A new stability and sensitivity design and diagnosis approach

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(Received January 25, 2016, Revised July 04, 2016, Accepted February 21, 2017)

Abstract. In the stability and sensitivity design and diagnosis approaches, there are various methodologies available. Bond graph modeling by lumping technique is one of the universal methodologies in methodical analysis used by many researchers in all over the world. The accuracy of the method is validated in different arenas. Bond graphs are a concise, pictorial representation of the energy storage, dissipation and exchange mechanisms of interacting dynamic systems, subsystems and components. This paper proposes a bond graph modeling for distributed parameter systems using lumping techniques. Therefore, a steel frame structure was modeled to analyze employing bond graph modeling of distributed system using lumping technique. In the analytical part, the effectiveness of bond graphs to model this system is demonstrated. The dynamic responses of the system were computed and compared with those computed from the finite element analysis. The calculated maximum deflection time histories were found to be comparable. The sensitivity and the stability of the steel frame structure was also studied in different aspects. Thus, the proposed methodology, with its simplicity, can be used for stability and sensitivity analyses as alternative to finite element method for steel structures. The major value brought in the practical design is the simplicity of the proposed method for steel structures.

Keywords: bond graph modeling; lumping techniques; steel structures; stability; sensitivity

1. Introduction

For the stability and sensitivity design and diagnosis approaches, some methodologies have been developed with determining the structural characteristics. Bond graphs have been devised by in late fifties. Researchers have elaborated this graphical model representation into a methodology that has experienced a considerable progress over the decades due to the steady work of many researchers all over the world. Since the early days, many researchers have been published on bond graph modeling with the aim of reflecting a part of contemporary research on and application of bond graph modeling in various areas in engineering (He et al. 2015, Gawthrop et al. 2005, Samantaray et al. 2006, Behzadipour and Khajepour 2006, Banerjee et al. 2009, Tsai and Gero 2010, Moustafa et al. 2010, Hroncova et al. 2012). However, the bond graph method is not well-known and understood in structural engineering, therefore, this method is rarely used to analyze a structural system. There are a limited number of studies on formulating structures using bond graphs and lumping techniques for distributed systems. Hybrid systems are also another approach in design procedures (Rahal et al. 2016, Borutzky 2012a, b, Margetts et al. 2013).

Margolis had studied the bond graphs to construct finite mode, long wavelength models of multidimensional structures in 1980 (Margolis 1980). A physically complex

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Copyright © 2017 Techno-Press, Ltd. http://www.techno-press.org/?journal=scs&subpage=6 steel structure 40 m long, 10 m wide, and 7 m tall was modeled using bond graphs. The resulting model required only 40 equations to compute the system natural frequencies and corresponding mode shapes. Margolis studied bond graph modeling for interacting lumped and distributed systems in another research work in 1985 (Margolis 1985). In this research work, a history has been given for the use of bond graphs in modeling distributed system dynamics. It started with the bond graph microelement which exactly represents the governing partial differential equations. This led directly to the cascading of elements to obtain a lumped representation for the distributed system. The delay bond concept was also discussed. A research paper developed by Orlikowski and Hein (2011) presents the use of bond graph and DTFM method for modeling of beam/bar systems and trusses in the structural engineering.

Moustafa *et al.* (2007) developed a fault diagnosis methodology for civil engineering structures based on the bond graph approach in 2007. According to this study, the bond graph theory provides a modeling framework that includes parametric models of the physical system and the sensors. Structural faults were modeled as abrupt or gradual damage in structural components. Sensor faults were modeled as biases or drifts from true measurements. A statistical method was used to identify significant deviations of measurements from nominal behavior of the structure. Fault isolation was carried out by comparing predicted effects of hypothesized faults with observed behavior of the structure. In the study, physical dynamic systems such as frame structures were modeled using shear frame models. The effectiveness and accuracy of bond graph models of structures as compared to traditional structural dynamics modeling techniques were examined. The determined displacement, velocity and acceleration responses for the frame structure were computed using the derived simulation models and by the dynamic analysis by using Duhamel's integral.

