Steel and Composite Structures, *Vol. 25, No. 4 (2017) 415-426*
DOI: https://doi.org/10.12989/scs.2017.25.4.415

Thermal vibration of two-dimensional functionally graded (2D-FG) porous Timoshenko nanobeams

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(*Received March 14, 2017, Revised July 27, 2017, Accepted August 01, 2017*)

Abstract. The thermo-mechanical vibration behavior of two dimensional functionally graded (2D-FG) porous nanobeam is reported in this paper. The material properties of the nanobeam are variable along thickness and length of the nanobeam according to the power law function. The nanobeam is modeled within the framework of Timoshenko beam theory. Eringen's nonlocal elasticity theory is used to develop the governing equations. Using the generalized differential quadrature method (GDQM) the governing equations are solved. The effect of porosity, temperature distribution, nonlocal value, L/h, FG power indexes along thickness and length and are investigated using parametric studies.

Keywords: thermal vibration; Eringen theory; porous; imperfect nanobeam; 2D-FGM

1. Introduction

The applications and usages of nanostructures are increasing every year from energy harvesting to biomedical diagnosis. Thus, it is needed to understand the nature of the behavior of these nanostructures before starting the expensive manufacturing process. So, many scientists have worked on the different characteristics of nanosystems.

Among these studies, many papers are published regarding the vibration behavior of nanobeams. Şimşek (2014) used nonlocal beam model to study the nonlinear vibration of nanobeams with axially immovable ends. Eltaher *et al.* (2013a) studied the vibration of a nonlocal Euler–Bernoulli nanobeams using finite element analysis. Malekzadeh and Shojaee (2013) studied the surface and nonlocal effects on the nonlinear flexural vibration of non-uniform nanobeams. Thai and Vo (2012) studied a nonlocal sinusoidal shear deformation beam theory to study the bending, buckling, and vibration behaviors of nanobeams. Using nonlocal theory, Hayati *et al.* (2017) studied the static and dynamic behavior of a curved single-walled carbon nanotube under twist–bending couple. Hosseini and Rahmani (2017) studied the axial and transverse dynamic response of a functionally graded nanobeam under a moving constant load. The free vibration model of a cantilever FG nanobeam with an attached mass at tip and under thermal loading is discussed by Rahmani *et al.* (2017b). buckling and free vibration of a curved Timoshenko FG microbeam is studied by Rahmani et al. (2017a) based on strain gradient theory (SGT) theory. In another study, Eltaher *et al.* (2013b) performed analysis on the coupled effects of surface properties and nonlocal effects on vibration behavior of nonlocal nanobeams by utilizing FEM. These were only a few examples of the expanded studies in the field of nanobeams and so many other studies have been focused on the vibrational analysis on nanobeams (Murmu and Adhikari 2010, Berrabah *et al.* 2013, Zhang *et al.* 2009, Lei *et al.* 2013, LI *et al.* 2011), etc.

Many nanosystems are subjected to thermal effects as the working temperature of these systems might rise up or come down. Hence, many researches on vibrational analysis of the nanobeams have considered the thermal effects. Youssef and Elsibai (2011) studied the thermal vibrational behavior of nanobeams made of gold. They studied the non-Fourier effect in heat conduction and also the coupling effect between temperature and strain rate. In addition, Ke and Wang (2012) analyzed the thermoelectric vibration of the piezoelectric Timoshenko nanobeams based on the nonlocal theory. Amirian *et al.* (2014) investigated the free vibration of nanobeams made of circular alumina considering the surface and thermal effects. Elsibai and Youssef (2011) analyzed the vibration of gold nano-beam resonator induced by ramp type heating.

Functionally graded materials (FGMs) are a special branch of composite materials which have varying material composition along its direction(s). As the functionally graded materials (FGMs) can possess the different material characteristics, their applications are wide-range. Optoelectronic devices (Wośko *et al.* 2005), advanced thermal barrier coating (Lee *et al.* 1996), sensor and energy applications (Müller *et al.* 2003), bio-medical applications (Pompe *et al.* 2003, Watari *et al.* 2004) and also solid

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http://www.techno-press.org/?journal=scs&subpage=6
ISSN: 1229-9367 (Print), 1598-6233 (Online)

insulator in gaseous insulation system (Kato *et al.* 2006) are just a few applications of FGMs. Importance of FGMs in future technology have made many researchers to conduct some wats for manufacturing these materials. Hassanin and Jiang (2010) have utilized infiltration to fabricate microceramic elements. They have utilized soft lithography, colloidal powder processing and the infiltration of yttria stabilized zirconia with a pre-ceramic polymer solution in their technique. Olevsky *et al*. (2007) have shown that electrophoretic deposition (EPD) sintering is an applicable method to produce net-shape bulk FGM. In addition, Anné *et al.* (2005) have fabricated a complex shaped functionally graded alumina and zirconia components for biomedical applications using EPD. For the reason that these materials have high temperature resistance, many studies are conducted to investigate the vibration of those nanobeams which are made of FGMs, with or without the thermal effects. Ansari *et al.* (2015) studied the nonlinear forced vibration of FG nanobeams in a thermal environment and by including the surface effects. Nazemnezhad and Hosseini-Hashemi (2014) have studied the nonlinear free vibration of Euler–Bernoulli FG nanobeams using the nonlocal elasticity theory. Hosseini-Hashemi *et al.* (2014) have studied the free vibration of FG nanobeams by considering surface effects and the piezoelectric field using nonlocal elasticity theory. Rahmani and Pedram (2014) studied vibration of Timoshenko nanobeam made of functionally graded material (FGM). Li (2013) studied the transverse vibrations of axially traveling nanobeams including strain gradient and thermal effects. Ahouel et al. (2016) studied bending, buckling, and vibration of functionally graded (FG) nanobeams using the nonlocal differential constitutive relations of Eringen. Ebrahimi and Salari (2015) studied the size-dependent thermal buckling and vibration of FG nanobeams using nonlocal elasticity theory of Eringen. Functionally graded materials were also considered in so many other studies on nanobeams, nanoplates, etc., such as (Natarajan *et al.* 2012, Belkorissat *et al*. 2015, Şimşek and Yurtcu 2013a, Shafiei *et al.* 2016b, Rafiee *et al*. 2013), etc.

Porous materials which contain pores are a type of material with different usages from oil industry to micro- and nano-technologies. In addition to FGMs, porous materials are also considered to study the behavior of beams and nanobeams. Renault *et al.* (2011) studied the visco-elastic parameters of soft, highly damped porous materials from beam bending vibrations. Della and Shu (2015) studied the vibration behavior of porous beams with embedded piezoelectric sensors and actuators. Leclaire *et al.* (2001) studied the vibrations of a rectangular porous plate. Zhou and Ma (2014) presented analysis on the dynamic of the 2D fluid-saturated porous beam based on the linear elastic theory and the Biot's model for saturated porous media. In one of the first studies on 2D-FGMs Nemat-Alla (2003) studied the ability of these materials to withstand super high temperatures and to give more reduction in thermal stresses. In another study, Nemat-Alla *et al*. (2009) proposed a 3D finite element model to study the elastic-plastic stress of 2D-FGM plates.

It is seen that no one has ever studied the vibration behavior of a nanobeam made of porous two dimensional functionally graded (2D-FG) materials even without consideration of thermal effects. Thus, here this problem is analyzed taking the thermal effects into the consideration.

The nanobeam is considered to be in the framework of Timoshenko beam theory and the material is changing along both thickness and length according to the power law function. Using the Eringen's nonlocal theory, the governing equations are developed and the generalized differential quadrature method (GDQM) is used to solve the equations. The effects of temperature change, FG and AFG power indexes, L/h, temperature distribution and nonlocal parameter are studied by presenting figures and tables.

