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Probabilistic computation of the structural performance of moment resisting steel frames

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Abstract. This study investigates the reliability of the performance levels of moment resisting steel frames subjected to lateral loads such as wind and earthquake. The reliability assessment has been performed with respect to three performance levels: serviceability, damageability, and ultimate limit states. A four-story moment resisting frame is used as a typical example. In the reliability assessment the uncertainties in the loadings and in the capacity of the frame have been considered. The wind and earthquake loads are assumed to have lognormal distribution, and the frame resistance is assumed to have a normal distribution. In order to obtain an appropriate limit state function a linear relation between the loading and the deflection is formed. For the reliability analysis an algorithm has been developed for determination of limit state functions and iterations of the first order reliability method (FORM) procedure. By the method presented herein the multivariable analysis of a complicated reliability problem is reduced to an S-R problem. The procedure for iterations has been tested by a known problem for the purpose of avoiding convergence problems. The reliability indices for many cases have been obtained and also the effects of the coefficient of variation of load and resistance have been investigated.

Keywords: steel frames; FORM; performance; reliability; safety

1. Introduction

For safety, economy, insurance and similar concerns it is very essential to evaluate the performance and reliability of structures. However, the lack of practical and feasible modern probabilistic methods limits the solutions to deterministic methods. Another major obstacle is the lack of probabilistic data, namely probability density functions (PDF) of loads, materials, workmanship quality, and environmental effects. The probabilistic evaluation for existing structures or for design of newly built structures is a new concept and it has been broadly investigated for two decades. An integrated view of the techniques and theory for reliability of structures and related basic probabilistic concepts are presented by Melchers (1999/2002) and Ang and Tang (2007). Also, many clarifying uncomplicated examples to improve understanding the problem are presented by Ghali *et al*. (2009) and Schneider (2006). Additionally, Ellingwood and Kanda (2005) have introduced a monograph which addresses major issues pertinent to meeting the performance goals of tall buildings related to safety and serviceability. Kanda *et al*. (1997) proposed a probabilistic second-moment seismic safety measure for existing buildings. They analyzed eleven buildings using lumped-mass models and utilized an available probability-based seismic map to estimate the site-specific hazards. Low and Hao (2000) incorporated the probabilistic properties of the material geometry and the loading into the analysis of reinforced concrete slabs under blast loading to obtain a realistic estimation of the structural response. The authors represented the structural system by a single degree of freedom (SDOF) system and they used performance criteria which are based on maximum strain limit.

Structural reliability techniques have been utilized in a wide range of civil and mechanical engineering problems. Tandjiria *et al*. (2000) employed response surface methods for the reliability analysis of laterally loaded piles. The authors used the pile head displacement and the maximum bending moment in the pile as the performance criteria of the pile. Basha and Babu (2014, 2009) studied stability of reinforced soil walls. The authors utilized a FORM to determine appropriate ranges for the values of the load and resistance factors. They investigated the effects and interrelations of coefficients of variation of soil parameters in detail.

A probabilistic seismic analysis of a reinforced concrete building was performed by Faggella *et al*. (2013). The authors modeled that building in 3-D and they used the performance based earthquake engineering methods. That study involved a probabilistic seismic hazard analysis and probabilistic structural analysis. They investigated the inter-story drift ratios and they also have observed that the maximum inter-story drift ratios occur at the mid stories, similar to this study. The obtained deformation shapes are typical drift profiles of the framed structures under that kind of horizontal loadings; the mid-story drifts are relatively larger compared to the higher and lower stories. Mahsuli and Haukaas (2013) proposed a regional damage model for risk analysis by employing a collection of interacting

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probabilistic models. For analysis they employed two approaches one of which was a straightforward scenario sampling and the other an algorithm for first and second order reliability methods. They implemented that model to assess the seismic risk of Vancouver region in Canada.

The degradation of structural members due to immediate or time dependent effects is an important phenomenon which can cause misleading about reliability of the structure. Bigaud and Ali (2014) investigated the flexural reliability change in the externally bonded reinforced concrete members and they reported the effect of CFRP strengthening. Muscolino *et al*. (2015) modelled the excitation as a Gaussian random process and presented a procedure for the analytical derivation of interval reliability sensitivity. And also, they analyzed a wind-excited truss structure to show the effectiveness of that procedure. Lately, Kozak and Liel (2015) inspected the reliability of open web steel joists under snow loads through Monte Carlo simulations. The authors modeled the problem in a typical load-resistance (S-R) form for safety and serviceability limit states. In that study lognormal, logistic, log-logistic and type III distributions were assumed for snow loads of different sites. Resistance was assumed to have normal and lognormal distributions and accordingly, reliability indices were obtained for various cases.

Component level reliability of structures has been broadly investigated; however system level reliability studies are quite limited. Choe *et al*. (2008) performed a probabilistic capacity model for RC columns. Quan and Gengwei (2002) investigated reliability index of RC beams for serviceability limit state and Neves *et al*. (2006) performed a reliability analysis of RC beam grids. Additionally, Petryna *et al*. (2002) and Sudret (2008) focused on reliability analysis of degrading RC elements.

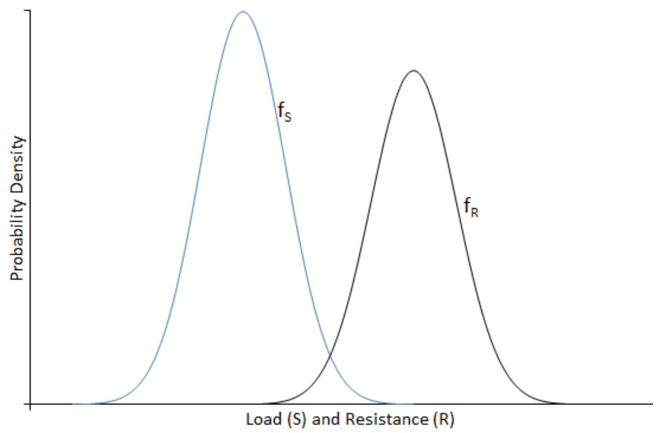
Recently, earthquake performance of buckling restrained braced frames was studied by Asgarian *et al*. (2016). The authors evaluated the mean annual frequency of exceeding immediate occupancy and collapse prevention limit states. Kia and Banazadeh (2016) implemented a fragility analysis for evaluating the vulnerability of steel moment resisting frames. The authors employed the model for two buildings and compared the results to the ones obtained by an incremental dynamic analysis. Additionally, O'Reilly and Sullivan (2016) developed a set of fragility functions for eccentrically braced steel frame structures. In that article it is considered that the damage is directly linked to the inter-story drift demand at each story.

The collapse of steel frame structures under infrequent loadings may lead to unexpected human losses and repair costs. This study attempts to quantify the structural vulnerabilities by employing reliability methods. For the reliability analysis an algorithm has been developed for determination of limit state functions and iterations of the FORM procedure. By the method presented herein the multivariable analysis of a complicated reliability problem is reduced to an S-R problem. The problem is approached in a global structural system level. In order to achieve this, in the limit state function system level parameters (herein inter-story drift ratios) are utilized instead of component level parameters such as stresses, strains, or crack width which are commonly employed in component level analyses. The procedure for iterations has been tested by a known problem for the purpose of avoiding convergence problems. A four-story moment resisting frame is analyzed under characteristic conditions as a typical example.

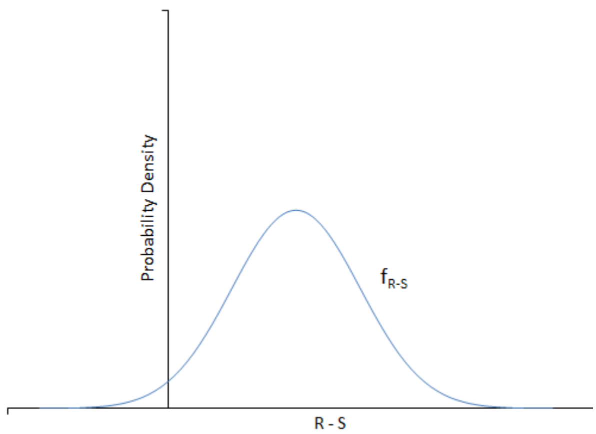
2. Fundamental variables and equations

For engineering structures it is difficult to estimate the failure probability precisely. This difficulty arises from several sources such as realistic sampling, mathematical modeling, performance criteria definition, and evaluation techniques.

