ &= Q_{66}\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial v}{\partial\alpha} + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial u}{\partial\beta} \\ &\quad +z\left[\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\phi_2}{\partial\alpha} + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\phi_1}{\partial\beta}\right) \\ &\quad +\frac{1}{2}\left(\frac{1}{R_1} - \frac{1}{R_2}\right)(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial v}{\partial\alpha} - \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial u}{\partial\beta}\Bigg]\Bigg\} \\ &D_\alpha = e_{15}\left\{\phi_1 + \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial w}{\partial\alpha} - \frac{u}{R_1}\right\} \\ &\quad -k_{11}\left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\Psi}{\partial\alpha}\cos\frac{\pi z}{h}\right\} \\ &D_\beta = e_{24}\left\{\phi_2 + \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial w}{\partial\beta} - \frac{v}{R_2}\right\} \\ &\quad -k_{22}\left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\Psi}{\partial\beta}\cos\frac{\pi z}{h}\right\} \\ &D_z = e_{31}\left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial u}{\partial\alpha} + \frac{w}{R_1} + z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\frac{\partial\phi_1}{\partial\alpha}\right\} \\ &\quad +e_{32}\left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial v}{\partial\beta} + \frac{w}{R_2} \right. \\ &\quad \left. +z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\frac{\partial\phi_2}{\partial\beta}\right\} \end{aligned} \quad (8)$$

$$-k_{33}\left\{-\frac{2}{h}\Psi_0-\frac{\pi}{h}\Psi(\alpha,\beta)sin\frac{\pi z}{h}\right\} \tag{8}$$

After completion of stress and electric displacement relations, the strain energy of doubly curved is defined as follows

$$U=\frac{1}{2}\int_\alpha\int_\beta\int_z[\sigma_\alpha\varepsilon_\alpha+\sigma_\beta\varepsilon_\beta+\tau_{yz}\gamma_{yz}+\tau_{xz}\gamma_{xz}+\tau_{xy}\gamma_{xy}-D_\alpha E_\alpha-D_\beta E_\beta-D_zE_z]dV \tag{9}$$

The work done by external forces including uniform transverse loads and reaction of Pasternak's foundation is calculated as

$$\delta W=\int\left\{-q\left[1+\frac{h}{2R_1}\right]\left[1+\frac{h}{2R_2}\right]+R_f\left[1-\frac{h}{2R_1}\right]\left[1-\frac{h}{2R_2}\right]\right\}R_1R_2\delta wd\beta d\alpha \tag{10}$$

in which reaction of foundation $R_f=K_1w-K_2\nabla^2w$.

Variation of kinetic energy is defined as follows

$$\delta T=\iiint\rho\{\dot{u}\delta\dot{u}+\dot{v}\delta\dot{v}+\dot{w}\delta\dot{w}\}\,dV \tag{11}$$

Arranging the variables after substitution of variation of strain energy and external works into Hamilton's principle $\delta W+\delta T-\delta U$ and arranging the variables leads to following governing equations

$$\begin{aligned}
&\delta u: && \frac{\partial}{\partial\alpha}\left(\frac{N_\alpha}{R_1}\right)+\frac{\partial}{\partial\beta}\left(\frac{N_{\beta\alpha}}{R_2}\right)+\frac{N_{\alpha z}^1}{R_1}-\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{\partial}{\partial\beta}\left(\frac{M_{\beta\alpha}}{R_2}\right)=I_1\ddot{u}+I_2\ddot{\phi}_1\\
&\delta\phi_1: && \frac{\partial}{\partial\alpha}\left(\frac{M_\alpha}{R_1}\right)-N_{\alpha z}^1+\frac{\partial}{\partial\beta}\left(\frac{M_{\beta\alpha}}{R_2}\right)=I_2\ddot{u}+I_3\ddot{\phi}_1\\
&\delta v: && +\frac{\partial}{\partial\beta}\left(\frac{N_\beta}{R_2}\right)+\frac{\partial}{\partial\alpha}\left(\frac{N_{\alpha\beta}}{R_1}\right)+\frac{N_{\beta z}^1}{R_2}+\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{\partial}{\partial\alpha}\left(\frac{M_{\alpha\beta}}{R_1}\right)=I_4\ddot{v}+I_5\ddot{\phi}_2\\
&\delta\phi_2: && \frac{\partial}{\partial\beta}\left(\frac{M_\beta}{R_2}\right)-N_{\beta z}^1+\frac{\partial}{\partial\alpha}\left(\frac{M_{\alpha\beta}}{R_1}\right)=I_5\ddot{v}+I_3\ddot{\phi}_2\\
&\delta w: && -\frac{N_\alpha^1}{R_1}-\frac{N_\beta^1}{R_2}+\frac{\partial}{\partial\beta}\left(\frac{N_{\beta z}}{R_2}\right)+\frac{\partial}{\partial\alpha}\left(\frac{N_{\alpha z}}{R_1}\right)=q\left[1+\frac{h}{2R_1}\right]\left[1+\frac{h}{2R_2}\right]-R_f\left[1-\frac{h}{2R_1}\right]\left[1-\frac{h}{2R_2}\right]+I_6\ddot{w}\\
&\delta\Psi: && +\frac{\partial}{\partial\alpha}\left(\frac{\bar{D}_\alpha}{R_1}\right)+\frac{\partial}{\partial\beta}\left(\frac{\bar{D}_\beta}{R_2}\right)+\bar{D}_z=0
\end{aligned} \tag{12}$$

in which the integration constants I_i are defined as

$$\begin{aligned}
\{I_1.I_2.I_3\}&=\int_{-h/2}^{+h/2}\rho\left\{\left(1+\frac{z}{R_1}\right)^2.z\left(1+\frac{z}{R_1}\right).z^2\right\}R_1\left[1+\frac{z}{R_1}\right]R_2\left[1+\frac{z}{R_2}\right]dz\\
\{I_4.I_5.I_6\}&=\int_{-h/2}^{+h/2}\rho\left\{\left(1+\frac{z}{R_2}\right)^2.z\left(1+\frac{z}{R_2}\right).1\right\}R_1\left[1+\frac{z}{R_1}\right]R_2\left[1+\frac{z}{R_2}\right]dz
\end{aligned} \tag{13}$$

