Steel and Composite Structures, *Vol. 24, No. 6 (2017) 659-670*
DOI: https://doi.org/10.12989/scs.2017.24.6.659

Static analysis of singly and doubly curved panels on rectangular plan-form

Rajendra Bahadur [a], A.K. Upadhyay [b] and K.K. Shukla [*]

Department of Applied Mechanics, MNNIT, Allahabad, U.P., 211004. India

(*Received August 13, 2016, Revised April 11, 2017, Accepted May 22, 2017*)

Abstract. In the present work, an analytical solution for the static analysis of laminated composites, functionally graded and sandwich singly and doubly curved panels on the rectangular plan-form, subjected to uniformly distributed transverse loading is presented. Mathematical formulation is based on the higher order shear deformation theory and principle of virtual work is applied to derive the equations of equilibrium subjected to small deformation. A solution methodology based on the fast converging finite double Chebyshev series is used to solve the linear partial differential equations along with the simply supported boundary condition. The effect of span to thickness ratio, radius of curvature to span ratio, stacking sequence, power index are investigated. The accuracy of the solution is checked by the convergence study of non-dimensional central deflection and moments. Present results are compared with those available in the literature.

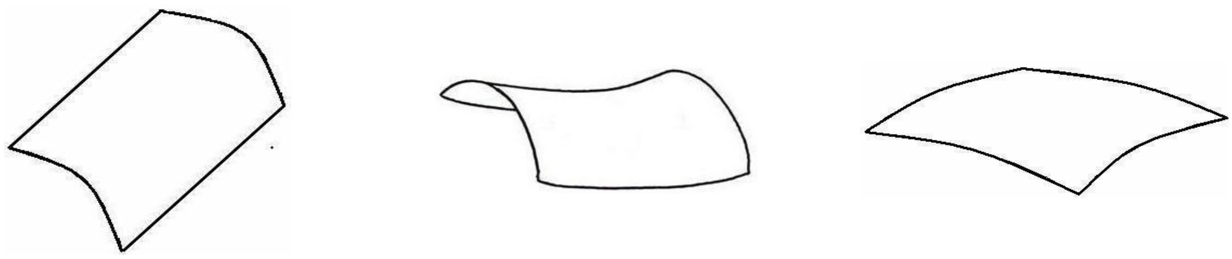
Keywords: analytical solution; higher order shear deformation theory; doubly curved panels; Chebyshev polynomial

1. Introduction

Singly and doubly curved panels of different materials are used as structural elements in high performance engineering structures, such as automobile, aerospace, chemical, naval, civil and many others. The tremendous increase in the use of composite and functionally graded material is due to their attractive properties, like enhanced corrosion resistance and the possibility to get the optimal design by changing the stacking sequence and fiber orientation. Plates and curved panels have been analyzed using numerical and analytical methods by many researchers. Utilizing Navier-type exact solution based on modified Sander's theory, Reddy and Liu (1985) presented static analysis of cylindrical and spherical shells under simply supported boundary condition. Utilizing double Fourier series and incorporating FSDT, Chaudhuri and Abu-Arja (1988) presented an exact solution to the boundary value problem of doubly curved anti-symmetric angle ply shells subjected to transverse loading. Chaudhuri and Kabir (1989) presented an analytical solution of shear-flexible doubly curved orthotropic shells of rectangular plan-form utilizing first order shear deformation theory. Employing higher order shear deformation theory with finite element formulation, Kant and Kommineni (1992) presented linear and geometrically non-linear responses of panels under transverse loading. Utilizing four classical shallow shell theories, Chaudhuri and Kabir (1993, 1994) presented static and dynamic responses of moderately thick, symmetric and anti-symmetric doubly curved panels with different boundary conditions. Ossadzow *et al*. (1995) presented theoretical modeling of laminated composite shells of arbitrary shape, accounting for continuity conditions of displacement and transverse shear stresses at layer interface. To and Liu (2001) developed a hybrid strain based lower order shell element for analysis of layer wise anisotropic shell structures undergoing large deformation. Utilizing higher order theory, Khare *et al*. (2005) presented closed form solution for thermo-mechanical and free vibration analysis of simply supported cross ply laminated composite and sandwich doubly curved panels on rectangular plan-form. Chakrabarti and Sheikh (2005) presented flexural response of sandwich plates with stiff laminated face sheets incorporating a refined higher order theory. Oktem and Chaudhuri (2007, 2009) presented Levy type analytical solution using double Fourier series for general cross-ply thick doubly curved panel of rectangular plan-form with different boundary conditions. Oktem and Soares (2011a, b) presented static analysis of general cross-ply laminated composite plates and doubly curved panels of rectangular plan-form with different boundary conditions utilizing boundary-discontinuous generalized double Fourier series. Utilizing Carrera's unified formulation, Carrera *et al*. (2011) presented the thickness stretching effects in the functionally graded plates and shells. Using higher order shear deformation theory, Oktem *et al*. (2012) presented the static response of functionally graded plates and doubly curved shells, using double Fourier series. Kiani *et al*. (2012) investigated static, dynamic and free vibration response of functionally graded doubly curved panels resting on Pasternak elastic foundation. Arefi (2014) presented general formulation for the thermo elastic analysis of a functionally graded cylindrical shell subjected to external loads, using shear deformation theory and energy method. Based on improved higher order sandwich panel theory, Fard *et al*. (2014) analyzed bending response of doubly curved

*Corresponding author, Professor,
E-mail: kkshukla@mnnit.ac.in
[a] Research Scholar, E-mail: rajendramech@gmail.com
[b] Assistant Professor, E-mail: ashutosh@mnnit.ac.in

http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)



(a) Cylindrical (b) Hyperboloid (c) Spherical

Fig. 1 Geometry of singly (a) and doubly (b and c) curved panels

sandwich panel subjected to different type of loadings. Utilizing double Fourier series, Alankaya and Oktem (2016) presented an analytical solution to the static analysis of laminated composite and sandwich doubly curved shallow shells. Sahoo *et al*. (2016) presented experimental results for the static, transient and free vibration response of laminated composite singly and doubly curved panels and compared the results with finite element simulation.

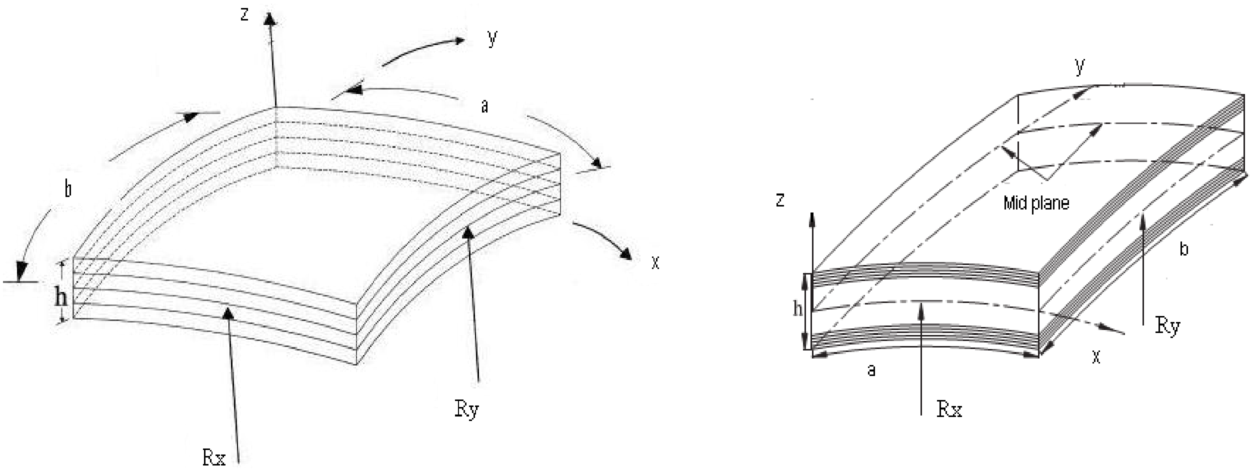
It is evident from the literature that most of the work related to static analysis of singly and doubly curved panels on rectangular plan-form is carried out using numerical method. It is also observed that relatively less attention is paid to the flexural analysis of hyperboloid and ellipsoidal panel. In the present study, Chebyshev series based, analytical approach is used for the static analysis of singly and doubly curved panels on rectangular plan-form, subjected to uniformly distributed transverse loading. Mathematical formulation is based on the higher order shear deformation theory and principle of virtual work is used to obtain the governing partial differential equations. Fast converging finite double Chebyshev series is used for spatial discretization of governing partial differential equations. The results are obtained for laminated composite, sandwich and functionally graded singly (cylindrical- 'C') and doubly curved (spherical-'S', hyperboloid-'H' and ellipsoid -'E') panels (Fig. 1). Some new results pertaining to the hyperboloid and ellipsoid are presented which can be used as a benchmark.