The present paper documents the development of bond graph modeling for distributed parameter systems using lumping techniques to develop a new stability and sensitivity design and diagnosis approach which was not published before in the area. The proposed methodology, with its simplicity, can be used for stability and sensitivity analyses as alternative to finite element method for steel structures. In some cases, finite element analysis can be complicated due to design details of the steel structures. The proposed approach is more practical and can be used by the practitioners in the structural design area including nonlinear problems. Nonlinear effects pose for steel structures are no particular problems with the proposed method. Principal emphasis is on the use of bond graphs and lumping techniques to predict the dynamic response of a structural system. The paper covers various analytical approaches for stability and sensitivity analysis issues in the structural systems.

2. Sensitivity and stability analysis

With the proposed approach, sensitivity analysis will be more realistic and easier comparing to existing ones. The bond graph method with lump mass technique predicts the dynamic response of steel frame structures very well. An equilibrium point always represents an equilibrium solution of the differential equation, that is, a solution x = constant for all *t*, time. The equation points are found here by setting energy variables (x_i , momentums and angular velocities) to zero in 20 state space equations. The calculated equilibrium point is given below

$$x_1 = F(t)/c_3, x_2 = 2F(t)/c_3, x_3 = 3F(t)/c_3, x_4 = 4F(t)/c_3, x_5 = 5F(t)/c_3, x_6 = 6F(t)/c_3, x_7 = 7F(t)/c_3, x_8 = 8F(t)/c_3, x_9 = 2F(t)/c_3, x_{10} = 10F(t)/c_3, \text{ and } x_{11} = \dots = x_{20} = 0$$

where $c_3 = EI/\Delta x^2$, *E* is young's modulus, *I* is the moment of inertia and Δx is the height of the lump masses. *F*(*t*) is the dynamic (time varying) blast load applied to the system.

3. Modeling structural system

For the structural steel frame structures, developed models for the current paper is very similar to the one analyzed by Moustafa *et al.* (2007). In both studies, bond graph method is employed to calculate the response of steel frame structures to dynamic loading. The main difference between two studies is the technique that is used to form the bond graphs. Moustafa *et al.* (2007)'s paper used massspring system technique and a lumping technique for distributed system is used in the present study. The lumping technique for distributed system is more complicated and accurate than the mass-spring system technique.

Using the information provided for two story structure in the paper by Moustafa *et al.* (2007), a bond graph is developed for the present study. Utilizing the bond graph, the state space equations are written. Finally, the response of the structure (floor displacements) is estimated. The Matlab was employed to calculate the floor deflections. Fig. 1 shows the floor deflections for the steel frame structure described by Moustafa *et al.* (2007). The story drift time histories in Fig. 1 match perfectly to the floor displacements reanalyzed in the present study (see Fig. 1).

4. Structural assessment

With the proposed methodology, in the analytical part, the structural system is analyzed utilizing bond graph method and a lumping technique for distributed structural system. Most often, a typical mechatronic system consists of different domain subsystems and when the structural analysis of such system is desired, the modeler will consider using one of the many available finite element codes (Orlikowski et al. 2009, Orlikowski and Hein 2011). These codes offer many advantages in that they are typically straightforward, although tedious, to use. A principal disadvantage to their use is that relatively large amounts of computer time and capacity are required for accurate system representation. In addition, physical insight into the physics of the problem is virtually lost in the mire of nodal points and junction constraints. The possibility of generating accurate lower order models for parameter and/or control studies is very small (Margolis 1985). Distributed parameter are given in terms of partial differential equations. However, similar to lumped parameter systems, they can







(b) Re-analysis of the steel frame structure described by Moustafa *et al.* (2007)

Fig. 1 Comparison of the model by Moustafa et al. (2007)

also be described by the transfer function method. In this case, the distributed transfer function is the corresponding mathematical model (Orlikowski *et al.* 2009).

Through use of normal modes, bond graphs can be used to construct perhaps the most accurate low order models for linear distributed systems. By requiring relatively few equations, when compared to finite difference and finite element models, a physically understandable model results for design and automatic control applications can be obtained. Distributed parameter systems are, analytically, those represented by partial rather than total differential equations. Distributed systems can be accurately approximated by the "lumped" assumptions.

