

# Feasibility study of bonding state detection of explosive composite structure based on nonlinear output frequency response functions

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**Abstract.** With the increasing application of explosive composite structure in many engineering fields, its interface bonding state detection is more and more significant to avoid catastrophic accidents. However, this task still faces challenges due to the complexity of the bonding interface. In this paper, the concept of nonlinear output frequency response functions (NOFRFs) is introduced to detect the bonding state of explosive composite structure. The NOFRFs can describe the nonlinear characteristics of nonlinear vibrating system. Because of the presence of the bonding interface, explosive composite structure itself is a nonlinear system; when bonding interface of the structure is damaged, its dynamic characteristics show enhanced nonlinear characteristic. Therefore, the NOFRFs-based detection index is proposed as indicator to detect the bonding state of explosive composite pipes. The experimental results verify the effectiveness of the detection approach.

**Keywords:** bonding state detection; explosive composite structure; nonlinear output frequency response functions

## 1. Introduction

Explosive composite of two or more different kinds of materials is accomplished by the deformation owing to high pressure and high temperature produced at the collision point (Findik 2011). With the performance of two or more metal materials such as high temperature resistance, high pressure resistance, corrosion resistance, etc., explosive composite structures have been increasingly applied in a variety of fields including oil and gas, aviation and aerospace, chemical industry and military industry. The ultimate mechanical and metallurgical characteristic of such structure is dominated by its interfacial bonding state (Rajani and Mousavi 2012). Poor bonding state may cause defects such as wrinkling, separation and bulge in service, which leads to the degradation of the service performance, failure and even catastrophic accidents. Thus, it is important to develop a detection method for the bonding state detection of explosive composite structures.

For the past few years, many destructive testing methods such as the tensile-shear test, Charpy impact test and bending test (Kaya and Kahraman 2013, Xia *et al.* 2014, Acarer and Demir 2008, Kahraman *et al.* 2005, Durgutlu *et al.* 2005) have been used to detect the bonding strength of explosive composite structure. However, those methods have the disadvantages that they are destructive inspection with high cost and low efficiency; they only can be used to detect the local area of the structure, and cannot be used for global and online detection. In the authors' previous studies (Si *et al.* 2015, 2014), the nondestructive detecting methods

based on the output signal of the explosive composite structures are proposed for the bonding state detection of such structures. But these methods only consider the output signal which may change caused by input signal not by the characteristic of the structure. Thus, detecting the bonding state of explosive composite structure only uses the change of output signal may lead to misdiagnosis.

Due to the presence of the bonding interface, the explosive composite structure itself is a nonlinear system. When bonding interface of the structure is damaged, its dynamic characteristics show enhanced nonlinear characteristic. So, nonlinear vibration-based method may be a promising solution for bonding state detection of such structure. With the development of the nonlinear theory, Volterra series has caused wide public concern in nonlinear system analysis (Rugh 1981). The multidimensional Fourier transformation of the Volterra kernels is called generalized frequency response functions (GFRFs) that extend the linear FRF concept to the nonlinear case. GFRFs, which represent the frequency domain characteristics of nonlinear system, gives intuitive explanations to many important nonlinear phenomena (Billings and Yusof 2007, Boaghe *et al.* 2002) and has been employed in damage identification fields. Surace *et al.* (2011) used generalized frequency response function to detect cracks in beam-like structures. Chatterjee (2010) assessed the damage in a cantilever beam based on higher order frequency response functions. However, the GFRF is multidimensional and difficult to measure, display and interpret in practice. To overcome this disadvantage, the nonlinear output frequency response functions (NOFRFs) is proposed by Lang and Billings (2007a, b). NOFRFs are regarded as another extension of the linear FRF concept to the nonlinear case, which is a complement to the GFRFs and has been used in structural

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damage detection. Peng *et al.* (2007, 2011) used NOFRFs based analysis method to detect the damage of beams and aluminum plates. Xia *et al.* (2015) diagnosed the fault of hydro generator based on NOFRFs. In this paper, the NOFRFs are introduced to detect the bonding state of explosive composite pipes which have been widely applied in oil and gas, chemical power and other fields. The aim of this paper is to establish a basis for the use of NOFRFs in bonding state detection of explosive composite structure in engineering practice.

