Steel and Composite Structures, *Vol. 24, No. 5 (2017) 569-578*
DOI: https://doi.org/10.12989/scs.2017.24.5.569

A new shear deformation plate theory with stretching effect for buckling analysis of functionally graded sandwich plates

S.R. Mahmoud [*1] and Abdelouahed Tounsi [2]

[1] *Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia*
[2] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*

(*Received August 21, 2016, Revised March 26, 2017, Accepted May 16, 2017*)

Abstract. In this research work, a simple and accurate hyperbolic plate theory for the buckling analysis of functionally graded sandwich plates is presented. The main interest of this theory is that, in addition to incorporating the thickness stretching effect ($\varepsilon_z \neq 0$), the displacement field is composed only of 5 unknowns as the first order shear deformation theory (FSDT), instead of 6 like in the well-known "*higher order shear and normal deformation theories*". Thus, the number of unknowns and governing equations for the present theory is reduced, significantly facilitating engineering analysis. Governing equations are obtained by employing the principle of minimum total potential energy. Comparison studies are performed to verify the validity of present results. A numerical investigation has been conducted considering and neglecting the thickness stretching effects on the buckling of sandwich plates with functionally graded skins. It can be concluded that the present theory is not only accurate but also simple in predicting the buckling response of sandwich plates with functionally graded skins.

Keywords: plate; computational modelling; buckling; functionally graded materials; stretching effect

1. Introduction

Functionally graded materials (FGMs) are generally metal-matrix composites (MMCs) that have a continuous variation of material properties from one surface to another. The ceramic constituent provides high-temperature resistance due to its low thermal conductivity. The ductile metal constituent on the other hand, prevents fracture caused by stresses due to high temperature gradient in a very short span of time. The concepts of FGMs were introduced by the Japanese Yamanouchi *et al*. (1990) and Koizumi (1993), and are used as thermal barrier materials for wide engineering applications such as space planes, space structures and nuclear reactors. The progress of FGM use in various engineering industries requires accurate models to predict their behaviours (Zidi *et al*. 2014, Kar and Panda 2015, Khelifa *et al*. 2015, Hadji *et al*. 2015, Arefi 2015a, b, Arefi and Allam 2015, Atmane *et al*. 2015, Al-Basyouni *et al*. 2015, Meradjah *et al*. 2015, Saidi *et al*. 2016, Bousahla *et al*. 2016, El-Hassar *et al*. 2016, Ebrahimi and Shafiei 2016, Hadji *et al*. 2016c). A critical review of more recent studies on the bending, dynamic and buckling investigation of functionally graded (FG) plates can be found in the work of Jha *et al*. (2013). Due to the transverse shear deformation effects that are more significant in thick plates or plates made of advanced composites like FGMs, shear deformation models that consider for shear deforma-tion effects are often employed to investigate the behaviours of FG plates. The first-order shear deformation theory (Mindlin 1951, Reissner 1945, Meksi *et al*. 2015, Bellifa *et al*. 2016, Hadji *et al*. 2016a, b, d, Bouderba *et al*. 2016) takes into consideration the shear deformation effects, but do not satisfy the equilibrium conditions at the top and bottom surfaces of the plate. A shear correction factor is therefore needed. To avoid the use of this parameter, many higher-order shear deformation theories (HSDTs) were proposed based on the assumption of quadratic, cubic or higher order distribution of in-plane displacements within the plate thickness, notable among them are Reddy (2000), Matsunaga (2008), Pradyumna and Bandyopadhyay (2008), Atmane *et al*. (2010), Benachour *et al*. (2011), Shahrjerdi *et al*. (2011), Fekrar *et al*. (2012), Bouderba *et al*. (2013), Meziane *et al*. (2014), Sallai *et al*. (2015), Akavci *et al*. (2015), Ait Yahia *et al*. (2015), Hassaine Daouadji and Hadji (2015), Mahi *et al*. (2015), Attia *et al*. (2015), Belkorissat *et al*. (2015), Laoufi *et al*. (2016), Benferhat *et al*. (2016), Bourada *et al*. (2016), Hadji *et al*. (2016e), Beldjelili *et al*. (2016), Boukhari *et al*. (2016), Eltaher *et al*. (2016), Bounouara *et al*. (2016), Houari *et al*. (2016), Chikh *et al*. (2016), Fahsi *et al*. (2017), Meksi *et al*. (2017) and Chikh *et al*. (2017). Most of these theories neglect the thickness stretching effect (i.e., $\varepsilon_z = 0$) due to considering a constant transverse displacement within the thickness direction. This assumption is suitable for thin or moderately thick FG plates, but is inadequate for thick FG plates (Qian *et al*. 2004). The interesting feature of the thickness stretching effect in FG plates has been proved in the study of Carrera *et al*. (2011). This effect has an important role in moderately thick and thick FG plates and should be taken into account (Hebali *et al*. 2014, Fekrar *et al*. 2014, Bousahla *et al*. 2014, Belabed *et al*. 2014, Hamidi *et al*. 2015, Larbi Chaht *et al*. 2015, Bourada *et al*. 2015, Draiche

*Corresponding author, Professor,
E-mail: srhassan@kau.edu.sa

http://www.techno-press.org/?journal=scs&subpage=6
ISSN: 1229-9367 (Print), 1598-6233 (Online)

et al. 2016, Bennoun *et al.* 2016, Ait Atmane *et al.* 2017, Benahmed *et al.* 2017, Bouafia *et al.* 2017).

Quasi-3D models are HSDTs in which the transverse displacement is expressed as a higher-order variation within the thickness of the plate, and consequently, thickness stretching effect is included. Swaminathan and Naveenkumar (2014) presented higher order refined computational models for the stability analysis of FG plates. There are many quasi-3D theories used in the scientific literature. Reddy (2011) developed quasi-3D models based on a cubic variation of axial displacements and a quadratic variation of transverse displacement. Recently, Neves *et al.* (2012) provided a hyperbolic shear deformation theory including the thickness stretching effect ($\varepsilon_z \neq 0$) for the buckling response of FG plates. It should be outlined that the abovementioned quasi-3D models are too cumbersome and computationally expensive since they use many variables (e.g., theories by Reddy (2011) with eleven parameters, and Neves *et al.* (2012) with nine parameters). Although some well-known quasi-3D models constructed by Zenkour (2007) and recently by Mantari and Guedes Soares (2012) have six unknowns, they are still more complicated than the FSDT. Thus, constructing a simple and easy quasi-3D theory is necessary.

This investigation aims to construct a simple quasi-3D hyperbolic shear deformation theory and extremely easy to implement for the buckling analysis of sandwich plates with functionally graded skins. Contrary to the well-known four-variable refined theories elaborated in (Benachour *et al.* 2011, Fekrar *et al.* 2012, Bouderba *et al.* 2013), where the stretching effect is neglected, in the present work, the proposed theory is enhanced via this so-called "stretching effect". By modeling the transverse displacement as a sum of three components namely: the bending, shear and thickness stretching parts, the number of variables of the present theory is reduced, and thus saving computational time.Governing equations obtained from the principle of minimum total potential energy are analytically solved for buckling problem of a simply supported sandwich plate. Numerical examples are presented to demonstrate and highlight the accuracy of the present theory.