In addition, the resultant components are defined as follows

$$\begin{aligned}
(1-\mu^2\nabla^2)N_\alpha&=A_1\frac{\partial u}{\partial\alpha}+A_2w+A_3\frac{\partial\phi_1}{\partial\alpha}+A_4\frac{\partial v}{\partial\beta}+A_5w+A_6\frac{\partial\phi_2}{\partial\beta}+A_7\Psi+N_\alpha{}^\Psi\\
(1-\mu^2\nabla^2)N_\alpha^1&=A_8\frac{\partial u}{\partial\alpha}+A_9w+A_{10}\frac{\partial\phi_1}{\partial\alpha}+A_{11}\frac{\partial v}{\partial\beta}+A_{12}w+A_{13}\frac{\partial\phi_2}{\partial\beta}+A_{14}\Psi+N_\alpha{}^{1\Psi}\\
(1-\mu^2\nabla^2)M_\alpha&=A_{15}\frac{\partial u}{\partial\alpha}+A_{16}w+A_{17}\frac{\partial\phi_1}{\partial\alpha}+A_{18}\frac{\partial v}{\partial\beta}+A_{19}w+A_{20}\frac{\partial\phi_2}{\partial\beta}+A_{21}\Psi+M_\alpha{}^\Psi\\
(1-\mu^2\nabla^2)N_\beta&=A_4\frac{\partial u}{\partial\alpha}+A_5w+A_6\frac{\partial\phi_1}{\partial\alpha}+A_{22}\frac{\partial v}{\partial\beta}+A_{23}w+A_{24}\frac{\partial\phi_2}{\partial\beta}+A_7\Psi+N_\beta{}^\Psi\\
(1-\mu^2\nabla^2)N_\beta^1&=A_{11}\frac{\partial u}{\partial\alpha}+A_{12}w+A_{13}\frac{\partial\phi_1}{\partial\alpha}+A_{25}\frac{\partial v}{\partial\beta}+A_{26}w+A_{27}\frac{\partial\phi_2}{\partial\beta}+A_{14}\Psi+N_\beta{}^{1\Psi}\\
(1-\mu^2\nabla^2)M_\beta&=A_{18}\frac{\partial u}{\partial\alpha}+A_{19}w+A_{20}\frac{\partial\phi_1}{\partial\alpha}+A_{28}\frac{\partial v}{\partial\beta}+A_{29}w+A_{30}\frac{\partial\phi_2}{\partial\beta}+A_{21}\Psi+M_\alpha{}^\Psi\\
(1-\mu^2\nabla^2)N_{\beta z}&=A_{31}\phi_2+A_{32}\frac{\partial w}{\partial\beta}-A_{33}v-A_{34}\frac{\partial\Psi}{\partial\beta},\\
(1-\mu^2\nabla^2)N_{\beta z}{}^1&=A_{39}\phi_2+A_{40}\frac{\partial w}{\partial\beta}-A_{41}v-A_{42}\frac{\partial\Psi}{\partial\beta}\\
(1-\mu^2\nabla^2)N_{\alpha z}&=A_{35}\phi_1+A_{36}\frac{\partial w}{\partial\alpha}-A_{37}u-A_{38}\frac{\partial\Psi}{\partial\alpha},\\
(1-\mu^2\nabla^2)N_{\alpha z}{}^1&=A_{43}\phi_1+A_{44}\frac{\partial w}{\partial\alpha}-A_{45}u-A_{46}\frac{\partial\Psi}{\partial\alpha}\\
(1-\mu^2\nabla^2)N_{\alpha\beta}&=A_{47}\frac{\partial v}{\partial\alpha}+A_{48}\frac{\partial u}{\partial\beta}+A_{49}\frac{\partial\phi_2}{\partial\alpha}+A_{50}\frac{\partial\phi_1}{\partial\beta}+A_{51}\frac{\partial v}{\partial\alpha}-A_{52}\frac{\partial u}{\partial\beta}\\
(1-\mu^2\nabla^2)N_{\beta\alpha}&=A_{53}\frac{\partial v}{\partial\alpha}+A_{54}\frac{\partial u}{\partial\beta}+A_{55}\frac{\partial\phi_2}{\partial\alpha}+A_{56}\frac{\partial\phi_1}{\partial\beta}+A_{57}\frac{\partial v}{\partial\alpha}-A_{58}\frac{\partial u}{\partial\beta}
\end{aligned} \tag{14}$$

$$
\begin{aligned}
(1-\mu^2\nabla^2)M_{\alpha\beta} &= A_{59}\frac{\partial v}{\partial\alpha}+A_{60}\frac{\partial u}{\partial\beta}+A_{61}\frac{\partial\phi_2}{\partial\alpha}\\
&\quad +A_{62}\frac{\partial\phi_1}{\partial\beta}+A_{63}\frac{\partial v}{\partial\alpha}-A_{64}\frac{\partial u}{\partial\beta}\\
(1-\mu^2\nabla^2)M_{\beta\alpha} &= A_{65}\frac{\partial v}{\partial\alpha}+A_{66}\frac{\partial u}{\partial\beta}+A_{67}\frac{\partial\phi_2}{\partial\alpha}\\
&\quad +A_{68}\frac{\partial\phi_1}{\partial\beta}+A_{69}\frac{\partial v}{\partial\alpha}-A_{70}\frac{\partial u}{\partial\beta}\\
\overline{D}_\alpha &= A_{71}\phi_1+A_{72}\frac{\partial w}{\partial\alpha}-A_{73}u-A_{74}\frac{\partial\Psi}{\partial\alpha}\\
\overline{D}_\beta &= A_{75}\phi_2+A_{76}\frac{\partial w}{\partial\beta}-A_{77}v-A_{78}\frac{\partial\Psi}{\partial\beta}\\
D_z &= A_{79}\frac{\partial u}{\partial\alpha}+A_{80}w+A_{81}\frac{\partial\phi_1}{\partial\alpha}+A_{82}\frac{\partial v}{\partial\beta}\\
&\quad +A_{83}w+A_{84}\frac{\partial\phi_2}{\partial\beta}+{D_z}^{\Psi}+A_{85}\Psi
\end{aligned}
\tag{14}
$$