The paper is organized as follows. In Section 2, the theory and algorithm of the frequency characteristic of nonlinear system and NOFRFs is introduced. In Section 3, the procedure of the detection method based on NOFRFs is given. In Section 4, the proposed method is applied to the bonding state detection of explosive composite pipes to validate its effectiveness. Conclusions are given in section 5.

## 2. Fundamental theory

### 2.1 The frequency response function of nonlinear system

For linear systems, it is well known that the relationship of the system input and output in time domain and frequency domain can be described as

$$\begin{aligned} y(t) &= \int_{-\infty}^{+\infty} h(t)u(t-\tau)d\tau \\ Y(j\omega) &= H(j\omega)U(j\omega) \end{aligned} \quad (1)$$

Where,  $U(j\omega)$  and  $Y(j\omega)$  are the system input and output spectra which are the Fourier transforms of the system time domain input  $u(t)$  and output  $y(t)$  respectively, and  $H(j\omega)$ , the Fourier transform of  $h(t)$ , is the system frequency response function (FRF).

For nonlinear systems, the relationship corresponding to Eq. (1) is much more complicated and is described in time domain by Volterra series

$$y(t) = \sum_{n=1}^N \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_n(\tau_1, \cdots, \tau_n) \prod_{i=1}^n u(t-\tau_i) d\tau_i \quad (2)$$

In which,  $u(t)$  and  $y(t)$  are the system input and output respectively,  $h_n(\tau_1, \cdots, \tau_n)$  is the  $n$ th-order Volterra kernel and  $N$  denotes the maximum order of the system nonlinearity. An frequency domain expression of this class of nonlinear systems under a general input was derived by Lang and Billings (2007a, b) as follows

$$\begin{cases} Y(j\omega) = \sum_{n=1}^N Y_n(j\omega), \quad \forall \omega \\ Y_n(j\omega) = \frac{1/\sqrt{n}}{(2\pi)^{n-1}} \times \int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1, \cdots, j\omega_n) \\ \quad \times \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega} \end{cases} \quad (3)$$

Where  $Y_n(j\omega)$  represents the  $n$ th order output frequency

response of the system.  $U(j\omega)$  and  $Y(j\omega)$  are the Fourier spectra of input  $u(t)$  and output  $y(t)$ .  $H_n(j\omega_1, \cdots, j\omega_n)$  is the  $n$ th order generalized frequency response function (GFRF), which is the multidimensional Fourier transformation of  $n$ th-order Volterra kernel  $h_n(\tau_1, \cdots, \tau_n)$  as follows.

$$H_n(j\omega_1, \cdots, j\omega_n) = \int_{-\infty}^{+\infty} \cdots \int_{-\infty}^{+\infty} h_n(\tau_1, \cdots, \tau_n) \times e^{-j(\omega_1\tau_1+\cdots+\omega_n\tau_n)} d\tau_1 \cdots d\tau_n \quad (4)$$

$$\int_{\omega_1+\cdots+\omega_n=\omega} H_n(j\omega_1, \cdots, j\omega_n) \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega}$$

denotes the integration of  $H_n(j\omega_1, \cdots, j\omega_n) \prod_{i=1}^n U(j\omega_i)$  over the  $n$ -dimensional hyperplane  $\omega_1 + \cdots + \omega_n = \omega$ .

If a non-linear systems subjected to a general input with a spectrum given by

$$U(j\omega) = \begin{cases} U(j\omega), & |\omega| \in [a, b] \\ 0 & \text{otherwise} \end{cases}$$

Where  $[a, b]$  represents the real frequency range of input signal. Under this assumption, the frequency range of the  $n$ th nonlinear output should be described as

$$\omega = \omega_l + \cdots + \omega_n, \quad \omega_l \in [-b, -a] \text{ or } [a, b], \quad l = 1, 2, \cdots, n$$

For linear systems  $Y(j\omega) = H(j\omega)U(j\omega)$ , the possible output frequencies are the same as the frequencies in the input. But for the nonlinear systems as shown in Eq. (3), the relationship between the input and output frequencies can be got by

$$f_Y = \bigcup_{n=1}^N f_{Y_n} \quad (5)$$

Where  $f_Y$  represents the non-negative frequency ranges of the system output.  $f_{Y_n}$  denotes the non-negative frequency range produced by the  $n$ th-order system nonlinearity. It is obviously that  $f_Y$  contains much richer frequency components than the range  $[a, b]$  of the input frequencies. Note that the  $f_Y$ , determined by Eq. (5), is the frequency range where frequency components may exist in the output of a nonlinear system, however, it does not mean that the output frequency components are definitely available over the whole range of  $f_Y$ .