2. Mathematical formulation

The laminated composite and sandwich doubly curved panels, consisting of finite number of layers are shown in Figs. 2(a)-(b), respectively. x and y are the lines on the mid surface of shell, z is a straight line normal to the mid surface. R_x and R_y are the principal radii of curvatures of mid surface of the panels. In the present work, governing differential equations are obtained using higher order shear deformation theory with third order variation of in-plane displacement and constant transverse displacement. At a point in the panel, the displacement field is given as (Kant and Kommineni 1992)

$$
\begin{aligned}
u(x,y,z) &= u_0(x,y) + z\psi_x(x,y) \\
&\quad + z^2 u_1(x,y) + z^3\phi_x(x,y) \\
v(x,y,z) &= v_0(x,y) + z\psi_y(x,y) \\
&\quad + z^2 v_1(x,y) + z^3\phi_y(x,y) \\
w(x,y,z) &= w_0(x,y)
\end{aligned}
\tag{1}
$$

where, u_0, v_0 are the in-plane displacements and w_0 is the transverse displacement of a point (x, y) on the middle plane of the panel, respectively. The functions ψ_x, ψ_y are the rotations of the normal to the middle plane about y and x-axes, respectively. u_1, v_1 and ϕ_x, ϕ_y are the higher order terms in the Taylor's series expansion, representing higher order transverse cross-sectional deformation modes.



(a) Laminated composite (b) Sandwich

Fig. 2 Geometry of doubly curved panel

Strain-displacement relations

Assuming thickness of the panel very much less than the radii of curvatures ($h << R_x$, R_y), the general strain-displacement relations are given as

$$\varepsilon_x = \frac{\partial u}{\partial x} + \frac{w}{R_x},\ \varepsilon_y = \frac{\partial v}{\partial y} + \frac{w}{R_y},$$
$$\gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},\ \gamma_{yz} = \frac{\partial v}{\partial z} + \frac{\partial w}{\partial y} - \frac{v}{R_y},\ \gamma_{xz} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x} - \frac{u}{R_x} \tag{2a}$$

Linear strains in terms of mid surface displacement can be obtained by substituting the displacement expressions from Eq. (1) into Eq. (2a), and given as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ \gamma_{yz}^0 \\ \gamma_{zx}^0 \end{Bmatrix} + z \begin{Bmatrix} \chi_x \\ \chi_y \\ \chi_{xy} \\ \chi_{yz} \\ \chi_{zx} \end{Bmatrix} + z^2 \begin{Bmatrix} \varepsilon_{x_0}^* \\ \varepsilon_{y_0}^* \\ \gamma_{xy_0}^* \\ \gamma_{yz_0}^* \\ \gamma_{zx_0}^* \end{Bmatrix} + z^3 \begin{Bmatrix} \chi_x^* \\ \chi_y^* \\ \chi_{xy}^* \\ \chi_{yz}^* \\ \chi_{zx}^* \end{Bmatrix} \tag{2b}$$

where

$$\begin{aligned}
&\varepsilon_x^0 = u_{0,x} + \frac{w_0}{R_x},\ \varepsilon_y^0 = v_{0,y} + \frac{w_0}{R_y},\ \gamma_{xy}^0 = u_{0,y} + v_{0,x}, \\
&\gamma_{yz}^0 = w_{0,y} - \frac{v_0}{R_y} + \psi_y,\ \gamma_{zx}^0 = w_{0,x} - \frac{u_0}{R_x} + \psi_x \\
&\chi_x = \psi_{x,x},\ \chi_y = \psi_{y,y},\ \chi_{xy} = \psi_{x,y} + \psi_{y,x}, \\
&\chi_{yz} = 2v_1 - \frac{\psi_y}{R_y},\ \chi_{zx} = 2u_1 - \frac{\psi_x}{R_x} \\
&\varepsilon_{x_0}^* = u_{1,x},\ \varepsilon_{y_0}^* = v_{1,y}, \\
&\gamma_{xy_0}^* = u_{1,y} + v_{1,x},\ \gamma_{yz_0}^* = 3\phi_y - \frac{v_1}{R_y},\ \gamma_{zx_0}^* = 3\phi_x - \frac{u_1}{R_x} \\
&\chi_x^* = \phi_{x,x},\ \chi_y^* = \phi_{y,y},\ \chi_{xy}^* = \phi_{x,y} + \phi_{y,x}, \\
&\chi_{yz}^* = -\frac{\phi_y}{R_y},\ \chi_{zx}^* = -\frac{\phi_x}{R_x}
\end{aligned} \tag{2c}$$

Effective material properties of FGM panels are obtained by the rule of mixture based on the power law grading and these properties are assumed to vary through the thickness. These properties are expressed as (Kiani *et al.* 2012)

$$P(z) = P_m + (P_c + P_m)V_c \tag{3a}$$

where, P_m and P_c are the effective properties of metal and ceramic respectively, and V_c is the volume fraction of ceramic and given as

$$V_c(z) = \left(\frac{z}{h} + \frac{1}{2}\right)^n \tag{3b}$$

where, 'n' is power law exponent and always greater than or equal to zero.

For k^{th} layer in the shell, considered as orthotropic, the stress-strain relationship is expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{zx} \end{Bmatrix}_k = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} & 0 & 0 \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} & 0 & 0 \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} & 0 & 0 \\ 0 & 0 & 0 & \overline{Q}_{44} & \overline{Q}_{45} \\ 0 & 0 & 0 & \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}_k \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}_k \tag{4}$$

where, $\overline{Q}_{ij}$'s (i, j = 1, 2, 4, 5, 6) are transformed reduced stiffness coefficients.

In-plane stresses and moments are expressed as

$$\begin{bmatrix} N_x & M_x & N_x^* & M_x^* \\ N_y & M_y & N_y^* & M_y^* \\ N_{xy} & M_{xy} & N_{xy}^* & M_{xy}^* \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} \left(1, z, z^2, z^3\right) dz \tag{5a}$$

Transverse shear stresses are written as

$$\begin{bmatrix} Q_x & S_x & Q_x^* & S_x^* \\ Q_y & S_y & Q_y^* & S_y^* \end{bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} \left(1, z, z^2, z^3\right) dz \tag{5b}$$

The stiffness coefficients are expressed as:

(a) For laminated composite panel

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}, J_{ij}\right) = \sum_{k=1}^{n} \int_{z_{k-1}}^{z_k} \overline{Q}_{ij}^{(k)} \left(1, z, z^2, z^3, z^4, z^5, z^6\right) dz; \quad \left(i, j = 1,2,4,5,6\right) \tag{6}$$