2. Problem formulation

In this work, a rectangular sandwich plate of length a, width b and thickness h is considered. The coordinate system is chosen such that the x-y plane coincides with the mid-plane of the plate

$$(z \in [-h/2, h/2]).$$

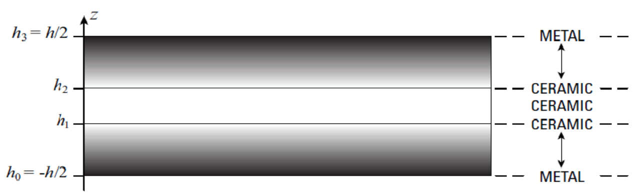

Fig. 1 Sandwich with isotropic core and FGM skins

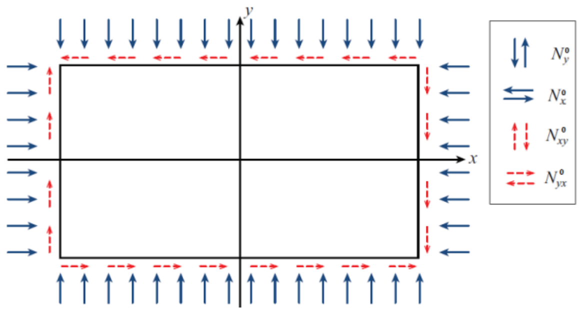

Fig. 2 Rectangular plate subjected to in-plane forces

The core of the sandwich plate is made of a ceramic material and skins are consisting of FGM within the thickness direction. In the lower skin, a mixture of ceramics and metals is changing from pure metal ($z = h_0 = -h/2$) to pure ceramic while the top skin face changes continuously from pure ceramic surface to pure metal surface ($z = h_3 = h/2$) as shown in Fig. 1. A simple power law in terms of the volume fraction of the ceramic phase is considered

$$V^{(1)} = \left(\frac{z-h_0}{h_1-h_0}\right)^k, \qquad z \in [h_0, h_1] \tag{1a}$$

$$V^{(2)} = 1, \qquad z \in [h_1, h_2] \tag{1b}$$

$$V^{(3)} = \left(\frac{z-h_3}{h_2-h_3}\right)^k, \qquad z \in [h_2, h_3] \tag{1c}$$

where $V^{(n)}$, (n = 1, 2, 3) represents the volume fraction function of layer n; k is the volume fraction index ($0 \leq k \leq +\infty$), which control the material distribution in the thickness direction.

The effective material properties, like Young's modulus E, and Poisson's ratio ν, can be mathematically expressed by the rule of mixture (Bessaim *et al.* 2013, Tounsi *et al.* 2013, 2016, Taibi *et al.* 2015, Abdelhak *et al.* 2016) as

$$P^{(n)}(z) = P_2 + (P_1 - P_2)V^{(n)} \tag{2}$$

where $P^{(n)}$ is the effective material property of FGM of layer n. P_1 and P_2 are the properties of the top and bottom faces of layer 1, respectively, and vice versa for layer 3 depending on the volume fraction $V^{(n)}$, (n = 1, 2, 3).

The sandwich plate loaded by a compressive in-plane forces acting on the mid-plane of the plate. N_x^0 and N_y^0 indicate the in-plane loads perpendicular to the edges x = 0 and y = 0 respectively, and N_{xy}^0 indicate the distributed shear force parallel to the edges x = 0 and y = 0 respectively (see Fig. 2).

3. A quasi-3D hyperbolic shear deformation theory plate theory

This section aims to derive the governing equations of the present hyperbolic shear deformation plate theory leading to the eigenvalue problem for the investigation of buckling plates.

3.1 Kinematics

The displacement field of the present theory is formulated based on the following hypotheses: (1) The transverse deflection is superposed into three parts namely: bending, shear and stretching components; (2) the in-plane displacements are superposed also into three parts namely: extension ,bending and shear components; (3) the bending components of the in-plane displacements are identical to those used in the classical plate theory (CPT); and (4) the shear parts of the in-plane displacements lead to the hyperbolic variations of shear strains as well as the shear stresses across the thickness of the plate in such a way that the shear stresses becomes zero on the top and bottom surfaces of the plate. Based on these assumptions, the following displacement field relations can be obtained

$$\begin{aligned}
u(x,y,z,t) &= u_0(x,y,t) - z\frac{\partial w_b}{\partial x} - f(z)\frac{\partial w_s}{\partial x} \\
v(x,y,z,t) &= v_0(x,y,t) - z\frac{\partial w_b}{\partial y} - f(z)\frac{\partial w_s}{\partial y} \\
w(x,y,z,t) &= w_b(x,y,t) + w_s(x,y,t) + g(z)\,\varphi(x,y,t)
\end{aligned} \tag{3}$$

where u_0 and v_0 denote the displacements along the x and y coordinate directions of a point on the mid-plane of the plate; w_b and w_s are the bending and shear components of the transverse deflection, respectively; and the additional displacement φ accounts for the effect of normal stress (stretching effect). The shape functions $f(z)$ and $g(z)$ are given as follows

$$f(z) = \frac{1}{[\cosh(\pi/2) - 1]}\left(\frac{h}{\pi}\sinh\left(\frac{\pi}{h}z\right) - z\right) \tag{4}$$

And

$$g(z) = 1 - f'(z) \tag{5}$$

3.2 Strains

For the displacement field in Eq. (3), the strain components become

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z)\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}$$

$$\begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z)\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \quad \varepsilon_z = g'(z)\,\varepsilon_z^0 \tag{6}$$

Where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix},$$

$$\begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \tag{7}$$

$$\begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} + \dfrac{\partial \varphi}{\partial y} \\ \dfrac{\partial w_s}{\partial x} + \dfrac{\partial \varphi}{\partial x} \end{Bmatrix}, \quad \varepsilon_z^0 = \varphi$$

and

$$g'(z) = \frac{dg(z)}{dz} \tag{8}$$

3.3 Elastic stress-strain relations

In the case of isotropic FG materials, the 3D constitutive equations can be written as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{22} & C_{23} & 0 & 0 & 0 \\ C_{13} & C_{23} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ \gamma_{yz} \\ \gamma_{xz} \\ \gamma_{xy} \end{Bmatrix} \tag{9}$$

where (σ_x, σ_y, σ_z, τ_{yz}, τ_{xz}, τ_{xy}) and (ε_x, ε_y, ε_z, γ_{yz}, γ_{xz}, γ_{xy}) are the stress and strain components, respectively.

