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Effects of triaxial magnetic field on the anisotropic nanoplates

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Abstract. In this study, the influences of triaxial magnetic field on the wave propagation behavior of anisotropic nanoplates are studied. In order to include small scale effects, nonlocal strain gradient theory has been implemented. To study the nanoplate as a continuum model, the three-dimensional elasticity theory is adopted in Cartesian coordinate. In our study, all the elastic constants are considered and assumed to be the functions of (x, y, z), so all kind of anisotropic structures such as hexagonal and trigonal materials can be modeled, too. Moreover, all types of functionally graded structures can be investigated. eigenvalue method is employed and analytical solutions for the wave propagation are obtained. To justify our methodology, our results for the wave propagation of isotropic nanoplates are compared with the results available in the literature and great agreement is achieved. Five different types of anisotropic structures are investigated in present paper and then the influences of wave number, material properties, nonlocal and gradient parameter and uniaxial, biaxial and triaxial magnetic field on the wave propagation analysis of anisotropic nanoplates are presented. From the best knowledge of authors, it is the first time that three-dimensional elasticity theory and nonlocal strain gradient theory are used together with no approximation to derive the governing equations. Moreover, up to now, the effects of triaxial magnetic field have not been studied with considering size effects in nanoplates. According to the lack of any common approximations in the displacement field or in elastic constant, present theory has the potential to be used as a bench mark for future works.

Keywords: triaxial magnetic field; anisotropic nanoplate; nonlocal strain gradient theory; three-dimensional elasticity theory

1. Introduction

Classical continuum models, like beam and shell theories, do not admit intrinsic size dependence such as in the elastic solutions of inclusions and inhomogeneities. The small size scales associated nanotechnology are often sufficiently small to call the applicability of classical continuum models into question. In structures with nanometer scales, size effects often become prominent, the cause of which need to be explicitly addressed with an increasing interest in the general area of nanotechnology (Sharma *et al*. 2003). The modeling of such a size-dependent structures has become an interesting subject of some researchers in this field (Sheehan and Lieber 1996, Yakobson and Smalley 1997). It is thus concluded that the classical continuum models at very small scales cannot be used and their applicability is questionable, since the material microstructure at small size, such as lattice spacing between individual atoms, becomes increasingly important and the discrete structure of the material can no longer be homogenized into a continuum. Therefore, newly proposed continuum model rather than the classical continuum models may be an alternative to taking into account the scale effect in the investigations of nanostructures.

In one of these newly continuum model, well-known as nonlocal elasticity theory, the scale effect was accounted in elasticity by assuming the stress at a reference point is considered to be a functional of the strain field at every point in the body (Eringen 1976). Up to now, several types of researches have been done on the nonlocal elasticity theory (Bounouara *et al*. 2016, Ahouel *et al*. 2016, Belkorissat *et al.* 2015, Shahsavari *et al*. 2017, Shahsavari and Janghorban 2017). In this theory, the internal size scale could be considered in the fundamental equations simply as a material parameter. (Peddieson *et al.* 2003) proposed the application of nonlocal elasticity models in nanostructures.

They developed the Euler–Bernoulli beam model based on nonlocal elasticity theory and concluded that nonlocal continuum mechanics could potentially play a useful role in nanotechnology applications. Further applications of the nonlocal continuum mechanics have been employed in studying the mechanical behavior of nanostructures. The free vibration analysis of rotating axially functionally graded nanobeams under an in-plane nonlinear thermal loading was provided by (Azimi *et al.* 2017). Their results showed that the fundamental frequency of AFG nanobeam decreases with nonlocal value. (Amara *et al.* 2010) proposed nonlocal elasticity model for investigation the buckling of multiwalled carbon nanotubes under temperature field. Their results showed that the small scale effect reduces the critical buckling strain. (Nami *et al*.

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2015) investigated the thermal buckling analysis of functionally graded rectangular nanoplates by using nonlocal elasticity theory and third-order shear deformation theory. In their studies, results showed that the critical buckling temperatures of piezoelectric nanoplates under nonlinear temperature distribution decreases with increase nonlocal parameter. (Chaht *et al.* 2015) provided the size-dependent bending and buckling behaviors of nanobeams made of functionally graded materials (FGMs) including the thickness stretching effect. In their studies a Navier-type solution is developed for simply-supported boundary conditions, and exact expressions were proposed for the deflections and the buckling load. (Heireche *et al*. 2010) investigated the nonlocal elasticity effect on vibration characteristics of protein microtubules. Also, the vibration characteristics of protein microtubules were examined based on a nonlocal Timoshenko beam model and using the wave propagation approach. (Murmu *et al.* 2012) proposed nonlocal elasticity model for investigation the vibration response of double-walled carbon nanotubes subjected to an externally applied longitudinal magnetic field. Also, (Murmu *et al.* 2013) examined the effect of in-plane magnetic field on the transverse vibration of single layer graphene sheets. In their study, governing equations of SLGS by considering the effect of in-plane magnetic field was obtained via nonlocal elasticity theory and Maxwell's relation. And, again (Murmu *et al.* 2014) on the basis of nonlocal elasticity theory have studied the influence of a transverse magnetic field on the axial vibration of nanorods such as carbon nanotubes. (Daneshmehr and Rajabpoor 2014) examined the stability analysis of size dependent functionally graded nanoplates on the basis of nonlocal elasticity and higher order plate theories including different boundary conditions.

One of the other theories that consider the small-scale effect is the gradient elasticity theory. For the most common form of strain gradient elasticity theory, stresses are explained by the kinematic parameters effective on the strain density (Lei *et al.* 2013). Strain gradient theory with one gradient parameter used by (Papargyri-Beskou and Beskos 2008) for gradient plates is another form of gradient theory with higher industrial application. In the past decade, researchers have had remarkable studies on the static and dynamic analysis of gradient nanostructures. As an example, the wave propagation behavior in rectangular nanoplates by using strain gradient theory with one gradient parameter was studied by Nami and Janghorban (Nami and Janghorban 2014b). Then, (Karami and Janghorban 2016) examined the effect of magnetic field on the wave propagation in rectangular nanoplates based on mentioned theory. They concluded that wave frequency will increase with increasing the gradient parameter. Furthermore, the bending analysis of rectangular nanoplates subjected to mechanical loading based on the strain gradient elasticity theory with one gradient parameter was investigated by (Nami and Janghorban 2014a). It was the first time that the exponential shear deformation formulation based on strain gradient elasticity theory was carried out.

(Askes and Aifantis 2009) provided the strain gradient elasticity theory with two length scale parameters which contained the nonlocal elasticity theory and strain gradient elasticity theory with one parameter to consider the effects of nonlocal and gradient parameter. This theory has been used by various authors for different structures in recent years (Li *et al.* 2016a, Li and Hu 2016, Mehralian *et al*. 2017, Zhu and Li 2017, Zeighampour *et al*. 2017, Xiao *et al.* 2017). The effect of the in-plane magnetic field on the wave propagation in rectangular nanoplates based on nonlocal strain gradient theory was presented by (Janghorban and Nami 2015). They used this theory for the first time in order to investigate the wave propagation in nanoplates. (Li *et al.* 2016a) examined the behavior of wave propagation in viscoelastic single-walled carbon nanotubes with surface effect under magnetic field based on nonlocal strain gradient theory. (Ebrahimi *et al.* 2016) investigated the wave propagation in temperature-dependent inhomogeneous nanoplates based on nonlocal strain gradient four variable refined shear deformation plate theory. By implementing Hamilton's principle, the governing differential equations was derived. It was observed that the frequencies and phase velocities with the increase of gradient parameter will increase, and will decrease with the increase of nonlocal parameter. The nonlinear free vibration of functionally graded nanobeam by using nonlocal strain gradient theory was studied by (Şimşek 2016). A closed-form solution was obtained for the nonlinear frequencies by the novel Hamiltonian approach. (Li *et al*. 2016b) examined the free vibration analysis of functionally graded beams based on the nonlocal strain gradient theory. The equations of motion and boundary conditions were deduced by employing the Hamilton principle. The model contains a material length scale parameter introduced to consider the significance of strain gradient stress field and a nonlocal parameter introduced to consider the significance of nonlocal elastic stress field. Recently, based on the nonlocal strain gradient theory, (Karami *et al.* 2017) investigated the effects of magnetic field on the wave propagation characteristics of rectangular FG nanoplate, (Zhu and Li 2017) studied a small-scaled rod in tension and the governing equations and boundary conditions for the nonlocal strain gradient rod in tension are using the principle of virtual work, wave propagation in fluid-conveying viscoelastic carbon nanotubes examined by (Li and Hu 2016), and wave propagation in double-walled carbon nanotube conveying fluid considering slip boundary condition and shell model presented by (Zeighampour *et al*. 2017).