(b) For FGM panel

$$\left(A_{ij}, B_{ij}, D_{ij}, E_{ij}, F_{ij}, H_{ij}, J_{ij}\right) = \int_{-h/2}^{h/2} \left\{ Q_{ij}^m + \left(Q_{ij}^c - Q_{ij}^m\right)\left(\frac{z}{h} + \frac{1}{2}\right)^n \right\} \left(1, z, z^2, z^3, z^4, z^5, z^6\right) dz; \quad \left(i, j = 1,2,4,5,6\right) \tag{7}$$

where

$$Q_{11}^m = Q_{22}^m = \frac{E_m}{1-\nu^2};\ Q_{12}^m = \frac{\nu E_m}{1-\nu^2};$$
$$Q_{44}^m = Q_{55}^m = Q_{66}^m = \frac{E_m}{2(1+\nu)};\ for\ metals \tag{8a}$$

$$Q_{11}^c = Q_{22}^c = \frac{E_c}{1-\nu^2};\ Q_{12}^c = \frac{\nu E_c}{1-\nu^2};$$
$$Q_{44}^c = Q_{55}^c = Q_{66}^c = \frac{E_c}{2(1+\nu)};\ for\ ceramic \tag{8b}$$

Governing equations of equilibrium

Governing partial differential equations of equilibrium are obtained using principle of virtual work

$$\delta\pi = \delta(U - W) = 0 \tag{9}$$

Where, U is strain energy and W is the work done by the external forces. These terms can be expressed as

$$\delta U = \int_x \int_y \int_z \Big(\sigma_x \delta\varepsilon_x + \sigma_y \delta\varepsilon_y + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz} \Big) dx\, dy\, dz \tag{10a}$$

$$\delta W = -\int_x \int_y q\,\delta w\, dx\, dy \tag{10b}$$

Integrating through the panel thickness and substituting the strains and introducing the stresses and moments from Eqs. (5a)-(5b), Eq. (9) is transformed into the following form

$$\begin{aligned}\delta\pi = \int_x \int_y (& N_x \delta\varepsilon_x^0 + N_y \delta\varepsilon_y^0 + N_{xy}\delta\gamma_{xy}^0 \\ &+ N_x^*\delta\varepsilon_{x_0}^* + N_y^*\delta\varepsilon_{y_0}^* + N_{xy}^*\delta\gamma_{xy_0}^* + M_x\delta\chi_x \\ &+ M_{xy}\delta\chi_{xy} + M_x^*\delta\chi_x^* + M_y^*\delta\chi_y^* + M_{xy}^*\delta\chi_{xy}^* \\ &+ Q_x\delta\gamma_{zx}^0 + Q_y\delta\gamma_{yz}^0 + Q_x^*\delta\gamma_{zx_0}^* + Q_y^*\delta\gamma_{yz_0}^* \\ &+ S_x\delta\chi_{zx} + S_y\delta\chi_{yz} - q\delta w_0) dx\, dy = 0\end{aligned} \tag{11}$$

'q' is the uniformly distributed transverse loading over the panel surface.

Governing equation of equilibrium are obtained from Eq. (11) and expressed as

$$\frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} + \frac{Q_x}{R_x} = 0; \tag{12a}$$

$$\frac{\partial N_y}{\partial y} + \frac{\partial N_{xy}}{\partial x} + \frac{Q_y}{R_y} = 0 \tag{12b}$$

$$\begin{aligned}&\frac{\partial Q_x}{\partial x} + \frac{\partial Q_y}{\partial y} + N_x\frac{\partial^2 w}{\partial x^2} + N_y\frac{\partial^2 w}{\partial y^2} \\ &+ 2N_{xy}\frac{\partial^2 w}{\partial x \partial y} - \frac{N_x}{R_x} - \frac{N_y}{R_y} + q = 0\end{aligned} \tag{12c}$$

$$\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} + \frac{S_x}{R_x} - Q_x = 0; \tag{12d}$$

$$\frac{\partial M_y}{\partial y} + \frac{\partial M_{xy}}{\partial x} + \frac{S_y}{R_y} - Q_y = 0 \tag{12e}$$

$$\frac{\partial N_x^*}{\partial x} + \frac{\partial N_{xy}^*}{\partial y} + \frac{Q_x^*}{R_x} - 2S_x = 0; \tag{12f}$$

$$\frac{\partial N_y^*}{\partial y} + \frac{\partial N_{xy}^*}{\partial x} + \frac{Q_y^*}{R_y} - 2S_y = 0 \tag{12g}$$

$$\frac{\partial M_x^*}{\partial x} + \frac{\partial M_{xy}^*}{\partial y} + \frac{S_x^*}{R_x} - 3Q_x^* = 0; \tag{12h}$$

$$\frac{\partial M_y^*}{\partial y} + \frac{\partial M_{xy}^*}{\partial x} + \frac{S_y^*}{R_y} - 3Q_y^* = 0 \tag{12i}$$

Utilizing Eqs. (4) and (5), the Eqs. (12a)-(12i) are cast in displacement form. Variationally consistent boundary condition are obtained and expressed as:

Simply supported immovable

$$u = v = w = \psi_y = u_1 = v_1 = \phi_y = 0 \quad \& \; M_x = M_x^* = 0 \; at \; x = \pm 1 \tag{13a}$$

$$u = v = w = \psi_x = u_1 = v_1 = \phi_x = 0 \quad \& \; M_y = M_y^* = 0 \; at \; y = \pm 1 \tag{13b}$$

Simply supported movable SS1 type

$$v = w = \psi_y = u_1 = v_1 = \phi_y = 0 \quad \& \; N_y = M_y = M_y^* = 0 \; at \; y = \pm 1 \tag{13c}$$

$$u = w = \psi_x = u_1 = v_1 = \phi_x = 0 \quad \& \; N_y = M_y = M_y^* = 0 \; at \; y = \pm 1 \tag{13d}$$

Non dimensional parameters used in present analysis are as follows

$$u = \frac{u_o}{h}; v = \frac{v_o}{h}; w = \frac{w_o}{h}; \quad X = \frac{2x}{a}; Y = \frac{2y}{b}; \lambda = \frac{a}{b}; \beta = \frac{a}{h}; \bar{Q} = \frac{qa^4}{E_2 h^4} \tag{14}$$

Governing differential equations are finally cast in non dimensional form using Eq. (14).

3. Solution methodology

The governing equations of equilibrium along with the appropriate boundary conditions are solved using fast converging double Chebyshev series. The i^{th} term in a Chebyshev polynomial is given as (Fox and Parker 1968)

$$T_i(x) = Cos(i\theta); \quad Cos\theta = x; \quad -1 \le x \le 1 \tag{15}$$

The recurrence relation can be found from the above equation and expressed as

$$T_{p+1}(x) + T_{p-1}(x) = 2xT_p(x) \tag{16}$$

The general displacement functions η (x, y) and the loadings are approximated as

$$\eta(x,y) = \sum_{i=0}^{M}\sum_{j=0}^{N} \delta_{ij}\,\eta_{i,j}\,T_i(x)T_j(y) \; ; \; -1 \le x, y \le 1 \tag{17}$$

where, M and N are the number of terms in finite degree double Chebyshev series. The values $\delta_{i,j}$ for different values of i & j are taken from Nath and Shukla (2001) and expressed as

$$\delta_{i,j} = \begin{cases} 0.25; & if\ i\ \&\ j\ both\ are\ zero \\ 0.5; & if\ i=0\ \&\ j\neq 0\ or\ i\neq 0\ \&\ j=0 \\ 1.0; & otherwise \end{cases} \tag{18}$$

The spatial derivative of the function is expressed as

$$\frac{\partial^{r+s}u(x,y)}{\partial x^r y^s} = \sum_{i=0}^{M-r}\sum_{j=0}^{N-s}\delta_{ij}\left(\frac{\partial^{r+s}u}{\partial x^r y^s}\right)_{ij} T_i(x)T_j(y)\ ; \quad -1\leq x,y\leq 1 \tag{19}$$

where, 'r and s' are the orders of derivatives with respect to x and y, respectively.

