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Vibration analysis of functionally graded nanocomposite plate moving in two directions

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Abstract. In the present study, vibration analysis of functionally graded carbon nanotube reinforced composite (FGCNTRC) plate moving in two directions is investigated. Various types of shear deformation theories are utilized to obtain more accurate and simplest theory. Single-walled carbon nanotubes (SWCNTs) are selected as a reinforcement of composite face sheets inside Poly methyl methacrylate (PMMA) matrix. Moreover, different kinds of distributions of CNTs are considered. Based on extended rule of mixture, the structural properties of composite face sheets are considered. Motion equations are obtained by Hamilton's principle and solved analytically. Influences of various parameters such as moving speed in x and y directions, volume fraction and distribution of CNTs, orthotropic viscoelastic surrounding medium, thickness and aspect ratio of composite plate on the vibration characteristics of moving system are discussed in details. The results indicated that thenatural frequency or stability of FGCNTRC plate is strongly dependent on axially moving speed. Moreover, a better configuration of the nanotube embedded in plate can be used to increase the critical speed, as a result, the stability is improved. The results of this investigation can be used in design and manufacturing of marine vessels and aircrafts.

Keywords: vibration\vibration analysis; plate; fiber reinforced polymers (FRP); sandwich composite; composite structures

1. Introduction

Moving materials are employed in different industrial applications such as magnetic tapes, paper and textile webs during production, processing and printing, plastic sheets, films, and the like. In general, although the mechanical characteristics of moving structures has been investigated up to now, much information is not available on the two dimensional moving systems. The effect of moving structures may be enough to change the dynamic behavior of the system even at low velocities. Higher than a special speed that is called "critical speed", the behavior of the system is changed. In the other words, system becomes unstable and may leads to irreparable damages, such as loss of raw material, low surface quality and unsatisfactory performance. Prediction of the dynamic behavior of moving systems, and ensuring that they remain stable, are requirements for the optimal design of these systems.

As the one of the first researches in the field of moving plates, Lin (1997) studied stability and vibration characteristics of axially moving plates. He demonstrated that the critical speed is equal to the speed at the onset of instability predicted by static and dynamic analyses. Also, it increases by reducing the length to width ratio and increasing the flexural stiffness of the plate. Wang (1999) investigated numerical analysis of moving orthotropic thin plates. He utilized a mixed finite element formulation based on the Mindlin-Reissner plate theory for his work and proved that it is reliable for both frequency and transient dynamics analyses. The modal spectral element formulation for moving thin plates was used by Kim *et al*. (2003). They assumed that plate moving with constant speed and subjected to a uniform in-plane axial tension. Then, they formulated the modal spectral element matrix in the frequency-domain by using the Kantorovich method. Hatami *et al*. (2007) developed free vibration analysis of axially moving symmetrically laminated plates subjected to in-plane forces with classical plate theory. At first, they employed an exact method to analyze vibration of multi-span traveling cross-ply laminates, and then, utilized a semi-analytical finite strip method for moving symmetric laminated plates. In the other research, Hatami *et al*. (2008) considered exact free vibration analysis of axially moving viscoelastic plates with a constant axial speed. By using the rheological models, they obtained the stiffness matrix of viscoelastic plate and showed the influences of axial speed and viscoelastic parameters on the free vibration of moving plates. Vibration analysis of axially moving viscoelastic plate with parabolically varying thickness was analyzed by Zho and Wang (2008). They considered the Kelvin–Voigt model to simulate the viscoelastic behavior of materials and calculated the dimensionless complex frequencies of their system with various boundary conditions. Yang *et al*. (2009) investigated approximate solutions of axially moving viscoelastic beams subject to multi-frequency excitations. They studied about superharmonic, subharmonic, and