The collected information constitutes the sample spaces which are usually described by probability density functions. In the classical approach the statistical parameters of the random variables are assumed to be unknown, but the parameters obtained by sampling are assumed to be constant estimators of those parameters. In order to identify a risk concept for structures, generally a reliability index (or safety index), β, is employed for quantification of the reliability. In this study, this index is calculated via a first-order estimate. Firstly, the perform-



(a) PDFs of *R* and *S*



(b) PDF of a function of *R* and *S*

Fig. 1 The PDFs of the random variables

ance of the structure needs to be explained in terms of structural random variables in a limit state equation form

$$Y = g(X_1, X_2, ..., X_n) \tag{1}$$

This equation signifies that the performance Y is a function of the random variables $X_1, X_2, ..., X_n$ which may represent loads, materials, and any variable that can affect the performance. However, in many cases the exact relation between the performance and the random variables is not known. Therefore, it is much practical to reduce the system to a load-resistance system whenever it is possible.

In Fig. 1(a) the load (S) and resistance (R) are separately shown on a probability density space. A new function *R-S* can be represented by a new PDF as shown in Fig. 1(b).

Traditional deterministic design approaches attain structural safety by shifting the positions of the curves. In that case the safety (the distance between the curves) is obtained in terms of the number of the standard deviations of f_S and f_R which are PDFs of load and resistance, respectively. The load curve is shifted to the right at an amount of $k_S\sigma_S$ and the resistance curve is shifted to the left at an amount of $k_R\sigma_R$, where σ_S and σ_R are the standard deviations and k_S and k_R are the safety coefficients of the standard deviations of load and resistance, respectively. However, by the deterministic safety approach the failure probability is not quantified. A more rational approach would be to compute the risk and select the design variables such that an acceptable risk of failure is achieved. Herein, this is attained through an *S-R* model.

In an *S-R* model a new random variable Z (Fig. 1(b)) can be defined as

$$Z = R - S \tag{2}$$

Negative values of Z simply show that the resistance of the system is less than the load, so these cases indicate the failure of the system. Thus, the failure probability, p_f, of the system can be identified as

$$p_f = P(Z < 0) \tag{3}$$

and assuming that R and S are normal variables and Φ is the cumulative distribution function of the standard normal variable, p_f can be calculated as

$$p_f = 1 - \Phi\left[\frac{\mu_R - \mu_S}{\sqrt{\mu_R^2 + \mu_S^2}}\right] \tag{4}$$

where μ_R and μ_S are the means of resistance and load, respectively. By using the inverse function of Φ it can be obtained that

$$\frac{\mu_R - \mu_S}{\sqrt{\mu_R^2 + \mu_S^2}} = \Phi^{-1}(1 - p_f) \tag{5}$$

Both sides of the above equation are defined as the reliability index, β.

Apart from the above mentioned procedure, for design of new structures, there is also a reverse procedure in which the reliability index is formerly decided. In that kind of approaches, as the target reliability is decided, the aim is to determine the load factor γ and capacity reduction factor ϕ to satisfy the necessary condition

$$\phi R_N > \gamma S_N \tag{6}$$

where R_N and S_N are nominal values of resistance and load, respectively.

In this study the problem is approached in the way that directly searches for the evaluation of reliability index of a structure and the details are presented in the succeeding sections.

3. Load-resistance relations and the statistical parameters

In a typical structural design various combinations of load types are used in order to prevent a possible failure case. It is extremely unlikely that all possible loads act together; therefore, they are fractioned as load combinations to avoid uneconomic design. That fractioning is quite understandable, but if the failure probabilities of the combinations were checked it would be seen that all of the combinations result in different failure probabilities. Even the same combination will result in different failure probabilities when it is checked for different structures. This inconvenience clearly results from the lack of probabilistic concerns during the design process.

In terms of deterministic design, there is a huge canon of both ASD and LRFD methods for design of steel structures (Englekirk 1994, Geschwindner 2008, ENV1993-1-1 1993). There exists a large variety of load combinations and safety factors, depending on the type, region, function, and vitality of the structure. The nominal loads for a wide variety of structures and cases are presented in ASCE/SEI 7-10 (2010).

In the framework of performance estimation by probabilistic methods, the statistical characteristics of load and resistance values, in addition to the nominal values, are very essential. There is a wide variety of distribution types and distribution parameters reported in the open literature (Ellingwood *et al*. 1980, Romao *et al*. 2011) and a summary is presented in Table 1. It should be underlined that the presented loads are code and site dependent, and in different studies they have been reported differently in terms of both intensities and statistical characteristics. Therefore, this set is just a selection from a huge set of conceivable values.

Similar to the load characteristics, the resistance characteristics should also be determined in terms of statistical parameters in order to be able to perform a reliability analysis. For resistance parameters a demon-trative data set from Ellingwood *et al*. (1980) is presented in Table 2. The values presented in Table 2 are component-level data and there is no particular data for a system of moment resisting frames. As the limit functions of the analyses performed for this study depend on lateral deflection of the frames, it is reasonable to use the data reported for steel columns.

Table 1 Typical load statistics

Load parameter	Nominal value (S)	$\bar{S}/S$	V_S	Distribution type
Dead (*D*), (pressure)	3 kN/m^2	1.05	0.07	normal
Live (*L*), (pressure)	2 kN/m^2	0.24	0.80	normal
Snow (*S*), (pressure)	0.75 kN/m^2	0.82	0.26	normal
Wind (*W*), (pressure)	0.5 kN/m^2 (see Appendix)	0.33	0.59	lognormal
Earthquake (*E*), (acceleration, g)	2.5 g (see Appendix)	0.64	1.38	lognormal

* $\bar{S}$ is the mean value of the load and V_S is the coefficient of variation of the load)

Table 2 Typical resistance statistics for steel structures [Data from Ellingwood *et al.* (1980)]

Resisting system	$\bar{R}/R$	V_R
Tension members (limit state: yielding)	1.05	0.11
Tension members (limit state: tensile strength)	1.10	0.11
Compact beam (uniform moment)	1.07	0.13
Axially loaded column	1.08	0.14

* $\bar{R}$ is the mean value of the resistance and V_R is the coefficient of variation of the resistance)

4. The limit state functions

In reality, it is very difficult to predict what kind of a behavior a building will exhibit under a certain level of forces. This is because there are many factors that may affect the behavior and response of a building such as the stiffness of structural elements, the strength of building components, and even the quality of construction that cannot be precisely defined. Furthermore, the analysis procedures used to predict building response are not totally perfect. Under these conditions, it is not appropriate to indicate that the performance can be predicted in an absolute sense. Thus, the reliability-based probabilistic approach presented herein for performance evaluation, explicitly recognizes these inherent uncertainties.

Theoretically the reliability of a structure is calculated through already-known performance functions. However, for real structures the response is computed through a numerical modelling, mostly by a finite element method. Therefore, for realistic cases there is no available performance function $g(X)$ in a closed-form function as a function of input random variables.

One of the most initial steps of evaluating the reliability or probability of failure of a structure is to decide on the specific performance criteria and the relevant load and resistance parameters. The basic variables, X_i, and the functional relationships among them corresponding to each performance criterion form the performance function as expressed in Eq. (1) in a generalized form and expressed in Eq. (2) in a specified form. Respectively, the failure surface (or the limit state) can be defined as

$$Z = 0 \tag{7}$$

For a conventional resistance-load problem the failure surface, $Z = g(S, R) = 0$, can be represented by a typical *S*-*R* curve as in Fig. 2. The space divided by the $g(S, R) = 0$ curve has two separate regions which represent safe and unsafe regions and the economically safe engineering design point lies on that curve.