Similar to the FRF of linear systems shown in Fig. 1, the GFRF of non-linear systems can be illustrated as shown in Fig. 2, in which the GFRFs  $H_1, H_2, \cdots, H_n$  is a natural extension of the FRF of linear case to the nonlinear case. The GFRFs reflect the nonlinear system properties independently from the system input and are useful to analyze the change of nonlinear system. Unfortunately, the GFRFs are multidimensional and difficult to measure, display and interpret in practice.

### 2.2 The nonlinear output frequency response functions (NOFRFs)

Given a static linear system and a static nonlinear system as follows

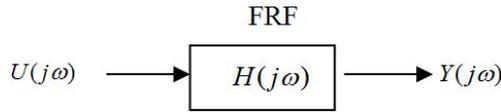


Fig. 1 The FRF of linear system

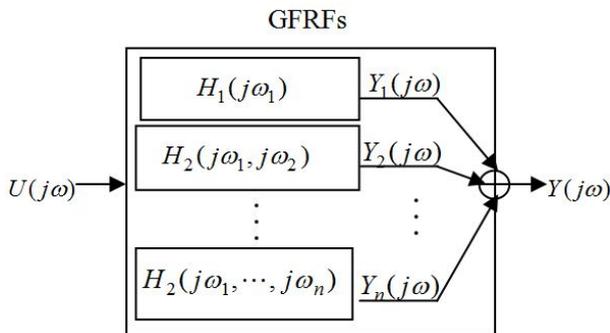


Fig. 2 The GFRF of nonlinear system

$$y(t) = ku(t) \tag{6}$$

$$y(t) = ku^2(t) \tag{7}$$

The frequency domain representation of Eq. (6) and Eq. (7) are

$$Y(j\omega) = kU(j\omega) \tag{8}$$

$$Y(j\omega) = kU_2(j\omega) = kU_2(j(\omega_1 + \omega_2)) = kU(j\omega_1) * U(j\omega_2) \tag{9}$$

In which,  $U_2(j\omega)$  is the Fourier transform of  $u^2(t)$ . Compared Eq. (8) with Eq. (9),  $U_2(j\omega)$  can be viewed as the natural extension of the input spectrum  $U(j\omega)$  to the second order nonlinear case, and can be expressed in terms of the input spectrum  $U(j\omega)$  as

$$U_2(j\omega) = \frac{1}{2\pi} \int_{\omega_1} U(j\omega_1) \cdot U(j(\omega - \omega_1)) d\omega_1 = \frac{1/\sqrt{2}}{2\pi} \int_{\omega_1} \sqrt{2}U(j\omega_1) \cdot U(j(\omega - \omega_1)) d\omega_1 \tag{10}$$

$\sqrt{2}$  can be interpreted as the result of

$$\left[1 + \left(\frac{\partial \omega}{\partial \omega_1}\right)^2\right]^{1/2} = \sqrt{2} \tag{11}$$

Substituting Eq. (11) into Eq. (10), and rewriting Eq. (10) as follows

$$U_2(j\omega) = \frac{1/\sqrt{2}}{2\pi} \int_{\omega} U(j\omega_1) \cdot U(j(\omega - \omega_1)) \left[1 + \left(\frac{\partial \omega}{\partial \omega_1}\right)^2\right]^{1/2} d\omega_1 = \frac{1/\sqrt{2}}{(2\pi)^{2-1}} \int_{\omega_1 + \omega_2 = \omega} \prod_{i=1}^2 U(j\omega_i) d\sigma_{2\omega} \tag{12}$$

By that analogy, the natural extension of input spectrum  $U(j\omega)$  to the  $n$ th order homogeneous nonlinear situation should be given as

$$U_n(j\omega) = \frac{1/\sqrt{n}}{(2\pi)^{n-1}} \int_{\omega_1 + \dots + \omega_n = \omega} \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega} \tag{13}$$

Where  $U_n(j\omega)$  is the Fourier transformation of the system input  $u(t)$  to the  $N$ th power.