Substitution of resultant components into governing equations leads to final governing equations in terms of primary displacement field as follows

$$
\begin{aligned}
\delta u:\quad & \frac{A_1}{R_1}\frac{\partial^2 u}{\partial\alpha^2}+\left(\frac{\chi[A_{70}-A_{66}]+A_{54}-A_{58}}{R_2}\right)\frac{\partial^2 u}{\partial\beta^2}\\
&-\frac{A_{45}}{R_1}u+\frac{A_3}{R_1}\frac{\partial^2\phi_1}{\partial\alpha^2}+\left(\frac{A_{56}-\chi A_{68}}{R_2}\right)\frac{\partial^2\phi_1}{\partial\beta^2}\\
&+\frac{A_{43}}{R_1}\phi_1+\frac{A_4}{R_1}\frac{\partial^2 v}{\partial\alpha\partial\beta}\\
&+\left(\frac{A_{57}+A_{53}-\chi[A_{65}+A_{69}]}{R_2}\right)\frac{\partial^2 v}{\partial\alpha\partial\beta}\\
&+\left(\frac{A_6}{R_1}+\frac{A_{55}-\chi A_{67}}{R_2}\right)\frac{\partial^2\phi_2}{\partial\alpha\partial\beta}\\
&+\left(\frac{A_5+A_2+A_{44}}{R_1}\right)\frac{\partial w}{\partial\alpha}+\left(\frac{A_7-A_{46}}{R_1}\right)\frac{\partial\Psi}{\partial\alpha}\\
&=-\frac{\partial}{\partial\alpha}\left(\frac{{N_\alpha}^{\Psi}}{R_1}\right)+(1-\mu^2\nabla^2)(I_1\ddot{u}+I_2\ddot{\phi}_1)
\end{aligned}
\tag{15a}
$$

$$
\begin{aligned}
\delta\phi_1:\quad & \frac{A_{15}}{R_1}\frac{\partial^2 u}{\partial\alpha^2}+\frac{(A_{66}-A_{70})}{R_2}\frac{\partial^2 u}{\partial\beta^2}\\
&+\frac{A_{17}}{R_1}\frac{\partial^2\phi_1}{\partial\alpha^2}+\frac{A_{68}}{R_2}\frac{\partial^2\phi_1}{\partial\beta^2}-A_{43}\phi_1\\
&+\left(\frac{A_{65}+A_{69}}{R_2}+\frac{A_{18}}{R_1}\right)\frac{\partial^2 v}{\partial\alpha\partial\beta}+A_{45}u\\
&+\left(\frac{A_{20}}{R_1}+\frac{A_{67}}{R_2}\right)\frac{\partial\phi_2}{\partial\alpha\partial\beta}+\left(\frac{A_{19}+A_{16}}{R_1}-A_{44}\right)\frac{\partial w}{\partial\alpha}\\
&+\left(\frac{A_{21}}{R_1}+A_{46}\right)\frac{\partial\Psi}{\partial\alpha}\\
&=-\frac{\partial}{\partial\alpha}\left(\frac{{M_\alpha}^{\Psi}}{R_1}\right)+(1-\mu^2\nabla^2)(I_2\ddot{u}+I_3\ddot{\phi}_1)
\end{aligned}
\tag{15b}
$$

$$
\begin{aligned}
\delta v:\quad & \left(\frac{A_4}{R_2}+\frac{\chi[A_{60}-A_{64}]+A_{48}-A_{52}}{R_1}\right)\frac{\partial^2 u}{\partial\alpha\partial\beta}\\
&+\left(\frac{A_{50}+\chi A_{62}}{R_1}+\frac{A_6}{R_2}\right)\frac{\partial^2\phi_1}{\partial\alpha\partial\beta}\\
&+\left(\frac{A_{47}+A_{51}+\chi[A_{59}+A_{63}]}{R_1}\right)\frac{\partial^2 v}{\partial\alpha^2}\\
&+\frac{A_{22}}{R_2}\frac{\partial^2 v}{\partial\beta^2}-\frac{A_{41}}{R_2}v+\left(\frac{A_{49}+\chi A_{61}}{R_1}\right)\frac{\partial^2\phi_2}{\partial\alpha^2}\\
&+\frac{A_{24}}{R_2}\frac{\partial^2\phi_2}{\partial\beta^2}+\frac{A_{39}}{R_2}\phi_2\\
&+\left(\frac{A_{40}+A_5+A_{23}}{R_2}\right)\frac{\partial w}{\partial\beta}+\left(\frac{A_7-A_{42}}{R_2}\right)\frac{\partial\Psi}{\partial\beta}\\
&=-\frac{\partial}{\partial\beta}\left(\frac{{N_\beta}^{\Psi}}{R_2}\right)+(1-\mu^2\nabla^2)(I_4\ddot{v}+I_5\ddot{\phi}_2)
\end{aligned}
\tag{15c}
$$

$$
\begin{aligned}
\delta\phi_2:\quad & \left(\frac{A_{18}}{R_2}+\frac{A_{60}-A_{64}}{R_1}\right)\frac{\partial^2 u}{\partial\alpha\partial\beta}+A_{41}v\\
&+\left(\frac{A_{62}}{R_1}+\frac{A_{20}}{R_2}\right)\frac{\partial^2\phi_1}{\partial\alpha\partial\beta}+\left(\frac{A_{59}+A_{63}}{R_1}\right)\frac{\partial^2 v}{\partial\alpha^2}\\
&+\frac{A_{28}}{R_2}\frac{\partial^2 v}{\partial\beta^2}+\left(\frac{A_{30}}{R_2}+\frac{A_{61}}{R_1}\right)\frac{\partial^2\phi_2}{\partial\alpha^2}-A_{39}\phi_2\\
&+\left(\frac{A_{29}+A_{19}}{R_2}-A_{40}\right)\frac{\partial w}{\partial\beta}+\left(\frac{A_{21}}{R_2}+A_{42}\right)\frac{\partial\Psi}{\partial\beta}\\
&=-\frac{\partial}{\partial\beta}\left(\frac{{M_\alpha}^{\Psi}}{R_2}\right)+(1-\mu^2\nabla^2)(I_5\ddot{v}+I_3\ddot{\phi}_2)
\end{aligned}
\tag{15d}
$$