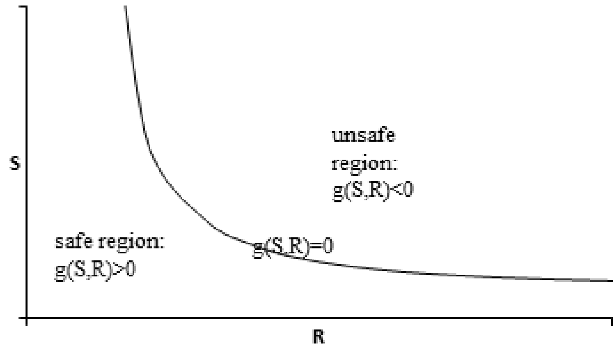


Fig. 2 The limit state function for a conventional *S*-*R* problem

5. Structural performance criteria as a limit for the performance function

For structures a set of plausible ultimate limit states must be identified in order to have a basis for reliability measurements. However, it is difficult to identify exactly at which mechanical state a structure or a structural member can be assumed as failed. Fortunately, in literature there are available, although limited, studies for various cases. Kanda *et al.* (1997) presented limit states for steel, reinforced concrete, and steel – reinforced concrete structures in terms of shear strain, interstory deflection angle, and cumulative plastic deformation ratio as follows:

- Reinforced concrete wall: Shear strain = 5.0 × 10-3
- Reinforced concrete moment resisting frame: Interstory deflection angle = 1/50
- Steel – reinforced concrete moment resisting frame: Interstory deflection angle = 1/30
- Steel moment resisting frame: Cumulative plastic deformation ratio = 14

The above mentioned limitations consider the ultimate state as the state that the structure loses its structural integrity. This approach is the widest one used for deterministic design purposes. But, considering the variety of the service levels, it is also a quite deficient one. As the performance based design methods have been widespread, the classification of limit states (performance level) has also been varied (Grecea *et al*. 2004, Ghobarah 2001, ATC 1997, FEMA-350 2000). Generally, the performance of MR steel frames is evaluated according to three limit states; serviceability limit state (SL), damageability limit state (DL), and ultimate limit state (UL).

The serviceability limit state is associated with light damage in structural and non-structural elements. In this level of damage the continuity of building service is essential. After slight restorations the building is expected to be ready to function normally. This level of performance is assumed to be achieved as long as the drift ratio does not exceed 0.6%. In damageability limit state the damage to the structure is considerably high, but the structure is repairable. The structure can be conserved after performing strengthening facilities. A drift ratio of 1% is a commonly accepted estimator for this level of damage. The ultimate limit state is defined as the state that the structure cannot be repaired and demolition is unavoidable, but life safety is still guaranteed. A drift ratio of 3% is accepted as an estimator for this level of damage.

The design objectives in current building codes are commonly associated with the abovementioned performance criteria. However, there are differing view-points on the meaning of performance limit, and additionally, the actual reliability of the design is not known (Grecea *et al*. 2004, Ghobarah 2001). The safety factors of the design codes are roughly considered as measures of reliability. Yet, a particular mechanical limit of a performance criterion can be used as an ultimate limit for forming a performance function. In this study, the drift ratio limitations are converted to case-specific limit drifts and are used as limit states for performance functions. The details of using the limit drifts in a performance function are presented under the heading 8. The performance functions.

The reliability of a structure can be investigated at two different levels: individual performance and overall system performance which are also referred to as component-level reliability and system-level reliability, respectively. The measures given in Table 3 are for system-level reliability. Initially they are given in a drift ratio form which is an indicator of overall structural performance. Multiplying those values by the story height the case-specific drifts are obtained for the frame and these drifts are used as performance limits.

6. The structural model

For the analyses a typical frame with three bays and four stories is considered. The bay width is 5000 mm and the

Table 3 The inter-story drift limits for corresponding performance levels

	SL	DL	UL
Limit drift ratio (%)	0.006	0.01	0.03
Limit drift Δ (mm)	18	30	90

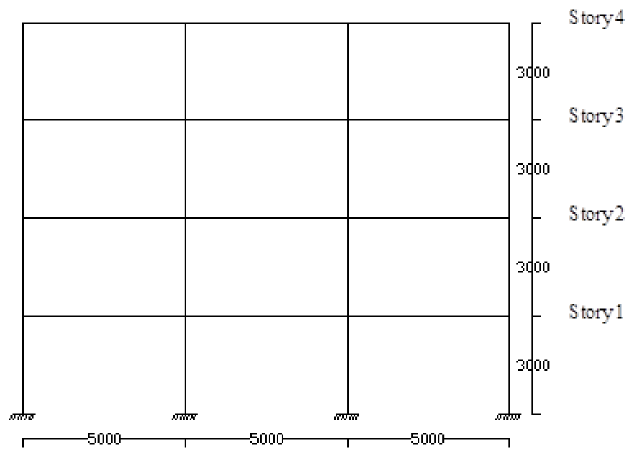


Fig. 3 The geometry of the analyzed frame (unit: mm)

Table 4 Cross-sectional properties of the column and beam members (unit: mm)

	W16X36	W12X30
Total depth	403.9	312.4
Top flange width	177.5	165.6
Top flange thickness	10.9	11.2
Web thickness	7.5	6.6
Bottom flange width	177.5	165.6
Bottom flange thickness	10.9	11.2

presented in Fig. 3. This frame is selected with a purpose such that the reliability analyses give distinctive results for different performance levels. If it were very safe the reliability for all performance levels would be very high and similar.

Wide flange steel sections are used as beam and column elements. The cross-section of the column elements is W16X36 and of the beam elements is W12X30. The cross-sectional properties of the columns and beams are presented in Table 4. The modulus of elasticity of the considered steel material, E, is 200 GPa and the Poisson's ratio, ν, is 0.3. Also, the minimum yield stress, F_y, is 344 MPa, and the minimum tensile strength, F_u, is 448 MPa.

7. Methodology: FOSM and AFOSM (Hasofer-Lind) methods

First order methods are basically extensions of mean value methods which are FOSM (first order second moment) MVFOSM (mean value first order second moment). Assuming that R and S are normally distributed independent variables, the variable $Z(R, S)$ is also normally distributed. As the failure event is defined by $R < S$ or $Z < 0$, the reliability index β can be calculated by

$$\beta = \frac{\mu_Z}{\sigma_Z} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{8}$$

and accordingly the probability of failure can be determined by

$$p_f = \Phi(-\beta) = 1 - \Phi(\beta) \tag{9}$$

A similar procedure may also be utilized for lognormal random variables. In that case the performance function can be expressed by a new random variable Y

$$Y = \frac{R}{S} \tag{10}$$

and the performance function can be constructed as

$$Z = \ln Y = \ln R - \ln S \tag{11}$$

The probability of failure for lognormal variables is defined as

$$p_f = 1 - \Phi\left(\frac{\lambda_R - \lambda_S}{\sqrt{\zeta_R^2 + \zeta_S^2}}\right) \tag{12}$$

where

$$\lambda = \ln \mu - 0.5\zeta^2 \tag{13}$$

and

$$\zeta^2 = \ln\left(1 + \delta^2\right) \tag{14}$$

Using Eqs. (13) and (14), Eq. (12) can be rewritten as follows

$$p_f = 1 - \Phi\left[\frac{\ln\left[\left(\frac{\mu_R}{\mu_S}\right)\sqrt{\frac{1+\delta_S^2}{1+\delta_R^2}}\right]}{\sqrt{\ln\left[\left(1+\delta_R^2\right)\left(1+\delta_S^2\right)\right]}}\right] \tag{15}$$