Wave propagation is an exciting field in many engineering branches. In the field of smart structures and structural engineering, wave propagation especially in the area active control of vibrations and noise and structural health monitoring based tools have found increasing applications. Besides, tremendous development occurred in the area of material science, wherein a new classification of structural materials is proposed to meet the specific application. In real life, the materials are generally anisotropic but in some cases they can be approximated with more simple structures. Study of these structures is more complex in comparison with isotropic structures. Flexural wave propagation analysis of nanoplates resting on

Table 1 Orthotropic nanoplates in our study. For each nanoplate, the dimensions and nonlocal parameter are shown, along with the material properties (Shen *et al.* 2010)

Nanoplate type	$h\,(nm)$	$E_1\,(GPa)$	$E_2\,(GPa)$	$G_{12}\,(GPa)$	υ_{12}	$\rho\left(kg/m^3\right)$	$\sqrt{\mu}\,(nm)$
Armchair sheet I	0.129	2434	2473	1039	0.197	6316	0.67
Armchair sheet II	0.143	2154	2168	923	0.202	5727	0.47
Armchair sheet III	0.156	1949	1962	846	0.201	5295	0.27
Zigzag sheet IV	0. 145	2145	2097	938	0.223	5624	0.47
Zigzag sheet V	0.149	2067	2054	913	0.204	5482	0.32
Zigzag sheet VI	0.154	1987	1974	857	0.205	5363	0.22

elastic medium performed by Wang *et al.* (Wang *et al*. 2010a). Also, (Wang *et al*. 2010b) studied the size effects on axial wave propagation of nanoplate. The wave propagation characteristics of a piezoelectric nanoplate were studied by (Zhang *et al.* 2014a). (Zhang *et al.* 2014b) examined wave propagation behavior of nanoplates incorporating surface stress effects. Also, (Zang *et al.* 2014) investigated the size effects on the longitudinal wave propagation of a piezoelectric nanoplate considering surface effects. Wave propagation analysis of size-dependent structures based on nonlocal elasticity and strain gradient theory proposed by (Lim *et al.* 2015).

In this paper, three-dimensional elasticity theory in conjunction with nonlocal strain gradient theory, known as strain gradient theory with two parameters, is derived in Cartesian coordinate with considering all elastic constants. This comprehensive theory with no approximation in displacements has the ability to study different models such as size-dependent structures, hexagonal and trigonal materials and multi-directional functionally graded materials. Present theory has only two length scale parameters which seems to be accurate and somehow simple for various problems. Moreover, the influence of triaxial magnetic field on the wave propagation in anisotropic nanoplates on the basis of Maxwell's relations is investigated.

2. Anisotropic materials

For almost all types of elastic materials such as isotropic and anisotropic materials, Hook's law usually represents the material behavior and relates the unknown stresses and strains. The general equation for Hooke's law is

$$\sigma = C\,\varepsilon \tag{1}$$

in above equation σ and ε denote stress and strain components, respectively and C is the elastic constant that is different in various structures. As a consequence, the above stress-strain relations are governed by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & C_{14} & C_{15} & C_{16} \\ C_{21} & C_{22} & C_{23} & C_{24} & C_{25} & C_{26} \\ C_{31} & C_{32} & C_{33} & C_{34} & C_{35} & C_{36} \\ C_{41} & C_{42} & C_{43} & C_{44} & C_{45} & C_{46} \\ C_{51} & C_{52} & C_{53} & C_{54} & C_{55} & C_{56} \\ C_{61} & C_{62} & C_{63} & C_{64} & C_{65} & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{2}$$

One of the main topics in mechanical systems is anisotropic structures analysis, which has absorbed considerable attention in classical continuum mechanics. However, the emergence of modern technologies considering to small-scale elements highlights the importance of length scale effect on the miniature systems. Although plenty of studies have been adopted for the analysis of micro- and nano-structures with isotropic material properties, the analysis of size effect on structures with general anisotropy has not been studied much in the literature so far. According to the anisotropic characters of many nano-engineered materials, the analysis of anisotropic nanostructures is essential. In continue, some examples of materials with anisotropic structure are provided.

2.1 Orthotropic materials

In material science and solid mechanics, orthotropic materials have material properties that differ along three mutually-orthogonal twofold axes of rotational symmetry.

They are a subset of anisotropic materials, because their properties change when measured from different directions.

A familiar example of an orthotropic material is wood. In wood, one can define three mutually perpendicular directions at each point in which the properties are different.

For orthotropic nanoplates, the thickness and material properties used are tabulated in Table 1. The behaviors of bulk waves in orthotropic rectangular nanoplates considering different values of wave numbers, "size effect" and triaxial magnetic field are listed in Table 3 and 8.

2.2 Hexagonal materials

In materials with hexagonal crystallinity the crystal is conventionally described by a right rhombus prism unit cell with two equal axes. The hexagonal crystal family consists of the 12 point groups such that at least one of their space groups has the hexagonal lattice as underlying lattice, and is the union of the hexagonal crystal system and the trigonal crystal system.

One of the materials with hexagonal system is beryllium crystal. It has an axis of symmetry such that a rotation of the crystal through 60° about that axis brings the space lattice into coincidence with its original configuration (Batra *et al.* 2004). The elastic constants are

$$\begin{bmatrix} 298.2 & 27.7 & 11.0 & 0 & 0 & 0 \\ & 298.2 & 11.0 & 0 & 0 & 0 \\ & & 340.8 & 0 & 0 & 0 \\ & \text{sym.} & & 165.5 & 0 & 0 \\ & & & & 165.5 & 0 \\ & & & & & 135.3 \end{bmatrix} \times \text{GPa} \tag{3}$$

and the mass density of beryllium equals to $\rho = 1850\dfrac{\text{kg}}{\text{m}^3}$.

The trend of wave frequencies for hexagonal rectangular nanoplates with considering nonlocal and gradient parameters are listed Table 4.

2.3 Trigonal materials

The components of the rotated elastic stiffness tensor used in the present study for a trigonal material are given in matrix form as (Batra *et al*. 2004)

$$\begin{bmatrix} 86.74 & 6.99 & 11.91 & -17.91 & 0 & 0 \\ & 86.74 & 11.91 & 17.91 & 0 & 0 \\ & & 107.2 & 0 & 0 & 0 \\ & \text{sym.} & & 57.94 & 0 & 0 \\ & & & & 57.94 & -17.91 \\ & & & & & 39.88 \end{bmatrix} \times \text{GPa} \tag{4}$$

and $\rho = 2649\dfrac{\text{kg}}{\text{m}^3}$.

The behaviors of bulk waves in trigonal rectangular nanoplates with different values of wave numbers and “size effect” are listed in Table 5.

2.4 Monoclinic materials

A crystal system of monoclinic materials is described by three vectors. Also, the crystal system is defined by vectors of unequal lengths, as in the orthorhombic system. The monoclinic crystal system has different names such as Hermann–Mauguin notation, Schoenflies notation and point groups (Prince *et al.* 2006).