The calculation of the elastic constants C_{ij} depends on which assumption of ε_z we consider. If $\varepsilon_z = 0$, then C_{ij} are the plane stress reduced elastic constants, defined as:

$$C_{11} = C_{22} = \frac{E(z)}{1-\nu^2}, \quad C_{12} = \nu\, C_{11} \tag{10a}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \frac{E(z)}{2(1+\nu)}, \tag{10b}$$

If $\varepsilon_z \neq 0$ (thickness stretching), then C_{ij} are the three-dimensional elastic constants, given by:

$$\begin{aligned} C_{11} = C_{22} = C_{33} &= \frac{(1-\nu)}{\nu}\lambda(z), \\ C_{12} = C_{13} = C_{23} &= \lambda(z) \end{aligned} \tag{11a}$$

$$C_{44} = C_{55} = C_{66} = G(z) = \mu(z) = \frac{E(z)}{2(1+\nu)}, \tag{11b}$$

where $\lambda(z) = \dfrac{\nu\, E(z)}{(1-2\nu)(1+\nu)}$ and $\mu(z) = G(z) = \dfrac{E(z)}{2(1+\nu)}$ are Lamé's coefficients. The moduli E, G and the elastic coefficients C_{ij} vary through the thickness according to Eq. (2).

3.4 Governing equations

The governing equations appropriate for the displacement field Eq. (3) and constitutive Eq. (9) are derived from the principle of minimum total potential energy. It states that

$$\delta U + \delta V = 0 \tag{12}$$

Where δU is the variation of strain energy; δV is the variation of work done by applied forces.

The variation of strain energy of the plate is expressed by

$$\begin{aligned}\delta U &= \int_{-h/2}^{h/2}\int_A \big[\sigma_x \delta\,\varepsilon_x + \sigma_y \delta\,\varepsilon_y + \sigma_z \delta\,\varepsilon_z + \tau_{xy}\delta\,\gamma_{xy} \\ &\quad + \tau_{yz}\delta\,\gamma_{yz} + \tau_{xz}\delta\,\gamma_{xz}\big]\mathrm{dAdz} \\ &= \int_A \big[N_x \delta\,\varepsilon_x^0 + N_y \delta\,\varepsilon_y^0 + N_z \delta\,\varepsilon_z^0 + N_{xy}\delta\,\gamma_{xy}^0 \\ &\quad + M_x^b \delta\,k_x^b + M_y^b \delta\,k_y^b + M_{xy}^b \delta\,k_{xy}^b \\ &\quad + M_x^s \delta\,k_x^s + M_y^s \delta\,k_y^s + M_{xy}^s \delta\,k_{xy}^s \\ &\quad + S_{yz}^s \delta\,\gamma_{yz} + S_{xz}^s \delta\,\gamma_{xz}\big]\,dA = 0\end{aligned} \tag{13}$$

where A is the top surface and the stress resultants N, M, and S are defined by

$$\begin{Bmatrix} N_x,\ N_y,\ N_{xy} \\ M_x^b,\ M_y^b,\ M_{xy}^b \\ M_x^s,\ M_y^s,\ M_{xy}^s \end{Bmatrix} = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \left(\sigma_x, \sigma_y, \tau_{xy}\right)\begin{Bmatrix} 1 \\ z \\ f(z) \end{Bmatrix} dz, \tag{14a}$$

$$N_z = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \sigma_z g'(z)\,dz, \tag{14b}$$

$$\left(S_{xz}^s, S_{yz}^s\right) = \sum_{n=1}^{3}\int_{h_{n-1}}^{h_n} \left(\tau_{xz}, \tau_{yz}\right) g(z)\,dz. \tag{14c}$$

The external virtual work due to external loads applied to the plate is given as:

$$\begin{aligned}\delta V = -\int_A \Bigg[& N_x^0 \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial x} + N_{xy}^0 \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial x} + \\ & N_{yx}^0 \frac{\partial w}{\partial x}\frac{\partial \delta w}{\partial y} + N_y^0 \frac{\partial w}{\partial y}\frac{\partial \delta w}{\partial y}\Bigg] dA\end{aligned} \tag{15}$$

being N_x^0 and N_y^0 the in-plane loads perpendicular to the edges $x = 0$ and $y = 0$, respectively, and N_{xy}^0 and N_{yx}^0 the distributed shear forces parallel to the edges $x = 0$ and $y = 0$, respectively.

Substituting the expressions for δU and δV from Eqs.(13)and (15) into Eq. (12) and integrating by parts, and collecting the coefficients of δu_0, δv_0, δw_b, δw_s and $\delta\varphi$, the following governing equations are obtained

$$\begin{aligned}
\delta\,u_0 &: \quad \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
\delta\,v_0 &: \quad \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
\delta\,w_b &: \quad \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} - \overline{N} = 0 \\
\delta\,w_s &: \quad \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - \overline{N} = 0 \\
\delta\,\phi &: \quad \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} - N_z - \overline{N} = 0
\end{aligned} \tag{16}$$

with

$$\overline{N} = \left[N_x^0 \frac{\partial^2 w}{\partial x^2} + 2N_{xy}^0 \frac{\partial^2 w}{\partial x \partial y} + N_y^0 \frac{\partial^2 w}{\partial y^2} \right] \tag{17}$$

By substituting Eq. (6) into Eq. (9) and the subsequent results into Eq. (14), the stress resultants are obtained as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix} = \begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix} + \begin{bmatrix} L \\ L^a \\ R \end{bmatrix}\varepsilon_z^0, \qquad S = A^s\gamma, \tag{18a}$$

$$\begin{aligned}N_z &= R^a\phi + L\left(\varepsilon_x^0 + \varepsilon_y^0\right) + L^a\left(k_x^b + k_y^b\right) \\ &\quad + R\left(k_x^s + k_y^s\right),\end{aligned} \tag{18b}$$

where

$$N = \left\{N_x, N_y, N_{xy}\right\}, \qquad M^b = \left\{M_x^b, M_y^b, M_{xy}^b\right\}, \qquad M^s = \left\{M_x^s, M_y^s, M_{xy}^s\right\}, \tag{19a}$$

$$\varepsilon = \left\{\varepsilon_x^0, \varepsilon_y^0, \gamma_{xy}^0\right\}, \qquad k^b = \left\{k_x^b, k_y^b, k_{xy}^b\right\}, \qquad k^s = \left\{k_x^s, k_y^s, k_{xy}^s\right\}, \tag{19b}$$

$$A = \begin{bmatrix} A_{11} & A_{12} & 0 \\ A_{12} & A_{22} & 0 \\ 0 & 0 & A_{66} \end{bmatrix}, \quad B = \begin{bmatrix} B_{11} & B_{12} & 0 \\ B_{12} & B_{22} & 0 \\ 0 & 0 & B_{66} \end{bmatrix}, \quad D = \begin{bmatrix} D_{11} & D_{12} & 0 \\ D_{12} & D_{22} & 0 \\ 0 & 0 & D_{66} \end{bmatrix}, \tag{19c}$$