For practical engineering purposes, assuming that δ_R and δ_S are small, Eq. (15) can be simplified as

$$p_f = 1 - \Phi\left[\frac{\ln\left(\frac{\mu_R}{\mu_S}\right)}{\sqrt{\delta_R^2 + \delta_S^2}}\right] \tag{16}$$

and accordingly the reliability index β is defined as

$$\beta = \frac{\ln\left(\frac{\mu_R}{\mu_S}\right)}{\sqrt{\delta_R^2 + \delta_S^2}} \tag{17}$$

This procedure can be generalized for multiple random variable cases. The performance function can be denoted by a vector X

$$Z = g(X) = g(X_1, X_2, X_3, ..., X_n) \tag{18}$$

By using the Taylor series expansion of the performance function Z and truncating the series at the linear terms the first-order mean, μ_z, and variance, σ_Z^2, of Z can be approximated as

$$\mu_Z = g\left(\mu_{X_1}, \mu_{X_2}, \mu_{X_3}, ..., \mu_{X_n}\right) \tag{19}$$

and

$$\sigma_Z^2 = \sum_{i=1}^{n}\sum_{j=1}^{n} \frac{\partial g}{\partial X_i}\frac{\partial g}{\partial X_j} COV\left(X_i, X_j\right) \tag{20}$$

respectively. If the variables are statistically independent, which is the case herein, the last expression can be reduced to

$$\sigma_Z^2 = \sum_{i=1}^{n}\left(\frac{\partial g}{\partial X_i}\right)^2 VAR\left(X_i\right) \tag{21}$$

By utilizing Eqs. (19)-(21) the reliability of the system can be determined simply by using Eq. (8).

Although the FOSM method is relatively simple, it has some deficiencies. First of all, it does not use the statistical distribution characteristics. Secondly, the basic reliability expression, Eq. (8), doesn't give the same result for mechanically equivalent performance functions. Therefore, an Advanced-FOSM (AFOSM or HL) method was proposed by Hasofer and Lind (1974) to overcome that deficiency. According to the method offered by Hasofer and Lind (1974) the assessment of the reliability index, β_{HL}, is based on the reduction of the problem standardized coordinate system. The Hasofer Lind reliability index measures the distance from the expectations of the resistance and load (R-S) random variables to the unsafe region in a way that is independent of a particular choice of the performance function. Hence, the invariance can be accomplished, and the reliability index for different formulations of the same problem will not be different.

The direct form of a HL method can be applied to normal random variables after initiating the reduced variables

$$X_i' = \frac{X_i - \mu_{X_i}}{\sigma_{X_i}} \tag{22}$$

and by using this equation the limit state equation

$$g(X) = 0 \tag{23}$$

is transformed into a reduced limit state equation

$$g(X') = 0 \tag{24}$$

Hasofer Lind Method utilizes analytical equations and uses directional cosines to determine the shortest distance to the multi-dimensional failure surface. That shortest distance corresponds to the reliability index β. In literature this is widely referred to as Hasofer-Lind's reliability and is shown by β_{HL}. In the reduced system β_{HL} is defined as the minimum distance from the origin to the line or surface identified by the limit state equation. That distance is expressed by

$$\beta_{HL} = \sqrt{\left(x'^*\right)^T \left(x'^*\right)} \tag{25}$$

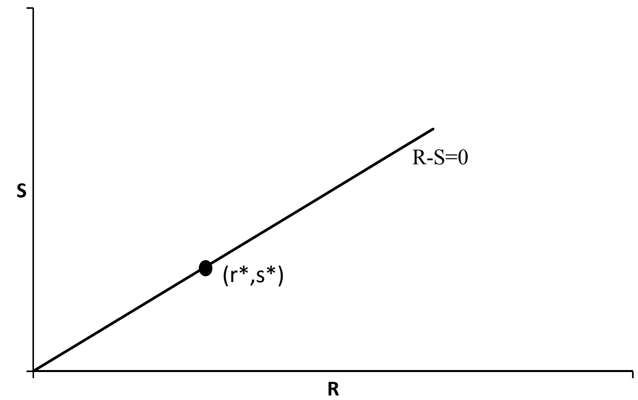


Fig. 4 The original limit state function

For an *S*-*R* problem the point at which the minimum distance is obtained is called the design point (r^*, s^*) and located on the limit state function, $Z = R - S = 0$ line (Fig. 4). Its position vector x^* is transformed into the new position vector x'^* in the reduced system which is obtained by utilizing Eq. (22) for R and S respectively as

$$R' = \frac{R - \mu_R}{\sigma_R} \tag{26}$$

$$S' = \frac{S - \mu_S}{\sigma_S} \tag{27}$$

The length of x'^* vector is equal to the reliability index β_{HL} as shown in Fig. 5.

$$\beta_{HL} = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{28}$$

If the method is generalized, then the probability function should consider the possibility of many random variables in original and transformed coordinates as in Eq. (29) and Eq. (30), respectively.

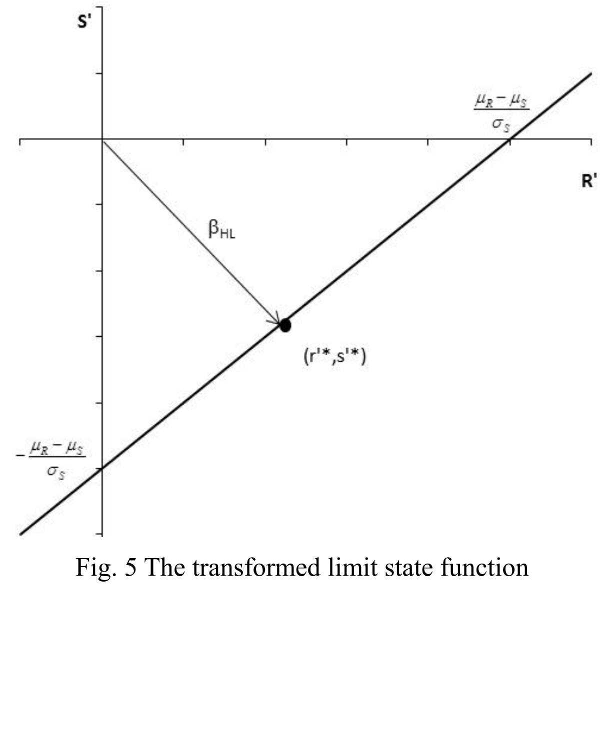


Fig. 5 The transformed limit state function

$$\mathrm{X} = (\mathrm{X}_1, \mathrm{X}_2, \mathrm{X}_3, ..., \mathrm{X}_n) \tag{29}$$

$$\mathrm{X}' = (\mathrm{X}'_1, \mathrm{X}'_2, \mathrm{X}'_3, ..., \mathrm{X}'_n) \tag{30}$$

Also, nonlinear cases of the limit state function similar to Fig. 6 are very likely in structural problems. However, the length of the shortest position vector is not as simple as the linear case presented in Fig. 5. For such functions Haldar and Mahadevan (2000) proposes the use of Lagrange multipliers and via that method the reliability index can be obtained as

$$\beta_{HL} = -\frac{\sum_{i=1}^{n} x_i'^* \left(\frac{\partial g}{\partial X_i'}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'}\right)^{2*}}} \tag{31}$$

All the partial derivatives in Eq. (31) are evaluated at the design point in the transformed coordinate system, and the coordinates of the design point are

$$x_i'^* = -\alpha_i \beta_{HL} \qquad (i = 1, 2, 3, ..., n) \tag{32}$$

where the direction cosines, α_i, are defined as

$$\alpha_i = \frac{\left(\frac{\partial g}{\partial X_i'}\right)^*}{\sqrt{\sum_{i=1}^{n} \left(\frac{\partial g}{\partial X_i'}\right)^{2*}}} \tag{33}$$

Correspondingly, employing Eq. (22) the coordinates of the design point in the original system can be determined as

$$x_i^* = \mu_{X_i} - \alpha_i \sigma_{X_i} \beta_{HL} \tag{34}$$

In order to utilize the above-mentioned Hasofer-Lind procedure all the random variables should be statistically independent with a normal distribution. If not all the random variables are normally distributed the nonnormal random variables need to be transformed into equivalent normal variables. For this purpose several transformation

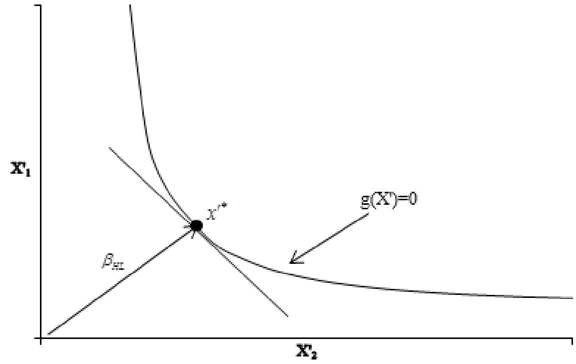


Fig. 6 The transformed two-variable nonlinear limit state function

methods have been suggested in the literature. Rosenblatt (1952), Nataf (1962), Chen and Lind (1983), and Rackwitz and Fiessler 1978) have been widely recognized for their contributions in this area. In this paper, for the transformation of log-normal random variables to equivalent normal random variables the Rackwitz and Fiessler method is employed.