When  $U_n(j\omega) = \int_{\omega_1 + \dots + \omega_n = \omega} \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega} \neq 0$ , com-

bined Eqs. (3) and (13), Lang and Billings (2007a, b) defined the NOFRFs as

$$U_n(j\omega) = \frac{Y_n(j\omega)}{U_n(j\omega)} = \frac{\int_{\omega_1 + \dots + \omega_n = \omega} H_n(j\omega_1, \dots, j\omega_n) \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega}}{\int_{\omega_1 + \dots + \omega_n = \omega} \prod_{i=1}^n U(j\omega_i) d\sigma_{n\omega}} \tag{14}$$

$G_n(j\omega)$  is a weighted sum of  $H_n(j\omega_1, L, j\omega_n)$  over the hyperplane  $\omega_1 + \dots + \omega_n = \omega$ . The weights depending on the input signal. Therefore, the NOFRF can be used as an alternative representation of the dynamical properties of any structure determined by the GFRFs. The GFRFs of nonlinear system in Fig. 2 can be expressed as another form based on NOFRFs as shown in Fig. 3, Eq. (3) can be rewritten as

$$Y(j\omega) = \sum_{n=1}^N Y_n(j\omega) = \sum_{n=1}^N G_n(j\omega) U_n(j\omega) \tag{15}$$

Compared with GFRFs in Fig. 2, the NOFRFs in Fig. 3 are one-dimensional, thus allows the analysis of nonlinear systems in the frequency domain to be implemented in a manner similar to the analysis of the FRF for linear systems. The algorithm to evaluate the NOFRFs value has been given in Ref. (Lang and Billings 2007a, b).

### 3. Bonding state detection of explosive composite structure based on NOFRFs

Explosive composite structure is viewed as a dynamic

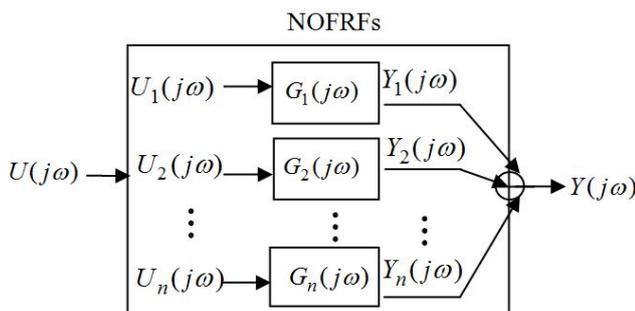


Fig. 3 The NOFRFs of nonlinear system

system which combined of stiffness, mass and damping. Due to the presence of the bonding interface, the dynamic system itself is a nonlinear system, and the nonlinear characteristic will be enhanced if there is a defect in the bonding interface. Based on above, the NOFRFs, which can describe the nonlinear characteristics of nonlinear system, are introduced to assess the bonding state of the explosive composite structure.

For any dynamic system, given the input spectra  $U(j\omega)$ , and thus its natural extension to the  $n$  order nonlinear cases is  $U_n(j\omega)$ , ( $n = 1, K, N$ ). The output spectra  $Y(j\omega)$  under  $U(j\omega)$  is expressed as

$$U_n(j\omega), (n = 1, \dots, N)$$

The NOFRFs  $[G_1(j\omega), \dots, G_N(j\omega)]$  represent the dynamical property of the system. If the system is ideal linear, the NOFRFs are  $[G_1(j\omega), \dots, G_N(j\omega)]^T = [G_1(j\omega), 0, \dots, 0]^T$ , the input and output spectra of the system is

$$Y(j\omega) = U_1(j\omega)G_1(j\omega)$$

Explosive composite structure itself is a nonlinear system, so all NOFRFs  $[G_1(j\omega), \dots, G_N(j\omega)]$  of the structure are not zero. If bonding interface of the structure is damaged, its dynamic characteristics show enhanced nonlinear characteristic, and thus the percentages of  $[G_2(j\omega), \dots, G_N(j\omega)]$  in all NOFRFs will increase while the percentage of the value  $G_1(j\omega)$  in all NOFRFs will decrease. Based on these, a NOFRFs-based detection index is defined as

$$DI(i) = \frac{\int_{-\infty}^{+\infty} G_n(j\omega)d\omega}{\int_{-\infty}^{+\infty} G_1(j\omega)d\omega} \quad (2 \leq n \leq N, i = n-1) \quad (16)$$