Using the above described procedure, the linear partial differential equations are discretized in the space domain. Finally the equation of equilibriums are reduced to a set of linear simultaneous equations by collocating the zeros of Chebyshev polynomials and expressed as

$$\sum_{i=0}^{M-2}\sum_{j=0}^{N-2}F_k\left(u_{0ij}, v_{0ij}, w_{0ij}, \psi_{xij}, \psi_{yij}, u_{1ij}, v_{1ij}, \phi_{xij}, \phi_{yij}, Q_{ij}\right)T_i(x)T_j(x) = 0; \quad (k = 1-9) \tag{20}$$

Similarly, the sets of boundary condition are discretized and expressed in linear simultaneous equations. Finally the set of Eq. (20) along with the boundary conditions are expressed as

$$A\eta = Q \tag{21}$$

where A is ($p\times q$) coefficient matrix, η is $q\times 1$ displacement coefficient matrix, and Q is ($p\times 1$) load vector.

Total number of equations obtained from the Eq. (21) is more than the total number of unknowns. To get unique and compatible solution, the multiple regression technique based on least square error norms is used. The multiple regression analysis gives

$$\eta = \left(A^T A\right)^{-1} A^T Q = BQ \tag{22}$$

The displacement at any location on the mid plane of the panel can be evaluated by putting the displacement vector in the Eq. (17).

4. Results and discussion

Static response of laminated composite, FGM and sandwich cylindrical ($R_x = R$ and $R_y = \infty$), spherical ($R_x = R_y = R$), hyperboloid ($R_x = -R_y = R$) and ellipsoidal ($R_x = 2R$ and $R_y = R$) panels are computed analytically using the fast conversing double Chebyshev series. The effects of the different parameters such as lamination scheme, span to thickness ratio, radius of curvature to span ratio, power index on non-dimensional central deflection and bending moments of singly and doubly curved panels are obtained. The accuracy and validity of the present method is examined by detailed convergence study. The present results are also compared with those available in the literature.

Laminated composite curved panels

Static responses of laminated composite singly (cylindrical-'C') and doubly (spherical-'S', hyperboloid-'H' and ellipsoid-'E') curved panels on the square plan-form are computed for SS1 type simply supported boundary condition. The following material properties are considered for the analysis

$$E_1 = 175.78\text{ GPa},\ E_2 = 7.0312\text{ GPa},$$

$$G_{12}/E_2 = G_{13}/E_2 = 0.5,\ G_{23}/E_2 = 0.2,\ \nu_{12} = 0.25$$

The normalized quantities used in the present analysis are

$$w_0^* = \frac{10^3 E_2 h^4 w_0}{qa^4};\quad \overline{M}_x = \frac{10^3 D_{11} M_x}{qa^2};\quad \overline{M}_y = \frac{10^3 D_{22} M_y}{qa^2}$$

where, w_0^* is non dimensional transverse central deflection, $\overline{M}_x$ and $\overline{M}_y$ are non dimensional central moments about y and x axes, respectively and w_0, M_x, M_y are the corresponding dimensional quantities. In the above normalized quantities, 'a' is the panel length, 'h' is panel thickness and 'q' is the load applied uniformly on the surface of the panel.

Fig. 3 shows the convergence of non-dimensional transverse central deflection (w_0^*) and bending moment ($\overline{M}_x, \overline{M}_y$) of a moderately thick (a/h = 10), symmetric (0/90/0) cross-ply doubly curved panel (R/a = 10) on the square plan-form. Deflection and moments show good convergence at 9-10 terms expansion of the variables. In further analysis, 9 terms expansion of the displacement variables in Chebyshev series is taken. To reduce the gap between $\overline{M}_x$ and $\overline{M}_y$, a reduced scale ($\overline{M}_x/6$) is taken in place of $\overline{M}_x$. Non-dimensional transverse central displace-ment of symmetric (0/90/0) and anti-symmetric (0/90) and bending moment ($\overline{M}_x$) of symmetric (0/90/0 and 0/90/90 /0) cross-ply doubly curved panel on the square plan-form (a/b = 1) is carried out. Non-dimensional central displace-ment is obtained for thin (a/h = 100) and moderately thick (a/h = 10) doubly curved panel at various R/a ratio. These results are compared with those due to Reddy and Liu (1985) and shown in Table 1. The bending moment of doubly curved panel (R/a = 10) is computed for different a/h ratio. These results are compared with those due to Oktem and Chaudhuri (2009) and shown in Table 2. Non-dimensional central displacement and bending moment both are in good agreement.

Variation of non-dimensional central deflection (w_0^*) with span to thickness ratio (a/h) for anti-symmetric (0/90) and symmetric (0/90/0) cross-ply laminated composite singly and doubly curved panel on square plan-form are shown in Figs. 4(a)-(b), respectively. It is observed that the non-dimensional transverse central deflection of the hyperboloid is the highest and that of spherical panel is the lowest for both symmetric (0/90/0) and anti-symmetric (0/90)

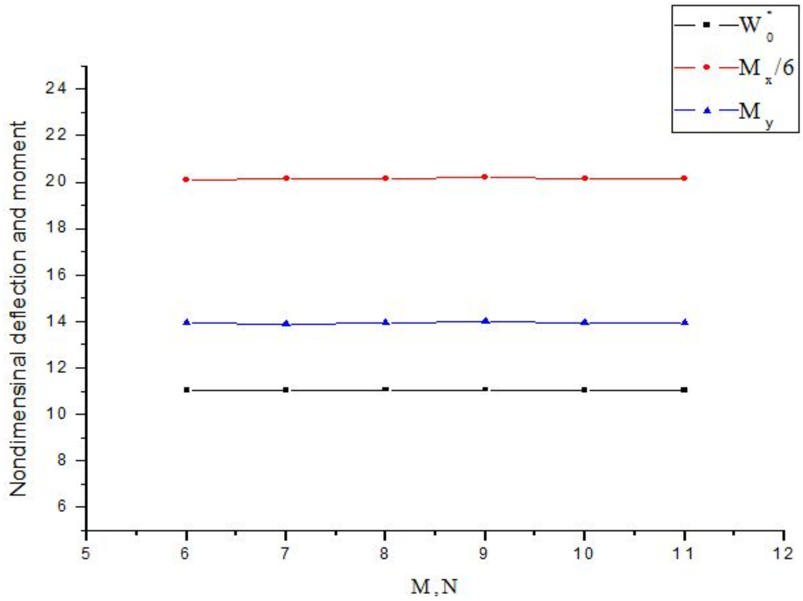


Fig. 3 Convergence of non-dimensional central deflection and moment of moderately thick (a/h = 10) symmetric, (0/90/0) laminated composite doubly curved panel (R/a = 10)

Table 1 Validation of non-dimensional central deflection (w_0^*) of symmetric (0/90/0) and anti-symmetric (0/90) laminated composite doubly curved panel for different R/a and a/h ratio

Lamination	a/h	R/a	Present	Reddy and Liu (1985)
0/90	10	10	18.91	18.74
		50	19.26	19.15
		100	19.26	19.16
	100	10	5.54	5.53
		50	15.72	15.71
		100	16.65	16.64
0/90/0	10	10	11.05	10.75
		50	11.01	10.89
		100	11.00	10.89
	100	10	3.64	3.64
		50	6.49	6.48
		100	6.65	6.64

Table 2 Validation of non-dimensional central moment ($\overline{M}_x$) of symmetric (0/90/0) and (0/90/90/0) laminated composite doubly curved panel (R/a = 10) for different a/h ratio