8. The performance functions

In this study the performance function is formed in an R-S problem character. Firstly, it is assumed that the performance function is a function of the inter-story drift, δ_i (subscript i is for the story number), and the inter-story drift limit, Δ. For example, for a certain limit state such as serviceability limit state (*SL*) the limit performance is Δ_{SL}. Now, S is fixed as Δ_{SL}, and the probability of failure corresponding to the SL state is the probability that the inter-story drift ratio, δ_i, passes Δ_{SL}. Hence, the performance criterion for the reliability of the system can be set as

$$Z = g(R,S) = R - S > 0 \Rightarrow (\delta - \Delta) = (\delta_i - \Delta_{SL}) > 0 \quad (35)$$

The limit states and corresponding drift limits regarding to the performance levels considered in this study are given in Table 3. However, the function expressed by Eq. (35) cannot be instantly proceeded with because the statistical information about the inter-story drift, δ_i, is not available. However, it is a function of the load, *F*, and the structural displacement stiffness, *K*. Fortunately, there is statistical information, though limited, both for loads and structural members reported in literature. The details of the forces and their statistical parameters are presented in Table 1 and the Appendix. The details of the resistance *K* are presented in Table 5.

Additionally, it should be indicated that *K* is assumed to be a normally distributed random variable having a coefficient of variation of 0.14.

Hence, putting δ_i in the form

$$\delta_i = F \times K_i \quad (36)$$

will make it possible to perform a reliability analysis.

To obtain K_i, the intensity of the nominal load is replaced by a unit load (1) and correspondingly the story displacements, d_i, are obtained (Fig. 7). Hence, $d_i - d_{i-1}$ values represent the structural inter-story drift stiffnesses, K_i.

Now, Eq. (36) can directly be used in the reliability analysis in the form of Eq. (37), because the statistical data of all the variables are available.

$$F \times K_i - \Delta = 0 \quad (37)$$

In order to clarify the algorithm performed herein all the employed procedure is summarized in a flowchart (Fig. 8).

Table 5 Nominal inter-story drift ratio stiffnesses for the earthquake (*E*) and wind (*W*) load distributions

Story	*E* stiffness	*W* stiffness
K_1	2.64	0.72
K_2	4.00	0.96
K_3	3.40	0.76
K_4	2.12	0.36

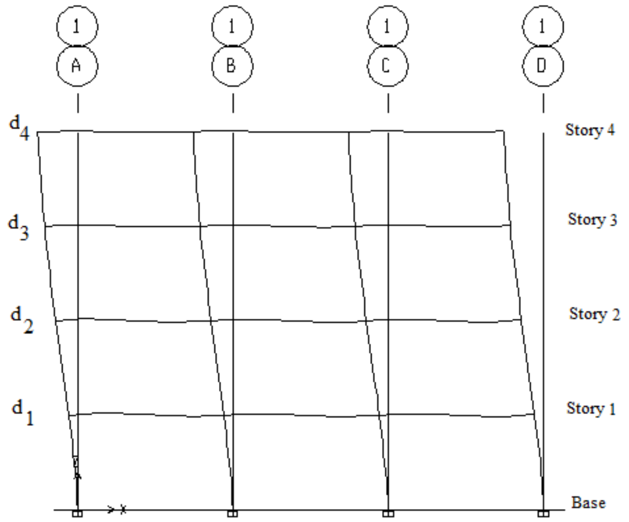


Fig. 7 A representational displacement shape of the frame

9. Results and discussion

The reliability analyses have been performed for a moment resisting frame under two typical load configurations: earthquake and wind loads. The selection of these forces basically results from the fact that the failure or performance criteria of framed structures are dominantly governed by horizontal forces and that the structural limits presented in Table 3 theoretically depend on horizontal forces.

The structural performance of a frame is expressed in terms of inter-story drifts. For determining the performance functions regarding to the performance levels, which are presented in Table 3, structural analyses have been performed. The obtained inter-story drifts are converted to inter-story drift ratios and presented in Fig. 9. Faggella *et al*. (2013) obtained very similar inter-story drift profiles for a framed RC building subjected to earthquake loads. An instant investigation of Table 5 and Fig. 9 (as a hint for reliability index trends) shows that a failure regarding to earthquake is much probable and that the most critical reliability indices will be obtained for second story for both cases.

After obtaining the performance function, the procedure for reliability analysis is an iterative process, therefore a problem which was previously investigated by Haldar and Mahadevan (2000) and Ellingwood *et al*. (1980) is also analyzed in order to understand if there is a convergence or calculation problem in the algorithm (Table 6). The results of the present study are in a step-by-step agreement with the results of Haldar and Mahadevan (2000). The procedure of Ellingwood *et al*. (1980) is different and it is completed in