The  $DI(i), (1 \leq i \leq N-1)$  represent the feature information of explosive composite structure, and can reflect the

different bonding state of such structure. The higher values of  $DI(i)$  indicate the bigger damage of the structure.

### 4. Experimental validations

Explosive composite pipe is a typical explosive composite structure and has been widely used for subsea pipelines. It is important to detect the bonding state of the pipes to avoid leakage. Thus, the explosive composite pipe is used to validate the feasibility of the proposed method. Due to bonding strength shortage and delamination damage are two major bonding defects of explosive composite pipe. So, the two types of pipes are detected by using the proposed method. It is well known that explosive load is one of the most important parameters affecting the quality of the bonds in explosive composite. With the increase of explosive load, the bonding quality is improved (Rajani and Mousavi 2012, Kahraman *et al.* 2005). In order to obtain the explosive composite pipes with different bonding strength and different damage area, the explosive loads is changed for different pipes in the composite when other composite parameters remain constant.

#### 4.1 Different bonding strength detection

In this section, three explosive composite pipes with different bonding strength are assessed by the NOFRFs-based detection indexes. The major dimensions and parameters of the pipes are given in Table 1, in which explosive loads represent the weight of explosive in unit length. Pipe 1 is the standard pipe with good bonding strength, pipe 2 is a damaged pipe with slight strength deficiency, pipe 3 is a damaged pipe with serious strength deficiency. The experimental setup is shown in Fig. 4, which mainly consists of explosive composite pipes, a hammer, an accelerometer and a data acquisition system. The pipe is freely supported at both ends. Hammer excitation is performed manually at a point 300 mm away

Table 1 Major dimensions and parameters of explosive composite pipes

Number	Size of the pipes				Explosive loads (g)
	Length (mm)	Pipe O.D. (mm)	Thickness of stainless steel (mm)	Thickness of carbon steel (mm)	
1	2500	114	2	10	5
2	2500	114	2	10	10
3	2500	114	2	10	15

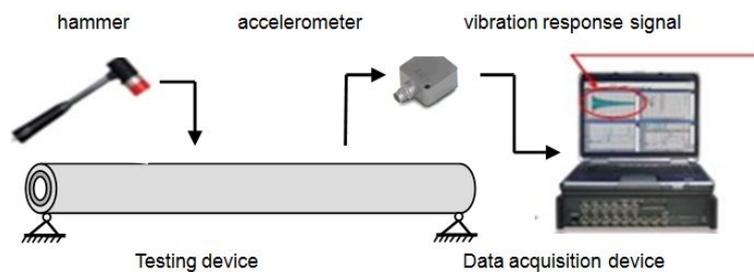


Fig. 4 Experimental setup of explosive composite pipe

from the left pipe end. The accelerometer is mounted at a point 250 mm away from the right pipe end. The sampling frequency is 3200 Hz. The sampling number is 2000. The impulse input and vibration response are measured to calculate the NOFRFs-based detection indexes.

Because of the higher the order of system nonlinearity is, the smaller the amplitudes of NOFRF are. Moreover, the greater the amount of calculation for the higher NOFRF identification is. Therefore, the NOFRFs up to fourth order are considered to detect the bond state of the pipes in this study. Four impulse input with different intensities are used to excite the pipe. According to Eq. (15), the frequency components of the output can be written as

$$\begin{bmatrix} Y^1(if) \\ Y^2(if) \\ Y^3(if) \\ Y^4(if) \end{bmatrix} = \begin{bmatrix} U_1^1(if), U_2^1(if), U_3^1(if), U_4^1(if) \\ U_1^2(if), U_2^2(if), U_3^2(if), U_4^2(if) \\ U_1^3(if), U_2^3(if), U_3^3(if), U_4^3(if) \\ U_1^4(if), U_2^4(if), U_3^4(if), U_4^4(if) \end{bmatrix} \begin{bmatrix} G_1(if) \\ G_2(if) \\ G_3(if) \\ G_4(if) \end{bmatrix} \quad (17)$$