Here the elastic constants used in present work for monoclinic materials are as follow (Batra *et al.* 2004),

$$\begin{bmatrix} 86.74 & 6.99 & 11.91 & -17.91 & 0 & 0 \\ & 86.74 & 11.91 & 17.91 & 0 & 0 \\ & & 107.2 & 0 & 0 & 0 \\ & \text{sym.} & & 57.94 & 0 & 0 \\ & & & & 57.94 & -17.91 \\ & & & & & 39.88 \end{bmatrix} \times \text{GPa} \tag{5}$$

and set $\rho = 2649\dfrac{\text{kg}}{\text{m}^3}$.

Computed wave frequencies in monoclinic rectangular nanoplates for different values of wave numbers in conjunction with small scale effects are listed in Table 6.

2.5 Triclinic materials

The matrix of elastic constants of a triclinic material can be obtained from that of a transversely isotropic material by appropriate rotations about the *x*- and the rotated *y*-axis (Batra *et al*. 2004). Generally, the triclinic materials have 21 elastic constants and three components of the propagation vector leading. Here the elastic constants used in present study for triclinic materials are defied (Batra *et al*. 2004)

$$\begin{bmatrix} 98.84 & 53.92 & 50.78 & -0.10 & 1.05 & 0.03 \\ & 99.19 & 50.87 & -0.18 & 0.55 & 0.03 \\ & & 87.23 & -0.18 & 1.03 & 0.02 \\ & \text{sym.} & & 21.4 & 0.07 & 0.25 \\ & & & & 21.10 & -0.04 \\ & & & & & 22.55 \end{bmatrix} \times \text{GPa} \tag{6}$$

and $\rho = 7750\dfrac{\text{kg}}{\text{m}^3}$.

The variations of wave frequencies in triclinic rectangular nanoplates with respect to length scale parameter and wave number are also listed in Table 7.

3. Maxwell's relations

Denoting $\mathbf{J}$ as current density, $\mathbf{h}$ as distributing vector of the magnetic field, and $\mathbf{e}$ as strength vectors of the electric field, the Maxwell relation according to (Kraus 1984, Mahmoud *et al.* 2014, Mahmoud *et al*. 2015) is given as

$$\mathbf{J} = \nabla \times \mathbf{h}. \tag{7}$$

$$\nabla \times \mathbf{e} = -\eta \frac{\partial \mathbf{h}}{\partial \text{t}} \tag{8}$$

$$\nabla \cdot \mathbf{h} = 0 \tag{9}$$

$$\mathbf{e} = -\eta \left(\frac{\partial \mathbf{U}}{\partial \text{t}} \times \mathbf{H} \right) \tag{10}$$

$$\mathbf{h} = \nabla \times \left(\mathbf{U} \times \mathbf{H} \right) \tag{11}$$

in which η is the magnetic field permeability. Also, $\nabla = \dfrac{\partial}{\partial x}\mathbf{i} + \dfrac{\partial}{\partial y}\mathbf{j} + \dfrac{\partial}{\partial z}\mathbf{k}$ is the Hamilton operator, $\mathbf{U} = u\mathbf{i} + v\mathbf{j} + w\mathbf{k}$ is the displacement vector. For the present analysis, we consider the triaxial magnetic field as a vector $\mathbf{H}=\left(H_x\mathbf{i}, H_y\mathbf{i}, H_z\mathbf{k}\right)$ acting on the nanoplate. $\left(\mathbf{i}, \mathbf{j}, \mathbf{k}\right)$ are the unit vectors. We can obtain the distributing vector of the magnetic field by Eq. (11)

$$\mathbf{h}=\nabla\times(\mathbf{U}\times\mathbf{H})=\left\{-H_x\left(\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}\right)+H_y\frac{\partial u}{\partial y}+H_z\frac{\partial u}{\partial z}\right\}\mathbf{i}$$
$$+\left\{H_x\frac{\partial v}{\partial x}-H_y\left(\frac{\partial u}{\partial x}+\frac{\partial w}{\partial z}\right)+H_z\frac{\partial v}{\partial z}\right\}\mathbf{j}\qquad(12)$$
$$+\left\{H_x\frac{\partial w}{\partial x}+H_y\frac{\partial w}{\partial y}-H_z\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)\right\}\mathbf{k}$$

$$\mathbf{J}=\nabla\times\mathbf{h}=$$
$$\left\{H_x\left(-\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right)+H_y\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\right.$$
$$\left.-H_z\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\right\}\mathbf{i}+\left\{-H_x\left(+\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)\right.$$
$$\left.+H_y\left(\frac{\partial^2 u}{\partial y\partial z}-\frac{\partial^2 w}{\partial x\partial y}\right)+H_z\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial y}\right)\right\}\mathbf{j}\qquad(13)$$
$$+\left\{H_x\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)-H_y\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial x\partial z}\right)\right.$$
$$\left.+H_z\left(-\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}\right)\right\}\mathbf{k}$$

The Lorentz force induced by the triaxial magnetic field is given as

$$\mathbf{F}=F_x\mathbf{i}+F_y\mathbf{j}+F_z\mathbf{k}=\eta(\mathbf{J}\times\mathbf{H})$$
$$=\eta\left[\left\{-H_xH_y\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)-H_xH_z\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+H_y^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial x\partial z}\right)\right.\right.$$
$$\left.+H_yH_z\left(2\frac{\partial^2 u}{\partial y\partial z}-\frac{\partial^2 v}{\partial x\partial z}-\frac{\partial^2 w}{\partial x\partial y}\right)+H_z^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial y}\right)\right\}\mathbf{i}+\left\{H_x^2\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)\right.$$
$$-H_xH_y\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial x\partial z}\right)-H_xH_z\left(\frac{\partial^2 u}{\partial y\partial z}-2\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right)-H_yH_z\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\qquad(14)$$
$$\left.+H_z^2\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\right\}\mathbf{j}+\left\{H_x^2\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-H_xH_y\left(\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}-2\frac{\partial^2 w}{\partial x\partial y}\right)\right.$$
$$\left.\left.+H_y^2\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)-H_xH_z\left(\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 v}{\partial x\partial y}\right)-H_yH_z\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\right\}\mathbf{k}\right]$$

Therefore, the Lorentz forces along the x, y and z-directions are

$$F_x=\eta\left\{-H_xH_y\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)-H_xH_z\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)+H_y^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial x\partial z}\right)\right.\qquad(15)$$
$$\left.+H_yH_z\left(2\frac{\partial^2 u}{\partial y\partial z}-\frac{\partial^2 v}{\partial x\partial z}-\frac{\partial^2 w}{\partial x\partial y}\right)+H_z^2\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial y}\right)\right\}$$

$$F_y=\eta\left\{H_x^2\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 w}{\partial y\partial z}\right)-H_xH_y\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 w}{\partial x\partial z}\right)-H_xH_z\left(\frac{\partial^2 u}{\partial y\partial z}-2\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right)\right.\qquad(16)$$
$$\left.-H_yH_z\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)+H_z^2\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\right\}$$

$$F_z=\eta\left\{H_x^2\left(\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 w}{\partial z^2}\right)-H_xH_y\left(\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}-2\frac{\partial^2 w}{\partial x\partial y}\right)+H_y^2\left(\frac{\partial^2 u}{\partial x\partial z}+\frac{\partial^2 w}{\partial y^2}+\frac{\partial^2 w}{\partial z^2}\right)\right.\qquad(17)$$
$$\left.-H_xH_z\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 v}{\partial x\partial y}\right)-H_yH_z\left(\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial y^2}+\frac{\partial^2 v}{\partial z^2}\right)\right\}$$

It should be noted that in present paper, the effective Lorentz force is a function of magnetic permeability and (H_x, H_y, H_z). For the present wave propagation analysis, at the best knowledge of authors, it is the first time that triaxial magnetic field is used for investigating graphene sheets with considering size effects.

4. Theoretical formulation

In this section, to study the behaviors of anisotropic structures containing magnetic field, geometry and belongings of which is presented in Fig. 1, the governing equations with considering the both nonlocal and gradient parameter in Cartesian coordinate are proposed.