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combination resonances. In other work, Yang and Fang (2013) presented the non-linear creep vibration of an axially moving string constituted by a fractional differentiation law.Their results indicated that the amplitude predicted by the fractional model is larger than that predicted by the viscoelastic solid models. Marynowski (2010) applied the viscoelastic theory to study free vibration of the axially moving Levy-type plate with two simply supported and two free edges boundary conditions. He illustrated influences of transport speed and relaxation times modeled by two-parameter Kelvin-Voigt and three-parameter Zener rheological models on the dynamic behavior of the axially moving viscoelastic plate. Also, Marynowski and Grabski (2013) investigated dynamic analysis of an axially moving plate under thermal loading. They solved the differential equations of the transverse motion by employing the extended Galerkin method and demonstrated the effects of thermal critical loading, axial speed and tension on dynamic behavior of axially moving plate. The stability analysis of an axially moving elastic plate travelling at constant speed between two supports with a 2D formulation was done by Banichuk *et al*. (2010). They inserted a thin elastic plate under bending and tension and observed that the 2D formulation reduces to the classical 1D model for the limit of a narrow strip. In other work, Banichuk *et al*. (2011) presented the static stability problems of axially moving orthotropic membranes and plates. They solved the equations of motion using analytical techniques and showed that the buckling mode has a shape localized in the regions close to the free boundaries. Tang and Chen (2011) investigated nonlinear free transverse vibrations of in-plane moving plates subjected to plane stresses. They utilized the multiple scales method to solve the nonlinear partial differential equation and presented the relationship between the nonlinear frequencies and the initial amplitudes at various in-plane moving speeds and the nonlinear coefficients, respectively. Then, in other research, nonlinear forced vibrations of in-plane translating viscoelastic plates under plane stresses were presented by Tang and Chen (2012). They employed the Routh–Hurvitz criterion in order to calculate the stabilities of the steady-state responses and examined the influences of the in-plane axial speed, the viscosity coefficient, and the excitation amplitude on it. Yang *et al*. (2011) studied vibrations and stability of an axially moving rectangular antisymmetric cross-ply composite plate. They determined the natural frequencies for the in-plane and out-of-plane vibrations by both the Galerkin method and differential quadrature method. Then, by investigating the complex natural frequencies for constant axial speed, they studied the instability caused by divergence and flutter. Also, Yang *et al*. (2012) used finite difference method for dynamical analysis of axially moving plate. They considered the complex natural frequencies for linear free vibrations and bifurcation for forced nonlinear vibration of axially moving viscoelastic plate and derived the equations of out-of-plane motion by Newton's second law. The nonlinear dynamics of an axially moving plate for forced motions was numerically developed by Ghayesh *et al*. (2013). Based on Lagrange equations, they determined the equations of motion via Hamilton's principle and utilized the pseudo-arc length continuation technique to solve them. An and Su (2014) investigated dynamic analysis of axially moving orthotropic plates using the generalized integral transform technique (GITT). They assumed two types of boundary conditions and concluded that the amplitudes of moving orthotropic plate reduce with decreasing translating speed and aspect ratio for both CCCC and CCSS. At the one of last research, Ghorbanpour Arani and Haghparast (2016) studied vibration and instability of axially moving viscoelastic micro-plate. To discuss the size effect, they used the modified couple stress theory (MCST). They obtained the equations of motion based on Hamilton's principle. Also, the effects of translating speed, viscosity coefficient, size effect, thickness and aspect ratio on the vibration characteristics were expressed by them.

Despite mentioned researches, free vibration and instability analysis of FGCNTRC plate moving in two directions is a novel topic that cannot be found in literature. To the best of authors' knowledge, for the first time, analysis of moving composite plate based on various types of shear deformation theories is developed in this paper. Utilizing Hamilton's principle governing equations of motion are derived and solved by an analytical method for different boundary conditions. Effects of various parameters such as moving speed, orthotropic viscoelastic surrounding medium, aspect ratio, volume fraction and distributions of CNTs on instability and critical speed of moving composite plate are discussed in details. The results of this investigation can be used to improve the design and manufacturing of marine vessels.

2. Fundamental relations

As shown in Fig. 1, consider a rectangular composite plate with length (a), width (b) and thickness (h) which moves along the x and y directions with constant speed C_x and C_y, respectively. Also, composite plate is taken to rest on orthotropic visco-Pasternak foundation. CNTs reinforce-ments are distributed through the thickness of composite plate as uniform distribution (UD) and three functionally graded (FG) distributions (FG-V, FG-O and FG-X). As it can be observed in Fig. 1, in UD distribution, CNTs are

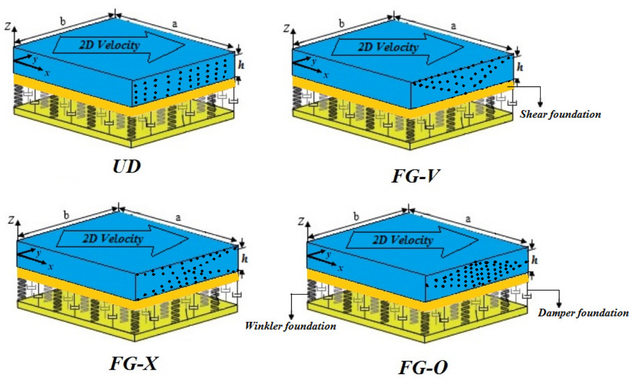

Fig. 1 Composite plate with different types of CNT distributions resting on orthotropic visco-Pasternak foundation moving in two directions

distributed in whole of plate uniformly, in FG-V distribution, CNTs are distributed from the bottom to top of plate linearly, in the type of FG-O, distribution of CNTs are decreased from the middle to outside of plate until it reaches zero and this case is vice versa for FG-X distribution.