Consequently, the NOFRFs  $[G_1(j\omega) \ G_2(j\omega) \ G_3(j\omega) \ G_4(j\omega)]$  are determined as

$$\begin{bmatrix} G_1(if) \\ G_2(if) \\ G_3(if) \\ G_4(if) \end{bmatrix} = \begin{bmatrix} U_1^1(if), U_2^1(if), U_3^1(if), U_4^1(if) \\ U_1^2(if), U_2^2(if), U_3^2(if), U_4^2(if) \\ U_1^3(if), U_2^3(if), U_3^3(if), U_4^3(if) \\ U_1^4(if), U_2^4(if), U_3^4(if), U_4^4(if) \end{bmatrix}^{-1} \begin{bmatrix} Y^1(if) \\ Y^2(if) \\ Y^3(if) \\ Y^4(if) \end{bmatrix} \quad (18)$$

The NOFRFs evaluation results of the three pipes are given in Figs. 5 to 7. Directly from the NOFRFs in Figs. 5 to 7, the three pipes cannot be distinguished effectively, so, the detection indexes are calculated based on the NOFRFs and given in Table 2. In can be seen from the Table 2 that the detection indexes  $DI(1)$ ,  $DI(2)$  and  $DI(3)$  all have shown an increasing trend with the change of the bonding strength of the three pipes from high to low. The NOFRFs-based detection indexes are quite sensitive to the change of the bonding strength of the pipe.