4.1 Governing equations

In this section, we derive the nonlocal strain gradient three-dimensional elasticity theory. The displacements for three-dimensional elasticity theory are given by (Sadd 2009)

$$\begin{aligned}u&=u(x,y,z,t)\\ v&=v(x,y,z,t)\\ w&=w(x,y,z,t)\end{aligned}\qquad(18)$$

where u, v, and w are the displacements in the x, y, and z-directions, respectively. For elastic materials, the general equation for strain-displacement can be defined by

$$\varepsilon=\frac{1}{2}\left[\nabla u+(\nabla u)^T\right]\qquad(19)$$

According to the above displacement field, the strain-displacement relations, based on Eq. (8) can be obtained as

$$\begin{array}{lll}\varepsilon_x=\dfrac{\partial u}{\partial x} & \varepsilon_y=\dfrac{\partial v}{\partial y} & \varepsilon_z=\dfrac{\partial w}{\partial z}\\ \gamma_{xy}=\left(\dfrac{\partial u}{\partial y}+\dfrac{\partial v}{\partial x}\right) & \gamma_{yz}=\left(\dfrac{\partial v}{\partial z}+\dfrac{\partial w}{\partial y}\right) & \gamma_{xz}=\left(\dfrac{\partial w}{\partial x}+\dfrac{\partial u}{\partial z}\right)\end{array}\qquad(20)$$

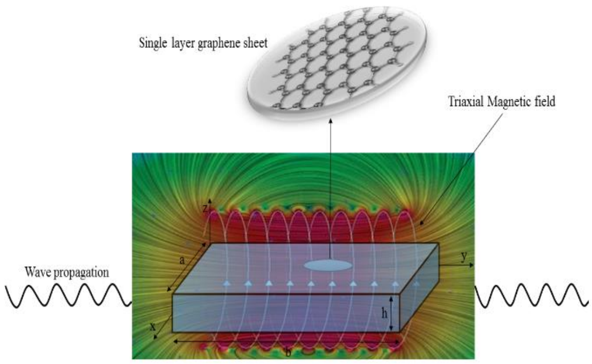

Fig. 1 Geometry of anisotropic structures under triaxial magnetic field

Equation of motion, which is an expression of Newton's second law can be expressed as follow

$$\nabla.\sigma+F=\rho\ddot{u} \tag{21}$$

where F denote body forces and ρ indicates the mass density.

The stress fields must satisfy the equilibrium condition, which yields the equilibrium equations obtained. According to Eq. (21) and the displacement field in Eq. (18) the following equilibrium equations can be obtained (Sadd 2009)

$$\begin{aligned}&\frac{\sigma_{xx}}{\partial x}+\frac{\tau_{yx}}{\partial y}+\frac{\tau_{zx}}{\partial z}+F_x=\rho\frac{\partial^2 u}{\partial t^2}\\&\frac{\tau_{xy}}{\partial x}+\frac{\sigma_{yy}}{\partial y}+\frac{\tau_{zy}}{\partial z}+F_y=\rho\frac{\partial^2 v}{\partial t^2}\\&\frac{\tau_{xz}}{\partial x}+\frac{\tau_{yz}}{\partial y}+\frac{\sigma_{zz}}{\partial z}+F_z=\rho\frac{\partial^2 w}{\partial t^2}\end{aligned} \tag{22}$$

4.2 The nonlocal strain gradient theory

Under consideration nonlocal strain gradient theory, the stress field takes into consider the effects of nonlocal elastic stress field and strain gradient stress field. Also, the stress can be defined as (Lim *et al*. 2015, Ebrahimi e*t al*. 2016)

$$t_{xx}=\sigma_{xx}-\frac{d\sigma_{xx}^{(1)}}{dx} \tag{23}$$

where the classical stress σ_{xx} and the higher-order stress $\sigma_{xx}^{(1)}$ are related to strain ε_x and strain gradient $\varepsilon_{x,x}$, respectively and can be stated as (Ebrahimi *et al. 2*016, Lim *et al*. 2015),

$$\sigma_{xx}=\int_0^L C\,\alpha_0\left(x',x,e_0a\right)C_{ijkl}:\varepsilon_x'\left(x'\right)dx' \tag{24}$$

$$\sigma^{(1)}{}_{xx}=l^2\int_0^L C\,\alpha_1\left(x',x,e_1a\right)\varepsilon_{x,x}'\left(x'\right)dx' \tag{25}$$

where L is the length of the plate, C_{ijkl} is the fourth-order elasticity tensor, l is the material length scale parameter introduced to consider the significance of strain gradient stress field, e_0a and e_1a, which are nonlocal parameters, are introduced to consider the significance of nonlocal elastic stress field (Nami and Janghorban 2015). $\alpha_0\left(x',x,e_0a\right)$ and $\alpha_1\left(x',x,e_1a\right)$ are the nonlocal functions for the classical stress tensor and the strain gradient stress tensor, respectively (Eringen 1983). The linear nonlocal differential operator which is written as follows, is applied to the both sides of Eq. (23); operator can be defined as

$$L_i=1-\left(e_ia\right)^2\nabla^2 \qquad for\ \ i=0,1 \tag{26}$$

in above relation ∇^2 is the Laplacian operator and can be defined as

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2} \tag{27}$$

Now, by applying Eq. (26) in Eq. (23) a clearer constitutive equation for the size-dependent can be derived as

$$\begin{aligned}&\left[1-\left(e_1a\right)^2\nabla^2\right]\left[1-\left(e_0a\right)^2\nabla^2\right]\sigma_{xx}\\&=C_{ijkl}\left[1-\left(e_1a\right)^2\nabla^2\right]\varepsilon_{kl}-C_{ijkl}\,l^2\left[1-\left(e_0a\right)^2\nabla^2\right]\nabla^2\varepsilon_{kl}\end{aligned} \tag{28}$$

Suppose $e_0=e_1=e$ and the nonlocal functions $\alpha_0\left(x',x,e_0a\right)$ and $\alpha_1\left(x',x,e_1a\right)$ satisfy the conditions in Eringen (Lim *et al*. 2015),

$$\left(1-\left(ea\right)^2\nabla^2\right)\sigma_{xx}=C_{ijkl}\,\varepsilon_{xx} \tag{29}$$

and the general constitutive for size-dependent plate can be simplified as (Askes and Aifantis 2009),

$$\left(\sigma_{ij}-\mu^2\sigma_{ij,mm}\right)=C_{ijkl}\left(\varepsilon_{ij}-l^2\varepsilon_{ij,mm}\right) \tag{30}$$

In this step, by differentiating from above equations, some of the terms in 3D elasticity theory with considering the both nonlocal and gradient parameters will appear. By considering the equilibrium equations, governing equations for investigating anisotropic nanoplates are achieved as follow