In the flow of defining the distributed representation of a dynamic component or subsystem, it is typical to start with spatially distributed finite lumps. However, a large number of lumps may be required to obtain accuracy at low frequencies. In addition, each new lump while improving low-frequency prediction, introduces new, totally inaccurate high frequencies. This approach to distributed system representation must be used cautiously, and with awareness on the part of the modeler. The following describes the basic methodology of lumping technique for distributed system. The beam is assumed to have a uniform crosssectional area A, mass density ρ , Young's modulus E, shear modulus G, area moment of inertia I, and length L. Fig. 2 shows the beam displaced at some instant of time. The spatial variable x defines a position along the beam, and w(x, t) is the transverse displacement of the position x at the time t.

In Fig. 3, the bond graph is shown. Here, the lumped model is reduced to a Bernoulli-Euler beam model. This graph was used for steel structure frame modeling in the present study as the Bernoulli-Euler model. The lumped



Fig. 2 Uniform beam in transverse motion (Karnopp *et al.* 1990)



Fig. 3 Bong graph finite lump model of Bernoulli-Euler beam (Karnopp *et al.* 1990)

parameter representations have the advantage that they can be straightforwardly combined into an overall system model. Also, nonlinear effects pose no particular problems. The disadvantage to the lumped parameter approach described so far is that the model will generate a large state space, and can cause severe computational problems due to large disparities in the time scales of the distributed portion of the model and the remainder of the system. However, true continuum models can yield insight into system behavior when analytical solutions are available, which typically restricts us to the linear case. Furthermore, it is usually difficult to incorporate a continuum model into an overall system model with interactions with complex lumped systems (or other continuous systems) external to the continuum. Thus, bond graph modeling of distributed system using lumping technique is utilized to analyze the structural steel frame system subjected to dynamic blast loading.

4.1 Description of structural system

A steel frame structure is analyzed employing bond graph modeling of distributed system using lumping technique. The structure consists of steel frame spaced at every 4.5 m. The height of the columns and the span of the floor beams are 4.5 m. A blast pressure of 40 MPa is acting on the wall of the structure. An individual blast load is calculated and applied to the system to represent the blast loading as shown in Fig. 4. The duration of the blast is 16 ms. A blast can be characterized by a peak pressure and an impulse. Impulse is the integration of the pressure-time history. Structural response to a blast is dependent upon both the peak pressure and blast impulse. Fig. 5 shows the idealized shape of the free-field blast load that was assumed for this analysis. The negative phase of the blast wave is usually ignored due to its small amplitude.

The structural frames are fixed to ground. Only one steel frame is analyzed to represent the response of the structure to dynamic load. In this example, the columns and beams are made up with W16×100 steel beams. As per TM5-1300 (1990), for blast-resistant design only the peak response, from the first cycle, of the structure is important. This first response cycle is minimally affected by damping in the system and damping effects are subsequently neglected in the theoretical procedure given in TM5-1300 (1990) for





Fig. 5 Idealized shape of the blast load

evaluating blast load response (ASCE 1997, Sabuwala *et al.* 2005). Thus damping effect is neglected in this study.

4.2 Analysis of structural system using bond graph method

The steel frame structure shown in Fig. 4 is analyzed employing bond graph modeling of distributed system using lumping technique. 10 lumped masses (the height of the each lumped mass is approximately 500 mm) were used to represent the column of the steel frame. The steel frame idealized as shown in Fig. 6. The cross-sectional area for



Fig. 6 The equivalent lump mass model used in bond graph modeling

each lump mass is equals the sum cross sectional areas of two column members (two identical columns are combined in one). Similarly, the moment of inertia of each or each lump mass is equals the sum moment of inertias of two column members. The top lump mass has the mass of the beam and the slab attached to beam. The system is nonautonomous system because a dynamic blast force is acting on the top lumped mass. The bond graph for the structural system is provided in Fig. 7. The bond graph model includes 20 variables (10 lumped mass rotations and 10 lumped mass moments).