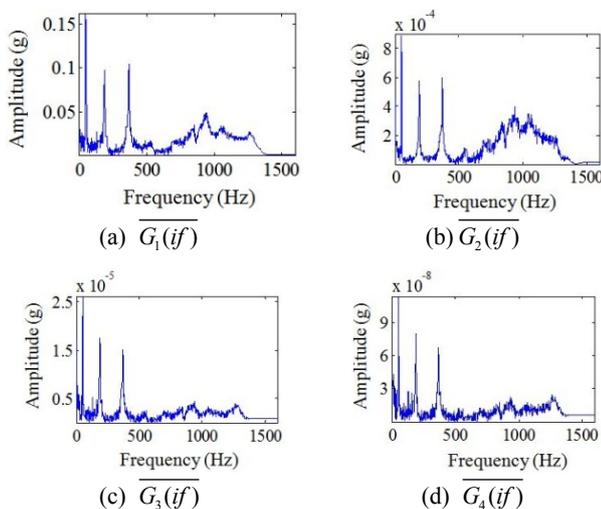


Fig. 5 The NOFRFs of standard pipe

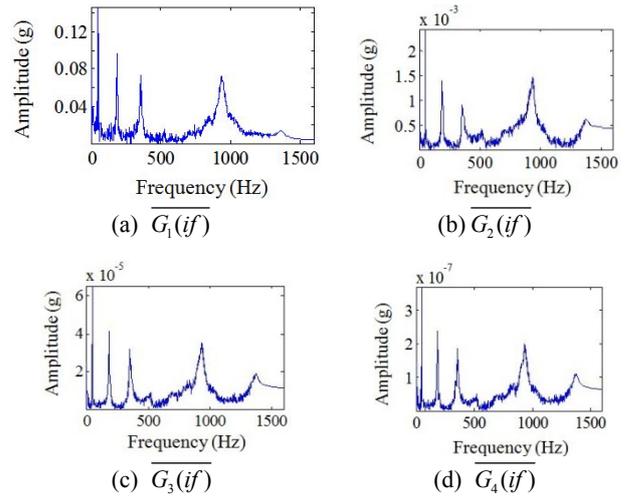


Fig. 6 The NOFRFs of slight damaged pipe

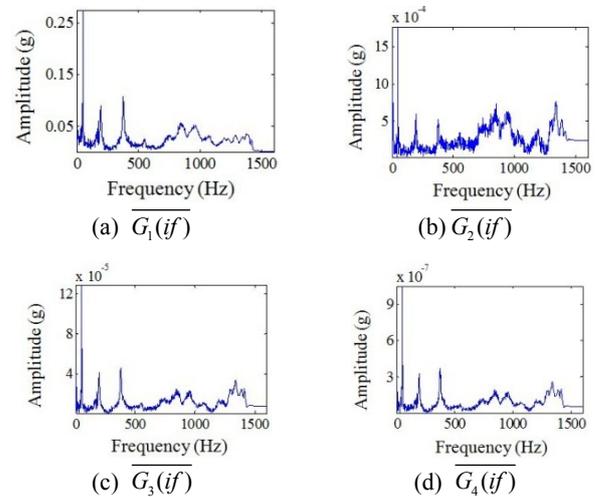


Fig. 7 The NOFRFs of serious damaged pipe

Table 2 The NOFRFs-based detection indexes  $DI(i)$  of the three pipes

$DI(i)$	Different bonding strength pipe		
	Standard pipe	Slight damaged pipe	Serious damaged pipe
$i = 1$	7.37E-03	1.01E-02	1.14E-02
$i = 2$	1.26E-04	3.12E-04	4.93E-04
$i = 3$	6.65E-07	1.83E-06	3.92E-06

#### 4.2 Delamination damage detection

Explosive composite pipes with different delaminated length are produced to verify the effectiveness of the NOFRFs-based detection indexes. For example, a 10 m pipe with 1m long damage is made by setting 1 m with explosive load 5 g while other 9 m with explosive load 25 g. Here are three 10 m pipes with delaminated length 1.5 m, 1 m and 0.5 m are made and the diagram are given in Fig. 8. The Explosive loads 5 g and 25 g represent the weight of explosive in unit length is 5 g and 25 g.

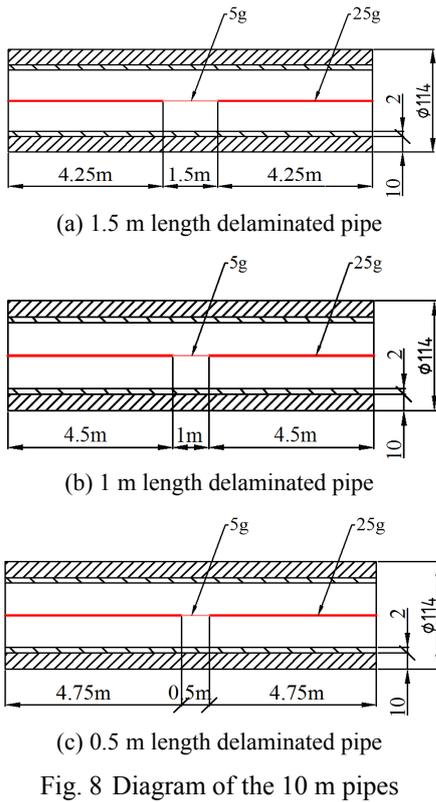


Fig. 8 Diagram of the 10 m pipes



Fig. 9 Experimental setup of explosive composite pipe

Fig. 9 shows the experimental setup, in which the pipe is freely supported at both ends. Hammer excitation is performed manually at a point 2.5 m away from the left pipe end. The accelerometer is mounted on the middle of the pipe. The impulse input and vibration response are measured at a sampling frequency of 2000 Hz. The sampling number is 8000.

Table 3 The NOFRFs-based detection indexes  $DI(i)$  of the three different delaminated length pipes

$DI(i)$	Different delaminated length pipe		
	0.5 m length delaminated pipe	1 m length delaminated pipe	1.5 m length delaminated pipe
$i = 1$	4.25E-04	4.36 E-04	5.20E-04
$i = 2$	7.52E-08	1.05E-07	1.21E-07
$i = 3$	4.75E-12	7.94E-12	9.40E-12