2. Problem and formulation

2.1 Functionally graded material

Consider a 2D-FG porous nanobeam which is composed of metal and ceramic with varying material composition along x and z directions (Fig. 1). The material composition can be varying along length (Fig. 1(a)), along thickness (Fig. 1(b)) and also along thickness and length simultaneously (Fig. 1(c)). It is clear that the mechanical properties of the nanobeam, i.e., Young's modulus 'E', Poisson's ratio 'ν', shear modulus 'G' and mass density 'ρ' vary with the material composition.

Considering two types of even (type I) and uneven (type II) of porosity distribution across the thickness, and with the porosity volume fraction of β (β <<1), the modified rule of mixture for the2D-FG porous nanobeam becomes

$$\mathrm{P(x,z)} = \mathrm{P_m}\left(\mathrm{V_m} - \frac{\beta}{2}\right) + \mathrm{P_c}\left(\mathrm{V_c} - \frac{\beta}{2}\right) \tag{1}$$

Where the subscripts of $()_c$ and $()_m$ are used to define the ceramic and metal, respectively. The power law of the volume fraction of ceramic is (Şimşek 2016, Shafiei *et al.* 2016b)

$$\mathrm{V_c(x,z)} = \left(\frac{1}{2} + \frac{\mathrm{z}}{\mathrm{h}}\right)^{\mathrm{nz}} \left(\frac{\mathrm{x}}{\mathrm{L}}\right)^{\mathrm{nx}} \tag{2}$$

Here, '*nx*' and '*nz*' are the AFG (axially functionally graded), and FG (functionally graded) power indexes respectively, which are related to the volume fraction change of the material composition. It should be noted that when both of *nz* and *nx* have non-negative values, the nanobeam is then 2D-FG. Hence, the material properties of the even porosity (FGM-I, Fig. 2) is obtained as (Wattanasakulpong and Chaikittiratana 2015, Şimşek 2016, Shafiei *et al.* 2016b)

$$\mathrm{P(x,z)} = \mathrm{P_m} + (\mathrm{P_c} - \mathrm{P_m})\left(\frac{1}{2} + \frac{\mathrm{z}}{\mathrm{h}}\right)^{\mathrm{nz}} \left(\frac{\mathrm{x}}{\mathrm{L}}\right)^{\mathrm{nx}} - \frac{\beta}{2}(\mathrm{P_c} + \mathrm{P_m}) \tag{3}$$

where *nz* and *nx* are FG and AFG power indexes and z and x are the distance from the mid-plane and left end of the FG beam respectively. The material of the beam is pure ceramic when *nx* and *nz* are set to be zero and increment of *nx* and *nz* increase the metal volume fraction. Thus, Young's

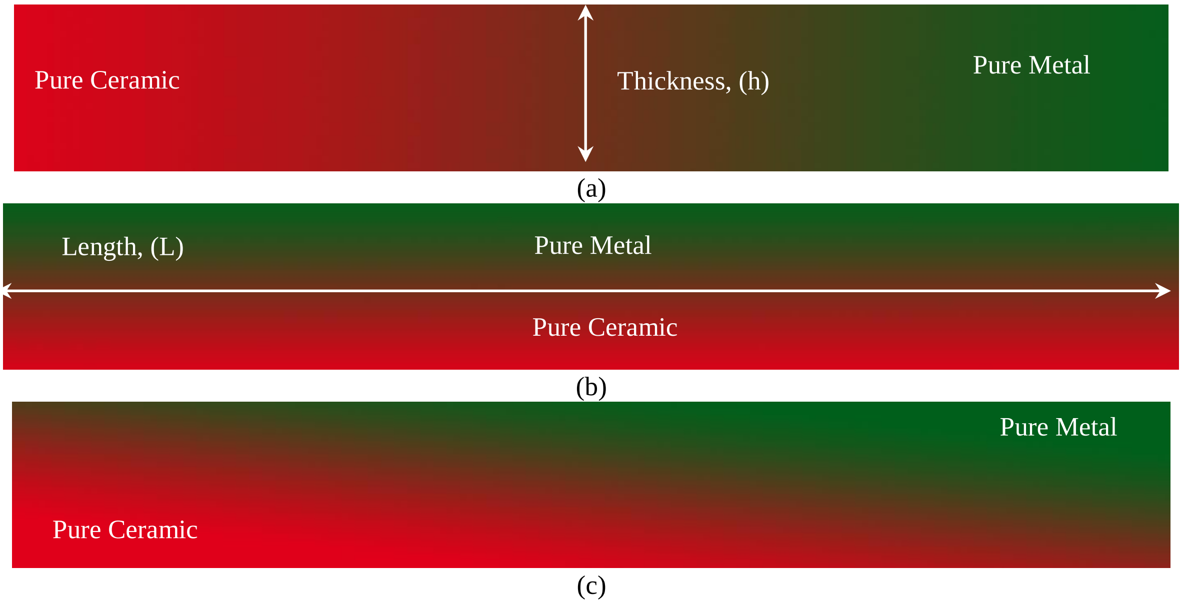

Fig. 1 Schematic of the different material distributions, (a) AFG, (b) FG and (c) 2D-FG

modulus ‘E’, Poisson’s ratio ‘ν’, the thermal distribution ‘α’, mass density ‘ρ’ and shear modulus ‘G’ equations of the imperfect FGM-I nanobeam is defined as

$$E(x,z)=E_m+(E_c-E_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(E_c+E_m) \tag{4a}$$

$$\nu(x,z)=\nu_m+(\nu_c-\nu_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\nu_c+\nu_m) \tag{4b}$$

$$\alpha(x,z)=\alpha_m+(\alpha_c-\alpha_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\alpha_c+\alpha_m) \tag{4c}$$

$$\rho(x,z)=\rho_m+(\rho_c-\rho_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\rho_c+\rho_m) \tag{4d}$$

$$G(x,z)=\frac{E(x,z)}{2+2\nu(x,z)} \tag{4e}$$

On the other hand, the mechanical properties of uneven porosity distribution of FG porous II (Fig. 2) can be obtained as

$$E(x,z)=E_m+(E_c-E_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(E_c+E_m)\left(1-\frac{2|z|}{h}\right) \tag{5a}$$

$$\nu(x,z)=\nu_m+(\nu_c-\nu_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\nu_c+\nu_m)\left(1-\frac{2|z|}{h}\right) \tag{5b}$$

$$\alpha(x,z)=\alpha_m+(\alpha_c-\alpha_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\alpha_c+\alpha_m)\left(1-\frac{2|z|}{h}\right) \tag{5c}$$

$$\rho(x,z)=\rho_m+(\rho_c-\rho_m)\left(\frac{1}{2}+\frac{z}{h}\right)^{nz}\left(\frac{x}{L}\right)^{nx}-\frac{\beta}{2}(\rho_c+\rho_m)\left(1-\frac{2|z|}{h}\right) \tag{5d}$$

$$G(x,z)=\frac{E(x,z)}{2+2\nu(x,z)} \tag{5e}$$

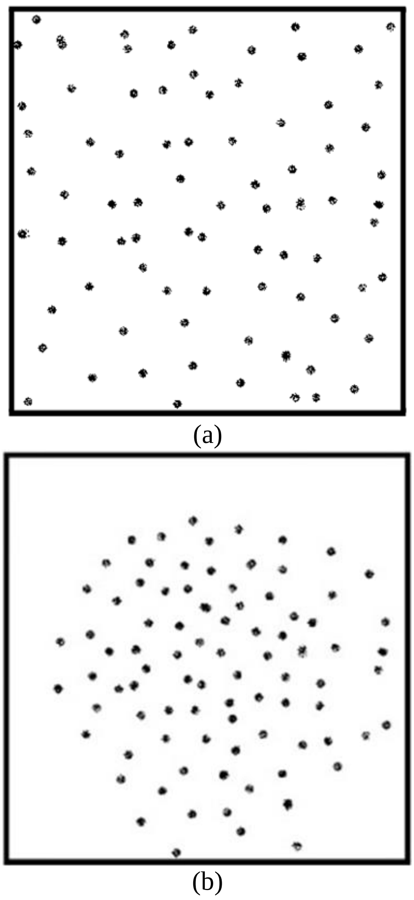

Fig. 2 Cross section area of FG porous beam. A: even distribution of porosities (FGM-I). B: uneven distribution of porosities (FGM-II)