The volume fraction of various distributions of CNTs in composite plate can be expressed as follows (Mohammadimehr *et al.* 2015)

$$V_{CNT}=\begin{cases} V_{CNT}^{*} & UD \\ (1+\dfrac{2z}{h})\,V_{CNT}^{*} & FG-V \\ 2(1-\dfrac{2|z|}{h})\,V_{CNT}^{*} & FG-O \\ 4(\dfrac{|z|}{h})\,V_{CNT}^{*} & FG-X \end{cases} \tag{1}$$

where

$$V_{CNT}^{*}=\frac{w_{CNT}}{w_{CNT}+(\rho_{CNT}/\rho_{M})(1-w_{CNT})},$$

in which, w and ρ are defined as the mass fraction and mass density, respectively. Also, the subscript M is used to describe matrix phase.

2.1 The extended rule of mixture

The mechanical properties of CNT reinforced composite plate including longitudinal (E_{11}), transversely elastic moduli (E_{22}) and shear modulus (G_{12}) can be obtained based on the extended rule of mixture as following form (Gibson 1994)

$$E_{11}=\eta_1 V_{CNT}E_{11CNT}+V_M E_{11M}, \tag{2a}$$

$$\frac{\eta_2}{E_{22}}=\frac{V_{CNT}}{E_{22CNT}}+\frac{V_M}{E_{22M}}, \tag{2b}$$

$$\frac{\eta_3}{G_{12}}=\frac{V_{CNT}}{G_{12CNT}}+\frac{V_M}{G_M}, \tag{2c}$$

$$\nu_{12}=V_{CNT}^{*}\nu_{12CNT}+V_M\nu_M, \tag{2d}$$

$$\nu_{21}=\nu_{12}E_{22}/E_{11}, \tag{2e}$$

$$\rho=V_{CNT}\rho_{CNT}+V_M\rho_M, \tag{2f}$$

where η_i (i = 1, 2, 3) and ν are the efficiency parameters and Poisson's ratio, respectively.

2.2 Displacement fields

In this study, to compare the accuracy of various shear deformation plate theories, six theories are selected. According to this, displacement fields which are predicted by four theories can be expressed as following form (Ghorbanpour Arani and Haghparast 2016)

$$\begin{aligned} U(x,y,z,t)&=u(x,y,t)-z\frac{\partial w(x,y,t)}{\partial x}+\psi(z)\phi_x(x,y,t),\\ V(x,y,z,t)&=v(x,y,t)-z\frac{\partial w(x,y,t)}{\partial y}+\psi(z)\phi_y(x,y,t),\\ W(x,y,z,t)&=w(x,y,t), \end{aligned} \tag{3}$$

in which u, v and w present the displacement components of composite plate along the x, y and z directions, respectively. Also, ϕ_x and ϕ_y are the rotation of middle surface in the x and y directions, respectively. It should be mentioned that by description of $\psi(z)$, the four type of theories can be specified. Classical plate theory (CPT), first order shear deformation theory (FSDT), third order shear deformation theory (TSDT) and sinusoidal shear deformation theory (SSDT) can be introduced as $\psi(z)$ = 0, z, $z - 4z^3/3h^2$ and $\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$, respectively.

According to higher order shear deformation theory (HSDT), displacement fields can be written as

$$\begin{aligned} U(x,y,z,t)&=u(x,y,t)+z\phi_x(x,y,t)+f(z)\left(\phi_x(x,y,t)+\frac{\partial w(x,y,t)}{\partial x}\right),\\ V(x,y,z,t)&=v(x,y,t)+z\phi_y(x,y,t)+f(z)\left(\phi_y(x,y,t)+\frac{\partial w(x,y,t)}{\partial y}\right),\\ W(x,y,z,t)&=w(x,y,t), \end{aligned} \tag{4}$$

where $f(z)=\frac{h}{\pi}\sin\left(\frac{\pi z}{h}\right)$. Also, the displacement fields are expressed based on new FSDT as (Yang *et al.* 2013)

$$\begin{aligned} U(x,y,z,t)&=u(x,y,t)-z\frac{\partial}{\partial x}\theta(x,y,t),\\ V(x,y,z,t)&=v(x,y,t)-z\frac{\partial}{\partial y}\theta(x,y,t),\\ W(x,y,z,t)&=w(x,y,t), \end{aligned} \tag{5}$$

where θ is an unknown displacement function of the middle of plate.

2.3 Kinematics and constitutive equations

The linear relations between the strain and displacement fields can be obtained by

$$\begin{Bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\end{Bmatrix}=\begin{Bmatrix}\dfrac{\partial U}{\partial x}\\ \dfrac{\partial V}{\partial y}\\ \dfrac{\partial W}{\partial z}\end{Bmatrix},\quad \begin{Bmatrix}\gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{Bmatrix}=\frac{1}{2}\begin{Bmatrix}\dfrac{\partial U}{\partial y}+\dfrac{\partial V}{\partial x}\\ \dfrac{\partial U}{\partial z}+\dfrac{\partial W}{\partial x}\\ \dfrac{\partial V}{\partial z}+\dfrac{\partial W}{\partial y}\end{Bmatrix}, \tag{6}$$