$$
\begin{aligned}
\delta w:\quad & \left(-\frac{A_{11}}{R_2}-\frac{A_8+A_{37}}{R_1}\right)\frac{\partial u}{\partial\alpha}\\
&+\left(\frac{A_{35}-A_{10}}{R_1}-\frac{A_{13}}{R_2}\right)\frac{\partial\phi_1}{\partial\alpha}\\
&+\left(-\frac{A_{33}+A_{25}}{R_2}-\frac{A_{11}}{R_1}\right)\frac{\partial v}{\partial\beta}-\frac{A_{27}}{R_2}\frac{\partial\phi_2}{\partial\beta}\\
&+\left(-\frac{A_9+A_{12}}{R_1}-\frac{A_{12}+A_{26}}{R_2}\right)w\\
&+\left(-\frac{A_{13}}{R_1}-\frac{A_{31}}{R_2}\right)\frac{\partial\phi_2}{\partial\beta}+\frac{A_{36}}{R_1}\frac{\partial^2 w}{\partial\alpha^2}\\
&+\left(\frac{A_{32}-A_{34}}{R_2}\right)\frac{\partial^2\Psi}{\partial\beta^2}-\frac{A_{38}}{R_1}\frac{\partial^2\Psi}{\partial\alpha^2}\\
&+\left(-\frac{A_{14}}{R_2}-\frac{A_{14}}{R_1}\right)\Psi\\
&=+\frac{{N_\alpha}^{1\Psi}}{R_1}+\frac{{N_\beta}^{1\Psi}}{R_2}\\
&+(1-\mu^2\nabla^2)\left\{q\left[1+\frac{h}{2R_1}\right]\left[1+\frac{h}{2R_2}\right]\right.\\
&\left.-R_f\left[1-\frac{h}{2R_1}\right]\left[1-\frac{h}{2R_2}\right]\right\}+(1-\mu^2\nabla^2)(I_6\ddot{w})
\end{aligned}
\tag{15e}
$$

$$
\begin{aligned}
\delta\Psi:\quad & \left(-\frac{A_{73}}{R_1}+A_{79}\right)\frac{\partial u}{\partial\alpha}+\left(\frac{A_{71}}{R_1}+A_{81}\right)\frac{\partial\phi_1}{\partial\alpha}\\
&+\left(A_{82}-\frac{A_{77}}{R_2}\right)\frac{\partial v}{\partial\beta}+\left(\frac{A_{75}}{R_2}+A_{84}\right)\frac{\partial\phi_2}{\partial\beta}\\
&+\frac{A_{72}}{R_1}\frac{\partial^2 w}{\partial\alpha^2}+\frac{A_{76}}{R_2}\frac{\partial^2 w}{\partial\beta^2}+(A_{80}+A_{83})w\\
&-\frac{A_{78}}{R_2}\frac{\partial^2\Psi}{\partial\beta^2}-\frac{A_{74}}{R_1}\frac{\partial^2\Psi}{\partial\alpha^2}+A_{85}\Psi=-{D_z}^{\Psi}
\end{aligned}
\tag{15f}
$$

3. Solution procedure

In this section, solution of the governing equations is proposed. The simply-supported boundary conditions are assumed for doubly curved piezoelectric shell. In addition, homogeneous electrical boundary conditions are considered for piezoelectric doubly curved nano shell. For this type of boundary conditions, the Navier's solution is proposed for six variables as follows (Arefi *et al*. 2017)

$$\begin{Bmatrix} u \\ \phi_1 \\ v \\ \phi_2 \\ w \\ \Psi \end{Bmatrix} = e^{i\omega t} \begin{Bmatrix} U\cos\lambda_m\alpha\sin\mu_n\beta \\ \Phi_1\cos\lambda_m\alpha\sin\mu_n\beta \\ V\sin\lambda_m\alpha\cos\mu_n\beta \\ \Phi_2\sin\lambda_m\alpha\cos\mu_n\beta \\ W\sin\lambda_m\alpha\sin\mu_n\beta \\ \Psi\sin\lambda_m\alpha\sin\mu_n\beta \end{Bmatrix} \tag{16}$$

The solution presented by Eq. (16) is applicable for simply-supported doubly curved shell with homogeneous electric potential distribution. It can be recognized that the boundaries ($\alpha = 0, \frac{L_1}{R_1}$) has no transverse displacement while can be moved freely along the α direction. The similar conclusion can be presented for the boundaries ($\beta = 0, \frac{L_2}{R_2}$) has no transverse displacement while can be moved freely along the β direction. In which the $\{X\} = \{U.\Phi_1.V.\Phi_2.W.\Psi\}^T$ are unknown amplitudes and $\lambda_m = \frac{mR_1}{L_1}.\mu_n = \frac{nR_2}{L_2}$. Substitution of proposed solution from Eq. (16) into governing equations of electro-elastic bending leads to following well-known format as follows

$$[K]\{X\} + \omega^2[M]\{X\} = \{F\} \tag{17}$$

Elements of stiffness matrix are derived as

$$K_{11} = -\frac{A_1}{R_1}\lambda_m{}^2 - \frac{\chi[A_{70} - A_{66}] + A_{54} - A_{58}}{R_2}\mu_n{}^2 - \frac{A_{45}}{R_1},$$

$$K_{12} = -\frac{A_3}{R_1}\lambda_m{}^2 - \left(\frac{A_{56} - \chi A_{68}}{R_2}\right)\mu_n{}^2 + \frac{A_{43}}{R_1},$$

$$K_{13} = -\left(\frac{A_4}{R_1} + \frac{A_{57} + A_{53} - \chi[A_{65} + A_{69}]}{R_2}\right)\lambda_m\mu_n,$$

$$K_{14} = -\left(\frac{A_6}{R_1} + \frac{A_{55} - \chi A_{67}}{R_2}\right)\lambda_m\mu_n,$$

$$K_{15} = \left(\frac{A_5 + A_2 + A_{44}}{R_1}\right)\lambda_m, \qquad K_{16} = (\frac{A_7 - A_{46}}{R_1})\lambda_m$$

$$M_{11} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_1,$$

$$M_{12} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_2. F_1 = -\frac{\partial}{\partial\alpha}\left(\frac{N_\alpha{}^\Psi}{R_1}\right)$$

$$K_{21} = -\frac{A_{15}}{R_1}\lambda_m{}^2 - \left(\frac{A_{66} - A_{70}}{R_2}\right)\mu_n{}^2 + A_{45}.,$$

$$K_{22} = -\frac{A_{17}}{R_1}\lambda_m{}^2 - \frac{A_{68}}{R_2}\mu_n{}^2 - A_{43}$$

$$K_{23} = -\left(\frac{A_{65} + A_{69}}{R_2} + \frac{A_{18}}{R_1}\right)\lambda_m\mu_n,$$

$$K_{24} = -\left(\frac{A_{20}}{R_1} + \frac{A_{67}}{R_2}\right)\lambda_m\mu_n,$$

$$K_{25} = +\left(\frac{A_{19} + A_{16}}{R_1} - A_{44}\right)\lambda_m,$$

$$K_{26} = \left(\frac{A_{21}}{R_1} + A_{46}\right)\lambda_m,$$

$$M_{21} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_2,$$

$$M_{22} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_3,$$

$$F_2 = -\frac{\partial}{\partial\alpha}\left(\frac{M_\alpha{}^\Psi}{R_1}\right),$$