Table 1 Temperature dependent coefficients of Young's modulus, thermal expansion coefficient and mass density (Yang and Shen 2002)

Material	Properties	P_0	P_{-1}	P_1	P_2	P_3
SUS304	E (Pa)	**2.0104e+11**	**0**	**0.000308**	**-6.53e-07**	**0**
	α (K-1)	**1.23e-05**	**0**	**0.000809**	**0**	**0**
	ρ (Kg/m3)	**8166**	**0**	**0**	**0**	**0**
	ν	**0.3262**	**0**	**-0.0002**	**3.80e-07**	**0**
Al_2O_3	E (Pa)	**3.4955e+11**	**0**	**-0.0003853**	**4.027e-07**	**-1.673e-11**
	α (K-1)	**6.8269e-06**	**0**	**0.0001838**	**0**	**0**
	ρ (Kg/m3)	**3750**	**0**	**0**	**0**	**0**
	ν	**0.26**	**0**	**0**	**0**	**0**

For the Using the nonlinear thermo-elasticity equation, the temperature-dependent material properties can be obtained at temperature T as (Touloukian and Ho 1970)

$$P = P_0\left(P_{-1}T^{-1} + P_1T + P_2T^2 + P_3T^3 + 1\right) \tag{6}$$

here, P_0, P_{-1}, P_1, P_2 *and* P_3 are the temperature-dependent coefficients of material properties which are given in Table 1.

2.2 Mathematical modeling

2.2.1 Nonlocal theory

The classic continuum theory considers every point in a body dependent on the small neighborhood of that point. Besides, the classic theories neglect the effect small distances between the atoms of a mass. For these reasons, the classic theories can't yield perfect results in studying the nano-sized materials as the small distances are considerable comparing to length, width and thickness of a nanobeam.

The Eringen's nonlocal elasticity theory defines the state of a point as a function of the region around that point (Eringen and Edelen 1972). In fact, the nonlocal elasticity theory not only considers the long range interactions in the material, but also considers the effect of tiny distances between the atoms. These exhaustive assumptions in the development of nonlocal theory, have made it a powerful tool in study of nanosystems (Juntarasaid *et al*. 2012, Ke and Wang 2012, Lee and Chang 2011). Here, using the nonlocal theory, the vibration equation of 2D-FG porous Timoshenko nanobeam is expressed as (Şimşek and Yurtcu, 2013b, Rahmani and Pedram 2014)

$$\delta u: \quad \frac{\partial}{\partial x}\left(A_{xx}\frac{\partial u}{\partial x} + B_{xx}\frac{\partial \varphi}{\partial x} - \bar{N}\right) = m_0\frac{\partial^2 u}{\partial t^2} + m_1\frac{\partial^2 \varphi}{\partial t^2} - (e_0a)^2\frac{\partial^2}{\partial x^2}\left(m_0\frac{\partial^2 u}{\partial t^2} + m_1\frac{\partial^2 \varphi}{\partial t^2}\right) \tag{7a}$$

$$\begin{aligned}\delta\varphi: \quad & \frac{\partial}{\partial x}\left(B_{xx}\frac{\partial u}{\partial x} + D_{xx}\frac{\partial \varphi}{\partial x} - \bar{M}\right) - C_{xz}\left(\varphi + \frac{\partial w}{\partial x}\right) \\ &= m_1\frac{\partial^2 u}{\partial t^2} + m_2\frac{\partial^2 \varphi}{\partial t^2} - (e_0a)^2\frac{\partial^2}{\partial x^2}\left(m_1\frac{\partial^2 u}{\partial t^2} + m_2\frac{\partial^2 \varphi}{\partial t^2}\right)\end{aligned} \tag{7b}$$

$$\begin{aligned}\delta w: \quad & \frac{\partial}{\partial x}\left[C_{xz}\left(\varphi + \frac{\partial w}{\partial x}\right)\right] + \frac{\partial}{\partial x}\left(-\bar{N}\frac{\partial w}{\partial x}\right) - (e_0a)^2\frac{\partial^2}{\partial x^2}\left[\frac{\partial}{\partial x}\left(-\bar{N}\frac{\partial w}{\partial x}\right)\right] \\ &= m_0\frac{\partial^2 w}{\partial t^2} - (e_0a)^2\frac{\partial^2}{\partial x^2}\left(m_0\frac{\partial^2 w}{\partial t^2}\right)\end{aligned} \tag{7c}$$

where

$$\left(A_{xx}, B_{xx}, D_{xx}\right) = \int_A E\left(x, z, T\right)\left(1, z, z^2\right)dA \tag{8a}$$

$$C_{xz} = K_S\int_A \frac{E\left(x, z, T\right)}{2\left(1 + \nu\left(x, z, T\right)\right)}dA \tag{8b}$$

$$m_i = \int_A \rho\left(x, z, T\right)z^i dA \tag{8c}$$

$\bar{N}$ and $\bar{M}$ are the external axial thermal load and thermal moment, respectively.

$$\left(\bar{N}, \bar{M}\right) = \int_A E\left(x, z, T\right)\alpha\left(x, z, T\right)\left(T - T_0\right)\left(1, z\right)dA \tag{9}$$

where T_0 is the reference temperature and here e_0a is the nonlocal parameter in Eringen's nonlocal elasticity theory. "K_S=5/6" is the shear correction factor. The boundary conditions for fully clamped are

$$u = 0 \qquad at \qquad x = 0 \; and \; x = L \tag{10a}$$

$$w = 0 \qquad at \qquad x = 0 \; and \; x = L \tag{10b}$$

$$\varphi = 0 \qquad at \qquad x = 0 \; and \; x = L \tag{10c}$$

2.3 Type of temperature rise

The temperature gradient nonlinear and along thickness. T_{UL} and T_{DR} define the temperature of the upper (at x=L) and lower (at x=0) surfaces respectively. According to the power function along thickness, the nonlinear temperature gradient is as follows (De Pietro *et al*. 2016, Mirjavadi *et al*. 2017)

$$T = T_0 + \Delta T\left(\frac{1}{2} + \frac{z}{h}\right)^{\alpha z}\left(\frac{x}{L}\right)^{\alpha x} \tag{11}$$

T_0=300(K) is reference point, where αx and αz is the power index of temperature variation function along length and thickness of beam and also, $\Delta T = T_{UL} - T_{DR}$.