in which, ε and γ describe normal and shear strains, respectively. Substituting Eqs. (3)-(5) in Eq. (6), kinematics equations are determined and it should be noted that ε_{zz} will be calculated equal to zero for six theories. Therefore, the constitutive equations can be written as follows

$$\begin{Bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{21} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \gamma_{xy} \end{Bmatrix}, \tag{7}$$

$$\begin{Bmatrix} \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = K_s \begin{bmatrix} Q_{44} & 0 \\ 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix},$$

where

$$Q_{11} = \frac{E_{11}}{1-\nu_{12}\nu_{21}}, Q_{12} = \frac{\nu_{21}E_{11}}{1-\nu_{12}\nu_{21}}, Q_{21} = \frac{\nu_{12}E_{11}}{1-\nu_{12}\nu_{21}},$$

$$Q_{22} = \frac{E_{22}}{1-\nu_{12}\nu_{21}},\ Q_{66} = G_{12},\ Q_{44} = G_{23},\ Q_{55} = G_{13}$$

σ and τ are normal and shear stresses, respectively and Q_{ij} (i, j = 1, 2 and 44, 55, 66) indicate the terms of engineering constants. Also, K_s is the shear correction factor. For FSDT, K_s = 5/6, and for the other five theories, K_s = 1.

2.4 Two dimensional moving of composite plate

The composite plate moving along the x and y directions with constant speed C_x and C_y, respectively. The velocity vector of plate can be obtained based on continuum mechanics as following relation (Marynowski and Grabski 2013)

$$\vec{V} = \frac{D\vec{r}}{Dt} = \frac{\partial \vec{r}}{\partial t} + V_i \frac{\partial \vec{r}}{\partial x_i}, \tag{8}$$

$$(i = x, y, z \text{ and } V_x = C_x,\ V_y = C_y,\ V_z = 0)$$

in which $\vec{r} = (U + C_x t)\vec{i} + (V + C_y t)\vec{j} + W\vec{k}$ present the displacement vector. Thus, the velocity vector $(\vec{V})$ can be written by Eq. (8)

$$\vec{V} = \left(\frac{\partial U}{\partial t} + C_x + C_x\frac{\partial U}{\partial x} + C_y\frac{\partial U}{\partial y}\right)\vec{i} + \left(\frac{\partial V}{\partial t} + C_y + C_x\frac{\partial V}{\partial x} + C_y\frac{\partial V}{\partial y}\right)\vec{j} + \left(\frac{\partial W}{\partial t} + C_x\frac{\partial W}{\partial x} + C_y\frac{\partial W}{\partial y}\right)\vec{k}. \tag{9}$$

2.5 Visco-Pasternak foundation

In order to simulate normal load, the simplest way is the use of Winkler type of elastic foundation. But, visco-Pasternak type of elastic foundation is one of the most comprehensive model that is able to consider both normal and transverse shear loads. Moreover, it takes into account the effects of damping loads on the system. In this research, the bottom surface of composite plate continuously rests on a visco-Pasternak foundation. So, based on this model, the force applied to the plate can be written as (Ghorbanpour Arani *et al*. 2016)

$$F = K_w W + C_d\frac{\partial W}{\partial t} - K_{gx}\frac{\partial^2 W}{\partial x^2} - K_{gy}\frac{\partial^2 W}{\partial y^2}, \tag{10}$$

where K_w and C_d represent Winkler and damping coefficients. Also, K_{gx} and K_{gy} are shear foundation parameters in x and y directions.

2.6 Hamilton's principle

The equations of motion are obtained based on Hamilton's principle. The analytical form of Hamilton's principle can be expressed as follows

$$\delta\Pi = \delta\int_{t_1}^{t_2}(U - K - \Sigma)dt = 0, \tag{11}$$

in which, U, K and Σ are strain energy, kinetic energy and external work, respectively, and each terms can be expressed according to the following relations

$$\delta U = \int_A\int_{-\frac{h}{2}}^{\frac{h}{2}}(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \tau_{xy}\delta\gamma_{xy} + \tau_{xz}\delta\gamma_{xz} + \tau_{yz}\delta\gamma_{yz})dzdA, \tag{12a}$$

$$\delta K = \delta(\int_A\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{1}{2}\rho_c(\vec{V})^2 dzdA), \tag{12b}$$

$$\delta\Sigma = \delta(\int_A\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{1}{2}F\cdot W\ dzdA). \tag{12c}$$

By substituting Eqs. (7), (9) and (10) into Eqs. (12a), (12b) and (12c), respectively, and subsequent results into Eq. (11), the governing equations of motion can be derived.