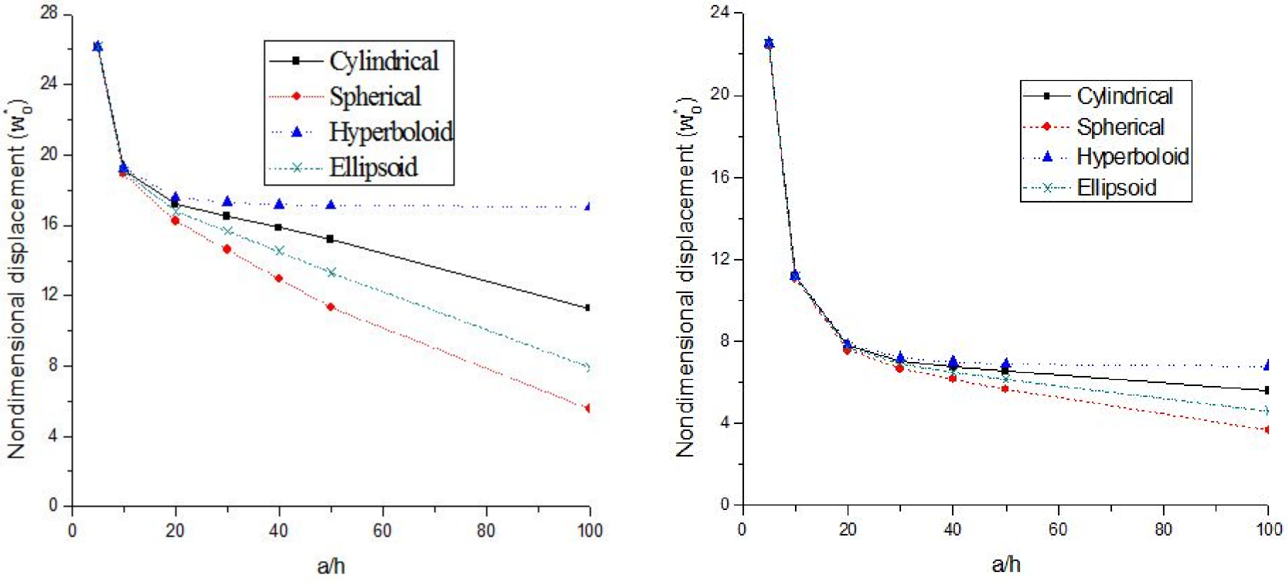
a/h	0/90/0		0/90/90/0	
	Present	Oktem and Chaudhuri (2009)	Present	Oktem and Chaudhuri (2009)
5	106.50	107.80	88.22	89.90
10	120.86	121.60	106.39	107.89
20	123.06	124.20	112.58	114.31
30	119.15	121.40	110.09	112.38
40	113.20	116.80	105.00	108.53
50	106.13	111.40	98.63	104.03

laminated composite panels. The non-dimensional transverse central deflection of hyperboloid is almost constant for the thin panels (a/h > 40) whereas for the ellipsoid, cylindrical and spherical panels, it continuously decreases with increase in the span to thickness ratio (a/h). Moreover, for the thick panels (a/h < 10) the non-dimensional central deflection is almost constant for ellipsoid, hyperboloid, cylindrical and spherical panels. Variation of non-dimensional central deflection of anti-symmetric (0/90) and symmetric (0/90/0) cross-ply laminated composite singly and doubly curved panels on square plan-form with R/a ratio are shown in Figs. 5(a)-(b), respectively. It is observed that R/a ratio has pronounced effect on non-dimensional central deflection of cylindrical and spherical deep panels (R/a < 20) and this effect is insignificant on shallow panels (R/a > 40). Hyperboloid has almost constant non-dimensional central deflection for all radii of curvature. It is also observed that R/a ratio is effective on anti-symmetric (0/90) panels than that on symmetric (0/90/0) curved panels.

Variation of non-dimensional central bending moments ($\overline{M}_x$) and ($\overline{M}_y$) with span to thickness ratio (a/h) of symmetric (0/90/0) cross-ply laminated composite singly and doubly (R/a = 10) curved panels is shown in Fig. 6(a)-(b), respectively. It is observed that non-dimensional central moments ($\overline{M}_x, \overline{M}_y$) of the hyperboloid is the highest and that of spherical panel is the lowest. The effect of a/h ratio on non-dimensional central bending moment of hyperboloid in thinner regime (a/h > 20) is negligible. It is clearly visible that the magnitude of non-dimensional central bending moment $\overline{M}_x$ is higher than that of $\overline{M}_y$.

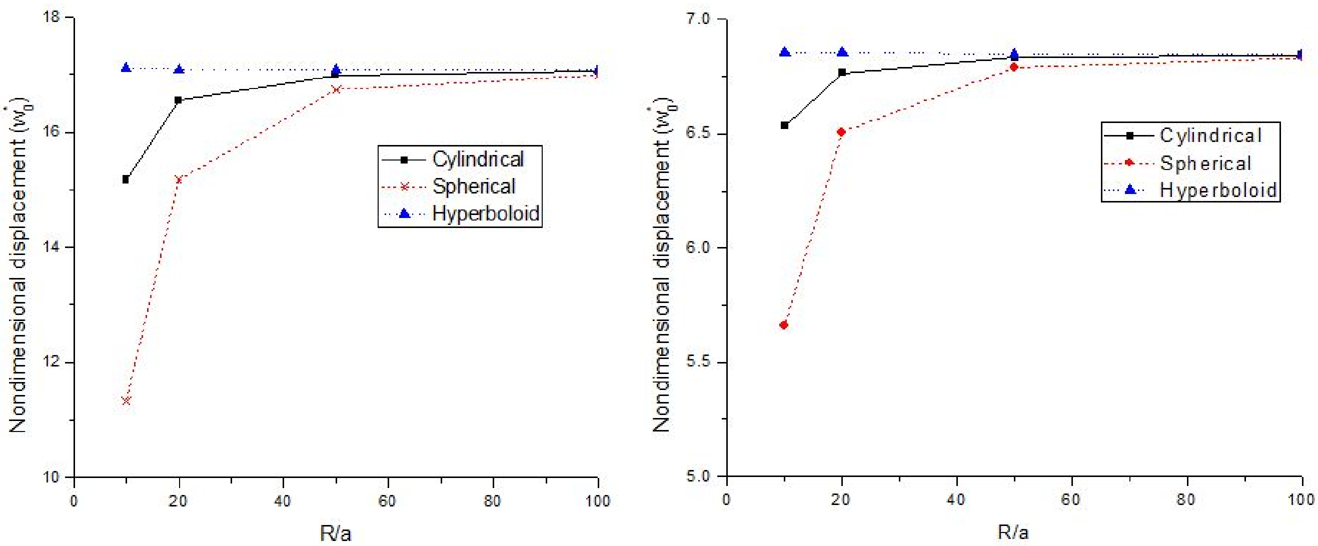
Functionally graded curved panels

Numerical results of functionally graded curved panels on square plan form are presented for simply supported boundary condition under uniformly distributed transverse load. The following material properties are considered for the analysis (Oktem *et al*. 2012)



(a) Anti-symmetric (0/90) (b) Symmetric (0/90/0)

Fig. 4 Variation of non-dimensional central deflection (w_0^*) of cross-ply laminated composite singly and doubly curved panels (R/a = 10) with a/h ratio

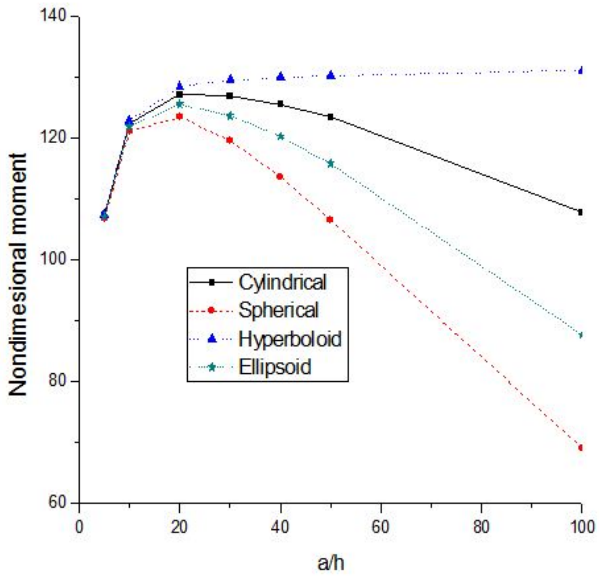


(a) Anti-symmetric (0/90) (b) Symmetric (0/90/0)