$$\begin{aligned}
&\frac{\partial C_{11}}{\partial x}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{11}\left(\frac{\partial^2 u}{\partial x^2}-\eta\left\{\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{12}}{\partial x}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{12}\left(\frac{\partial^2 v}{\partial x\partial y}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{13}}{\partial x}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{13}\left(\frac{\partial^2 w}{\partial x\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{14}}{\partial x}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{14}\left(\left\{\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial z^3}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{15}}{\partial x}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{15}\left(\left\{\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{16}}{\partial x}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{16}\left(\left\{\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}+\frac{\partial^4 v}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{61}}{\partial y}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{61}\left(\frac{\partial^2 u}{\partial x\partial y}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{62}}{\partial y}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{62}\left(\frac{\partial^2 v}{\partial y^2}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^4}+\frac{\partial^4 v}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{63}}{\partial y}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{63}\left(\frac{\partial^2 w}{\partial y\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{64}}{\partial y}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{64}\left(\left\{\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial y^2}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y^4}+\frac{\partial^4 v}{\partial y\partial z^3}+\frac{\partial^4 w}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{65}}{\partial y}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{65}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 u}{\partial y\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}+\frac{\partial^4 u}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{66}}{\partial y}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{66}\left(\left\{\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial y^4}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{51}}{\partial z}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{51}\left(\frac{\partial^2 u}{\partial x\partial z}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{52}}{\partial z}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{52}\left(\frac{\partial^2 v}{\partial y\partial z}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{53}}{\partial z}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{53}\left(\frac{\partial^2 w}{\partial z^2}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{54}}{\partial z}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{54}\left(\left\{\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 w}{\partial z\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial z^4}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{55}}{\partial z}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{55}\left(\left\{\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial^2 u}{\partial z^2}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial z^3}+\frac{\partial^4 u}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{56}}{\partial z}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{56}\left(\left\{\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y\partial z^3}+\frac{\partial^4 v}{\partial x\partial z^3}\right\}\right)\\
&=\left(\rho\frac{\partial^2 u}{\partial t^2}-\rho\mu^2\left\{\frac{\partial^4 u}{\partial t^2\partial x^2}+\frac{\partial^4 u}{\partial t^2\partial y^2}+\frac{\partial^4 u}{\partial t^2\partial z^2}\right\}\right)-F_x\left(1-\mu^2\nabla^2\right)
\end{aligned} \tag{31}$$

The second governing equation of 3D elasticity theory for anisotropic nanoplates including size effect can be calculated by following equation

$$
\begin{aligned}
&\frac{\partial C_{61}}{\partial x}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{61}\left(\frac{\partial^2 u}{\partial x^2}-\eta\left\{\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{62}}{\partial x}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{62}\left(\frac{\partial^2 v}{\partial x\partial y}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{63}}{\partial x}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{63}\left(\frac{\partial^2 w}{\partial x\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{64}}{\partial x}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{64}\left(\left\{\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial z^3}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{65}}{\partial x}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{65}\left(\left\{\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{66}}{\partial x}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{66}\left(\left\{\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}+\frac{\partial^4 v}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{21}}{\partial y}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{21}\left(\frac{\partial^2 u}{\partial x\partial y}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{22}}{\partial y}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{22}\left(\frac{\partial^2 v}{\partial y^2}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^4}+\frac{\partial^4 v}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{23}}{\partial y}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{23}\left(\frac{\partial^2 w}{\partial y\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{24}}{\partial y}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{24}\left(\left\{\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial y^2}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y^4}+\frac{\partial^4 v}{\partial y\partial z^3}+\frac{\partial^4 w}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{25}}{\partial y}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{25}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 u}{\partial y\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}+\frac{\partial^4 u}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{26}}{\partial y}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{26}\left(\left\{\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial y^4}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{41}}{\partial z}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{41}\left(\frac{\partial^2 u}{\partial x\partial z}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{42}}{\partial z}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{42}\left(\frac{\partial^2 v}{\partial y\partial z}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{43}}{\partial z}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{43}\left(\frac{\partial^2 w}{\partial z^2}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{44}}{\partial z}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{44}\left(\left\{\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 w}{\partial z\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial z^4}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{45}}{\partial z}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{45}\left(\left\{\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial^2 u}{\partial z^2}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial z^3}+\frac{\partial^4 u}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{46}}{\partial z}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{46}\left(\left\{\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y\partial z^3}+\frac{\partial^4 v}{\partial x\partial z^3}\right\}\right)\\
&=\left(\rho\frac{\partial^2 v}{\partial t^2}-\rho\mu^2\left\{\frac{\partial^4 v}{\partial t^2\partial x^2}+\frac{\partial^4 v}{\partial t^2\partial y^2}+\frac{\partial^4 v}{\partial t^2\partial z^2}\right\}\right)-F_y\left(1-\mu^2\nabla^2\right)
\end{aligned}
\tag{32}
$$

Finally, according to equilibrium equations, the third governing equation of nonlocal strain gradient 3D elasticity theory can be directly written in terms of displacements *u*, *v* and *w* as

$$
\begin{aligned}
&\frac{\partial C_{51}}{\partial x}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{51}\left(\frac{\partial^2 u}{\partial x^2}-\eta\left\{\frac{\partial^4 u}{\partial x^4}+\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{52}}{\partial x}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{52}\left(\frac{\partial^2 v}{\partial x\partial y}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{53}}{\partial x}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{53}\left(\frac{\partial^2 w}{\partial x\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{54}}{\partial x}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{54}\left(\left\{\frac{\partial^2 v}{\partial x\partial z}+\frac{\partial^2 w}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x\partial z^3}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{55}}{\partial x}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{55}\left(\left\{\frac{\partial^2 w}{\partial x^2}+\frac{\partial^2 u}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{56}}{\partial x}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{56}\left(\left\{\frac{\partial^2 u}{\partial x\partial y}+\frac{\partial^2 v}{\partial x^2}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 v}{\partial x^4}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}+\frac{\partial^4 v}{\partial x^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{41}}{\partial y}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{41}\left(\frac{\partial^2 u}{\partial x\partial y}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{42}}{\partial y}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{42}\left(\frac{\partial^2 v}{\partial y^2}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^4}+\frac{\partial^4 v}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{43}}{\partial y}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{43}\left(\frac{\partial^2 w}{\partial y\partial z}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{44}}{\partial y}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{44}\left(\left\{\frac{\partial^2 v}{\partial y\partial z}+\frac{\partial^2 w}{\partial y^2}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial y^4}+\frac{\partial^4 v}{\partial y\partial z^3}+\frac{\partial^4 w}{\partial y^2\partial z^2}\right\}\right)\\
&+\frac{\partial C_{45}}{\partial y}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{45}\left(\left\{\frac{\partial^2 w}{\partial x\partial y}+\frac{\partial^2 u}{\partial y\partial z}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 w}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 w}{\partial x\partial y\partial z^2}+\frac{\partial^4 u}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{46}}{\partial y}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{46}\left(\left\{\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 v}{\partial x\partial y}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y^2}+\frac{\partial^4 v}{\partial x^3\partial y}+\frac{\partial^4 u}{\partial y^4}+\frac{\partial^4 v}{\partial x\partial y^3}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 v}{\partial x\partial y\partial z^2}\right\}\right)\\
&+\frac{\partial C_{31}}{\partial z}\left(\frac{\partial u}{\partial x}-\eta\left\{\frac{\partial^3 u}{\partial x^3}+\frac{\partial^3 u}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial x\partial z^2}\right\}\right)+C_{31}\left(\frac{\partial^2 u}{\partial x\partial z}-\eta\left\{\frac{\partial^4 u}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial x\partial z^3}\right\}\right)\\
&+\frac{\partial C_{32}}{\partial z}\left(\frac{\partial v}{\partial y}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^3}+\frac{\partial^3 v}{\partial y\partial z^2}\right\}\right)+C_{32}\left(\frac{\partial^2 v}{\partial y\partial z}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{33}}{\partial z}\left(\frac{\partial w}{\partial z}-\eta\left\{\frac{\partial^3 w}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial z^3}\right\}\right)+C_{33}\left(\frac{\partial^2 w}{\partial z^2}-\eta\left\{\frac{\partial^4 w}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{34}}{\partial z}\left(\left\{\frac{\partial v}{\partial z}+\frac{\partial w}{\partial y}\right\}-\eta\left\{\frac{\partial^3 v}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial y^3}+\frac{\partial^3 v}{\partial z^3}+\frac{\partial^3 w}{\partial y\partial z^2}\right\}\right)\\
&+C_{34}\left(\left\{\frac{\partial^2 v}{\partial z^2}+\frac{\partial^2 w}{\partial z\partial y}\right\}-\eta\left\{\frac{\partial^4 v}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial z^4}+\frac{\partial^4 w}{\partial y\partial z^3}\right\}\right)\\
&+\frac{\partial C_{35}}{\partial z}\left(\left\{\frac{\partial w}{\partial x}+\frac{\partial u}{\partial z}\right\}-\eta\left\{\frac{\partial^3 w}{\partial x^3}+\frac{\partial^3 u}{\partial x^2\partial z}+\frac{\partial^3 w}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y^2\partial z}+\frac{\partial^3 w}{\partial x\partial z^2}+\frac{\partial^3 u}{\partial z^3}\right\}\right)\\
&+C_{35}\left(\left\{\frac{\partial^2 w}{\partial x\partial z}+\frac{\partial^2 u}{\partial z^2}\right\}-\eta\left\{\frac{\partial^4 w}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial x^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y^2\partial z^2}+\frac{\partial^4 w}{\partial x\partial z^3}+\frac{\partial^4 u}{\partial z^4}\right\}\right)\\
&+\frac{\partial C_{36}}{\partial z}\left(\left\{\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right\}-\eta\left\{\frac{\partial^3 u}{\partial x^2\partial y}+\frac{\partial^3 v}{\partial x^3}+\frac{\partial^3 u}{\partial y^3}+\frac{\partial^3 v}{\partial x\partial y^2}+\frac{\partial^3 u}{\partial y\partial z^2}+\frac{\partial^3 v}{\partial x\partial z^2}\right\}\right)\\
&+C_{36}\left(\left\{\frac{\partial^2 u}{\partial y\partial z}+\frac{\partial^2 v}{\partial x\partial z}\right\}-\eta\left\{\frac{\partial^4 u}{\partial x^2\partial y\partial z}+\frac{\partial^4 v}{\partial x^3\partial z}+\frac{\partial^4 u}{\partial y^3\partial z}+\frac{\partial^4 v}{\partial x\partial y^2\partial z}+\frac{\partial^4 u}{\partial y\partial z^3}+\frac{\partial^4 v}{\partial x\partial z^3}\right\}\right)\\
&=\left(\rho\frac{\partial^2 w}{\partial t^2}-\rho\mu^2\left\{\frac{\partial^4 w}{\partial t^2\partial x^2}+\frac{\partial^4 w}{\partial t^2\partial y^2}+\frac{\partial^4 w}{\partial t^2\partial z^2}\right\}\right)-F_z\left(1-\mu^2\nabla^2\right)
\end{aligned}
\tag{23}
$$