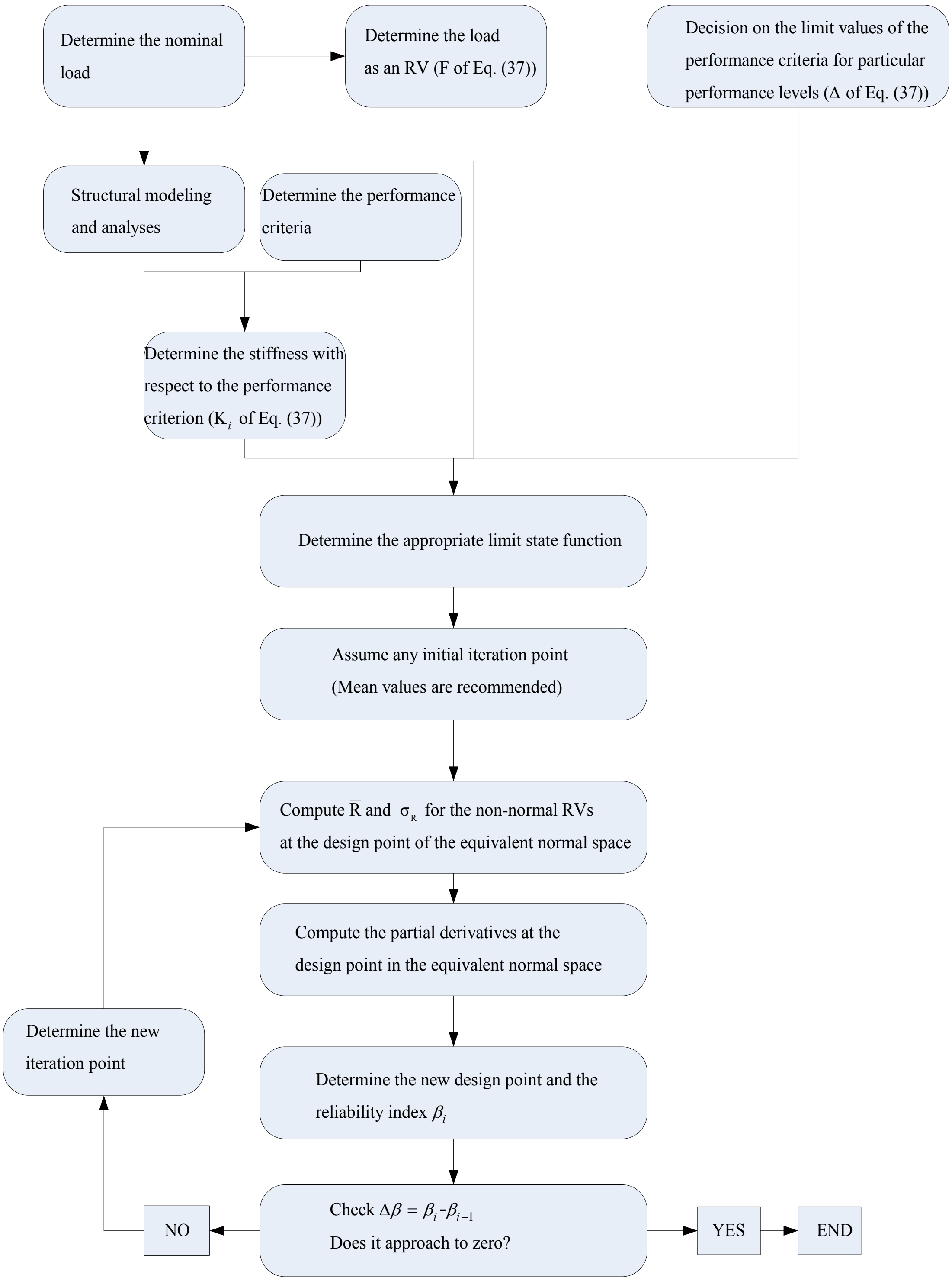


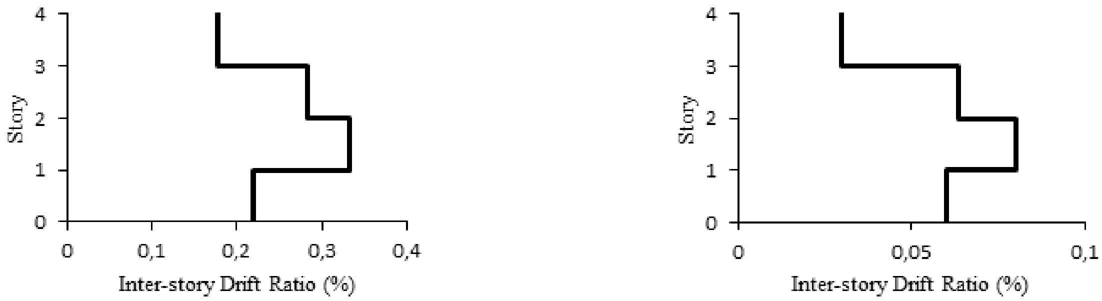
Fig. 8 The algorithm flowchart for the procedure

three steps; therefore there is no stepwise agreement. But, the final steps of all three studies are very close and the difference of the result of Ellingwood *et al*. (1980) is 0.132%.

The reliability analyses were initially performed for two load cases, three performance levels, and four different story levels. The reliability indices of the wind loading case are 5.5 to 6 times higher than that of the earthquake loading. This result is in agreement with the fact that the failures

Table 6 Comparison of the performance of the algorithm for the problem $g(F, Z) = FZ - 1140$, which was investigated by Haldar and Mahadevan (2000) and Ellingwood *et al.* (1980)

Step number	1	2	3	4
β, Present	3.9386	5.1455	5.1509	5.1508
β, Haldar and Mahadevan (2000)	3.939	5.145	5.151	5.151
β, Ellingwood *et al*. (1980)	5.001	5.136	5.144	-



(a) Earthquake load (b) Wind load

Fig. 9 The inter-story drift ratios obtained for the nominal loads

caused by earthquakes are much more probable compared to the failures caused by wind loads. Most critical results are obtained from the second story and the safest results are obtained from the fourth story; this issue is valid for both earthquake and wind load cases. The ratio of highest to the lowest reliability indices is 1.43 which is calculated for earthquake load performance case at serviceability limit (SL) state. The same ratio for wind load performance case is relatively low (1.21) and it is again obtained for the SL state. From these results it can be deduced that for low

Table 7 Results of the reliability analyses for the earthquake loading

	Limit (mm)	β (story 1)	β (story 2)	β (story 3)	β (story 4)
SL	18	1,8298	1,4305	1,5865	2,0401
DL	30	2,3204	1,9209	2,0770	2,5306
UL	90	3,3756	2,9760	3,1321	3,5859

Table 8 Results of the reliability analyses for the wind loading

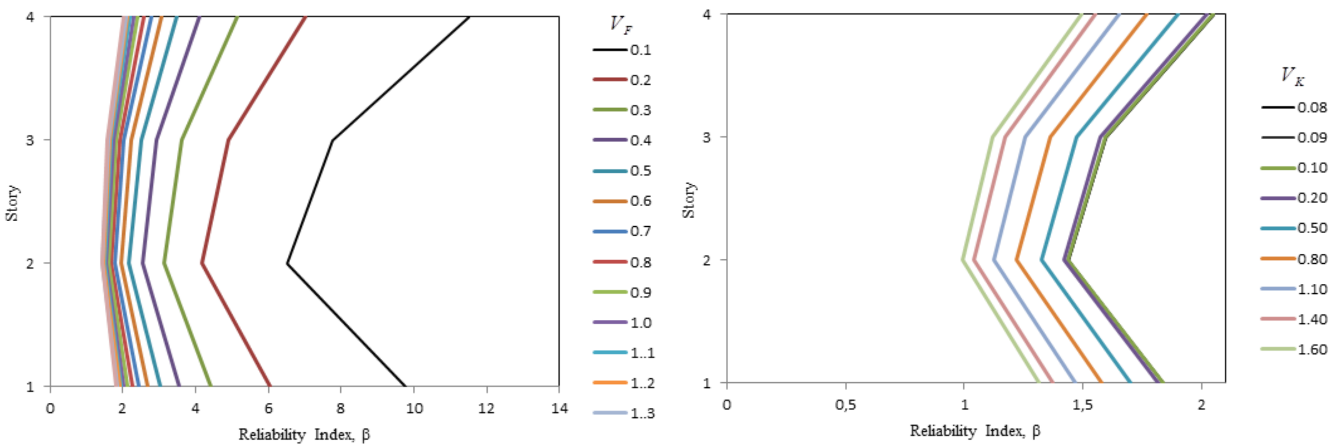
	Limit (mm)	β (story 1)	β (story 2)	β (story 3)	β (story 4)
SL	18	9,0877	8,5721	8,9908	10,3309
DL	30	10,0038	9,4878	9,9068	11,2478
UL	90	11,9759	11,4592	11,8788	13,2213

reliability index cases the drift stiffness differences between the stories is more critical. The reliability indices obtained from the analyses are tabulated in Tables 7 and 8.

Another set of analyses has been performed for investigating the effect of the coefficient of variation of the variables of the performance function (Eq. (37)). The coefficient of variation of the lognormal random variable F, V_F, was considered for a range between 0.1 and 1.3 (Fig. 10(a)). The calculations were performed only for serviceability limit – SL performance level (Δ = 18 mm), and in order to easily monitor the effect of V_F, during the calculations V_K was kept constant as 0.14. Similarly, the effect of the coefficient of variation of the normally distributed random variable K, V_K, was considered for a range between 0.08 and 1.60 (Fig. 10(b)), and the calculations were performed for SL performance level. This time V_F was kept constant as 1.38. The analyses show that the variation of V_F is significantly more effective between 0.1 and 0.3. For the variation V_K that kind of a range is not observed. Additionally, it should be noted that the effect of coefficient of variation is similar at all of the stories and for both cases.