By setting the coefficients of δu, δv, δw, $\delta\phi_x$ and $\delta\phi_y$ to zero, separately, the governing equations of motion based on CPT, FSDT, TSDT and SSDT can be obtained as follows

δu:

$$\begin{aligned}
&-A_{120}\frac{\partial^2 v}{\partial y\partial x} - A_{660}\frac{\partial^2 v}{\partial y\partial x} - B_{121}\frac{\partial^2 \phi_y}{\partial y\partial x} - B_{111}\frac{\partial^2 \phi_x}{\partial x^2} \\
&-B_{661}\frac{\partial^2 \phi_x}{\partial y^2} - B_{661}\frac{\partial^2 \phi_y}{\partial y\partial x} + A_{121}\frac{\partial^3 w}{\partial y^2\partial x} + A_{111}\frac{\partial^3 w}{\partial x^3} \\
&+2A_{661}\frac{\partial^3 w}{\partial y^2\partial x} - A_{660}\frac{\partial^2 u}{\partial y^2} - A_{110}\frac{\partial^2 u}{\partial x^2} 2I_0 C_x\frac{\partial^2 u}{\partial x\partial t} \\
&+2I_0 C_y\frac{\partial^2 u}{\partial y\partial t} + I_0 C_x^2\frac{\partial^2 u}{\partial x^2} + I_0 C_y^2\frac{\partial^2 u}{\partial y^2} \\
&-2I_1 C_x\frac{\partial^3 w}{\partial x^2\partial t} - 2I_1 C_y\frac{\partial^3 w}{\partial y\partial x\partial t} - I_1 C_x^2\frac{\partial^3 w}{\partial x^3} \\
&-I_1 C_y^2\frac{\partial^3 w}{\partial y^2\partial x} + 2I_3 C_x\frac{\partial^2 \phi_x}{\partial x\partial t} + 2I_3 C_y\frac{\partial^2 \phi_x}{\partial y\partial t} + I_3 C_x^2\frac{\partial^2 \phi_x}{\partial x^2} \\
&+I_3 C_y^2\frac{\partial^2 \phi_x}{\partial y^2} + I_0\frac{\partial^2 u}{\partial t^2} + 2I_0 C_x C_y\frac{\partial^2 u}{\partial y\partial x} + 2I_3 C_x C_y\frac{\partial^2 \phi_x}{\partial y\partial x} \\
&-2I_1 C_x C_y\frac{\partial^3 w}{\partial y\partial x^2} + I_3\frac{\partial^2 \phi_x}{\partial t^2} - I_1\frac{\partial^3 w}{\partial x\partial t^2} = 0,
\end{aligned} \tag{13a}$$

δv: (13b)

$$
\begin{aligned}
&-A_{120}\frac{\partial^2 u}{\partial y\partial x}-A_{660}\frac{\partial^2 u}{\partial y\partial x}-B_{221}\frac{\partial^2 \phi_y}{\partial y^2}-B_{121}\frac{\partial^2 \phi_x}{\partial y\partial x}\\
&-B_{661}\frac{\partial^2 \phi_y}{\partial x^2}-B_{661}\frac{\partial^2 \phi_x}{\partial y\partial x}+A_{221}\frac{\partial^3 w}{\partial y^3}+A_{121}\frac{\partial^3 w}{\partial y\partial x^2}\\
&+2A_{661}\frac{\partial^3 w}{\partial y\partial x^2}-A_{660}\frac{\partial^2 v}{\partial x^2}-A_{220}\frac{\partial^2 v}{\partial y^2}2I_0C_y\frac{\partial^2 v}{\partial y\partial t}\\
&+2I_0C_x\frac{\partial^2 v}{\partial x\partial t}+I_0C_y^2\frac{\partial^2 v}{\partial y^2}+I_0C_x^2\frac{\partial^2 v}{\partial x^2}\\
&-2I_1C_y\frac{\partial^3 w}{\partial y^2\partial t}-2I_1C_x\frac{\partial^3 w}{\partial y\partial x\partial t}-I_1C_y^2\frac{\partial^3 w}{\partial y^3}\\
&-I_1C_x^2\frac{\partial^3 w}{\partial y\partial x^2}+I_3C_y^2\frac{\partial^2 \phi_y}{\partial y^2}+2I_3C_y\frac{\partial^2 \phi_y}{\partial y\partial t}+I_3C_x^2\frac{\partial^2 \phi_y}{\partial x^2}\\
&+2I_3C_x\frac{\partial^2 \phi_y}{\partial x\partial t}+I_0\frac{\partial^2 v}{\partial t^2}+2I_0C_yC_x\frac{\partial^2 v}{\partial y\partial x}-2I_1C_yC_x\frac{\partial^3 w}{\partial y^2\partial x}\\
&+2I_3C_xC_y\frac{\partial^2 \phi_y}{\partial y\partial x}+I_3\frac{\partial^2 \phi_y}{\partial t^2}-I_1\frac{\partial^3 w}{\partial y\partial t^2}=0,
\end{aligned}
\tag{13b}
$$