$$B^s = \begin{bmatrix} B_{11}^s & B_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & B_{66}^s \end{bmatrix}, \quad D^s = \begin{bmatrix} D_{11}^s & D_{12}^s & 0 \\ D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & D_{66}^s \end{bmatrix}, \quad H^s = \begin{bmatrix} H_{11}^s & H_{12}^s & 0 \\ H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & H_{66}^s \end{bmatrix}, \tag{19d}$$

$$S = \left\{S_{xz}^s, S_{yz}^s\right\}, \qquad \gamma = \left\{\gamma_{xz}, \gamma_{yz}\right\}, \qquad A^s = \begin{bmatrix} A_{44}^s & 0 \\ 0 & A_{55}^s \end{bmatrix}, \tag{19e}$$

$$\begin{Bmatrix} L \\ L^a \\ R \\ R^a \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \lambda(z) \begin{Bmatrix} 1 \\ z \\ f(z) \\ g'(z)\dfrac{1-\nu}{\nu} \end{Bmatrix} g'(z) dz \tag{19e}$$

Here the stiffness coefficients A_{ij} and B_{ij},… etc., are defined as

$$\begin{Bmatrix} A_{11} & B_{11} & D_{11} & B_{11}^s & D_{11}^s & H_{11}^s \\ A_{12} & B_{12} & D_{12} & B_{12}^s & D_{12}^s & H_{12}^s \\ A_{66} & B_{66} & D_{66} & B_{66}^s & D_{66}^s & H_{66}^s \end{Bmatrix} = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \lambda(z)\left(1, z, z^2, f(z), z\, f(z), f^2(z)\right) \begin{Bmatrix} \dfrac{1-\nu}{\nu} \\ 1 \\ \dfrac{1-2\ \nu}{2\ \nu} \end{Bmatrix} dz \tag{20a}$$

and

$$\left(A_{22}, B_{22}, D_{22}, B_{22}^s, D_{22}^s, H_{22}^s\right) = \left(A_{11}, B_{11}, D_{11}, B_{11}^s, D_{11}^s, H_{11}^s\right), \tag{20b}$$

$$A_{44}^s = A_{55}^s = \sum_{n=1}^{3} \int_{h_{n-1}}^{h_n} \mu(z)\left[g(z)\right]^2 dz, \tag{20c}$$

3.4 Governing equations in terms of displacements

Introducing Eq. (18) into Eq. (16), the governing equations can be expressed in terms of displacements (δu_0, δv_0, δw_b, δw_s, $\delta\varphi$) and the appropriate equations take the form

$$\begin{aligned} &A_{11} d_{11} u_0 + A_{66} d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 - \\ &B_{11} d_{111} w_b - \left(B_{12} + 2B_{66}\right) d_{122} w_b - \left(B_{12}^s + 2B_{66}^s\right) d_{122} w_s \\ &-B_{11}^s d_{111} w_s + L d_1 \phi = 0 \end{aligned} \tag{21a}$$

$$\begin{aligned} &A_{22} d_{22} v_0 + A_{66} d_{11} v_0 + \left(A_{12} + A_{66}\right) d_{12} u_0 - \\ &B_{22} d_{222} w_b - \left(B_{12} + 2B_{66}\right) d_{112} w_b - \left(B_{12}^s + 2B_{66}^s\right) d_{112} w_s \\ &-B_{22}^s d_{222} w_s + L d_2 \phi = 0 \end{aligned} \tag{21b}$$

$$\begin{aligned} &B_{11} d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0 + \left(B_{12} + 2B_{66}\right) d_{112} v_0 \\ &+B_{22} d_{222} v_0 - D_{11} d_{1111} w_b - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_b \\ &-D_{22} d_{2222} w_b - D_{11}^s d_{1111} w_s - 2\left(D_{12}^s + 2D_{66}^s\right) d_{1122} w_s \\ &-D_{22}^s d_{2222} w_s + L^a \left(d_{11}\phi + d_{22}\phi\right) - \overline{N} = 0 \end{aligned} \tag{21c}$$

$$\begin{aligned} &B_{11}^s d_{111} u_0 + \left(B_{12}^s + 2B_{66}^s\right) d_{122} u_0 + \left(B_{12}^s + 2B_{66}^s\right) d_{112} v_0 \\ &+B_{22}^s d_{222} v_0 - D_{11}^s d_{1111} w_b - 2\left(D_{12}^s + 2D_{66}^s\right) d_{1122} w_b \\ &-D_{22}^s d_{2222} w_b - H_{11}^s d_{1111} w_s - 2\left(H_{12}^s + 2H_{66}^s\right) d_{1122} w_s \\ &-H_{22}^s d_{2222} w_s + A_{44}^s d_{11} w_s + A_{55}^s d_{22} w_s + R\left(d_{11}\phi + d_{22}\phi\right) \\ &+ A_{44}^s d_{11}\phi + A_{55}^s d_{22}\phi - \overline{N} = 0 \end{aligned} \tag{21d}$$

$$\begin{aligned} &L\left(d_1 u_0 + d_2 v_0\right) - L^a\left(d_{11} w_b + d_{22} w_b\right) + \\ &\left(R - A_{44}^s\right) d_{11} w_s + \left(R - A_{55}^s\right) d_{22} w_s + R^a \phi \\ &-A_{44}^s d_{11}\phi - A_{55}^s d_{22}\phi - \overline{N} = 0 \end{aligned} \tag{21e}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1, 2). \tag{22}$$

4. Analytical solutions

The Navier solution procedure is employed to obtain the analytical solutions for a simply supported sandwich plate. The solution is assumed to be of the form

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \\ \varphi \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\lambda\, x)\sin(\mu\, y) \\ V_{mn}\sin(\lambda\, x)\cos(\mu\, y) \\ W_{bmn}\sin(\lambda\, x)\sin(\mu\, y) \\ W_{smn}\sin(\lambda\, x)\sin(\mu\, y) \\ \Phi_{mn}\sin(\lambda\, x)\sin(\mu\, y) \end{Bmatrix} \tag{23}$$

where U_{mn}, V_{mn}, W_{bmn}, W_{smn} and Φ_{mn} are arbitrary coefficients to be determined, and $\lambda = m\pi / a$ and $\mu = n\pi / b$.

By substituting Eq. (23) into Eqs. (21) we obtain some results that concern the buckling of FG sandwich plates subjected to a system of uniform in-plane compressive loads N_x^0 and N_y^0 ($N_{xy}^0 = 0$).