3. Solution methodology

The solving procedure is processed using the generalized differential quadrature method (GDQM). For this purpose, the r-th order derivative of function $f(x_i)$ is

$$\left.\frac{\partial^r f(x)}{\partial x^r}\right|_{x=x_P} = \sum_{j=1}^{k} C_{ij}^{(r)} f(x_i) \tag{12}$$

k is the number of grid points along x direction and C_{ij} is

$$\begin{aligned} C_{ij}^{(1)} &= \frac{M(x_i)}{(x_i - x_j)M(x_j)}; \qquad i, j = 1,2,\ldots,\mathrm{n} \qquad \textit{and} \qquad \mathrm{i} \neq \mathrm{j} \\ C_{ij}^{(1)} &= -\sum_{j=1, i\neq j}^{n} C_{ij}^{(1)}; \qquad i = j \end{aligned} \tag{13}$$

where $M(x)$ is:

$$M(x_i) = \prod_{j=1, j\neq i}^{k} (x_i - x_j) \tag{14}$$

The weighting coefficient $C^{(r)}$, as

$$\begin{aligned} C_{ij}^{(r)} &= r\left[C_{ij}^{(r-1)}C_{ij}^{(1)} - \frac{C_{ij}^{(r-1)}}{(x_i - x_j)}\right]; \qquad i, j = 1,2,\ldots,k, \mathrm{i}\neq \mathrm{j} \qquad \textit{and} \qquad 2 \leq \mathrm{r} \leq k-1 \\ C_{ii}^{(r)} &= -\sum_{j=1, i\neq j}^{n} C_{ij}^{(r)}; \qquad i, j = 1,2,\ldots,k \qquad \textit{and} \qquad 1 \leq \mathrm{r} \leq k-1 \end{aligned} \tag{15}$$

The distribution of the mesh points is obtained using the Chebyshev-Gauss-Lobatto technique as

$$x_i = \frac{L}{2}\left(1 - \cos\left(\frac{(i-1)}{(N-1)}\pi\right)\right) \qquad i = 1,2,3,\ldots,k \tag{16}$$

The motion equations and boundary conditions of the bi-directional FG nanobeam (Eqs. (7) and (10)) are the combination of two matrixes. Then the stiffness matrixes can be calculated as

$$\left\{[K] - \omega^2[M]\right\}\{\lambda\} = 0 \tag{17}$$

Then, the motion equation is solved by GDQM and applying the ,the weighting coefficients (Eq. (15)) to the linear motion equation yields

$$\sum_{s=1}^{n} C_{rs}{}^{(1)}\left(A_{xx}\sum_{s=1}^{n} C_{rs}{}^{(1)}u_s + B_{xx}\sum_{s=1}^{n} C_{rs}{}^{(1)}\varphi_s\right) - \frac{d\bar{N}}{dx} = \omega^2\left[m_0 u_s + m_1\varphi_s - (e_0 a)^2\sum_{s=1}^{n} C_{rs}{}^{(2)}(m_0 u_s + m_1\varphi_s)\right] \tag{18a}$$

$$\begin{aligned} &\sum_{s=1}^{n} C_{rs}{}^{(1)}\left(B_{xx}\sum_{s=1}^{n} C_{rs}{}^{(1)}u_s + D_{xx}\sum_{s=1}^{n} C_{rs}{}^{(1)}\varphi_s\right) - \frac{d\bar{M}}{dx} - C_{xz}\left(\varphi_s + \sum_{s=1}^{n} C_{rs}{}^{(1)}w_s\right) \\ &= \omega^2\left[m_1 u_s + m_2\varphi_s - (e_{.0}a)^2\sum_{s=1}^{n} C_{rs}{}^{(2)}(m_1 u_s + m_2\varphi_s)\right] \end{aligned} \tag{18a}$$

$$\begin{aligned} &\sum_{s=1}^{n} C_{rs}{}^{(1)}\left[C_{xz}\left(\varphi_s + \sum_{s=1}^{n} C_{rs}{}^{(1)}w_s\right)\right] + \sum_{s=1}^{n} C_{rs}{}^{(1)}\left(-\bar{N}\sum_{s=1}^{n} C_{rs}{}^{(1)}w_s\right) \\ &-(e_0 a)^2\sum_{s=1}^{n} C_{rs}{}^{(2)}\left[\sum_{s=1}^{n} C_{rs}{}^{(1)}\left(-\bar{N}\sum_{s=1}^{n} C_{rs}{}^{(1)}w_s\right)\right] = \omega^2\left[m_0 w_s - (e_0 a)^2\sum_{s=1}^{n} C_{rs}{}^{(2)}(m_0 w_s)\right] \end{aligned} \tag{18c}$$

Using the boundary conditions of nanobeam (Eq. (10)) and by assembling the related matrixes to the boundary conditions and governing equations, the linear fundamental vibration of nanobeam can be calculated as below (Shafiei *et al.* 2016a)

$$\begin{bmatrix} [K_{dd}] & [K_{db}] \\ [K_{bd}] & [K_{bb}] \end{bmatrix}\begin{Bmatrix} \{\lambda_d\} \\ \{\lambda_b\} \end{Bmatrix} = \omega^2\begin{bmatrix} [M_{dd}] & [M_{db}] \\ [M_{bd}] & [M_{bb}] \end{bmatrix}\begin{Bmatrix} \{\lambda_d\} \\ \{\lambda_b\} \end{Bmatrix} \tag{19}$$

where b and d indexes indicate the boundary and domain, respectively and λ is the mode shape.

4. Numerical result

The evaluation of the thermo-mechanical vibrational behavior of 2D-FG porous nano-beam is performed through illustration of different figures and tables. The results are derived for FG, AFG and 2D-FG Timoshenko nano-beam considering two different types of porous materials. The boundary condition is clamped in the presented figures and tables. The results show the effect of porosity volume fraction, FG and AFG power indexes, L/h, temperature gradient, temperature change and nonlocal value on the non-dimensional frequency of 2D-FG porous nanobeams.

To have better illustration of the results, the non-dimensional parameters are defined as below

$$\mu = \frac{\mathrm{e_0 a}}{\mathrm{L}} \tag{20a}$$

$$\Psi^2 = \left.\frac{m_0 L^4}{D_{xx}}\right|_{pure-ceramic} \omega^2 \tag{20b}$$

where μ and Ψ are the non-dimensional nonlocal parameter and non-dimensional frequency, respectively. The validity of the derived equations and the solution procedure are shown through the comparison of the results with the results of Wang *et al.* (2007) in Table 2.

Fig. 3 is served to depict the effect of FG and AFG power indexes (*nz* and *nx*) on the non-dimensional frequency of perfect and porous local and nano-beams. It is seen that the increment of FG and AFG power indexes decrease the non-dimensional frequency. Besides, it can be said that the effect of *nx* is more than that of *nz* and also, increasing FG or AFG power indexes, decrease the effect of AFG or FG power indexes respectively. It should be noted that the decrement of the non-dimensional frequency due to the increment of *nx* or *nz* happens because as the FG or AFG power indexes increase, the volume portion of metal which have lower stiffness increases. And this decreases the total stiffness of the nanobeam, which leads to the decrement of the frequency. Comparing Figs. 3(a) and 3(c) respectively with Figs. 3(b) and (d) shows that the frequency of local beam is higher than nanobeam, which is due to the lower stiffness of the nonlocal nanobeams.

The non-dimensional frequency of FG and AFG porous types I and II local beams is shown with respect to the FG and AFG power indexes and for different values of porosity volume fractions (β). Similar to Fig. 3, it is seen in Fig. 4 that

Table 2 Comparison of the results of non-dimensional frequency ($\sqrt{\Psi}$) with Wang *et al*. (2007)

		Simply supported	Clamped	Clamped-Simply supported	Clamped-Free
μ=0	Wang *et al.* (2007)	**3.1416**	**4.73**	**3.9266**	**1.8751**
	Present	**3.141589**	**4.730024**	**3.926595**	**1.875102**
μ=0.1	Wang *et al.* (2007)	**3.0685**	**4.5945**	**3.8209**	**1.8792**
	Present	**3.068528**	**4.594441**	**3.820884**	**1.879161**
μ=0.3	Wang *et al.* (2007)	**2.68**	**3.9184**	**3.2828**	**1.9154**
	Present	**2.680000**	**3.918355**	**3.282835**	**1.915367**
μ=0.5	Wang *et al.* (2007)	**2.3022**	**3.3153**	**2.7899**	**2.0219**
	Present	**2.302229**	**3.315312**	**2.789921**	**2.02192**