$$K_{31} = -\left(\frac{A_4}{R_2} + \frac{\chi[A_{60} - A_{64}] + A_{48} - A_{52}}{R_1}\right)\lambda_m\mu_n.,$$

$$K_{23} = -\left(\frac{A_{50} + \chi A_{62}}{R_1} + \frac{A_6}{R_2}\right)\lambda_m\mu_n,$$

$$K_{33} = -\left(\frac{A_{47} + A_{51} + \chi[A_{59} + A_{63}]}{R_1}\right)\lambda_m{}^2 - \frac{A_{22}}{R_2}\mu_n{}^2 - \frac{A_{41}}{R_2},$$

$$K_{34} = -\left(\frac{A_{49} + \chi A_{61}}{R_1}\right)\lambda_m{}^2 - \frac{A_{24}}{R_2}\mu_n{}^2 + \frac{A_{39}}{R_1},$$

$$K_{35} = +\left(\frac{A_5 + A_{23} + A_{40}}{R_2}\right)\mu_n,$$

$$K_{36} = +\left(\frac{A_7 - A_{42}}{R_2}\right)\mu_n,$$

$$M_{33} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_4,$$

$$M_{34} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_5,$$

$$F_3 = -\frac{\partial}{\partial\beta}\left(\frac{N_\beta{}^\Psi}{R_2}\right),$$

$$K_{41} = -\left(\frac{A_{18}}{R_2} + \frac{A_{60} - A_{64}}{R_1}\right)\lambda_m\mu_n,$$

$$K_{42} = -\left(\frac{A_{62}}{R_1} + \frac{A_{20}}{R_2}\right)\lambda_m\mu_n,$$

$$K_{43} = -\left(\frac{A_{59} + A_{63}}{R_1}\right)\lambda_m{}^2 - \frac{A_{28}}{R_2}\mu_n{}^2 + A_{41},$$

$$K_{44} = -\frac{A_{61}}{R_1}\lambda_m{}^2 - \frac{A_{30}}{R_2}\mu_n{}^2 - A_{39}$$

$$K_{45} = \left(\frac{A_{29} + A_{19}}{R_2} - A_{40}\right)\mu_n,$$

$$K_{46} = \left(\frac{A_{21}}{R_2} + A_{42}\right)\mu_n,$$

$$M_{43} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_5,$$

$$M_{44} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_3,$$

$$F_4 = -\frac{\partial}{\partial\beta}\left(\frac{M_\alpha{}^\Psi}{R_2}\right),$$

$$K_{51} = \left(\frac{A_{11}}{R_2} + \frac{A_8 + A_{37}}{R_1}\right)\lambda_m.,$$

$$K_{52} = -\left(\frac{A_{35} - A_{10}}{R_1} - \frac{A_{13}}{R_2}\right)\lambda_m,$$

$$K_{53} = \left(\frac{A_{33} + A_{25}}{R_2} + \frac{A_{11}}{R_1}\right)\mu_n,$$

$$K_{54} = \left(\frac{A_{27} - A_{31}}{R_2} + \frac{A_{13}}{R_1}\right)\mu_n,$$

Table 2 The variation of fundamental natural frequencies of doubly curved shell in terms of nonlocal parameter $\mu(nm)$ for various radius to thickness ratio R/h

$\mu(nm)$	$R/h = 10$	$R/h = 20$	$R/h = 50$	$R/h = 100$
0	529/2047	294/8370	123/8012	62/7763
0/1	529/0742	294/7643	123/7707	62/7608
0/2	528/6832	294/5465	123/6792	62/7144

$$K_{55} = -\frac{A_{36}}{R_1}\lambda_m{}^2 - \frac{A_{32}}{R_2}\mu_n{}^2 - \left(\frac{A_9 + A_{12}}{R_1} + \frac{A_{12} + A_{26}}{R_2}\right) + \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)\left[1 - \frac{h}{2R_1}\right]\left[1 - \frac{h}{2R_2}\right]\left\{K_w + K_G\left(\frac{\lambda_m{}^2}{(R_1 - \frac{h}{2})^2} + \frac{\mu_n{}^2}{(R_2 - \frac{h}{2})^2}\right)\right\},$$

$$K_{56} = \frac{A_{38}}{R_1}\lambda_m{}^2 + \frac{A_{34}}{R_2}\mu_n{}^2 - \left(\frac{A_{14}}{R_2} + \frac{A_{14}}{R_1}\right),$$

$$M_{55} = \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)I_6,$$

$$F_5 = +\frac{N_\alpha{}^{1\Psi}}{R_1} + \frac{N_\beta{}^{1\Psi}}{R_2} + \left(1 + \mu^2\{\lambda_m{}^2 + \mu_n{}^2\}\right)q\left[1 + \frac{h}{2R_1}\right]\left[1 + \frac{h}{2R_2}\right]$$

$$K_{61} = \left(\frac{A_{73}}{R_1} - A_{79}\right)\lambda_m, \qquad K_{62} = -\left(\frac{A_{71}}{R_1} + A_{81}\right)\lambda_m,$$

$$K_{63} = \left(\frac{A_{77}}{R_2} - A_{82}\right)\mu_n, \qquad K_{64} = -\left(\frac{A_{75}}{R_2} + A_{84}\right)\mu_n,$$

$$K_{65} = -\frac{A_{72}}{R_1}\lambda_m{}^2 - \frac{A_{76}}{R_2}\mu_n{}^2 + (A_{80} + A_{83}),$$

$$K_{65} = +\frac{A_{78}}{R_2}\mu_n{}^2 + \frac{A_{74}}{R_1}\lambda_m{}^2 + A_{85},$$

$$F_6 = -D_z{}^{\Psi}$$

4. Numerical results and discussion

In this section the numerical results of the problem is presented. Before presentation of full numerical results, the material properties of piezoelectric doubly curved shell are presented.

$$Q_{11} = 138.499\,GPa, \qquad Q_{22} = 138.499\,GPa, \qquad Q_{33} = 114.745\,GPa$$
$$Q_{12} = 77.371\,GPa, \qquad Q_{13} = 73.643\,GPa, \qquad Q_{23} = 73.643\,GPa$$
$$Q_{44} = 25.6\,GPa, \qquad Q_{55} = 25.6\,GPa, \qquad Q_{66} = 30.6\,GPa$$
$$e_{13} = e_{31} = -5.2\,C/m^2, \qquad e_{23} = e_{32} = -5.2\,C/m^2,$$
$$e_{33} = 15.8\,C/m^2, \qquad e_{15} = 12.72\,C/m^2, \qquad e_{24} = 12.72\,C/m^2$$
$$k_{11} = 1.306 \times 10^{-8}\,F/m, \qquad k_{22} = 1.306 \times 10^{-8}\,F/m, \qquad k_{33} = 1.151 \times 10^{-8}\,F/m$$