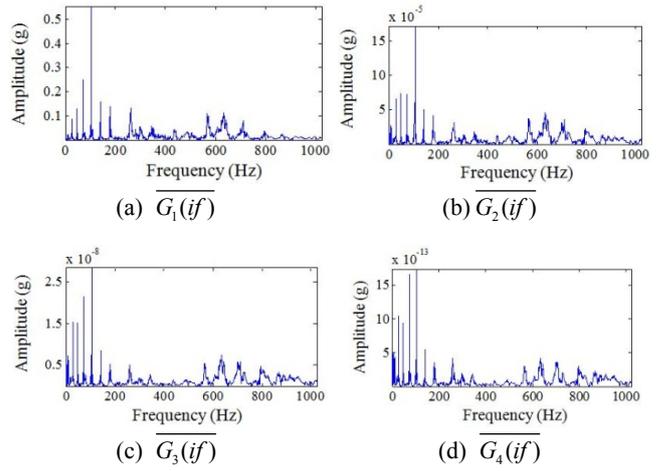


Fig. 10 The NOFRFs of 0.5 m length delaminated

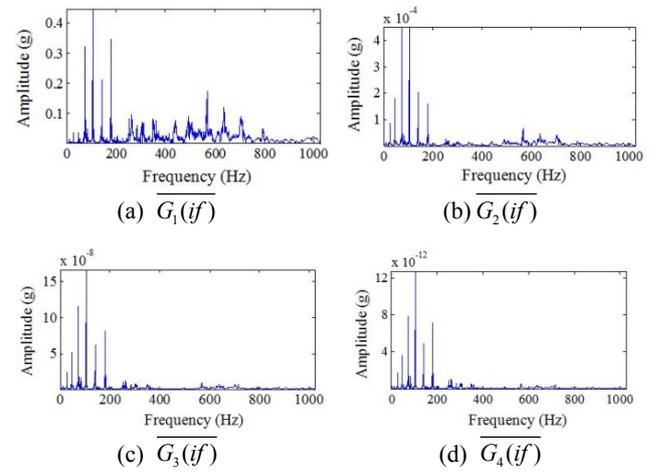


Fig. 11 The NOFRFs of 1 m length delaminated

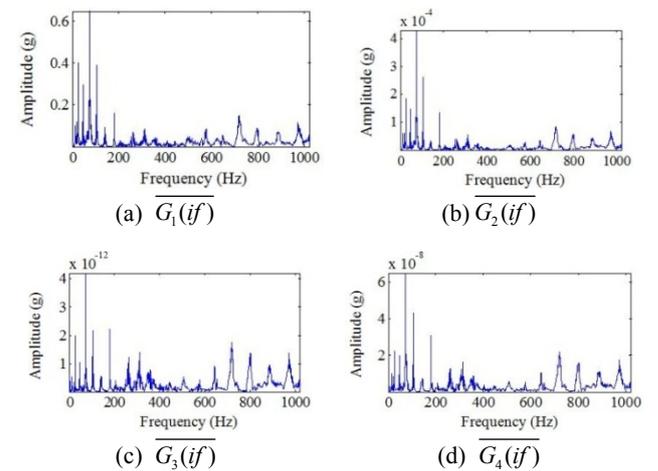


Fig. 12 The NOFRFs of 1.5 m length delaminated pipe

Similar to Section 4.1, the first four NOFRFs of the three pipes are identified and given in Figs. 10 to 12, and the detection indexes  $DI(i)$  ( $i = 1, 2, 3$ ) are calculated and shown in Table 3. It is obviously that the  $DI(i)$  ( $i = 1, 2, 3$ ) values of the pipes in Table 3 are increased with the

delaminated length, and the larger the delaminated length is, the bigger the detection indexes  $DI(i)$  ( $i = 1, 2, 3$ ) values are. The NOFRFs-based detection indexes are sensitive indicators for the delaminate damage of the pipe.

## 5. Conclusions

Explosive composite structure can be viewed as a nonlinear system. The presence of damage may lead to enhancing the nonlinear characteristics of the structure. NOFRFs are excellent tools to reflect the nonlinear characteristics of nonlinear system. The stronger the nonlinear characteristic of the nonlinear system is, the larger the percentages of high order NOFRFs are and the smaller the percentage of  $G_1(j\omega)$  is. Thus, the NOFRFs-based detection indexes are proposed to detect the bonding state of explosive composite structure in this article. The proposed method is applied to bonding strength detection and delamination damage detection of explosive composite pipes. The results show that the NOFRFs-based detection indexes are sensitive indicators of the damage of the explosive composite pipe.

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