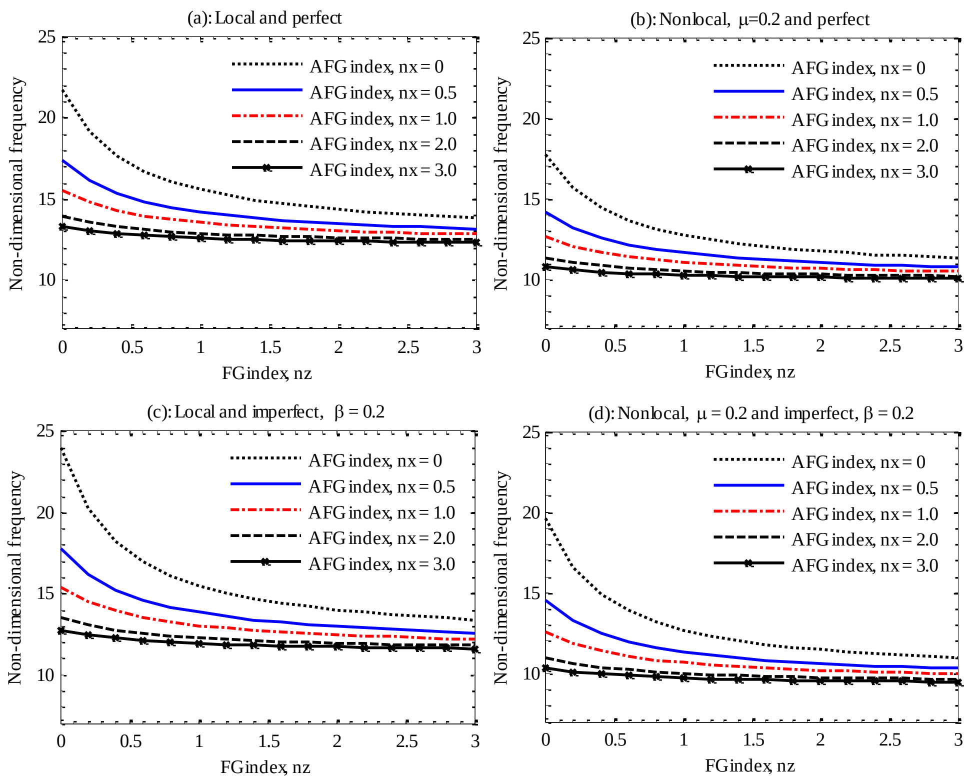

Fig. 3 Non-dimensional frequency of clamped FG-Imperfect and porous nanobeams versus the FG power index when L/h=15

the increment of FG and AFG power indexes decreases the non-dimensional frequency of local beam.Figs. 4(c) and 4(d) show that the non-dimensional frequency of local beam increases with the porosity volume fraction (β) and the increment of FG or AFG power indexes decrease the dependency of frequency on the porosity volume fraction (β). The more complex behavior of FGM-I local beam is shown in Figs. 4(a) and 4(b) where the effect of β depends on the value of FG and AFG power indexes. The reason of this behavior is that the dependency of the non-dimensional frequency of FGM-I on the FG or AFG power indexes is highly affected by β and as β increases, the effect of FG and AFG power indexes increases. Finally, this changes the effect of β in higher values of FG/AFG power indexes.

Non-dimensional frequency of two dimensional FG (2D-FG) nanobeam is shown in Fig. 5 versus FG and AFG power indexes simultaneously. It is observed that increasing the porosity volume fraction (β) increases the frequency of FGM-II nanobeams as the stiffness of the nanobeam increases with the porosity volume fraction. In addition, increment of FG/AFG power indexes and nonlocal parameter decreases the non-dimensional frequency as it decreases the stiffness.

Figs. 6-8 show the non-dimensional frequency of nanobeam versus the temperature change respectively for different porosity volume fractions, different nonlocal values and different power indexes of temperature variation function. It is seen in Figs. 6-8 that the non-dimensional frequency decreases with the increment of temperature change, FG and AFG power indexes.

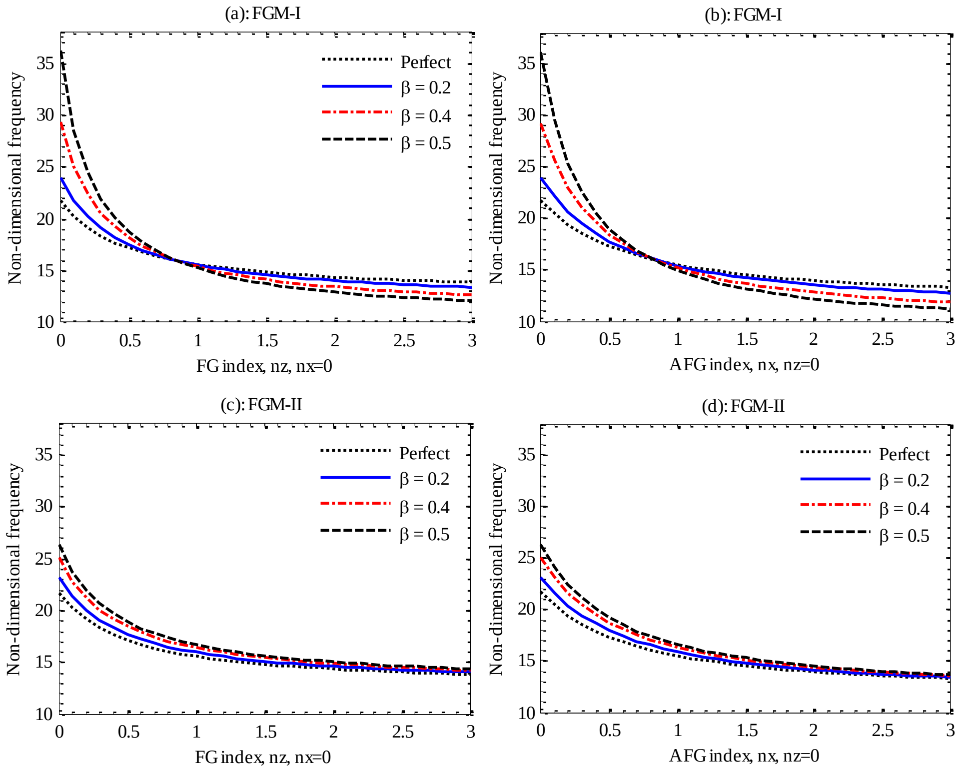

Fig. 4 Non-dimensional frequency of FG and AFG porous-I and II clamped local beams (μ=0) when L/h=15