$\delta\phi_x$:

$$
\begin{aligned}
&K_sC_{552}\phi_x+D_{121}\frac{\partial^3 w}{\partial y^2\partial x}+D_{111}\frac{\partial^3 w}{\partial x^3}+2D_{661}\frac{\partial^3 w}{\partial y^2\partial x}\\
&-B_{122}\frac{\partial^2 \phi_y}{\partial y\partial x}-B_{121}\frac{\partial^2 v}{\partial y\partial x}-B_{111}\frac{\partial^2 u}{\partial x^2}-B_{661}\frac{\partial^2 u}{\partial y^2}\\
&-B_{662}\frac{\partial^2 \phi_y}{\partial y\partial x}-B_{661}\frac{\partial^2 v}{\partial y\partial x}-B_{112}\frac{\partial^2 \phi_x}{\partial x^2}-B_{662}\frac{\partial^2 \phi_x}{\partial y^2}\\
&+2I_5C_x\frac{\partial^2 \phi_x}{\partial x\partial t}+2I_5C_y\frac{\partial^2 \phi_x}{\partial y\partial t}+I_5C_x^2\frac{\partial^2 \phi_x}{\partial x^2}+I_5C_y^2\frac{\partial^2 \phi_x}{\partial y^2}\\
&+2I_3C_x\frac{\partial^2 u}{\partial x\partial t}+2I_3C_y\frac{\partial^2 u}{\partial y\partial t}+I_3C_x^2\frac{\partial^2 u}{\partial x^2}+I_3C_y^2\frac{\partial^2 u}{\partial y^2}\\
&-2I_4C_y\frac{\partial^3 w}{\partial y\partial x\partial t}-2I_4C_x\frac{\partial^3 w}{\partial x^2\partial t}-I_4C_y^2\frac{\partial^3 w}{\partial y^2\partial x}\\
&-I_4C_x^2\frac{\partial^3 w}{\partial x^3}+I_5\frac{\partial^2 \phi_x}{\partial t^2}+2I_5C_xC_y\frac{\partial^2 \phi_x}{\partial y\partial x}-2I_4C_xC_y\frac{\partial^3 w}{\partial y\partial x^2}\\
&+2I_3C_xC_y\frac{\partial^2 u}{\partial y\partial x}+I_3\frac{\partial^2 u}{\partial t^2}-I_4\frac{\partial^3 w}{\partial x\partial t^2}=0,
\end{aligned}
\tag{13c}
$$

δw:

$$
\begin{aligned}
&-D_{221}\frac{\partial^3 \phi_y}{\partial y^3}-D_{121}\frac{\partial^3 \phi_y}{\partial y\partial x^2}-D_{121}\frac{\partial^3 \phi_x}{\partial y^2\partial x}-D_{111}\frac{\partial^3 \phi_x}{\partial x^3}\\
&-2D_{661}\frac{\partial^3 \phi_x}{\partial y^2\partial x}-2D_{661}\frac{\partial^3 \phi_y}{\partial y\partial x^2}-A_{221}\frac{\partial^3 v}{\partial y^3}-A_{121}\frac{\partial^3 u}{\partial y^2\partial x}\\
&-A_{111}\frac{\partial^3 u}{\partial x^3}-A_{121}\frac{\partial^3 v}{\partial y\partial x^2}-2A_{661}\frac{\partial^3 u}{\partial y^2\partial x}-2A_{661}\frac{\partial^3 v}{\partial y\partial x^2}\\
&+4A_{662}\frac{\partial^4 w}{\partial y^2\partial x^2}+A_{112}\frac{\partial^4 w}{\partial x^4}+A_{222}\frac{\partial^4 w}{\partial y^4}+2A_{122}\frac{\partial^4 w}{\partial y^2\partial x^2}\\
&-2I_2C_xC_y\frac{\partial^4 w}{\partial y\partial x^3}-2I_2C_yC_x\frac{\partial^4 w}{\partial y^3\partial x}+2I_0C_xC_y\frac{\partial^2 w}{\partial y\partial x}\\
&+2I_4C_xC_y\frac{\partial^3 \phi_y}{\partial y^2\partial x}+2I_1C_yC_x\frac{\partial^3 v}{\partial y^2\partial x}+2I_1C_yC_x\frac{\partial^3 u}{\partial y\partial x^2}\\
&+2I_4C_xC_y\frac{\partial^3 \phi_x}{\partial y\partial x^2}+I_4\frac{\partial^3 \phi_y}{\partial y\partial t^2}+I_1\frac{\partial^3 v}{\partial y\partial t^2}+I_0\frac{\partial^2 w}{\partial t^2}+I_1\frac{\partial^3 u}{\partial x\partial t^2}\\
&-I_2\frac{\partial^4 w}{\partial y^2\partial t^2}+I_4\frac{\partial^3 \phi_x}{\partial x\partial t^2}-I_2\frac{\partial^4 w}{\partial x^2\partial t^2}+2I_0C_x\frac{\partial^2 w}{\partial x\partial t}\\
&+2I_0C_y\frac{\partial^2 w}{\partial y\partial t}-I_2C_x^2\frac{\partial^4 w}{\partial x^4}-I_2C_x^2\frac{\partial^4 w}{\partial y^2\partial x^2}-I_2C_y^2\frac{\partial^4 w}{\partial y^2\partial x^2}\\
&-I_2C_y^2\frac{\partial^4 w}{\partial y^4}+I_0C_x^2\frac{\partial^2 w}{\partial x^2}+I_0C_y^2\frac{\partial^2 w}{\partial y^2}-2I_2C_x\frac{\partial^4 w}{\partial x^3\partial t}\\
&-2I_2C_y\frac{\partial^4 w}{\partial y\partial x^2\partial t}-2I_2C_y\frac{\partial^4 w}{\partial y^3\partial t}-2I_2C_x\frac{\partial^4 w}{\partial y^2\partial x\partial t}+2I_4C_y\frac{\partial^3 \phi_y}{\partial y^2\partial t}\\
&+2I_4C_x\frac{\partial^3 \phi_y}{\partial y\partial x\partial t}+I_4C_y^2\frac{\partial^3 \phi_y}{\partial y^3}+2I_1C_x\frac{\partial^3 u}{\partial x^2\partial t}+2I_1C_y\frac{\partial^3 u}{\partial y\partial x\partial t}\\
&+I_1C_x^2\frac{\partial^3 u}{\partial x^3}+I_1C_y^2\frac{\partial^3 u}{\partial y^2\partial x}+2I_1C_y\frac{\partial^3 v}{\partial y^2\partial t}+2I_1C_x\frac{\partial^3 v}{\partial y\partial x\partial t}\\
&+I_1C_y^2\frac{\partial^3 v}{\partial y^3}+I_1C_x^2\frac{\partial^3 v}{\partial y\partial x^2}+I_4C_x^2\frac{\partial^3 \phi_y}{\partial y\partial x^2}+2I_4C_y\frac{\partial^3 \phi_x}{\partial y\partial x\partial t}\\
&+I_4C_x^2\frac{\partial^3 \phi_x}{\partial x^3}+2I_4C_x\frac{\partial^3 \phi_x}{\partial x^2\partial t}+I_4C_y^2\frac{\partial^3 \phi_x}{\partial y^2\partial x}\\
&-K_wW-\mathrm{C}_d\frac{\partial W}{\partial t}+K_{gx}\frac{\partial^2 W}{\partial x^2}+K_{gy}\frac{\partial^2 W}{\partial y^2}=0,
\end{aligned}
\tag{13d}
$$