Table 1 The variation of fundamental natural frequencies of doubly curved shell in terms of nonlocal parameter μ (nm) for various angles θ_1

$\mu(nm)$	θ_1= 0.5 Rad	θ_1= 1 Rad	θ_1= 1.5 Rad
0	107/532	107/462	107/449
0/1	107/267	107/238	107/231
0/2	106/486	106/572	106/587
0/3	105/220	105/490	105/539
0/4	103/522	104/028	104/122

The natural frequencies of doubly curved piezoelectric nano shell are presented in GHz. To present a parametric study on the vibration analysis of doubly curved piezoelectric nano shell, two dimensionless parameters are employed as follows

$$\theta_1 = L_1/R_1 \qquad \theta_2 = L_2/R_2$$

In addition the dimensionless R/h is employed for presentation of numerical results.

Table 1 lists the fundamental natural frequencies of doubly curved nano shell in terms of nonlocal parameter $\mu(nm)$ for various angles θ_1. The numerical results indicates that the natural frequencies are decreased with increase of nonlocal parameter and opening angles θ_1. Listed in Table 2 is variation of fundamental natural frequencies of doubly curved shell in terms of nonlocal parameter $\mu(nm)$ for various radius to thickness ratio R/h. One can conclude that with increase of radius to thickness ratio R/h, the stiffness of doubly curved nano shell is decreased and consequently the natural frequencies are decreased significantly.

Fig. 2 shows natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer μ(nm) for various values of θ_1 for $\theta_2 = 0.25\,Rad$. It is observed that with increase of nonlocal parameter μ(nm) , the fundamental frequencies of doubly curved nano shell are decreased significantly. One can conclude that this decreasing is due to decrease of stiffness of nano shell. These results are in accordance with this fact that based on Eringen's nonlocal elasticity theory, with increase of small scale parameter, the stiffness of nano structure is decreased. In addition, with increase of θ_1 the natural frequencies are increased smoothly. Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer μ(nm) for various values of θ_1 for $\theta_2 = 0.5\,Rad$ are presented in

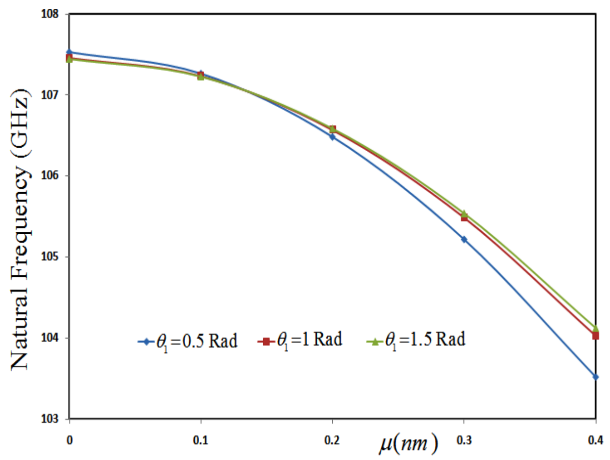

Fig. 2 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 0.25\,Rad$

Fig. 3. These results show that the natural frequencies are decreased with increase of θ_1 for nonlocal parameter less than 0.25 nm and for one greater than 0.25 nm, the natural frequencies are increased with increase of θ_1.

Figs. 4, 5, 6 and 7 show variation of natural frequencies in terms of nonlocal parameter $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 1.$ 1.5. 2 and 5 Rad. These results show that with increase of θ_1 the natural frequencies are decreased significantly. One can conclude that with increase of $\theta_1 = {}^{L_1}/_{R_1}$ the length of doubly curved along the α direction is increased and consequently the stiffness and natural frequencies of nano shell are decreased.

Influence of ratio of radius to thickness on the natural frequencies of doubly curved nano shell is presented in Fig. 8. This figure shows variation of natural frequencies in terms of nonlocal parameter $\mu(\mathrm{nm})$ for various dimensionless values ${}^{\mathrm{R}}/_{\mathrm{h}} = 10, 20,$ 50 and 100. The numerical results indicate that with increase of radius to