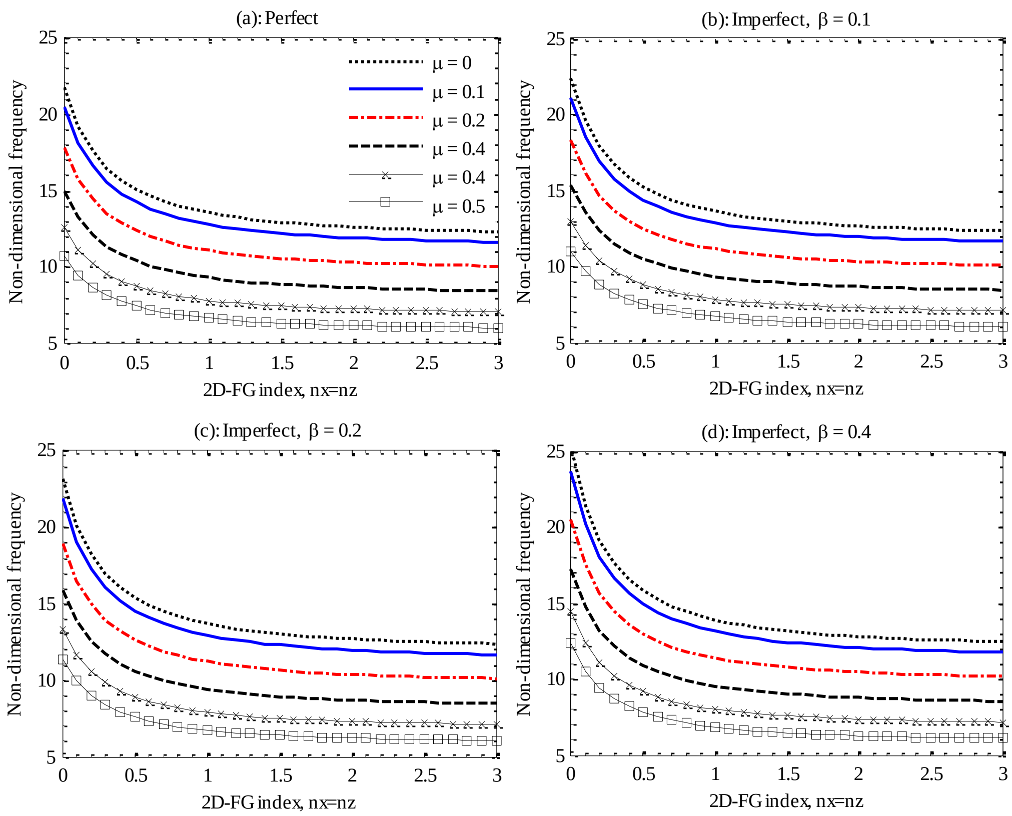

Fig. 5 Non-dimensional frequency of 2D-FG porous-II clamped nanobeam versus FG/AFG power indexes when L/h=15, FG-II

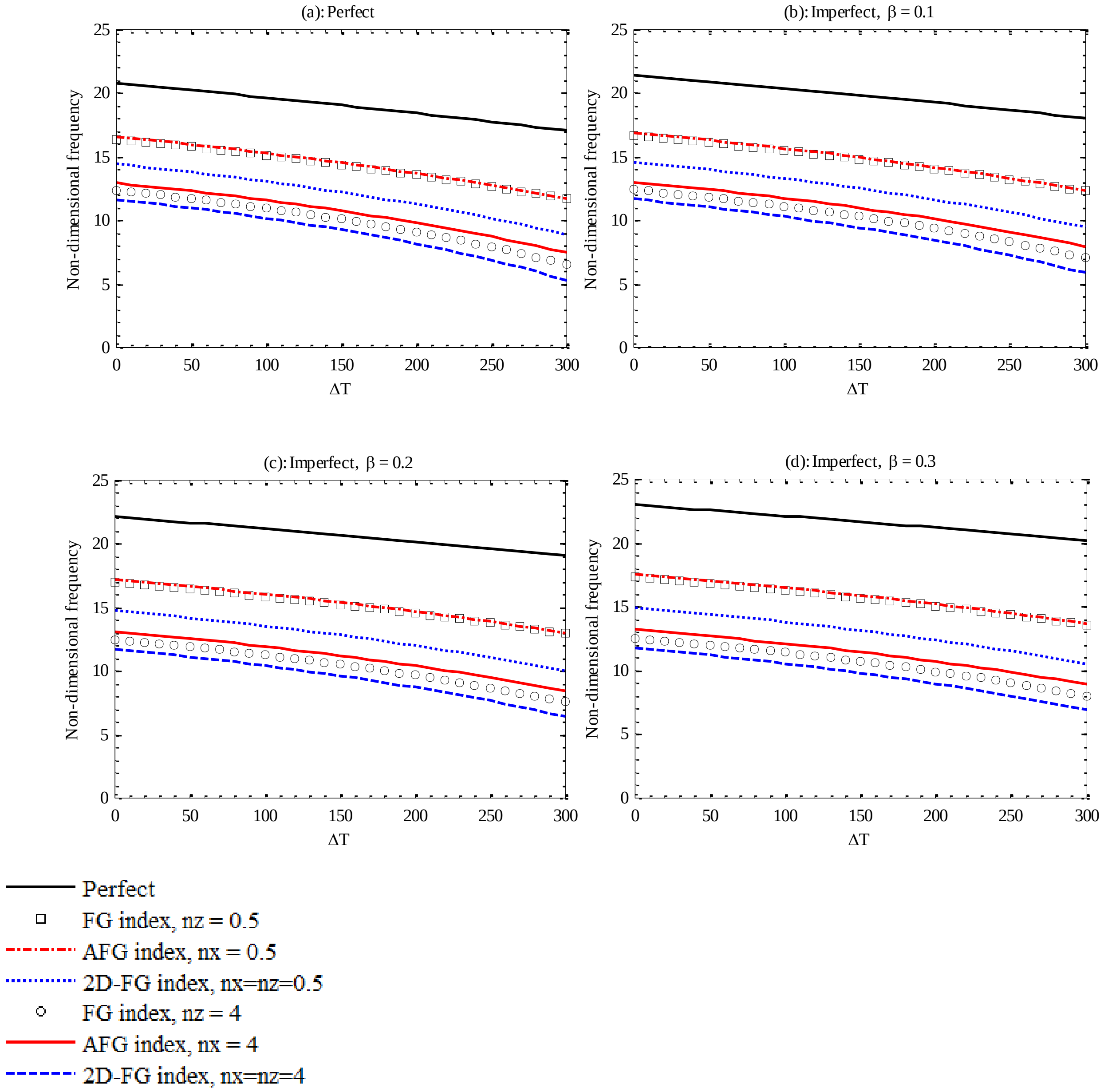

Fig. 6 Non-dimensional frequency of FG, AFG, and 2D-FG porous-II clamped nanobeams versus temperature change when μ=0.1, L/h=20, αz=αx=0.1, with uniform temperature gradient for different values of porosity

Besides, it is seen in Fig. 6 that the increment of porosity volume fraction increases the non-dimensional frequency due to the increment of stiffness.

Figs. 7 and 8 also depict the effect of αz, αx and nonlocal parameter on the dependency of the non-dimensional frequency on temperature change. It is seen in Fig. 7 that increasing the nonlocal parameter increases the effect of temperature change on the frequency. In fact, it can be observed that the temperature have no significant effect on the frequency of local beam. On the other hand, it is shown if Fig. 8 that the effect of temperature change decreases with the increment of power index of temperature (αz=αx=1) .

The non-dimensional frequency of 2D-FG nanobeams in different values of FG and AFG power indexes, L/h and temperature change are given in Tables 3 and 4. Table 3 shows the values when μ=β=0.1, αz=αx=1 and Table 4 shows the non-dimensional frequencies of nanobeam when μ=0.2 and β=0.1 and αz=αx=1. It is shown that the increment of L/h increases the non-dimensional frequency. Also, the non-dimensional frequency decreases slightly with the increment of temperature change. And also, the effect of temperature change increases with the value of L/h. The effects of *nz* and *nx* are shown in Tables 3 and 4. It is seen that the increment of *nx* and/or *nz* decreases the non-dimensional frequency as it decreases the stiffness. Also, increment of *nx* or *nz*, reduces the effect of the other one. It is also noted that the effect of *nx* is a bit more than that of *nz*.