5. Solution procedure

In this section, the equations of motion are solved using eigenvalue method to calculate the behaviors of the wave frequency in anisotropic materials. In the present study, wave propagation in anisotropic nanoplates is investigated away from any boundary conditions (simply supported, free, clamped, etc.) similar to many other studies on macro and nanostructures.

In order to find the wave propagation behavior of anisotropic rectangular nanoplates, the displacement fields of the waves propagating in the *x-y-z* coordinate are assumed with the following terms

$$u = A_1 e^{i\left(xk_x + yk_y + zk_z - \omega t\right)}$$
$$v = A_2 e^{i\left(xk_x + yk_y + zk_z - \omega t\right)} \quad (34)$$
$$w = A_3 e^{i\left(xk_x + yk_y + zk_z - \omega t\right)}$$

where A_1 ; A_2 and A_3 are the coefficients of wave amplitude, k_x , k_y and k_z are the wave numbers of wave propagation along *x*, *y* and *z*- directions respectively and ω is frequency.

By substituting Eq. (34) in Eqs. (31)-(33), gives

$$\left(\left[K\right] - \omega^2\left[M\right]\right)\left\{X\right\} = 0 \quad (35)$$

in above equation the unknown parameters are $\left\{X\right\} = \left\{A_1, A_2, A_3\right\}^T$.

The dispersion relations of wave propagation in the anisotropic rectangular nanoplate can be developed by setting the following determinant to zero

$$\left|\left[K\right] - \omega^2\left[M\right]\right| = 0 \quad (36)$$

6. Numerical results

In this paper, the wave propagation of anisotropic nanoplates under triaxial magnetic field is presented by some numerical examples. The effects of various parameters such as nonlocal and gradient parameter, uniaxial, biaxial and triaxial magnetic field and wave number on the wave propagation of anisotropic nanoplates are investigated. It is mentioned that the thickness, material properties and elastic constants of five different anisotropic structures such as orthotropic, hexagonal, trigonal, monoclinic, and triclinic used in present paper are given in Table 2 and relations (3-6) which can be found here (Shen *et al*. 2010, Batra *et al.* 2004). In some numerical examples, according to the lack of elastic constant for different types of anisotropic nanostructures, the elastic constants for macro structures are used.

6.1 Verification

For isotropic nanoplates, following material properties and thickness are used, E=1.06 TPa, υ=0.25, ρ=2250 (kg/m^3) and h=0.34 nm (Karami and Janghorban 2016) and (e_0a) varies from 0 to 1.0 nm. Wave number is handled in offering the results in graphical and tabular forms as below

$$\mathrm{k} = \sqrt{k_x^2 + k_y^2 + k_z^2} \quad (37)$$

In Table 2, the wave frequencies of isotropic nanoplates based on nonlocal strain gradient three-dimensional elasticity theory are compared with the results of Janghorban and Nami (Janghorban and Nami 2016). One can find great agreement between the results. For this comparison, we derived the nonlocal strain gradient governing equations for the second-order shear deformation theory and solved them for wave propagation of isotropic nanoplates.

6.2 Wave propagation of anisotropic nanoplates excluding magnetic field

In this section, the wave propagation of anisotropic materials is presented in Tables 3-7. For this investigations, the effects of magnetic field are neglected. In Tables 3-7, the effects of nonlocal and gradient parameters and various wave numbers on the wave frequency of anisotropic materials such as orthotropic, hexagonal, trigonal, monoclinic and triclinic are presented.

Table 3, shows the effect of variation of wave numbers in three directions on the wave propagation of orthotropic nanoplates. It can be understood that inclusion of nonlocal parameter leads to reduction in wave frequency of orthotropic nanoplates. So, nonlocal parameter has a stiffness-softening influence on the plate structure. Also, nonlocal parameter has a significant influence on wave frequency when the gradient parameter effect is included in the model. From this table, it is obvious that the value of wave frequency increases by an increase in the gradient parameter. Furthermore, the effect of both length scale parameters on wave frequency becomes more significant at larger wave numbers. Moreover, as wave number increases, an increase in the value of wave frequency can be seen.

From Tables 4-7, it is concluded that with the increase of nonlocal and gradient parameter, the wave frequency of anisotropic materials will increase and decrease respectively. Moreover, it is noticed that the wave frequencies of anisotropic materials increase as the wave number increase. Also, the wave frequency of anisotropic materials is not very sensitive with length scale parameters changes at small wave numbers, but in higher wave numbers the variations of wave frequency is more noticeable. For all the values of the nonlocal and gradient parameters and wave numbers, hexagonal and monoclinic square plates have the highest and lowest wave frequencies, respectively.

6.3 Wave propagation of anisotropic nanoplates including triaxial magnetic field

In this section, wave propagation of anisotropic materials under different magnetic field changes are presented in Tables 8-11. Note, in Tables 8-11, the frequencies extracted from the equation of motions by using eigenvalue method and MATLAB software are sorted in three different modes named as M1, M2, and M3. It is mentioned that for biaxial and triaxial cases, the magnitude of magnetic field in the *y* and *z* directions are equal to the *x* direction. In Tables 8-11, the effects of triaxial magnetic field, nonlocal and gradient parameters and wave numbers on the wave frequency of anisotropic materials such as orthotropic, hexagonal, trigonal, monoclinic and triclinic are presented.