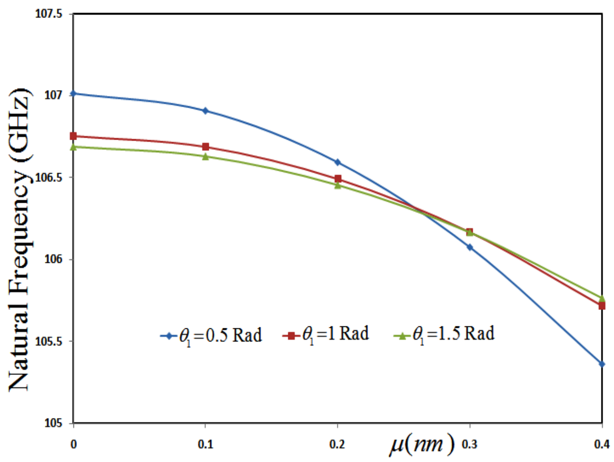

Fig. 3 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 0.5\,Rad$

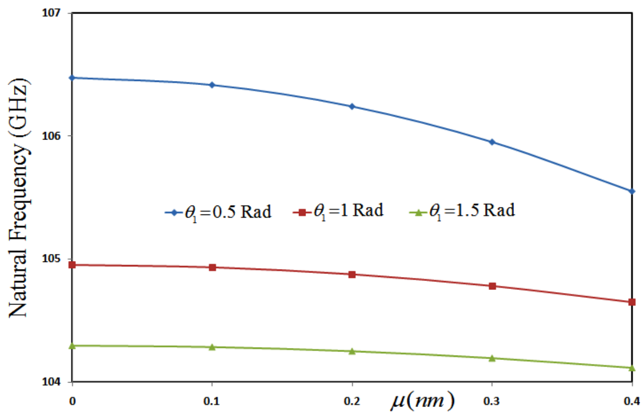

Fig. 5 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 1.5\,Rad$

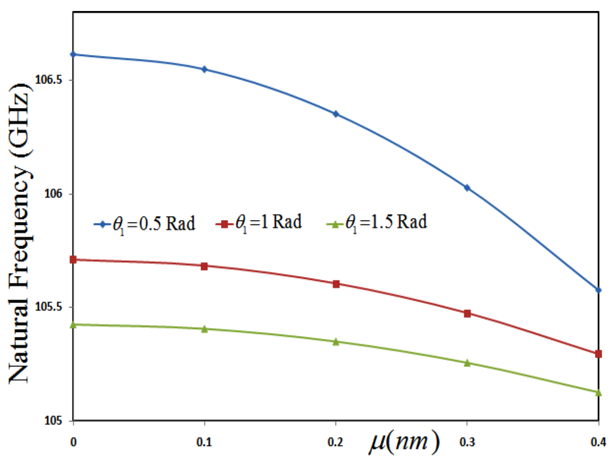

Fig. 4 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 1\,Rad$

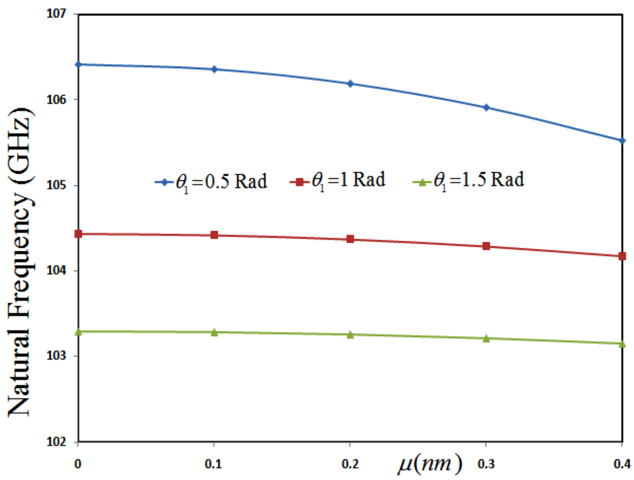

Fig. 6 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 2\,Rad$

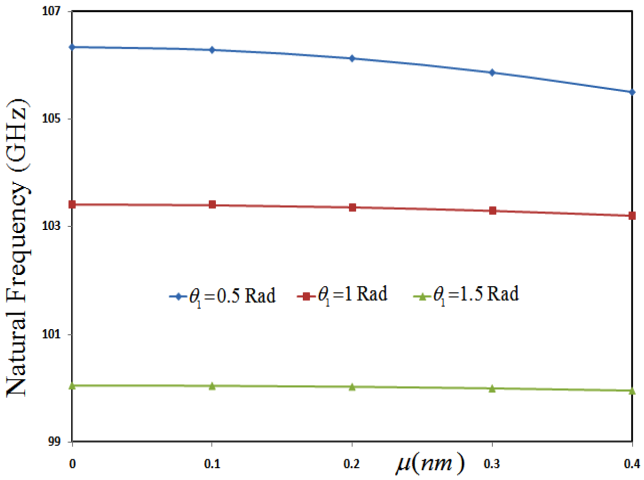

Fig. 7 Natural frequencies of doubly curved piezoelectric nano shell in terms of nonlocal parameter in nanometer $\mu(\mathrm{nm})$ for various values of θ_1 for $\theta_2 = 5\ Rad$

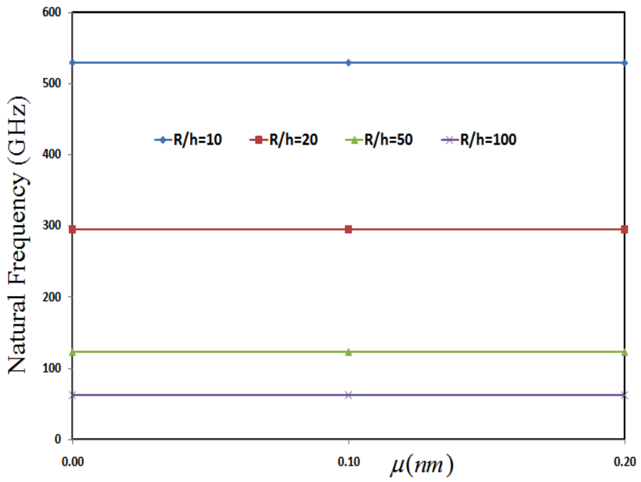

Fig. 8 Variation of natural frequencies in terms of nonlocal parameter $\mu(\mathrm{nm})$ for various dimensionless values $\mathrm{R}/_{\mathrm{h}} = 10.\ 20.\ 50.100$

thickness ratio $\mathrm{R}/_{\mathrm{h}}$, the fundamental natural frequencies are decreased significantly. One can conclude that with increase of radius to thickness ratio $\mathrm{R}/_{\mathrm{h}}$, the stiffness of structure is decreased that leads to decrease of fundamental frequencies of doubly curved nano shell.

5. Conclusions

The governing equations of motion for a doubly curved piezoelectric nano shell were derived in this paper based on Hamilton's principle. First order shear deformation theory and nonlocal piezo-elasticity relations were employed to derive the governing equations of motion. The size dependency was accounted in the governing equations of motion based on Eringen's nonlocal elasticity theory through employing the nonlocal parameter. The numerical results were obtained using Navier's method for a simply-supported nano shell. The numerical results show that some significant parameters such nonlocal parameter, radii of curvature and thickness of nano shell has important effects on the free vibrations responses.

The nonlocal parameter based on Eringen's nonlocal elasticity theory has significant influence on the free vibration responses of doubly curved piezoelectric nano shell. One can see that with increase of nonlocal parameter, the stiffness of material is decreased and consequently the natural frequencies are decreased.

The defined angles of doubly curved nano shell (θ_1, θ_2) have important effects on the free vibration response. The numerical results indicate that variation of natural frequencies in terms of θ_1 are strongly depend on the value of θ_1 and μ. One can concluded that for θ_2 more than 1 radian, the natural frequencies are decreased with increase of θ_1, however for θ_2 less than 1 radian, the natural frequencies are depending on both nonlocal parameter μ and θ_1 concurrently.

The influence of radius to thickness ratio $\mathrm{R}/_{\mathrm{h}}$ was studied on the variation of fundamental natural frequencies of doubly curved piezoelectric nano shell. The numerical results indicates that with increase of radius to thickness ratio $\mathrm{R}/_{\mathrm{h}}$, the stiffness of structure is decreased that leads to decrease of fundamental frequencies of doubly curved piezoelectric nano shell.