Assuming that there is a given ratio between these forces such that $N_x^0 = -N_0$ and $N_y^0 = -\gamma\, N_0$; $\gamma = N_y^0 / N_x^0$ (here γ is non-dimensional load parameter), we get

$$\left([K]\right)\{\Delta\} = \{0\} \tag{24}$$

where $\{\Delta\}$ denotes the column

$$\{\Delta\}^T = \{U_{mn}, V_{mn}, W_{bmn}, W_{smn}, \Phi_{mn}\}, \tag{25}$$

and

$$[K] = \begin{bmatrix} s_{11} & s_{12} & s_{13} & s_{14} & s_{15} \\ s_{12} & s_{22} & s_{23} & s_{24} & s_{25} \\ s_{13} & s_{23} & s_{33} + \alpha & s_{34} + \alpha & s_{35} + \alpha \\ s_{14} & s_{24} & s_{34} + \alpha & s_{44} + \alpha & s_{45} + \alpha \\ s_{15} & s_{25} & s_{35} + \alpha & s_{45} + \alpha & s_{55} + \alpha \end{bmatrix} \tag{26}$$

in which

$$
\begin{aligned}
s_{11} &= -\left(A_{11}\lambda^2 + A_{66}\mu^2\right)\\
s_{12} &= -\lambda\,\mu\left(A_{12} + A_{66}\right)\\
s_{13} &= \lambda\,[B_{11}\lambda^2 + (B_{12} + 2B_{66})\,\mu^2]\\
s_{14} &= \lambda\,[B_{11}^s\lambda^2 + (B_{12}^s + 2B_{66}^s)\,\mu^2]\\
s_{15} &= L\lambda\\
s_{22} &= -\left(A_{66}\lambda^2 + A_{22}\mu^2\right)\\
s_{23} &= \mu\,[(B_{12} + 2B_{66})\,\lambda^2 + B_{22}\mu^2]\\
s_{24} &= \mu\,[(B_{12}^s + 2B_{66}^s)\,\lambda^2 + B_{22}^s\mu^2]\\
s_{25} &= L\mu\\
s_{33} &= -\left(D_{11}\lambda^4 + 2(D_{12} + 2D_{66})\lambda^2\mu^2 + D_{22}\mu^4\right)\\
s_{34} &= -\left(D_{11}^s\lambda^4 + 2(D_{12}^s + 2D_{66}^s)\lambda^2\,\mu^2 + D_{22}^s\,\mu^4\right)\\
s_{35} &= -L^a\left(\lambda^2 + \mu^2\right)\\
s_{44} &= -\Big(H_{11}^s\lambda^4 + 2(H_{11}^s + 2H_{66}^s)\lambda^2\mu^2\\
&\qquad + H_{22}^s\mu^4 + A_{55}^s\lambda^2 + A_{44}^s\mu^2\Big)\\
s_{45} &= -\left[A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R\left(\lambda^2 + \mu^2\right)\right]\\
s_{55} &= -\left(A_{44}^s\lambda^2 + A_{55}^s\mu^2 + R^a\right)\\
\alpha &= N_0\left(\lambda^2 + \gamma\mu^2\right)
\end{aligned}
\tag{27}
$$

The critical buckling loads (N_{cr}) can be obtained from the stability problem $|K| = 0$.

5. Numerical results and discussions

In this section, various numerical examples solved are described and discussed for establishing the efficiency and the accuracy of the present theory for the buckling analysis of FGM sandwich plates. For all the problems a simply supported (diaphragm supported) plate is considered for the analysis. The core material of the present sandwich plate is fully ceramic. The bottom skin varies from a metal-rich surface to a ceramic-rich surface while the top skin face varies from a ceramic-rich surface to a metal-rich surface. The material properties are $E_m = 70E_0$ (aluminum) and $E_c = 380E_0$ (alumina) being $E_0 = 1$ GPa. Poisson's ratio is $v_m = v_c = v = 0.3$ for both aluminum and alumina. The non-dimensional parameter used is

$$
\overline{N}_{cr} = \frac{N_{cr}a^2}{100h^2E_0} \tag{28}
$$

The following four layer configurations are used for multi-layered FGM plates

(i) 1-2-1 configuration in which thickness of the core is twice the thickness of face sheets.
(ii) 1-1-1 configuration in which thickness of the core is same as the thickness of face sheets.
(iii) 2-1-2 configuration in which thickness of the core is half the thickness of face sheets.
(iv) 1-0-1 configuration in which is made of two layers of equal thickness without a core.

Tables 1 and 2 respectively list the non dimensionalized values of uniaxial and biaxial critical buckling loads in an FGM sandwich plate for various values of power law parameter and thickness of the core with respect to face sheets. The obtained results are compared with the quasi-3D hyperbolic sine shear deformation theory (Neves *et al.* 2012). In addition, the results of a third-order shear deformation plate theory (TSDPT) (Zenkour 2005) and a sinusoidal shear deformation plate theory (SSDPT) (Zenkour 2005) are also provided to show the importance of including the thickness-stretching effect. The TSDPT solution (Zenkour 2005) and the SSDPT solution (Zenkour 2005) are computed based on a cubic and sinusoidal variation of in-plane displacements, respectively, and a constant transverse displacement across the thickness (i.e., thickness-stretching effect is omitted, $\varepsilon_z = 0$). It can be observed that the obtained results are in good agreement with quasi-3D hyperbolic sine shear deformation theory (Neves *et al.* 2012). However, the TSDPT (Zenkour 2005) and the SSDPT (Zenkour 2005), which omit the thickness-stretching effect, slightly over estimate the critical buckling loads. It is worth noting that the developed theory consists of five unknowns, while the number of unknowns in the TSDPT (Reddy 2000), SSDPT (Zenkour 2005) and quasi-

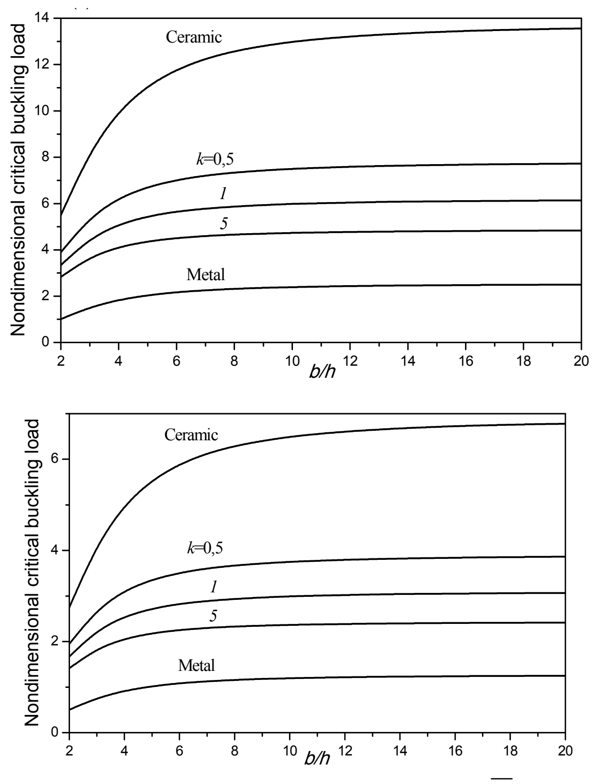

Fig. 3 Nondimensional critical buckling load ($\overline{N}_{cr}$) as a function of side-to-thickness ratio (b/h) of (1-2-1) FGM sandwich plates for various values of k; (a) Plate subjected to uniaxial compressive load ($\gamma = 0$) and (b) Plate subjected to biaxial compressive load ($\gamma = 1$)