Table 2 Comparison of solutions for isotropic wave propagation $\left(\omega_w = \omega_w \times 10^{14}, k_y = k_z = 0\right)$

		ω_w							
		$l = 0.0(\mathrm{nm})$		$l = 0.5(nm)$		$l = 1.0(\mathrm{nm})$		$l = 1.5(\mathrm{nm})$	
k	$\mu(nm)$	a	Present	a	Present	a	Present	a	Present
1×10^9	0.0	0.2241	0.2241	0.2506	0.2506	0.3170	0.3170	0.4041	0.4041
	0.5	0.2005	0.2005	0.2242	0.2242	0.2836	0.2836	0.3615	0.3615
	1.0	0.1585	0.1585	0.1772	0.1772	0.2242	0.2242	0.2858	0.2858

[a] Ref. (Janghorban and Nami 2016)

Table 3 Wave frequency $\left(\omega_w = \omega_w \times 10^{14}\right)$ in orthotropic nanoplates for various nonlocal and gradient parameters and wave numbers

	ω_w					
	$l = 0.0(nm)$			$l = 1.5(nm)$		
	$\mu(nm)$					
	0.0	0.75	1.5	0.0	0.75	1.5
$k_x = k_y = k_z = 1\times10^9$						
Armchair sheet I	0.1780	0.1098	0.0647	0.4472	0.2728	0.1607
Armchair sheet II	0.1771	0.1080	0.0636	0.4400	0.2684	0.1581
Armchair sheet III	0.1753	0.1069	0.0630	0.4356	0.2657	0.1565
Zigzag sheet IV	0.2860	0.1745	0.1027	0.4362	0.2661	0.1567
Zigzag sheet V	0.2837	0.1731	0.1019	0.4393	0.2680	0.1578
Zigzag sheet VI	0.2780	0.1708	0.1006	0.4352	0.2655	0.1563
$k_x = k_y = k_z = 3\times10^9$						
Armchair sheet I	0.5399	0.1342	0.0687	3.7172	0.9239	0.4730
Armchair sheet II	0.5313	0.1321	0.0676	3.6567	0.9089	0.4653
Armchair sheet III	0.5258	0.1307	0.0692	3.6203	0.8998	0.4607
Zigzag sheet IV	0.8580	0.2133	0.1092	3.6236	0.9006	0.4611
Zigzag sheet V	0.8512	0.2116	0.1083	3.6516	0.9076	0.4647
Zigzag sheet VI	0.8399	0.2088	0.1069	3.6165	0.8989	0.4602

Table 4 The effects of length scale parameters and wave number on the wave frequency of hexagonal square plate $\left(\omega_w = \omega_w \times 10^{14}, k_y = k_z = 0\right)$

		ω_w				
		$l\ (nm)$				
k	$\mu(nm)$	0.0	0.25	0.75	1.25	1.75
1×10^9	0.0	0.1270	0.1309	0.1587	0.2032	0.2559
	0.75	0.1016	0.1047	0.1270	0.1626	0.2047
	1.5	0.0704	0.0726	0.0880	0.1127	0.1419
2×10^9	0.0	0.2539	0.2839	0.4578	0.6837	0.9242
	0.75	0.1408	0.1575	0.2539	0.3792	0.5127
	1.5	0.0803	0.0898	0.1448	0.2162	0.2923

Table 5 The effects of length scale parameters and wave number on the wave frequency of trigonal square plate $\left(\omega_w = \omega_w \times 10^{14}, k_y = k_z = 0\right)$

		ω_w				
		$l\,(nm)$				
k	$\mu(nm)$	0.0	0.25	0.75	1.25	1.75
1×10^9	0.0	0.0572	0.0590	0.0715	0.0916	0.1153
	0.75	0.0458	0.0472	0.0572	0.0733	0.0923
	1.5	0.0317	0.0327	0.0397	0.0508	0.0640
2×10^9	0.0	0.1144	0.1280	0.2063	0.3082	0.4166
	0.75	0.0635	0.0710	0.1144	0.1709	0.2311
	1.5	0.0362	0.0405	0.0652	0.0974	0.1317

Table 6 The effects of length scale parameters and wave number on the wave frequency of monoclinic square plate $\left(\omega_w = \omega_w \times 10^{14}, k_y = k_z = 0\right)$

		ω_w				
		$l\,(nm)$				
k	$\mu(nm)$	0.0	0.25	0.75	1.25	1.75
1×10^9	0.0	0.0331	0.0341	0.0414	0.0530	0.0667
	0.75	0.0265	0.0273	0.0331	0.0424	0.0534
	1.5	0.0184	0.0189	0.0229	0.0294	0.0370
2×10^9	0.0	0.0662	0.0740	0.1193	0.1782	0.2409
	0.75	0.0367	0.0410	0.0662	0.0989	0.1336
	1.5	0.0209	0.0234	0.0377	0.0564	0.0762

Table 7 The effects of length scale parameters and eave number on the wave frequency of triclinic square plate $\left(\omega_w = \omega_w \times 10^{14}, k_y = k_z = 0\right)$

		ω_w				
		$l\,(nm)$				
k	$\mu(nm)$	0.0	0.25	0.75	1.25	1.75
1×10^9	0.0	0.0357	0.0368	0.0446	0.0572	0.0720
	0.75	0.0286	0.0295	0.0357	0.0457	0.0576
	1.5	0.0198	0.0204	0.0248	0.0317	0.0399
2×10^9	0.0	0.0714	0.0799	0.1288	0.1923	0.2600
	0.75	0.0396	0.0443	0.0714	0.1067	0.1442
	1.5	0.0226	0.0253	0.0407	0.0608	0.0822

Table 8, examines the effects of wave number, both length scale parameters, uniaxial, biaxial, and triaxial magnetic field on the wave propagation in orthotropic nanoplates for H_x=100, 500 and 1000. The nonlocal and gradient parameters are taken 1nm. It can be seen that with the increase of magnetic field, the wave frequency increases. Besides, for higher values of wave numbers, magnetic field shows less effect on the wave frequencies. Also, the effect f variations of biaxial magnetic field on the wave propagation in different types of orthotropic nanoplates with considering wave number in *x* direction is more than uniaxial and triaxial magnetic field. In realizing the influence of triaxial magnetic field, the wave frequency in anisotropic structures are shown in Tables 9-11. They are tabulated for three cases including uniaxial, biaxial, and triaxial magnetic field. Tables 9-11, demonstrates the effect of magnetic field on changes of wave frequency for four different anisotropic structures versus wave number at $l = \mu = 0$. It is seen that the wave frequency has a notable increasing trend versus wave number. Also, it can be

Table 8 Wave frequency $\left(\omega_{M3} = \omega_{M3} \times 10^{14}\right)$ in orthotropic nanoplates for different wave numbers under the variations of magnetic field, $\left(\mu = 1\times10^{-9}, l = 1\times10^{-9}, k_y = k_z = 0\right)$

	ω_w					
	$k = 1\times10^9$			$k = 2\times10^9$		
	Uniaxial	Biaxial	Triaxial	Uniaxial	Biaxial	Triaxial
$H_x = 100$						
Armchair sheet I	1.2829	2.0027	1.2829	2.5654	4.0054	2.5654
Armchair sheet II	1.2699	1.9806	1.2699	2.5393	3.9613	2.5393
Armchair sheet III	1.2644	1.9590	1.2644	2.5283	3.9180	2.5283
Zigzag sheet IV	1.2918	2.0038	1.2918	2.5832	4.0077	2.5832
Zigzag sheet V	1.2909	1.9840	1.2909	2.5813	3.9679	2.5813
Zigzag sheet VI	1.2645	1.9671	1.2645	2.5285	3.9342	2.5285
$H_x = 1000$						
Armchair sheet I	1.3131	2.0440	1.3094	2.5897	4.0879	2.5825
Armchair sheet II	1.3034	2.0269	1.2989	2.5664	4.0537	2.5574
Armchair sheet III	1.3008	2.0099	1.2955	2.5577	4.0197	2.5471
Zigzag sheet IV	1.3254	2.0504	1.3209	2.6103	4.1008	2.6013
Zigzag sheet V	1.3254	2.0324	1.3205	2.6092	4.0647	2.5995
Zigzag sheet VI	1.3005	2.0171	1.2953	2.5576	4.0340	2.5473