10. Conclusions

- The proposed reliability algorithm employs FORM methods which enable to emphasize the impact of



(a) Effect of V_F (b) Effect of V_K

Fig. 10 The effect of the coefficient of variations on the reliability index the considered random variables. Although the procedure presented herein is focused on moment resisting steel frames, it can be adapted for all framed structures for which there exists a drift based signifier of performance levels.

- For low reliability index cases the drift stiffness differences between the stories is more critical. Among the stories the highest reliability is obtained for the story 4 which has the lowest drift stiffness.
- The effect of coefficient of variation is similar to each other at all the stories for both of the loading conditions.
- If a single stiffness indicator is to be used for calculating reliability, then using the stiffness of the story which has the highest inter-story drift ratio would be on the safe side. Although, for framed structures that story is generally one of the mid-stories, it doesn't always have to be, because it highly depends on loading and structural element characteristics.
- Generally, reliability of a structure is treated for a single performance function. This arises from the idea of making reliability calculations practically applicable to engineering problems. But, it should be noted that there may exist multiple performance functions for the same problem depending on the point of view. For example, the reliability indexes that obtained for a strength limit and that for a serviceability limit would probably be different. Additionally, the reliability of the results highly depends on the available data sets, parameters of the distribution function, idealization of the load process, and failure mode idealization.
- By using the proposed procedure the reliability indices of steel frames can be calculated, however, extensive studies are needed for obtaining common agreements on target reliability indices. In order to have confident reliability approaches, target reliability indices are crucial. Yet, in the current literature there is no complete work for the broad variety of different cases.
- Another inadequate item in the literature is the relation between performance criteria and failure probability. Also, the probabilistic relation between structural element behavior and the total structural behavior requires extensive research.

References

Ang, A.H.S. and Tang, W.H. (2007), *Probability Concepts in Engineering Emphasis on Applications to Civil and Environmental Engineering*, (2nd Edition), John Wiley and Sons, New York, NY, USA.

Applied Technology Council (ATC) (1997), NEHRP Guidelines for Seismic Rehabilitation of Buildings; Report No. FEMA-273, Federal Emergency Management Agency, Washington, DC, USA.

ASCE/SEI 7-10 (2010), Minimum Design Loads for Buildings and Other Structures; Structural Engineering Institute of ASCE, Reston, VA, USA.

Asgarian, B., Golsefidi, E.S. and Shokrgozar, H.R. (2016), "Probabilistic seismic evaluation of buckling restrained braced frames using DCFD and PSDA methods", *Earthq. Struct., Int. J.*, **10**(1), 105-123.

Basha, B.M. and Babu, G.L.S. (2009), "Optimum design for external seismic stability of geosynthetic reinforced soil walls: Reliability based approach", *J. Geotech. Geoenviron. Eng. ASCE*, **136**(6), 797-812.

Basha, B.M. and Babu, G.L.S. (2014), "Reliability-based load and resistance factor design approach for external seismic stability of reinforced soil walls", *Soil Dyn. Earthq. Eng.*, **60**, 8-21.

Bigaud, D. and Ali, O. (2014), "Time-variant flexural reliability of RC beams with externally bonded CFRP under combined fatigue-corrosion actions", *Reliab. Eng. Syst. Saf.*, **131**, 257-270.

Chen, X. and Lind, N.C. (1983), "Fast probability integration by three-parameter normal tail approximation", *Struct. Saf.*, **1**(4), 269-276.

Choe, D.E., Gardoni, P., Rosowsky, D. and Haukaas, T. (2008), "Probabilistic capacity models and seismic fragility estimates for RC columns subject to corrosion", *Reliab. Eng. Syst. Saf.*, **93**(3), 383-393.

DBYBHY-2007 (2007), Turkish Seismic Design Code for Buildings, Specification for Structures to be built in Disaster Areas; Institute of Turkish Standard, Ankara, Turkey.

Ellingwood, B., Galambos, T.V., MacGregor, J.G. and Cornell, C.A. (1980), *Development of probability based load criterion for American National Standard A58*; NBS Special Publication 577, National Bureau of Standards, United States Department of Commerce, Washington, DC, USA.

Ellingwood, B.R. and Kanda, J. (Editors) (2005), *Structural Safety and its Quality Assurance*, ASCE, VA, USA.

Englekirk, R. (1994), *Steel Structures Controlling Behavior Through Design*; John Wiley and Sons, New York, NY, USA.

ENV1993-1-1 (1993), Eurocode 3, Design of steel structures – Part 1-1: General rules and rules for buildings.

Faggella, M., Barbosa, A.R., Conte, J.P., Spacone, E. and Restrepo, J.I. (2013), "Probabilistic seismic response analysis of a 3-D reinforced concrete building", *Struct. Saf.*, **44**, 11-27.

FEMA-350 (2000), Recommended seismic design criteria for new steel moment frame buildings; FEMA-273, Federal Emergency Management Agency, Washington, DC, USA.

Geschwindner, L.F. (2008), *Unified Design of Steel Structures*, John Wiley and Sons, NJ, USA.

Ghali, A., Neville, A.M. and Brown, T.G. (2009), *Structural Analysis a Unified Classical and Matrix Approach*, (6th Edition), Taylor and Francis, New York, NY, USA.

Ghobarah, A. (2001), "Performance-based design in earthquake engineering: State of development", *Eng. Struct.*, **23**(8), 878-884.

Grecea, D., Dinu, F. and Dubina, D. (2004), "Performance criteria for MR steel frames in seismic zones", *J. Constr. Steel Res.*, **60**(3), 739-749.

Haldar, A. and Mahadevan, S. (2000), *Probability, Reliability and Statistical Methods in Engineering Design*, John Wiley and Sons, New York, NY, USA.

Hasofer, A.M. and Lind, N.C. (1974), "An exact and invariant first-order reliability format", *J. Eng. Mech. Div.* (*ASCE*), **100**, 111-121.

Kanda, J., Iwasaki, R., Kobayashi, H. and Ellingwood, B.R. (1997), "Probability-based seismic safety evaluation of existing buildings", *Eng. Struct.*, **19**(9), 708-717.

Kia, M. and Banazadeh, M. (2016), "Closed-form fragility analysis of the steel moment resisting frames", *Steel Compos. Struct., Int. J.*, **21**(1), 93-107.

Kozak, D.L. and Liel, A.B. (2015), "Reliability of steel roof structures under snow loads", *Struct. Saf.*, **54**, 46-56.

Low, H.Y. and Hao, H. (2000), "Reliability analysis of reinforced concrete slabs under explosive loading", *Struct. Saf.*, **23**(2), 157-178.

Mahsuli, M. and Haukaas, T. (2013), "Seismic risk analysis with reliability methods, part I: Models", *Struct. Saf.*, **42**, 54-62.

Melchers, R.E. (1999/2002), *Structural Reliability Analysis and Prediction*, (2nd Edition), John Wiley and Sons, Chichester, UK.

Muscolino, G., Santoro, R. and Sofi, A. (2015), "Explicit reliability sensitivities of linear structures with interval uncertainties under stationary stochastic excitation", *Struct. Saf.*, **52**, 219-232.

Nataf, A. (1962), "Détermination des distributions de probabilités dont les marges sont données", *C. R. Acad. Sci.*, **225**, 42-43.

Neves, R.A., Chateauneuf, A., Venturini, W.S. and Lemaire, M. (2006), "Reliability analysis of reinforced concrete grids with nonlinear material behavior", *Reliab. Eng. Syst. Saf.*, **91**(6), 735-744.

O'Reilly, G.J. and Sullivan, T.J. (2016), "Fragility functions for eccentrically braced steel frame structures", *Earthq. Struct., Int. J.*, **10**(2), 367-388.

Petryna, Y.S., Pfanner, D., Stangenberg, F. and Kratzig, W.B. (2002), "Reliability of reinforced concrete structures under fatigue", *Reliab. Eng. Syst. Saf.*, **77**(3), 253-261.

Quan, Q. and Gengwei, Z. (2002), "Calibration of reliability index of RC beams for serviceability limit state of maximum crack width", *Reliab. Eng. Syst. Saf.*, **75**(3), 359-366.