Table 1 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to uniaxial compressive load (a/h = 10)

k	Theory	$\overline{N}_{cr}$			
		1-0-1	2-1-2	1-1-1	1-2-1
0	TSDPT[(a)]	6.50248	6.50248	6.50248	6.50248
	SSDPT[(a)]	6.50303	6.50303	6.50303	6.50303
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	6.47652	6.47652	6.47652	6.47652
	Present ($\varepsilon_{zz} \neq 0$)	6.49215	6.49215	6.49215	6.49215
0.5	TSDPT[(a)]	3.68219	3.97042	4.21823	4.60841
	SSDPT[(a)]	3.68284	3.97097	4.21856	4.60835
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	3.58096	3.85809	4.09641	4.47110
	Present ($\varepsilon_{zz} \neq 0$)	3.67770	3.96573	4.21340	4.60320
1	TSDPT[(a)]	2.58357	2.92003	3.23237	3.75328
	SSDPT[(a)]	2.58423	2.92060	3.23270	3.75314
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	2.53062	2.85563	3.15750	3.66013
	Present ($\varepsilon_{zz} \neq 0$)	2.58096	2.91732	3.22956	3.74998
5	TSDPT[(a)]	1.32910	1.52129	1.78978	2.36734
	SSDPT[(a)]	1.33003	1.52203	1.79032	2.36744
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	1.31829	1.50409	1.76507	2.32354
	Present ($\varepsilon_{zz} \neq 0$)	1.32699	1.52012	1.78936	2.36702
10	TSDPT[(a)]	1.24363	1.37316	1.59736	2.13995
	SSDPT[(a)]	1.24475	1.37422	1.59728	2.19087
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	1.23599	1.36044	1.57893	2.10275
	Present ($\varepsilon_{zz} \neq 0$)	1.24109	1.37150	1.59680	2.14001

Table 2 Comparison of nondimensional critical buckling load of square FG sandwich plates subjected to biaxial compressive load (γ = 1, h/b = 0.1)

k	Theory	$\overline{N}_{cr}$			
		1-0-1	2-1-2	1-1-1	1-2-1
0	TSDPT[(a)]	13.00495	13.00495	13.00495	13.00495
	SSDPT[(a)]	13.00606	13.00606	13.00606	13.00606
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	12.95304	12.95304	12.95304	12.95304
	Present ($\varepsilon_{zz} \neq 0$)	12.98429	12.98429	12.98429	12.98429
0.5	TSDPT[(a)]	7.36437	7.94084	8.43645	9.21681
	SSDPT[(a)]	7.36568	7.94195	8.43712	9.21670
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	7.16191	7.71617	8.19283	8.94221
	Present ($\varepsilon_{zz} \neq 0$)	7.35541	7.93147	8.42681	9.20640
1	TSDPT[(a)]	5.16713	5.84006	6.46474	7.50656
	SSDPT[(a)]	5.16846	5.84119	6.46539	7.50629
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	5.06123	5.71125	6.31501	7.32025
	Present ($\varepsilon_{zz} \neq 0$)	5.16191	5.83465	6.45911	7.49996
5	TSDPT[(a)]	2.65821	3.04257	3.57956	4.73469
	SSDPT[(a)]	2.66006	3.04406	3.58063	4.73488
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	2.63658	3.00819	3.53014	4.64707
	Present ($\varepsilon_{zz} \neq 0$)	2.65398	3.04023	3.57873	4.73404
10	TSDPT[(a)]	2.48727	2.74632	3.19471	4.27991
	SSDPT[(a)]	2.48928	2.74844	3.19456	4.38175
	Ref[(b)] ($\varepsilon_{zz} \neq 0$)	2.47199	2.72089	3.15785	4.20550
	Present ($\varepsilon_{zz} \neq 0$)	2.48217	2.74301	3.19359	4.28002

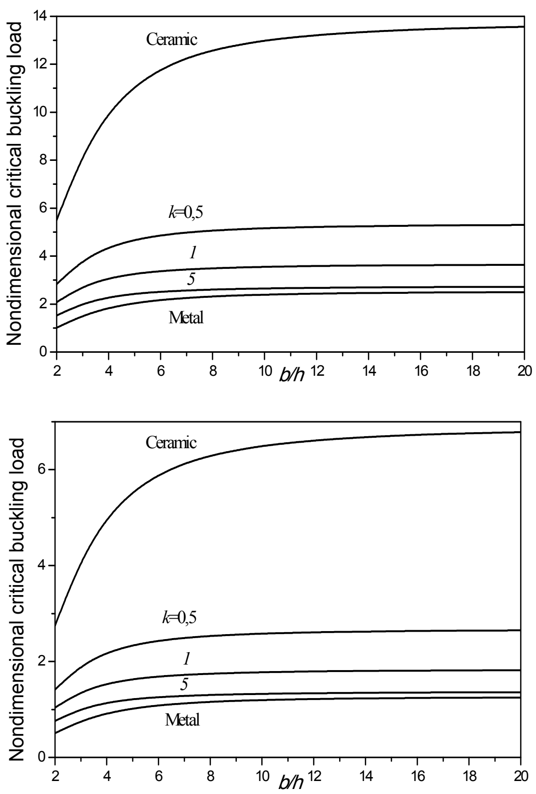

Fig. 4 Nondimensional critical buckling load ($\overline{N}_{cr}$) as a function of side-to-thickness ratio (b/h) of (1-0-1) FGM sandwich plates for various values of k; (a) Plate subjected to uniaxial compressive load (γ = 0) and (b) Plate subjected to biaxial compressive load (γ = 1)

3D theory (Neves *et al*. 2012) is five and six, respectively. Consequently, it may be concluded that the present quasi-3D theory is not only more accurate than the higher order shear deformation theory (TSDPT and SSDPT) having the same five unknowns, but also comparable with the quasi-3D theory having more number of unknowns.

Figs. 3 and 4 show the variation of the critical buckling loads of the (1-2-1) and (1-0-1) types of square FG sandwich plates versus side-to-thickness ratio using the present new simple quasi-3D hyperbolic shear deformation theory. It can be seen that the critical buckling loads become maximum for the ceramic plates and minimum for the metal plates. It is seen that the results increase smoothly as the amount of ceramic in the sandwich plate increases. Also, the buckling load of plate under uniaxial compression is almost the twice of that of the case of the plate under biaxial compression.