Table 9 Effects of uniaxial, biaxial and triaxial magnetic field on the wave frequency (M1) of anisotropic nanostructures, $\left(\omega_{M1} = \omega_{M1} \times 10^{13}, k_y = k_z = 0\right)$

	ω_{M1}								
	Uniaxial			Biaxial			Triaxial		
	100	500	1000	100	500	1000	100	500	1000
$k = 1\times10^8$									
Hexagonal	0.0858	0.0930	0.1125	0.0858	0.0919	0.1005	0.0858	0.0920	0.0994
Trigonal	0.0336	0.0450	0.0572	0.0336	0.0418	0.0468	0.0335	0.0394	0.0419
Monoclinic	0.0337	0.0451	0.0572	0.0336	0.0410	0.0448	0.0336	0.0417	0.0467
Triclinic	0.0169	0.0244	0.0357	0.0169	0.0227	0.0262	0.0169	0.0216	0.0238
$k = 1\times10^9$									
Hexagonal	0.8568	0.8938	1.0008	0.8568	0.8832	0.8874	0.8568	0.8838	0.8934
Trigonal	0.3337	0.4081	0.5680	0.3336	0.3739	0.3100	0.3335	0.3541	0.2823
Monoclinic	0.3338	0.3959	0.5461	0.3337	0.3557	0.2699	0.3337	0.3632	0.3197
Triclinic	0.1688	0.2127	0.3060	0.1688	0.1918	0.1501	0.1689	0.1902	0.1544
$k = 2\times10^9$									
Hexagonal	1.7116	1.7417	1.8324	1.7116	1.7207	1.6069	1.7116	1.7221	1.6280
Trigonal	0.6648	0.7553	0.9254	0.6647	0.6838	0.2230	0.6644	0.6481	0.1575
Monoclinic	0.6641	0.7166	0.8603	0.6639	0.6345	0.2313	0.6640	0.6497	0.2615
Triclinic	0.3376	0.3771	0.4686	0.3376	0.3328	0.1788	0.3377	0.3334	0.1131

concluded that the wave frequency in three different modes increased significantly for increasing all magnetic field changes. The influences of triaxial magnetic field on the wave frequency (M3) are more than other cases but for the wave frequency (M1) and (M2), the effects of biaxial and uniaxial magnetic field are more notable, respectively. Furthermore, the review of results indicates that hexagonal structure has the highest, and the triclinic structure, the lowest wave frequency in all magnetic field cases (uniaxial, biaxial, and triaxial) and modes (M1, M2, and M3). It should be noted that these results are independent of the wave number. Moreover, the variations of magnetic field have no sensible effects on the wave frequency at lower wave numbers.

7. Conclusions

In this work, for the first time, the effects of triaxial magnetic field on the wave propagation in anisotropic single-layered graphene sheets were presented. The equations of motion were determined by the three-dimensional elasticity theory in conjunction with the nonlocal strain gradient theory. Different types of anisotropic structures such as orthotropic, hexagonal, trigonal, monoclinic and triclinic materials were modeled and compared with each other. The effects of different parameters such as nonlocality, gradient parameter and uniaxial, biaxial and triaxial magnetic field were also studied. Hence, the following conclusions were notable,

Table 10 Effects of uniaxial, biaxial and triaxial magnetic field on the wave frequency (M2) of anisotropic nanostructures, $\left(\omega_{M2} = \omega_{M2} \times 10^{13}, k_y = k_z = 0\right)$

	ω_{M2}								
	Uniaxial			Biaxial			Triaxial		
	100	500	1000	100	500	1000	100	500	1000
$k = 1\times10^8$									
Hexagonal	0.0949	0.1015	0.1198	0.0949	0.1015	0.1198	0.0949	0.1006	0.1169
Trigonal	0.0514	0.0572	0.0695	0.0514	0.0580	0.0774	0.0514	0.0592	0.0794
Monoclinic	0.0513	0.0572	0.0695	0.0513	0.0595	0.0798	0.0513	0.0565	0.0752
Triclinic	0.0174	0.0247	0.0395	0.0174	0.0244	0.0395	0.0174	0.0246	0.0398
$k = 1\times10^9$									
Hexagonal	0.9487	1.0148	1.1980	0.9487	1.0148	1.1980	0.9487	1.0053	1.1445
Trigonal	0.5134	0.5722	0.5722	0.5134	0.5767	0.7740	0.5134	0.5801	0.7476
Monoclinic	0.5134	0.5722	0.5722	0.5134	0.5951	0.7983	0.5132	0.5631	0.7180
Triclinic	0.1725	0.2438	0.3570	0.1724	0.2439	0.3953	0.1724	0.2356	0.3639
$k = 2\times10^9$									
Hexagonal	1.8974	2.0295	2.3959	1.8974	2.0295	2.3959	1.8973	2.0104	2.2736
Trigonal	1.0263	1.1445	1.1445	1.0262	1.1502	1.5478	1.0262	1.1479	1.4530
Monoclinic	1.0267	1.1445	1.1445	1.0267	1.1902	1.5966	1.0263	1.1248	1.4093
Triclinic	0.3427	0.4879	0.7141	0.3426	0.4877	0.7906	0.3426	0.4671	0.7040

Table 11 Effects of uniaxial, biaxial and triaxial magnetic field on the wave frequency (M3) of anisotropic nanostructures, $\left(\omega_{M3} = \omega_{M3} \times 10^{13}, k_y = k_z = 0\right)$

	ω_{M3}								
	Uniaxial			Biaxial			Triaxial		
	100	500	1000	100	500	1000	100	500	1000
$k = 1\times10^8$									
Hexagonal	0.1270	0.1270	0.1270	0.1272	0.1329	0.1552	0.1274	0.1386	0.1744
Trigonal	0.0572	0.0595	0.0798	0.0576	0.0684	0.1004	0.0579	0.0753	0.1182
Monoclinic	0.0572	0.0595	0.0798	0.0576	0.0676	0.0994	0.0579	0.0762	0.1191
Triclinic	0.0357	0.0357	0.0396	0.0359	0.0412	0.0587	0.0361	0.0454	0.0697
$k = 1\times10^9$									
Hexagonal	1.2696	1.2696	1.2696	1.2717	1.3289	1.5383	1.2739	1.3853	1.7383
Trigonal	0.5722	0.5868	0.7829	0.5756	0.6785	0.9723	0.5789	0.7518	1.1745
Monoclinic	0.5722	0.5951	0.7983	0.5756	0.6723	0.9646	0.5790	0.7603	1.1833
Triclinic	0.3572	0.3572	0.3954	0.3590	0.4102	0.5724	0.3607	0.4529	0.6923
$k = 2\times10^9$									
Hexagonal	2.5392	2.5392	2.5392	2.5435	2.6572	3.0635	2.5478	2.7702	3.4714
Trigonal	1.1445	1.1660	1.5597	1.1512	1.3516	1.9140	1.1578	1.5024	2.3415
Monoclinic	1.1445	1.1902	1.5966	1.1511	1.3410	1.9009	1.1580	1.5191	2.3589
Triclinic	0.7143	0.7143	0.7975	0.7179	0.8190	1.1304	0.7215	0.9050	1.3800

- Based on a general conclusion, the three-dimensional elasticity theory in conjunction with the nonlocal strain gradient theory, as a single theory, can show accuracy consequence for the wave propagation characteristics of anisotropic structures including size effects.
- The wave frequency of anisotropic structures can be increased by an increase in the value of gradient parameter or a decrease in the nonlocal parameter’s magnitude. In other words, the presented model can estimate both stiffness-softening and stiffness-hardening influences generated by nonlocal and gradient parameters, respectively.
- The effect of nonlocal parameter and gradient parameter on the anisotropic structures with higher wave numbers is more significant than anisotropic structures with lower wave numbers.
- The wave frequencies of anisotropic structures depend on the magnitude of the magnetic field. So, with the increase of magnetic field changes, the wave frequencies grow significantly.

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