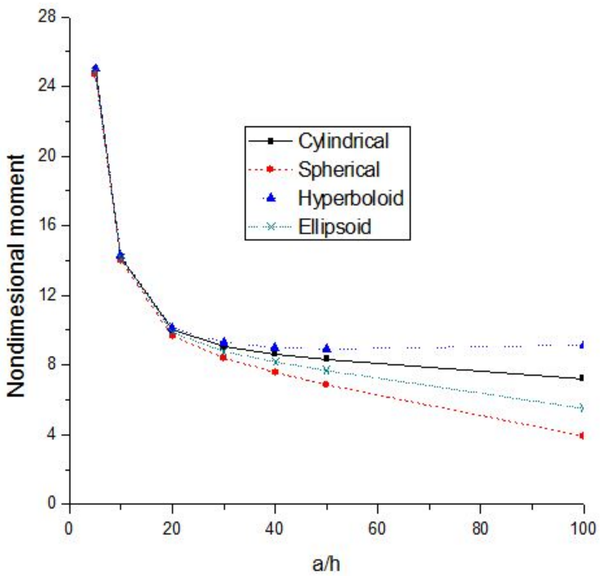
Fig. 5 Variation of non-dimensional central deflection (w_0^*) of thin (a/h = 50) anti-symmetric (0/90) and symmetric (0/90/0) cross-ply laminated composite curved panels with R/a ratio

Table 3 Convergence of non-dimensional central deflection of moderately thick (a/h = 10) and moderately deep (R/a = 10) functionally graded curved panels

M, *N*	*n* = 1		*n* = 2	
	Singly curved panel	Doubly curved panel	Singly curved panel	Doubly curved panel
6	4.0927E-03	3.8266E-03	4.1206E-03	3.8516E-03
7	4.0925E-03	3.8261E-03	4.1205E-03	3.8514E-03
8	4.0934E-03	3.8269E-03	4.1204E-03	3.8507E-03
9	4.0947E-03	3.8283E-03	4.1215E-03	3.8518E-03
10	4.0931E-03	3.8268E-03	4.1205E-03	3.8512E-03
11	4.0934E-03	3.8272E-03	4.1209E-03	3.8515E-03



(a) Bending moment at centre $(\overline{M}_x)$



(b) Bending moment at centre $(\overline{M}_y)$

Fig. 6 Variation of non-dimensional central bending moments $(\overline{M}_x, \overline{M}_y)$ of symmetric (0/90/0) cross-ply laminated composite singly and doubly curved panels (R/a = 10) with a/h ratio

Table 4 Validation of non-dimensional central deflection of functionally graded doubly curved panels (R/a = 10)

n	a/h = 10		a/h = 20		a/h = 50	
	Present	Oktem (2012)	Present	Oktem (2012)	Present	Oktem (2012)
0	4.002E-3	3.998E-3	3.261E-3	3.255E-3	1.537E-3	1.527E-3
0.5	3.849E-3	3.972E-3	3.042E-3	3.165E-3	1.407E-3	1.442E-3
1.0	3.828E-3	4.026E-3	3.002E-3	3.188E-3	1.394E-3	1.449E-3
2.0	3.851E-3	4.082E-3	3.021E-3	3.237E-3	1.426E-3	1.494E-3
Metal	4.002E-3	3.998E-3	3.261E-3	3.255E-3	1.537E-3	1.527E-3

E_c = 151 GPa, E_m = 70 GPa, $v_c = v_m$ = 0.3

The following normalized quantities are used for the present analysis

$$w_0^* = \frac{D_{22} h w_0}{q a^4}; \quad \overline{M}_x = \frac{D_{11} M_x}{q a^2}; \quad \overline{M}_y = \frac{D_{22} M_y}{q a^2}$$

In order to assess the accuracy of the present solution methodology convergence of non-dimensional transverse central deflection of a functionally graded singly and doubly curved panels (R/a = 10) on the square plan-form (a/b = 1) for different power index (n = 1, 2) are carried out and presented in Table 3. Central deflection shows good convergence at 9-10 terms expansion of the variables.

Variation of non-dimensional transverse central displace-ment of doubly curved panel for different a/h ratio and different power index (n = 0 to 2) is obtained and shown in Table 4. These results are compared with those due to Oktem *et al*. (2012) and found in good agreement.

An attempt has been made to study the influence of power index on non-dimensional central displacement and bending moment responses of functionally graded singly (cylindrical-'C') and doubly (spherical-'S', hyperboloid-'H' and ellipsoid-'E') curved panel. Variation of non-dimensional central displacement (w_0^*) and bending moment $(\overline{M}_x)$ of moderately deep (R/a = 10) and shallow (R/a=50) functionally graded singly and doubly curved panels is obtained for different power index (n) and presented in Tables 5-6, respectively. This analysis is carried out for thick and thin panels (a/h = 5 to 50). It is observed that non-dimensional transverse central displacement (w_0^*) and bending moment $(\overline{M}_x)$ decreases with increase in the a/h ratio. It is also observed that lager power index of functionally graded singly and doubly curved panel tends to lower down the bending moment.

Laminated composite sandwich panels

Numerical results for static analysis of laminated composite sandwich singly and doubly curved panels subjected to uniformly distributed load on the top face sheet with SS1 type simply supported boundary conditions are presented. Core thickness is considered to be 0.8h. Top and bottom face sheets are assumed to have the same thickness. The properties considered for the composite face sheet and the core are (Chakrabarti and Sheikh 2005)

Face – E_1/E_2 = 25, $G_{12} = G_{13}$ = 0.5 E_2,

G_{23} = 0.2 E_2, v_{12} = 0.25

Table 5 Variation of non-dimensional transverse central deflection (w_0^*) and central moment ($\overline{M}_x$) of moderately deep (R/a = 10) functionally graded singly and doubly curved panel

	n	Panel	a/h					
			5	10	20	30	40	50
w_0^*	0	H	0.004850	0.004112	0.003570	0.003044	0.002533	0.002082
		C	0.004868	0.004163	0.003730	0.003318	0.002887	0.002476
		E	0.004849	0.004107	0.003558	0.003025	0.002509	0.002058
		S	0.004812	0.004003	0.003261	0.002574	0.001990	0.001538
	0.5	H	0.004853	0.004132	0.003584	0.003046	0.002526	0.002069
		C	0.004813	0.004090	0.003596	0.003149	0.002711	0.002311
		E	0.004765	0.003991	0.003367	0.002811	0.002308	0.001886
		S	0.004701	0.003849	0.003043	0.002363	0.001818	0.001407
	1	H	0.004968	0.004173	0.003623	0.003087	0.002566	0.002107
		C	0.004852	0.004095	0.003576	0.003122	0.002686	0.002290
		E	0.004795	0.003982	0.003332	0.002772	0.002276	0.001863
		S	0.004722	0.003828	0.003002	0.002329	0.001795	0.001394
	2	H	0.005005	0.004213	0.003665	0.003140	0.002627	0.002170
		C	0.004935	0.004122	0.003592	0.003143	0.002714	0.002324
		E	0.004875	0.004006	0.003347	0.002794	0.002305	0.001895
		S	0.004799	0.003852	0.003021	0.002356	0.001828	0.001427
$\overline{M}_x$	0	H	0.009363	0.009101	0.008160	0.006944	0.005720	0.004633
		C	0.009399	0.009225	0.008565	0.007650	0.006649	0.005681
		E	0.009360	0.009091	0.008127	0.006890	0.005653	0.004562
		S	0.009286	0.008852	0.007431	0.005830	0.004440	0.003357
	0.5	H	0.007497	0.007291	0.006536	0.005557	0.004573	0.003701
		C	0.007424	0.007206	0.006559	0.005769	0.004961	0.004214
		E	0.007341	0.007008	0.006084	0.005054	0.004097	0.003288
		S	0.007234	0.006750	0.005481	0.004225	0.003197	0.002417
	1	H	0.006541	0.006664	0.005978	0.005090	0.004195	0.003401
		C	0.006749	0.006521	0.005883	0.005141	0.004403	0.003730
		E	0.006658	0.006316	0.005423	0.004474	0.003614	0.002895
		S	0.006547	0.006061	0.004863	0.003724	0.002808	0.002120
	2	H	0.006351	0.006183	0.005564	0.004762	0.003948	0.003218
		C	0.006245	0.006029	0.005430	0.004746	0.004070	0.003455
		E	0.006158	0.005835	0.005006	0.004136	0.003350	0.002692
		S	0.006052	0.005598	0.004492	0.003449	0.002609	0.001974