6. Conclusions

A new, simple and accurate hyperbolic plate theory with stretching effect for the buckling analysis of functionally graded sandwich plates is presented in this work.The developed model contains five unknowns, but considers both shear deformation and thickness-stretching effects without requiring any shear correction factor. The governing equations are deduced via the principle of minimum total potential energy. Results indicate that the present approach is able to provide very accurate results compared with the other HSDTs with higher number of unknowns and so deserve particular attention and offer potential for future research.

Acknowledgments

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, undergrant No. (254-130-D1435). The authors, therefore, acknowledge with thanks DSR technical and financial support.

References

Abdelhak, Z., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2016), "Thermal buckling response of functionally graded sandwich plates with clamped boundary conditions", *Smart Struct. Syst., Int. J.*, **18**(2), 267-291.

Ait Yahia, S., Atmane, H.A., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.

Akavci, S.S. (2015), "An efficient shear deformation theory for free vibration of functionally graded thick rectangular plates on elastic foundation", *Compos. Struct.*, **108**, 667-676.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), "Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position", *Compos. Struct.*, **125**, 621-630.

Arefi, M. (2015a), "Elastic solution of a curved beam made of functionally graded materials with different cross sections", *Steel Compos. Struct., Int. J.*, **18**(3), 659-672.

Arefi, M. (2015b), "Nonlinear electromechanical analysis of a functionally graded square plate integrated with smart layers resting on Winkler-Pasternak foundation", *Smart Struct. Syst., Int. J.*, **16**(1), 195-211.

Arefi, M. and Allam, M.N.M. (2015), "Nonlinear responses of an arbitrary FGP circular plate resting on the Winkler-Pasternak foundation", *Smart Struct. Syst., Int. J.*, **16**(1), 81-100.

Atmane, H.A., Tounsi, A., Mechab, I. and Adda Bedia, E.A. (2010), "Free vibration analysis of functionally graded plates resting on Winkler–Pasternak elastic foundations using a new shear deformation theory", *Int. J. Mech. Mater. Des.*, **6**(2), 113-121.

Atmane, H.A., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct., Int. J.*, **19**(2), 369-384.

Atmane, H.A., Tounsi, A and, Bernard, F. (2017), "Effect of thickness stretching and porosity on mechanical response of a functionally graded beams resting on elastic foundations", *Int. J. Mech. Mater. Des.*, **13**(1), 71-84

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories", *Steel Compos. Struct., Int. J.*, **18**(1), 187-212.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Bég, O.A. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.: Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), "Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory", *Smart Struct. Syst., Int. J.*, **18**(4), 755-786.

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model", *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), "Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position", *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Benachour, A., Tahar, H.D., Atmane, H.A., Tounsi, A. and Ahmed, M.S. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B Eng*, **42**(6), 1386-1394.

Benahmed, A., Houari, M.S.A., Benyoucef, S., Belakhdar, K., Tounsi, A. (2017), "A novel quasi-3D hyperbolic shear deformation theory for functionally graded thick rectangular plates on elastic foundation", *Geomech. Eng., Int. J.*, **12**(1), 9-34.

Benferhat, R., Hassaine Daouadji, T., Hadji, L. and Said, M. (2016), "Static analysis of the FGM plate with porosities", *Steel Compos. Struct., Int. J.*, **21**(1), 123-136.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), "A novel five-variable refined plate theory for vibration analysis of functionally graded sandwich plates", *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bessaim, A., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Adda Bedia, E.A. (2013), "A new higher-order shear and normal deformation theory for the static and free vibration analysis of sandwich plates with functionally graded isotropic face sheets", *J. Sandw. Struct. Mater.*, **15**, 671-703.

Bouafia, K., Kaci, A., Houari, M.S.A., Benzair, A. and Tounsi, A. (2017), "A nonlocal quasi-3D theory for bending and free flexural vibration behaviors of functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **19**(2), 115-126.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler-Pasternak elastic foundations", *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2016), "Thermal stability of functionally graded sandwich plates using a simple shear deformation theory", *Struct. Eng. Mech., Int. J.*, **58**(3), 397-422.

Boukhari, A., Atmane, H.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.

Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.

Bourada, F., Amara, K. and Tounsi, A. (2016), "Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory", *Steel Compos. Struct., Int. J.*, **21**(6), 1287-1306.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Comput. Meth.*, **11**(6), 1350082.

Bousahla, A.A., Benyoucef, S., Tounsi, A. and Mahmoud, S.R. (2016), "On thermal stability of plates with functionally graded coefficient of thermal expansion", *Struct. Eng. Mech., Int. J.*, **60**(2), 313-335.

Carrera, E., Brischetto, S., Cinefra, M. and Soave, M. (2011), "Effects of thickness stretching in functionally graded plates and shells", *Compos. Part B Eng*, **42**(2), 123-133.

Chikh, A., Bakora, A., Heireche, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2016), "Thermo-mechanical post-buckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(4), 617-639.

Chikh, A., Tounsi, A., Hebali, H. and Mahmoud, S.R. (2017), "Thermal buckling analysis of cross-ply laminated plates using a simplified HSDT", *Smart Struct. Syst., Int. J.*, **19**(3), 289-297.

Draiche, K., Tounsi, A. and Mahmoud, S.R. (2016), "A refined theory with stretching effect for the flexure analysis of laminated composite plates", *Geomech. Eng., Int. J.*, **11**(5), 671-690.

Ebrahimi, F. and Shafiei, N. (2016), "Application of Eringen's nonlocal elasticity theory for vibration analysis of rotating functionally graded nanobeams", *Smart Struct. Syst., Int. J.*, **17**(5), 837-857.

Eltaher, M.A., Khater, M.E., Park, S., Abdel-Rahman, E. and Yavuz, M. (2016), "On the static stability of nonlocal nanobeams using higher-order beam theories", *Adv. Nano Res., Int. J.*, **4**(1), 51-64.

El-Hassar, S.M., Benyoucef, S., Heireche, H. and Tounsi, A. (2016), "Thermal stability analysis of solar functionally graded plates on elastic foundation using an efficient hyperbolic shear deformation theory", *Geomech. Eng., Int. J.*, **10**(3), 357-386.

Fahsi, A., Tounsi, A., Hebali, H., Chikh, A., Adda Bedia, E.A. and Mahmoud, S.R. (2017), "A four variable refined *n*th-order shear deformation theory for mechanical and thermal buckling analysis of functionally graded plates", *Geomech. Eng., Int. J.*, **13**(3). [In Press]

Fekrar, A., El Meiche, N., Bessaim, A., Tounsi, A. and Adda Bedia, E.A. (2012), "Buckling analysis of functionally graded hybrid composite plates using a new four variable refined plate theory", *Steel Compos. Struct., Int. J.*, **13**(1), 91-107.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **4**(49), 795-810.

Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2015), "A refined exponential shear deformation theory for free vibration of FGM beam with porosities", *Geomech. Eng., Int. J.*, **9**(3), 361-372.