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CC

Appendix

$$\{A_1, A_2, A_3\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] Q_{11} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}, \frac{1}{R_1}, \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_4. A_5. A_6\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] Q_{12} \left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}. \frac{1}{R_2}. \frac{z}{R_2\left(1+\frac{z}{R_2}\right)}\right\} dz$$

$$\{A_7\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] e_{13} \frac{\pi}{h} sin\frac{\pi z}{h} dz. N_\alpha{}^{\Psi} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] \frac{2}{h} \Psi_0 e_{13}\, dz$$

$$\{A_8. A_9. A_{10}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right]\left[1+\frac{z}{R_1}\right] Q_{11} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_1}. \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_{11}. A_{12}. A_{13}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right]\left[1+\frac{z}{R_1}\right] Q_{12} \left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}. \frac{1}{R_2}. \frac{z}{R_2\left(1+\frac{z}{R_2}\right)}\right\} dz$$

$$\{A_{14}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right]\left[1+\frac{z}{R_1}\right] e_{13} \frac{\pi}{h} sin\frac{\pi z}{h} dz. N_\beta{}^{\Psi} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] \frac{2}{h} \Psi_0 e_{13}\, dz$$

$$N_\alpha{}^{1\Psi} = N_\beta{}^{1\Psi} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right]\left[1+\frac{z}{R_1}\right] \frac{2}{h} \Psi_0 e_{13}\, dz$$

$$\{A_{15}. A_{16}. A_{17}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] zQ_{11} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_1}. \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_{18}. A_{19}. A_{20}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] zQ_{12} \left\{\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}. \frac{1}{R_2}. \frac{z}{R_2\left(1+\frac{z}{R_2}\right)}\right\} dz$$

$$\{A_{21}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] ze_{13} \frac{\pi}{h} sin\frac{\pi z}{h} dz. M_\alpha{}^{\Psi} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] \frac{2}{h} z\Psi_0 e_{13}\, dz$$

$$\{A_{22}. A_{23}. A_{24}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] Q_{22} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_1}. \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_{25}. A_{26}. A_{27}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right]\left[1+\frac{z}{R_1}\right] Q_{22} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_1}. \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_{28}. A_{29}. A_{30}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] zQ_{22} \left\{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_1}. \frac{z}{R_1\left(1+\frac{z}{R_1}\right)}\right\} dz$$

$$\{A_{31}. A_{32}. A_{33}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] Q_{44} \left\{1. \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}. \frac{1}{R_1}\right\} dz. A_{34} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] e_{42} \frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \cos\frac{\pi z}{h} dz$$

$$\{A_{35}. A_{36}. A_{37}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] Q_{55} \left\{1. \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}. \frac{1}{R_2}\right\} dz.$$

$$A_{38} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] e_{51} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)} cos\frac{\pi z}{h} dz$$

$$\{A_{39}.A_{40}.A_{41}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right]\left[1+\frac{z}{R_2}\right] Q_{44} \left\{1.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.\frac{1}{R_1}\right\} dz.$$

$$A_{42} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right]\left[1+\frac{z}{R_2}\right] e_{42} \frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \cos\frac{\pi z}{h} dz$$

$$\{A_{43}.A_{44}.A_{45}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right]\left[1+\frac{z}{R_2}\right] Q_{55} \left\{1.\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2}\right\} dz.$$

$$A_{46} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right]\left[1+\frac{z}{R_2}\right] e_{51} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)} cos\frac{\pi z}{h} dz$$

$$\{A_{47}.A_{48}.A_{49}.A_{50}.A_{51}.A_{52}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] Q_{66} \left\{\begin{matrix} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \\ .z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\right).z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \end{matrix}\right\} dz$$

$$\{A_{53}.A_{54}.A_{55}.A_{56}.A_{57}.A_{58}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] Q_{66} \left\{\begin{matrix} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \\ .z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\right).z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \end{matrix}\right\} dz$$

$$\{A_{59}.A_{60}.A_{61}.A_{62}.A_{63}.A_{64}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] zQ_{66} \left\{\begin{matrix} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \\ .z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\left(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}\right).z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \end{matrix}\right\} dz$$

$$\{A_{65}.A_{66}.A_{67}.A_{68}.A_{69}.A_{70}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] zQ_{66} \left\{\begin{matrix} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.z\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.z\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \\ .z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)(\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.z\frac{1}{2}\left(\frac{1}{R_1}-\frac{1}{R_2}\right)\frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \end{matrix}\right\} dz$$

$$\{A_{71}.A_{72}.A_{73}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] e_{15} \left\{1.\frac{1}{R_1\left(1+\frac{z}{R_1}\right)}.\frac{1}{R_2}\right\} \cos\frac{\pi z}{h} dz.$$

$$\{A_{74}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_2}\right] k_{11} \frac{1}{R_1\left(1+\frac{z}{R_1}\right)} \frac{\partial\Psi}{\partial\alpha} \cos^2\frac{\pi z}{h} dz$$

$$\{A_{75}.A_{76}.A_{77}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] e_{24} \left\{1.\frac{1}{R_2\left(1+\frac{z}{R_2}\right)}.\frac{1}{R_1}\right\} \cos\frac{\pi z}{h} dz.$$

$$\{A_{78}\} = \int_{-h/2}^{+h/2} \left[1+\frac{z}{R_1}\right] k_{22} \frac{1}{R_2\left(1+\frac{z}{R_2}\right)} \frac{\partial \Psi}{\partial \alpha} \cos^2 \frac{\pi z}{h} dz$$

$$\{A_{79}.A_{80}.A_{81}.A_{82}.A_{83}.A_{84}\}$$
$$= \int_{-h/2}^{+h/2} \frac{\pi}{h} sin \frac{\pi z}{h} \left[1+\frac{z}{R_1}\right]\left[1+\frac{z}{R_2}\right] e_{31} \{\frac{1}{R_1\left(1+\frac{z}{R_1}\right)} . \frac{1}{R_1} . z \frac{1}{R_1\left(1+\frac{z}{R_1}\right)} . \frac{1}{R_2\left(1+\frac{z}{R_2}\right)} . \frac{1}{R_2} . z \frac{1}{R_2\left(1+\frac{z}{R_2}\right)}\} dz$$

$$A_{85} = \int_{-h/2}^{+h/2} (\frac{\pi}{h} sin \frac{\pi z}{h})^2 [1+\frac{z}{R_1}][1+\frac{z}{R_2}] k_{33}\, dz . D_z{}^{\Psi} = \int_{-h/2}^{+h/2} \frac{\pi}{h} sin \frac{\pi z}{h} [1+\frac{z}{R_1}][1+\frac{z}{R_2}] \frac{2}{h} \Psi_0 k_{33}\, dz$$