$\delta\phi_y$:

$$
\begin{aligned}
&K_sC_{442}\phi_y+D_{221}\frac{\partial^3 w}{\partial y^3}+D_{121}\frac{\partial^3 w}{\partial y\partial x^2}+2D_{661}\frac{\partial^3 w}{\partial y\partial x^2}\\
&-B_{221}\frac{\partial^2 v}{\partial y^2}-B_{122}\frac{\partial^2 \phi_x}{\partial y\partial x}-B_{121}\frac{\partial^2 u}{\partial y\partial x}-B_{661}\frac{\partial^2 v}{\partial x^2}-B_{662}\frac{\partial^2 \phi_x}{\partial y\partial x}\\
&-B_{661}\frac{\partial^2 u}{\partial y\partial x}-B_{222}\frac{\partial^2 \phi_y}{\partial y^2}-B_{662}\frac{\partial^2 \phi_y}{\partial x^2}+I_5C_y^2\frac{\partial^2 \phi_y}{\partial y^2}+I_5C_x^2\frac{\partial^2 \phi_y}{\partial x^2}\\
&+2I_5C_y\frac{\partial^2 \phi_y}{\partial y\partial t}+2I_5C_x\frac{\partial^2 \phi_y}{\partial x\partial t}-2I_4C_y\frac{\partial^3 w}{\partial y^2\partial t}-2I_4C_x\frac{\partial^3 w}{\partial y\partial x\partial t}\\
&+I_3C_y^2\frac{\partial^2 v}{\partial y^2}+2I_3C_y\frac{\partial^2 v}{\partial y\partial t}+2I_3C_x\frac{\partial^2 v}{\partial x\partial t}+I_3C_x^2\frac{\partial^2 v}{\partial x^2}\\
&-I_4C_x^2\frac{\partial^3 w}{\partial y\partial x^2}-I_4C_y^2\frac{\partial^3 w}{\partial y^3}+I_5\frac{\partial^2 \phi_y}{\partial t^2}+2I_5C_xC_y\frac{\partial^2 \phi_y}{\partial y\partial x}\\
&-2I_4C_xC_y\frac{\partial^3 w}{\partial y^2\partial x}+2I_3C_xC_y\frac{\partial^2 v}{\partial y\partial x}+I_3\frac{\partial^2 v}{\partial t^2}-I_4\frac{\partial^3 w}{\partial y\partial t^2}=0,
\end{aligned}
\tag{13e}
$$