Core – $E_{1C} = E_{2C} = 0.04\ E_2$,

$G_{12C} = 0.016\ E_2$, $G_{23C} = G_{13C} = 0\ .06\ E_2$, $\nu_{12C} = 0.25$

The normalized quantities used in the present analysis are

$$w_0^* = \frac{100 E_2 h^4 w_0}{q a^4};\quad \overline{M}_x = \frac{10^3 D_{11} M_x}{q a^2};\quad \overline{M}_y = \frac{10^3 D_{22} M_y}{q a^2}$$

Convergence of non-dimensional transverse central deflection (w_0^*) and moments ($\overline{M}_x, \overline{M}_y$) of a moderately thick (a/h = 10) and moderately deep (R/a = 10) laminated composite (0/90/C/90/0) singly and doubly curved sandwich panels is shown in Table 7 and validation of non-dimensional transverse central deflection of flat sandwich panel is shown in the Table 8. Deflection and moment shows good convergence at 9-10 terms expansion of the variables. Present non-dimensional transverse central deflection (w_0^*) of flat panel is in good agreement with those due to Pagano (1970) and Chakrabarti and Sheikh (2005). Table 9 shows the variation of non-dimensional transverse central deflection and moments with span to thickness ratio (a/h) of laminated composite (0/90/C/90/0) curved sandwich panels (H- hyperboloid, C- cylindrical, S- spherical) on square plan form for different R/a ratio. From the results it is observed that non-dimensional transverse central deflection (w_0^*) and bending moment ($\overline{M}_x$) for spherical panel is the

Table 6 Variation of non-dimensional transverse central deflection (w_0^*) and central moment ($\overline{M}_x$) of shallow (R/a = 50) functionally graded singly and doubly curved panel

	n	Panel	a/h					
			5	10	20	30	40	50
w_0^*	0	H	0.004903	0.004268	0.004091	0.004031	0.003980	0.003923
		C	0.004904	0.004270	0.004099	0.004049	0.004010	0.003969
		E	0.004862	0.004147	0.003679	0.003229	0.002770	0.002342
		S	0.004902	0.004263	0.004073	0.003993	0.003914	0.003824
	0.5	H	0.004907	0.004293	0.004122	0.004063	0.004010	0.003952
		C	0.004896	0.004275	0.004092	0.004024	0.003968	0.003910
		E	0.004796	0.004056	0.003520	0.003036	0.002575	0.002165
		S	0.004882	0.004248	0.004028	0.003915	0.003804	0.003687
	1	H	0.004969	0.004332	0.004155	0.004095	0.004042	0.003984
		C	0.004953	0.004306	0.004110	0.004035	0.003972	0.003908
		E	0.004832	0.004055	0.003492	0.003002	0.002544	0.002141
		S	0.004934	0.004272	0.004033	0.003907	0.003786	0.003660
	2	H	0.005058	0.004366	0.004175	0.004113	0.004062	0.004006
		C	0.005041	0.004338	0.004125	0.004046	0.003981	0.003917
		E	0.004914	0.004080	0.003507	0.003022	0.002571	0.002173
		S	0.004984	0.004301	0.004045	0.003914	0.003792	0.003666
$\overline{M}_x$	0	H	0.009470	0.009463	0.009413	0.009335	0.009232	0.009105
		C	0.009471	0.009468	0.009433	0.009380	0.009310	0.009224
		E	0.009387	0.009180	0.008412	0.007371	0.006263	0.005223
		S	0.009467	0.009452	0.009371	0.009244	0.009075	0.008868
	0.5	H	0.007583	0.007579	0.007545	0.007487	0.007407	0.007306
		C	0.007563	0.007544	0.007484	0.007410	0.007322	0.007223
		E	0.007392	0.007127	0.006366	0.005471	0.004587	0.003795
		S	0.007538	0.007491	0.007359	0.007193	0.007000	0.006782
	1	H	0.006928	0.006922	0.006883	0.006826	0.006752	0.006659
		C	0.006903	0.006876	0.006799	0.006713	0.006618	0.006514
		E	0.006714	0.006439	0.005697	0.004866	0.004066	0.003359
		S	0.006873	0.006814	0.006660	0.006481	0.006282	0.006066
	2	H	0.006421	0.006412	0.006368	0.006313	0.006244	0.006160
		C	0.006395	0.006365	0.006282	0.006194	0.006100	0.006001
		E	0.006212	0.005951	0.005261	0.004499	0.003770	0.003124
		S	0.006557	0.006305	0.006148	0.005972	0.005782	0.005579

Table 7 Convergence of non-dimensional transverse central deflection (w_0^*) and moment ($\overline{M}_x, \overline{M}_y$) of moderately thick ($a/h$ = 10) singly and doubly curved (R/a=10) sandwich (0/90/C/90/0) panels

M, N	Singly curved panel			Doubly curved panel		
	w_0^*	$\overline{M}_x$	$\overline{M}_y$	w_0^*	$\overline{M}_x$	$\overline{M}_y$
6	2.5690	73.9894	65.1759	2.6427	76.2879	61.8826
7	2.5749	73.7996	64.7325	2.6494	76.1380	61.2936
8	2.6031	74.3983	65.4548	2.6832	76.8184	62.1383
9	2.6021	74.8071	65.7812	2.6821	77.2302	62.4850
10	2.5887	74.3455	65.4155	2.6668	76.7431	62.1035
11	2.5896	74.3775	65.3357	2.6678	76.7779	62.0146

Table 8 Validation non-dimensional transverse central deflection (w_0^*) of a moderately thick (a/h = 10) laminated composite (0/90/C/90/0) flat sandwich panel

Simply supported	Pagano (1970)	2.63
	Chakrabarti and Sheikh (2005)	2.62
	Present	2.55
Clamped	Chakrabarti and Sheikh (2005)	1.48
	Present	1.63

highest and that of hyperboloid is the lowest for thick to moderately thick (a/h < 20) curved sandwich panel. However, for thin (a/h > 20) sandwich panels, non-dimensional transverse central deflection (w_0^*) and bending moment ($\overline{M}_x$) of hyperboloid is the highest and that of spherical panel is the lowest.

5. Conclusions

Chebyshev series based analytical solution to the static analysis of laminated composites, functionally graded and sandwich singly and doubly curved panels on the rectangular plan-form, subjected to uniformly distributed transverse load is presented. The effect of span to thickness ratio, radii of curvature to span ratio, stacking sequence and power index on the deflection and moments are investigated. Following observations are made:

- In laminated composite curved panels, radius of curvature to span ratio (R/a) has significant effect on the flexural response of cylindrical and spherical panels and it is more pronounced for R/a < 40.
- Span to thickness ratio (a/h) has pronounced effect on the non-dimensional central deflection and bending moment both and it is more pronounced for thick shells (a/h < 20).