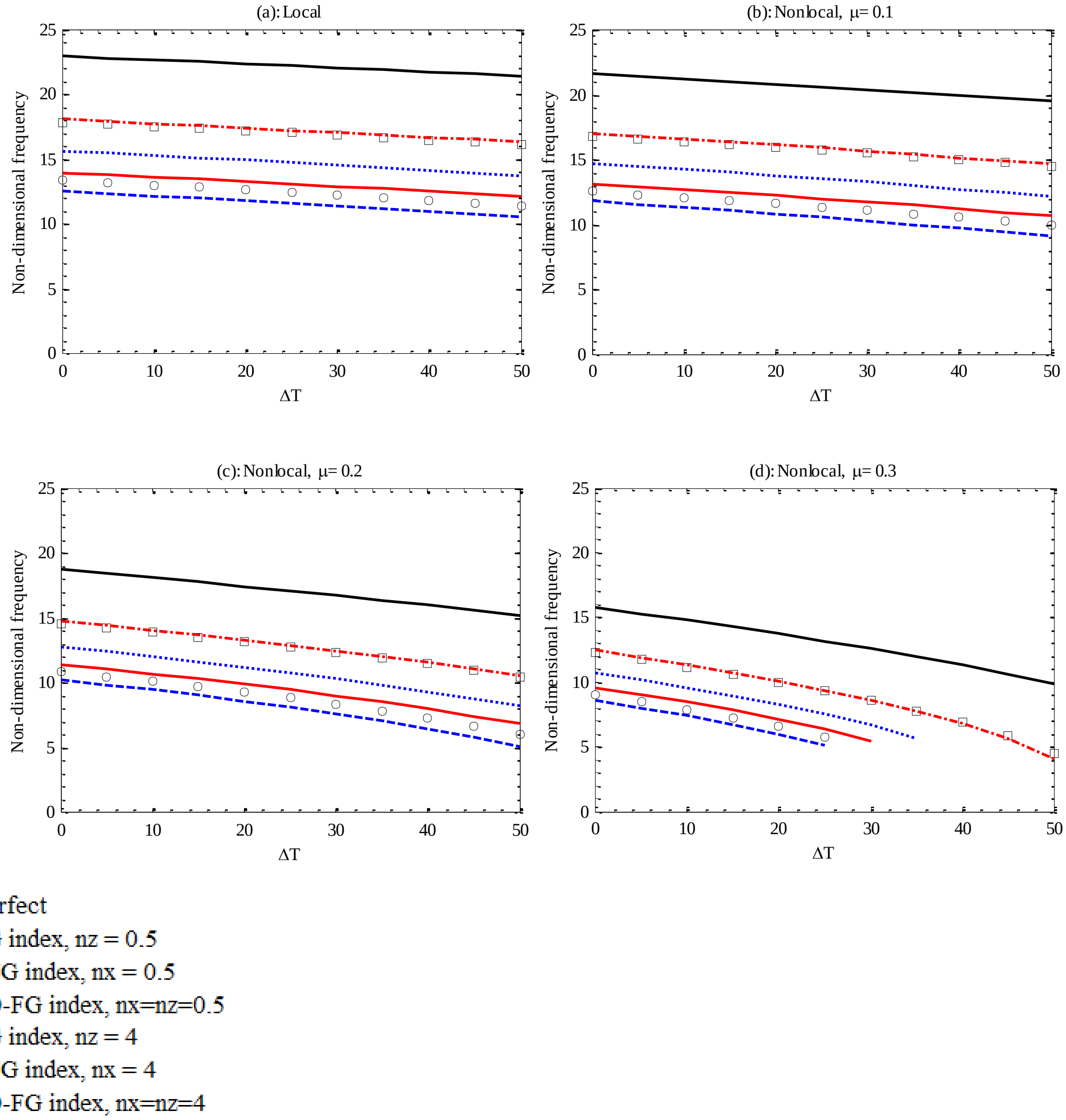

Fig. 7 Non-dimensional frequency of FG, AFG, and 2D-FG porous-II clamped nanobeams versus temperature change when β=0.1, L/h=40, αz=αx=0.1 with uniform temperature gradient for different values of nonlocal parameter

Table 3 Non-dimensional frequency of 2D-FG porous-II nanobeam in different values of L/h and temperature change with uniform temperature gradient when μ=0.1, β=0.1 and αz=αx=1

		ΔT = 0				ΔT = 20				ΔT = 40			
		nx=0	nx=0.1	nx=1	nx=10	nx=0	nx=0.1	nx=1	nx=10	nx=0	nx=0.1	nx=1	nx=10
L/h=5	nz=0	17.54208	16.44937	12.22371	9.711243	17.5286	16.43545	12.20835	9.695641	17.51502	16.42072	12.18961	9.675508
	nz=0.1	16.28193	15.41227	11.90479	9.673358	16.26811	15.39813	11.88918	9.657618	16.25353	15.38264	11.87	9.637284
	nz=0.5	13.67949	13.2049	11.1258	9.577812	13.66503	13.19031	11.10985	9.561877	13.64829	13.17308	11.08987	9.541179
	nz=1	12.3568	12.05092	10.65729	9.520231	12.34207	12.03612	10.64133	9.504282	12.32424	12.01799	10.62108	9.483468
	nz=2	11.30739	11.11866	10.24175	9.470203	11.29251	11.10373	10.22592	9.454323	11.27395	11.08498	10.2056	9.433495
	nz=10	9.974691	9.912385	9.638467	9.382344	9.959508	9.897166	9.622873	9.366627	9.939879	9.877423	9.602441	9.345825
L/h=10	nz=0	20.41425	19.15177	14.27557	11.32524	20.36408	19.09984	14.2189	11.26724	20.31352	19.04657	14.15744	11.20276
	nz=0.1	18.93205	17.92859	13.88903	11.27764	18.8805	17.8758	13.83109	11.21899	18.82762	17.82089	13.76804	11.15373
	nz=0.5	15.88893	15.34386	12.95911	11.16084	15.83474	15.28923	12.89934	11.10119	15.77709	15.23071	12.83373	11.03466
	nz=1	14.36716	14.01601	12.41463	11.09438	14.3119	13.96056	12.35466	11.03458	14.2521	13.90028	12.28847	10.96776
	nz=2	13.18713	12.96757	11.94548	11.04073	13.13139	12.91171	11.886	10.98112	13.07037	12.85039	11.82005	10.91439
	nz=10	11.68086	11.60207	11.25971	10.94264	11.62407	11.54514	11.20116	10.88353	11.56108	11.48191	11.13578	10.8172
L/h=15	nz=0	21.13043	19.82577	14.78944	11.72961	21.01817	19.70953	14.66253	11.59977	20.90466	19.59077	14.52838	11.46017
	nz=0.1	19.59232	18.5556	14.38528	11.67933	19.47694	18.43741	14.25542	11.54796	19.35904	18.31566	14.11781	11.40663
	nz=0.5	16.43882	15.87625	13.41669	11.55677	16.31742	15.75386	13.28256	11.42306	16.19067	15.62556	13.13957	11.27897
	nz=1	14.86826	14.50583	12.85343	11.48805	14.74443	14.38158	12.71882	11.35395	14.61377	14.25015	12.57483	11.20929
	nz=2	13.65772	13.43031	12.37178	11.43367	13.5328	13.30514	12.23829	11.29995	13.40003	13.1719	12.09511	11.15559
	nz=10	12.11042	12.0271	11.66632	11.33306	11.98315	11.89954	11.53498	11.20042	11.8467	11.76265	11.39353	11.05709