Rackwitz, R. and Fiessler, B. (1978), "Structural reliability under combined random load sequences", *Comput. Struct.*, **9**(5), 489-494.

Romao, X., Delgado, R. and Costa, A. (2011), "Assessment of the statistical distributions of structural demand under earthquake loading", *J. Earthq. Eng.*, **15**(5), 724-753.

Rosenblatt, M. (1952), "Remarks on a multivariate transformation", *Ann. Math. Stat.*, **23**(3), 470-472.

Schneider, J. (2006), *Introduction to Safety and Reliability of Structures. Structural Engineering Documents No. 5*; (2nd Ed.), IABSE, Zürich, Switzerland.

Sudret, B. (2008), "Probabilistic models for the extent of damage in degrading reinforced concrete structures", *Reliab. Eng. Syst. Saf.*, **93**(3), 410-422.

Tandjiria, V., The C.I., and Low, B.K. (2000), "Reliability analysis of laterally loaded piles using response surface methods", *Struct. Saf.*, **22**(4), 335-355.

TS498 (1997), Design loads for buildings, Institute of Turkish Standards; Ankara, Turkey.

CC

Appendix. Nominal earthquake and wind loads

Earthquake load

The nominal earthquake loads and their distribution on the structure depend on the seismic zone, mass, and dynamic properties of the structure. Similarly, the nominal wind load of the structure and its distribution on the structure depend on the local wind characteristics of the construction site and the area of the structure which is subjected to wind pressure. The earthquake load and wind load calculations for the considered frame are as follows:

The mass of each story was assumed as;

m_1 = 33 t, m_2 = 33 t, m_3 = 33 t, m_4 = 27.5 t, and the total mass M = 126.5 t.

The total base shear of the frame can be calculated as

$$V_b = Mg \frac{A_0 IS}{R_D} \tag{A1}$$

where g is the gravitational acceleration A_0 is the ground acceleration coefficient, I is a building importance factor, S is the spectral acceleration coefficient, and R_D is the ductility reduction factor.

The acceleration spectrum used for earthquake loads (E) is presented in Fig. A1. With a dynamic analysis the period of the frame was obtained as 0.79 s and therefore, the spectral acceleration coefficient was chosen as 2.5. The effective ground acceleration coefficient was chosen as 0.4, and the building importance factor was chosen as 1. For ductile steel frames the seismic load reduction factor can be chosen as 8 (DBYBHY-2007 2007). Therefore, the total base shear was V_b = 155.12 kN.

The distribution of the base shear to the stories is applied by the following approximate distribution rule

$$F_i = V_b \frac{w_i H_i}{\sum_{j=1}^{N} w_j H_j} \tag{A2}$$

where w is the weight of the story and H is the height of that story. Thus, the distribution which is presented in Fig. A2 was obtained.

The statistical properties indicated for the earthquake load intensity should be attributed to the acceleration shown in Fig. A1. Therefore, in order to obtain the inter-story drift ratio stiffnesses the earthquake load was normalized by 2.5, which is the spectral acceleration coefficient of the considered frame. Thus, for the earthquake load reliability the random variable F of Eq. (37) was assumed to be a lognormal distribution having a nominal value of 2.5 and a mean value of 1.6 (Table 1) with a coefficient of variation of 1.38. Accordingly, the inter-story drifts were obtained for a unit acceleration and these drifts are the stiffnesses for the

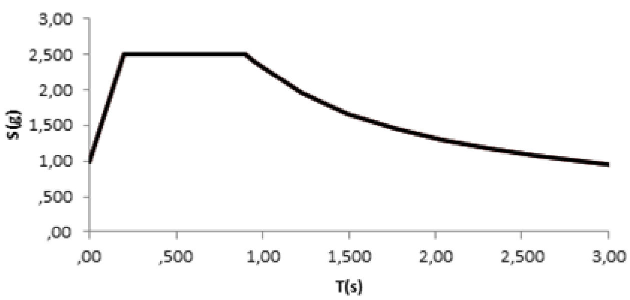


Fig. A1 Period-acceleration spectrum used for the considered frame

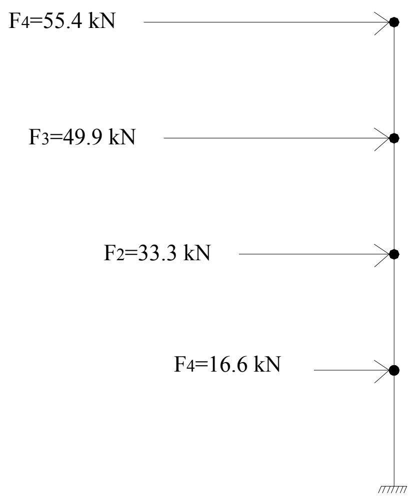
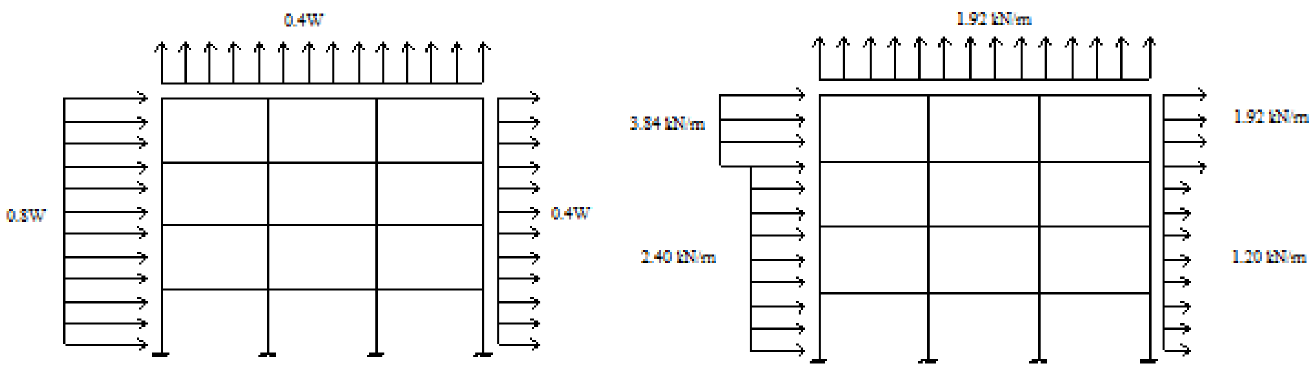


Fig. A2 Nominal earthquake load distributed to the stories



(a) Effect of V_F (b) Effect of V_K

Fig. A3 Wind load distribution

earthquake load distribution of the unit-intensity accelera-ion.

Wind load

In order to determine nominal wind load of the frame, the wind pressure W can approximately be determined as

$$W = \frac{v^2}{1600} \quad (\mathrm{kN/m^2}) \tag{A3}$$

where v is the wind velocity. Without considering the variation of the pressure W through the height, its distribution can be assumed as shown in Fig. A3(a) (TS498 1997). In a case that there is no specific data, the wind velocity can be assumed as 28 m/s for the height up to 9 m and 36 m/s for the height between 9 m and 12 m.

Thus, the wind pressure was calculated as W = 0.5 kN/m^2 for the first 9 meters and W = 0.8 kN/m^2 for the upper 3 m. As the attributed area of the frame was assumed to have a width of 6 m, the nominal wind load distribution was calculated as shown in Fig. A3(b).

For wind load it was assumed that the statistical information is attributable to the wind pressure W. Hence, the wind load was normalized by 0.5 for determining the inter-story drift ratio stiffness for the wind load distribution.

For the reliability of the wind load case the random variable F of Eq. (37) was assumed to be a lognormal distribution having a nominal value of 0.5 and a mean value of 0.165 (Table 1) with a coefficient of variation of 0.59. Accordingly, the inter-story drifts were obtained for a unit pressure and these drifts are the stiffnesses for the wind load distribution of the unit-intensity acceleration.