In the structural model, linear elastic material is assumed. The material non-linearity can be included in the model by replacing the Young's modulus E. However, it is believed that the stresses due to the blast loading in the steel frame system will be lower than the yield strength of the steel material. Thus, a linear elastic material is used in the analysis. The main interest of the current study is to



Fig. 7 The bond graph for the equivalent lump mass model



Fig. 8 Maximum steel frame deflection calculated using bond graph modeling with lump mass technique



Fig. 9 Deflected finite element model and Effective stress distribution

calculate the maximum deflection of the frame structure. The frame has the maximum deflection at the floor beam level. To calculate the maximum deflection of the frame structure is equal to sum of the 10 lumped mass deflections. The calculated maximum deflection time history using bond graph method with lump mass technique for the steel frame structure is provided in Fig. 8. The calculated peak deflection is approximately 20 mm.

Stretural effectiveness of bond graphs to model this system is demonstrated. To do this, the dynamic responses of this system are computed and compared with those computed from the finite element analysis. By doing this, we will ensure that the stresses are lower than the yield strength and the linear elastic material assumption is acceptable. In addition, the sufficiency of the number of the lump masses used in bond graph is tested. If the number of the lump masses in bond graph is not sufficient, the maximum deflection time histories are expected to be significantly different.

Finite element analysis is performed for the structural frame system. The deflected shape of the structure provided in Fig. 9 shows the effective stress distribution. According to Fig. 9, we can conclude that the elastic material assumption in the bond graph modeling is correct because

the response of the structure is in elastic material region. The finite element analyses are performed for two cases: with damping and without damping. The maximum deflection time history of the steel frame structure with the bond graph method is compared with those computed from the finite element analysis in Fig. 10. The maximum deflection time histories for three analyses are well compared. The bond graph method with 10 lump masses slightly underestimates the deflections. This might be because the bond graph is based on Bernoulli-Euler beam assumption which the shear deflections are neglected. It can be concluded that the number of lump masses used in bond graph method is sufficient and the bond graph method with lump mass technique can predict the response of the steel structure as well as finite element analysis method.

5. Sensitivity assessment

The equilibrium point is changed with respect to time, or in other words the structural system is in equilibrium at any time. At time = td (duration of the dynamic loading), the equilibrium point is the origin. There is only one equilibrium point at each time. The Jacobian matrix is same



deflection time histories

as the coefficient matrix that includes the coefficients of variables x_i in the state space matrix. The Jacobian matrix does not vary with time and consists of constant numbers. 20 eigenvalues are shown in Table 1. Ten of the eigenvalues are zero and the rest is negative real number. In the case of eigenvalues of negative real number and unequal, the equilibrium point is a stable point and/or the equilibrium point is stable. When one or more eigenvalues are zero, the Jacobian matrix has a nontrivial null space. Any vector in the null space is an equilibrium point for the system; that is, the system has an equilibrium subspace, rather than an equilibrium point.

5.1 Bifurcation analysis

The qualitative behavior of a system is determined by the pattern of its equilibrium points and periodic orbits, as well as by their stability properties. One issue of practical importance is whether the system maintains its qualitative behavior under infinitesimally small perturbations. When it does so, the system is said to be structurally stable. In particularly, with bifurcation analysis, the interest is the perturbations that will change the equilibrium points or periodic orbits of the system or change their stability properties (Loureiro *et al.* 2012). The effect of c_1 , c_2 and c_3 constant in state space equations on the stability properties where $c_1 = \rho_1 A \Delta x^2$, $c_2 = \rho_2 A \Delta x^2$, and $c_3 = EI/\Delta x^2$. The ρ represents mass densities, A is cross sectional area. Four cases are considered to analyze the effect of c_1 , (the effect of the c_2 is not considered since c_1 and c_2 are dependent), and c_3 on the stability properties. Case 1: c_3 is very small, Case 2: c_1 is very small, Case 3: c_1 and c_3 is very small, Case 4: c_3 is negative (not a realistic case).