Hadji, L., Khelifa, Z. and Adda Bedia, E.A. (2016c), "A new higher order shear deformation model for functionally graded beams", *KSCE J. Civil Eng.*, **20**(5), 1835-1841.

Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2016a), "Dynamic behavior of FGM beam using a new first shear deformation theory", *Earthq. Struct., Int. J.*, **10**(2), 451-461.

Hadji, L., Hassaine Daouadji, T., Meziane, M.A.A., Tlidji, Y. and Adda Bedia, E.A. (2016b), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(2), 315-325.

Hadji, L., Meziane, M.A.A., Abdelhak, Z., Hassaine Daouadji, T. and Adda Bedia, E.A. (2016d), "Static and dynamic behavior of FGM plate using a new first shear deformation plate theory", *Struct. Eng. Mech., Int. J.*, **57**(1), 127-140.

Hadji, L., Zouatnia, N. and Kassoul, A. (2016e), "Bending analysis of FGM plates using a sinusoidal shear deformation theory", *Wind Struct., Int. J.*, **23**(6), 543-558.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.

Hassaine Daouadji, T. and Hadji, L. (2015), "Analytical solution of nonlinear cylindrical bending for functionally graded plates", *Geomech. Eng., Int. J.*, **9**(5), 631-644.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**(2), 374-383.

Houari, M.S.A., Tounsi, A., Bessaim, A. and Mahmoud, S.R. (2016), "A new simple three-unknown sinusoidal shear deformation theory for functionally graded plates", *Steel Compos. Struct., Int. J.*, **22**(2), 257-276.

Jha, D.K., Kant, T. and Singh, R.K. (2013), "A critical review of recent research on functionally graded plates", *Compos. Struct.*, **96**, 833-849.

Kar, V.R. and Panda, S.K. (2015), "Nonlinear flexural vibration of shear deformable functionally graded spherical shell panel", *Steel Compos. Struct., Int. J.*, **18**(3), 693-709.

Khelifa, Z., Hassaine Daouadji, T., Hadji, L., Tounsi, A. and Adda Bedia, E.A. (2015), "A new higher order shear deformation model of functionally graded beams based on neutral surface position", *Trans. Indian Inst. Met.*, **69**(3), 683-691.

Koizumi, M. (1993), "The concept of FGM", *Ceram. Trans. Funct. Grad. Mater.*, **34**, 3-10.

Laoufi, I., Ameur, A., Zidi, M., Adda Bedia, E.A. and Bousahla, A.A. (2016), "Mechanical and hygrothermal behaviour of functionally graded plates using a hyperbolic shear deformation theory", *Steel Compos. Struct., Int. J.*, **20**(4), 889-912.

Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Bég, O.A. and Mahmoud, S.R. (2015), "Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect", *Steel Compos. Struct., Int. J.*, **18**(2), 425-442.

Mahi, A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Mantari, J.L. and Guedes Soares, C.G. (2012), "Generalized hybrid quasi-3D shear deformation theory for the static analysis of advanced composite plates", *Compos. Struct.*, **94**(8), 2561-2575.

Matsunaga, H. (2008), "Free vibration and stability of functionally graded plates according to a 2-D higher-order deformation theory", *Compos. Struct.*, **82**(4), 499-512.

Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), "A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations", *Struct. Eng. Mech., Int. J.*, **53**(6), 1215-1240.

Meksi, R., Benyoucef, S., Mahmoudi, A., Tounsi, A., AddaBedia, E.A. and Mahmoud, S.R. (2017), "An analytical solution for bending, buckling and vibration responses of FGM sandwich plates", *J. Sandw. Struct. Mater.* [In Press]

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(3), 793-809.

Meziane, M.A.A., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Mindlin, R.D. (1951), "Influence of rotatory inertia and shear on flexural motions of isotropic, elastic plates", *J. Appl. Mech.*, **18**(1), 31-38.

Neves, A.M.A., Ferreira, A.J.M., Carrera E., Cinefra M., Jorge R.M.N. and Soares, C.M.M. (2012), "Buckling analysis of sandwich plates with functionally graded skins using a new quasi-3D hyperbolic sine shear deformation theory and collocation with radial basis functions", *ZAMM – Journal of Applied Mathematics and Mechanics/Zeitschrift für Angewandte Mathematik und Mechanik*, **92**(9), 749-766.

Pradyumna, S. and Bandyopadhyay, J.N. (2008), "Free vibration analysis of functionally graded curved panels using a higher-order finite element formulation", *J. Sound Vib.*, **318**(1), 176-192.

Qian, L.F., Batra, R.C. and Chen, L.M. (2004), "Static and dynamic deformations of thick functionally graded elastic plates by using higher-order shear and normal deformable plate theory and meshless local Petrov-Galerkin method", *Compos. Part B Eng*, **35**(6), 685-697.

Reddy, J.N. (2000), "Analysis of functionally graded plates", *Int. J. Numer. Methods Eng.*, **47**(1-3), 663-684.

Reddy, J.N. (2011), "A general nonlinear third-order theory of functionally graded plates", *Int. J. Aerosp. Lightweight Struct.*, **1**(1), 1-21.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl.Mech.*, **12**(2), 69-72.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng., Int. J.*, **11**(2), 289-307.

Sallai, B., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2015), "Analytical solution for bending analysis of functionally graded beam", *Steel Compos. Struct., Int. J.*, **19**(4), 829-841.

Shahrjerdi, A., Mustapha, F., Bayat, M. and Majid, D.L. (2011), "Free vibration analysis of solar functionally graded plates with temperature-dependent material properties using second order shear deformation theory", *J. Mech. Sci. Tech.*, **25**(9), 2195-2209.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Solid.*, **47**, 349-361.

Taibi, F.Z., Benyoucef, S., Tounsi, A., Bachir Bouiadjra, R., Adda Bedia, E.A. and Mahmoud, S.R. (2015), "A simple shear deformation theory for thermo-mechanical behaviour of functionally graded sandwich plates on elastic foundations", *J. Sandw. Struct. Mater.*, **17**(2), 99-129.

Tounsi, A., Houari, M.S.A. and Benyoucef, S. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech., Int. J.*, **60**(4), 547-565.

Yamanouchi, M., Koizumi, M., Hirai, T. and Shiota, I. (1990), *Proceeding of the 1st International Symposium on Functionally Gradient Materials*, Sendai, Japan, October.

Zenkour, A.M. (2005), "A comprehensive analysis of functionally graded sandwich plates: Part 2—Buckling and free vibration", *Int. J. Solids Struct*, **42**(18), 5243-5258.

Zenkour, A.M. (2007), "Benchmark trigonometric and 3-D elasticity solutions for an exponentially graded thick rectangular plate", *Arch. Appl. Mech.*, **77**(4), 197-214.

Zidi, M., Tounsi, A., Houari, M.S.A. and Anwar Bég, O. (2014), "Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory", *Aerosp. Sci. Technol.*, **34**, 24-34.

CC