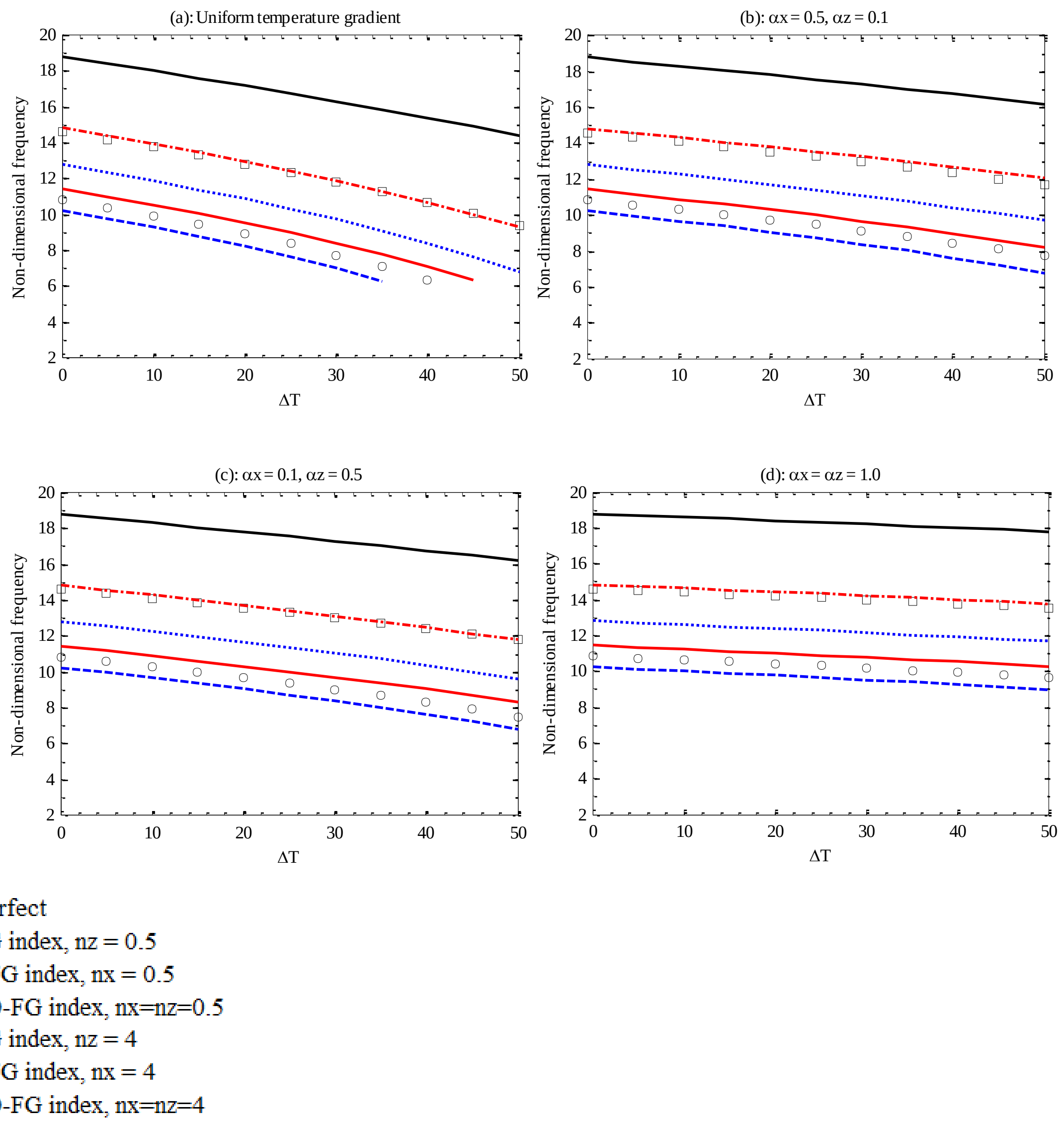

Fig. 8 Non-dimensional frequency of FG, AFG, and 2D-FG porous-II clamped nanobeams versus temperature change when β=0.1, μ=0.2, L/h=40 with different temperature gradients

Table 4 Non-dimensional frequency of 2D-FG porous-II nanobeam in different values of L/h and temperature change when μ=0.2, β=0.1 and αz=αx=1

		ΔT = 0				ΔT = 20				ΔT = 40			
		nx=0	nx=0.1	nx=1	nx=10	nx=0	nx=0.1	nx=1	nx=10	nx=0	nx=0.1	nx=1	nx=10
L/h=5	nz=0	15.28668	14.35044	10.62267	8.428844	15.26531	14.32832	10.59774	8.404003	15.24383	14.30552	10.56981	8.374965
	nz=0.1	14.18486	13.441	10.35072	8.399131	14.16289	13.41844	10.32563	8.374164	14.14019	13.39469	10.29736	8.344948
	nz=0.5	11.913	11.50893	9.683651	8.324495	11.88991	11.48555	9.658379	8.299336	11.8647	11.45977	9.629503	8.269788
	nz=1	10.76271	10.50292	9.28142	8.280073	10.73919	10.4792	9.256215	8.254895	10.71279	10.45245	9.227146	8.225242
	nz=2	9.854395	9.693997	8.924767	8.242174	9.830642	9.67011	8.899764	8.217065	9.803503	9.642735	8.870695	8.187415
	nz=10	8.700618	8.64689	8.404193	8.17438	8.676421	8.622614	8.379438	8.149423	8.648178	8.59423	8.35029	8.119818
L/h=10	nz=0	17.71591	16.64001	12.36655	9.790316	17.63504	16.556	12.27459	9.697233	17.55335	16.47017	12.17701	9.596639
	nz=0.1	16.42798	15.57359	12.03587	9.752816	16.34465	15.48785	11.94265	9.65907	16.2595	15.39946	11.8435	9.557705
	nz=0.5	13.78541	13.32239	11.23807	9.661078	13.69755	13.23333	11.14296	9.56619	13.60561	13.13974	11.0412	9.463437
	nz=1	12.46585	12.16794	10.76982	9.609369	12.37625	12.0776	10.67448	9.514166	12.28142	11.98175	10.5721	9.410973
	nz=2	11.44452	11.25828	10.36597	9.568196	11.35421	11.16746	10.27115	9.473008	11.25787	11.07044	10.16903	9.369759
	nz=10	10.14072	10.07356	9.7736	9.491681	10.04891	9.981432	9.679473	9.396613	9.950043	9.88215	9.577644	9.293395
L/h=15	nz=0	18.32116	17.2104	12.80258	10.13141	18.13974	17.0218	12.59637	9.922352	17.95534	16.82861	12.37903	9.69861
	nz=0.1	16.98672	16.10465	12.45673	10.09174	16.79968	15.91208	12.24747	9.88111	16.60803	15.71358	12.02648	9.655587
	nz=0.5	14.25163	13.77376	11.62565	9.995297	14.05415	13.57354	11.41174	9.781935	13.84841	13.36429	11.18477	9.553228
	nz=1	12.89042	12.58284	11.14109	9.941651	12.68894	12.37968	10.92653	9.727484	12.47724	12.1658	10.69823	9.497779
	nz=2	11.84219	11.64928	10.72608	9.899675	11.63908	11.44504	10.51265	9.685449	11.42435	11.22884	10.28505	9.455602
	nz=10	10.50229	10.43149	10.11635	9.820805	10.29579	10.22427	9.904464	9.60669	10.07567	10.00326	9.677743	9.376892

4. Conclusions

The thermal vibrational analysis is performed on two-dimensional functionally graded (2D-FG) porous nanobeams based on Timoshenko beam theory and Eringen's nonlocal elasticity theory. The results are obtained using the generalized differential quadrature method (GDQM) and the boundary condition is taken as clamped-clamped. The main results are briefly explained here:

- Increasing *nx* and *nz* decrease the frequency. Also, the effect of *nx* or *nz* decreases by increment of the other one.
- Increasing the 2D-FG power indexes, decreases the effect of porosity volume fraction in porous-II.
- The effect of porosity volume fraction of porous-I is vice versa in low and high values of 2D-FG power indexes.
- Increasing the power index of the temperature variation function (αz and αx), porosity volume fraction and also nonlocal parameter decrease the dependency of the non-dimensional frequency on the temperature change.
- Increment of L/h and nonlocal parameter increases the non-dimensional frequency.
- Increment of L/h does not have any effect on the dependency of the critical buckling temperature on 2D-FG power indexes.
- The non-dimensional frequency decreases with the increment of temperature change.

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