As shown in Table 2, the stability properties change only

Table 1 Calculated 20 eigenvalues

in Case 4 which is an unrealistic case as young's modulus or the moment of inertia of any section cannot be zero. The height of the lump mass should be positive real too. It can be concluded that no bifurcation is expected for this system in the given framework.

5.2 Sensitivity equations

In this procedure, we need to solve the nonlinear nominal state equation and the-linear time-varying sensitivity equation. Except for some trivial cases, we will be forced to solve these equations numerically. The sensitivities for 20 variables are calculated but the plots of sensitivities for only two variables (x5, and x10, rotations at the 5th and 10th lump mass, respectively) are shown in Figs. 12 and 13. According to the figures, the lump mass rotations are more sensitive to variations in the Young's modulus and the moment of inertia of section than to variations in the mass density and cross-sectional area.

5.3 Lyapunov stability analysis

Stability theory plays a central role in systems theory

Table 2 Calculated eigenvalues for four cases

	Original	Case 1	Case 2	Case 3	Case 4
1	0.000	0	0	0	0
2	0.000	0	0	0	0
3	-0.007	-6.7E-13	-0.01191	-1.2E-12	166.4497
4	-0.024	-2.4E-12	-1.4E+08	-0.01377	139.6852
5	-0.547	-5.5E-11	-5.4E+09	-0.53963	103.1389
6	-4.077	-4.1E-10	-4.1E+10	-4.06824	65.70026
7	-14.431	-1.4E-09	-1.4E+11	-14.421	34.85711
8	-34.857	-3.5E-09	-3.5E+11	-34.8473	14.43094
9	-65.700	-6.6E-09	-6.6E+11	-65.6919	4.077254
10	-103.139	-1E-08	-1E+12	-103.133	0.547242
11	-139.685	-1.4E-08	-1.4E+12	-139.682	0.024194
12	-166.450	-1.7E-08	-1.7E+12	-166.449	0.00666
13	0.000	0	0	0	0
14	0.000	0	0	0	0
15	0.000	0	0	0	0
16	0.000	0	0	0	0
17	0.000	0	0	0	0
18	0.000	0	0	0	0
19	0.000	0	0	0	0
20	0.000	0	0	0	0

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	Modes	1	2	3	4	5	6	7	8	9	10
	Eigenvalues	0.000	0.000	-0.007	-0.024	-0.547	-4.077	-14.431	-34.857	-65.700	-
_											
	Modes	11	12	13	14	15	16	17	18	19	20
	Eigenvalues	-	-	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000



Fig. 12 Sensitivities of the variable x5



Fig. 13 Sensitivities of the variable x10



Fig. 14 Variation of V(x, t) with respect to time

and engineering. There are different kinds of stability problems that arise in the study of dynamical systems. The stability of equilibrium points is concerned mainly. Stability of equilibrium points is usually characterized in the sense of Lyapunov. An equilibrium point is stable if all solutions starting at nearby points stay nearby; otherwise, it is unstable. It is asymptotically stable if all solutions starting at nearby points not only stay nearby, but also tend to the equilibrium point as time approaches infinity (Khalil 2002).

The analysis shows that V(x) = 0 at any time and at any equilibrium point as illustrated in Fig. 14. Thus, the system is stable at in all equilibrium points and at any time.

6. Conclusions

A Steel frame structure was analyzed employing the bond graph method and the lumping techniques. The dynamic response of the structure to the blast loading (time varying load) was studied. The effectiveness of bond graphs to model this system is demonstrated. The dynamic responses of this system were computed and compared with those computed from the finite element analysis. The maximum deflection time histories from two methods were found to be comparable. The stability and sensitivity of the structural system was studied in different aspects. Finally, the structural system was found to be stable.

With the study, given results make a new approach in the field. Especially the calculated maximum deflection time histories were close to what it should be. Also, the sensitivity and the stability of the steel frame structure was studied with the proposed method. Thus, the proposed methodology can be used for stability and sensitivity analyses of steel structures. That is also another important part in the study.

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