Table 9 Variation of non-dimensional transverse central deflection (w_0^*) and central moments ($\overline{M}_x, \overline{M}_y$) of laminated composite (0/90/C/90/0) singly and doubly curved sandwich panels

	R/a	Panel	a/h						
			5	10	20	30	40	50	100
w_0^*	10	H	5.954	2.553	1.694	1.534	1.478	1.452	1.417
		C	6.184	2.602	1.698	1.524	1.457	1.418	1.295
		S	6.547	2.682	1.699	1.494	1.398	1.329	1.029
	20	H	5.939	2.543	1.689	1.531	1.475	1.449	1.415
		C	6.064	2.572	1.694	1.530	1.471	1.441	1.382
		S	6.220	2.608	1.698	1.524	1.456	1.418	1.294
	50	H	5.948	2.544	1.688	1.530	1.474	1.449	1.414
		C	6.001	2.556	1.691	1.531	1.474	1.448	1.409
		S	6.058	2.570	1.694	1.531	1.472	1.444	1.393
$\overline{M}_x$	10	H	69.284	73.313	76.917	78.046	78.499	78.717	79.003
		C	72.159	74.807	77.090	77.518	77.316	76.801	71.805
		S	76.709	77.230	77.100	75.917	74.075	71.748	56.456
	20	H	71.964	74.839	77.437	78.257	78.583	78.735	78.895
		C	73.588	75.731	77.672	78.219	78.340	78.285	76.988
		S	75.616	76.847	77.863	77.899	77.540	76.949	71.859
	50	H	73.620	75.790	77.776	78.460	78.661	78.775	78.873
		C	74.320	76.185	77.908	78.450	78.649	78.720	78.567
		S	75.083	76.616	78.032	78.445	78.547	78.520	77.671
$\overline{M}_y$	10	H	70.331	67.672	65.427	64.741	64.455	64.307	64.086
		C	67.122	65.781	64.484	63.829	63.250	62.623	58.277
		S	61.936	62.485	62.466	61.505	59.992	58.079	45.542
	20	H	67.458	65.998	64.770	64.391	64.225	64.131	63.943
		C	65.717	64.989	64.340	64.072	63.873	63.676	62.392
		S	63.502	63.643	63.645	63.391	62.971	62.419	58.136
	50	H	65.715	64.992	64.388	64.198	64.107	64.049	63.897
		C	64.979	64.569	64.224	64.102	64.027	63.962	63.642
		S	64.168	64.092	64.017	63.949	63.857	63.742	62.894

- In FGM curved panels, both non-dimensional central deflection and bending moment are highest for the hyperboloid and lowest for the spherical panels.
- Non-dimensional central deflection and bending moment both decreases with increase in the a/h ratio.
- In the case of laminated composite sandwich curved panels, non-dimensional transverse central deflection and bending moment both are highest for spherical panel and lowest for hyperboloid panels, for thick to moderately thick (a/h < 20) curved sandwich panel.

References

Alankaya, V. and Oktem, A.S. (2016), "Static analysis of laminated and sandwich composite doubly-curved shallow shells", *Steel Compos. Struct., Int. J.*, **20**(5), 1043-1066.

Arefi, M. (2014), "Generalized shear deformation theory for thermo elastic analyses of the functionally graded cylindrical shells", *Struct. Eng. Mech., Int. J.*, **50**(3), 403-417.

Carrera, E., Brischetto, S., Cinefra, M. and Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos. Part B*, **42**(2), 123-133.

Chakrabarti, A. and Sheikh, A.H. (2005), "Analysis of laminated sandwich plates based on inter-laminar shear stress continuous plate theory", *J. Eng. Mech.*, **131**(4), 377-384.

Chaudhuri, R.A. and Abu-Arja, K.R. (1988), "Exact solution of shear-flexible doubly curved anti-symmetric angle-ply shells", *Int. J. Engng Sci.*, **26**(6), 587-604.

Chaudhuri, R.A. and Kabir, H.R.H. (1989), "On analytical solutions to boundary-value problems of doubly-curved moderately thick orthotropic shells", *Int. J. Eng. Sci.*, **27**(11), 1325-1336.

Chaudhuri, R.A. and Kabir, H.R.H. (1993), "Sensitivity of the response of moderately thick cross-ply doubly-curved panels to lamination and boundary constraint-II application", *Int. J. Solids Struct.*, **30**(2), 273-286.

Chaudhuri, R.A. and Kabir, H.R.H. (1994), "Static and dynamic Fourier analysis of finite cross-ply doubly curved panels using classical shallow shell theories", *Compos. Struct.*, **28**(1), 73-91.

Fard, K.M., Livani, M., Vaisi, A. and Gholami, M. (2014), "Improved high-order bending analysis of double curved sandwich panels subjected to multiple loading conditions", *Latin Am. J. Solid. Struct.*, **11**(9), 1591-1614.

Fox, L. and Parker, I.B. (1968), *Chebyshev Polynomials in Numerical Analysis*, Oxford University Press, London, UK.

Kant, T. and Fafard, M. (1992), "Transient response of isotropic, orthotropic and anisotropic composite sandwich shells with the super-parametric element", *Finite Elem. Anal. Design*, **12**(1), 63-73.

Kant, T. and Kommineni, J.R. (1992), "Geometrically non-linear analysis of doubly curved laminated and sandwich fibre reinforced composite shells with a higher order theory and C° finite elements", *J. Reinf. Plast. Compos.*, **11**(9), 1048-1076.

Khare, R.K., Rode, V., Garg, A.K. and John, S.P. (2005), "Higher-order closed-form solutions for thick laminated sandwich shells", *J. Sandw. Struct. Mater.*, **7**(4), 335-358.

Kiani, Y., Akbarzadeh, A.H., Chen, Z.T. and Eslami, M.R. (2012), "Static and dynamic analysis of an FGM doubly curved panel resting on the pasternak-type elastic foundation", *Compos. Struct.*, **94**(8), 2474-2484.

Nath, Y. and Shukla, K.K. (2001), "Non-linear transient analysis of moderately thick laminated composite plates", *J. Sound Vib.*, **247**(3), 509-526.

Oktem, A.S. and Chaudhuri, R.A. (2007), "Fourier analysis of thick cross-ply levy type clamped doubly-curved panels", *Compos. Struct.*, **80**(4), 489-503.

Oktem, A.S. and Chaudhuri, R.A. (2009), "Higher-order theory based boundary-discontinuous Fourier analysis of simply supported thick cross-ply doubly curved panels", *Compos. Struct.*, **89**(3), 448-458.

Oktem, A.S. and Soares, C.G. (2011a), "Boundary discontinuous Fourier solution for plates and doubly curved panels using a higher order theory", *Compos.: Part B*, **42**(4), 842-850.

Oktem, A.S. and Soares, C.G. (2011b), "Higher order theory based Fourier analysis of cross-ply plates and doubly curved panels". *J. Compos. Mater.*, **46**(21), 2675-2694.

Oktem, A.S., Mantari, J.L. and Soares, C.G. (2012), "Static response of functionally graded plates and doubly-curved shells based on a higher order shear deformation theory", *Eur. J. Mech. A/Solids*, **36**, 163-172.

Ossadzow, C, Mulier, P. and Touratier, M. (1995), "A general doubly curved laminate shell theory", *Compos. Struct.*, **32**(1-4), 299-312.

Pagano, N.J. (1970), "Exact solutions for rectangular bidirectional composite and sandwich plates", *J. Compos. Mater.*, **4**(1), 20-

Reddy, J.N. and Liu, C.F. (1985), "A higher-order shear deformation theory of laminated elastic shells", *Int. J. Eng. Sci.*, **23**(3), 319-330.

Sahoo, S.S., Panda S.K. and Mahapatra, T.R. (2016), "Static, free vibration and transient response of laminated composite curved shallow panel - An experimental approach", *Eur. J. Mech. A/Solids*, **59**, 95-113.

To, C.W.S. and Liu, M.L. (2001), "Geometrically nonlinear analysis of layerwise anisotropic shell structures by hybrid strain based lower order elements", *Finite Eleme. Anal. Des.*, **37**(1), 1-34.

BU