By putting the coefficients of δu, δv, δw and $\delta\theta$ to zero, separately, the governing equations of motion based on NFSDT can be calculated as follows

δu:

$$
\begin{aligned}
&A_{111}\frac{\partial^3 \theta}{\partial x^3}+A_{121}\frac{\partial^3 \theta}{\partial y^2\partial x}-A_{660}\frac{\partial^2 v}{\partial y\partial x}-A_{120}\frac{\partial^2 v}{\partial y\partial x}\\
&-A_{110}\frac{\partial^2 u}{\partial x^2}-A_{660}\frac{\partial^2 u}{\partial y^2}+2A_{661}\frac{\partial^3 \theta}{\partial y^2\partial x}-2I_1C_xC_y\frac{\partial^3 \theta}{\partial y\partial x^2}
\end{aligned}
\tag{14a}
$$

$$+2I_0 C_x C_y \frac{\partial^2 u}{\partial y \partial x} + 2I_0 C_x \frac{\partial^2 u}{\partial x \partial t} + 2I_0 C_y \frac{\partial^2 u}{\partial y \partial t} + I_0 C_x^2 \frac{\partial^2 u}{\partial x^2}$$
$$+I_0 C_y^2 \frac{\partial^2 u}{\partial y^2} - I_1 C_y^2 \frac{\partial^3 \theta}{\partial y^2 \partial x} - I_1 C_x^2 \frac{\partial^3 \theta}{\partial x^3} \tag{14a}$$
$$-2I_1 C_x \frac{\partial^3 \theta}{\partial x^2 \partial t} - 2I_1 C_y \frac{\partial^3 \theta}{\partial y \partial x \partial t} + I_0 \frac{\partial^2 u}{\partial t^2} - I_1 \frac{\partial^3 \theta}{\partial x \partial t^2} = 0,$$

δv:

$$A_{121} \frac{\partial^3 \theta}{\partial y \partial x^2} + A_{221} \frac{\partial^3 \theta}{\partial y^3} - A_{660} \frac{\partial^2 u}{\partial y \partial x} - A_{120} \frac{\partial^2 u}{\partial y \partial x}$$
$$-A_{220} \frac{\partial^2 v}{\partial y^2} - A_{660} \frac{\partial^2 v}{\partial x^2} + 2A_{661} \frac{\partial^3 \theta}{\partial y \partial x^2} - 2I_1 C_x C_y \frac{\partial^3 \theta}{\partial y^2 \partial x}$$
$$+2I_0 C_x C_y \frac{\partial^2 v}{\partial y \partial x} + 2I_0 C_x \frac{\partial^2 v}{\partial x \partial t} + 2I_0 C_y \frac{\partial^2 v}{\partial y \partial t} + I_0 C_x^2 \frac{\partial^2 v}{\partial x^2} \tag{14b}$$
$$+I_0 C_y^2 \frac{\partial^2 v}{\partial y^2} - I_1 C_x^2 \frac{\partial^3 \theta}{\partial y \partial x^2} - I_1 C_y^2 \frac{\partial^3 \theta}{\partial y^3}$$
$$-2I_1 C_x \frac{\partial^3 \theta}{\partial y \partial x \partial t} - 2I_1 C_y \frac{\partial^3 \theta}{\partial y^2 \partial t} + I_0 \frac{\partial^2 v}{\partial t^2} - I_1 \frac{\partial^3 \theta}{\partial y \partial t^2} = 0,$$

$\delta\theta$:

$$-K_s A_{550} \frac{\partial^2 \theta}{\partial x^2} - K_s A_{440} \frac{\partial^2 \theta}{\partial y^2} + K_s A_{550} \frac{\partial^2 w}{\partial x^2} + K_s A_{440} \frac{\partial^2 w}{\partial y^2}$$
$$-A_{111} \frac{\partial^3 u}{\partial x^3} - A_{121} \frac{\partial^3 v}{\partial y \partial x^2} - A_{121} \frac{\partial^3 u}{\partial y^2 \partial x} - A_{221} \frac{\partial^3 v}{\partial y^3} + 2A_{122} \frac{\partial^4 \theta}{\partial y^2 \partial x^2}$$
$$+4A_{662} \frac{\partial^4 \theta}{\partial y^2 \partial x^2} + A_{112} \frac{\partial^4 \theta}{\partial x^4} + A_{222} \frac{\partial^4 \theta}{\partial y^4} - 2A_{661} \frac{\partial^3 v}{\partial y \partial x^2} - 2A_{661} \frac{\partial^3 u}{\partial y^2 \partial x}$$
$$2I_1 C_x C_y \frac{\partial^3 v}{\partial y^2 \partial x} - 2I_2 C_x C_y \frac{\partial^4 \theta}{\partial y^3 \partial x} - 2I_2 C_x C_y \frac{\partial^4 \theta}{\partial y \partial x^3}$$
$$+2I_1 C_x C_y \frac{\partial^3 u}{\partial y \partial x^2} - I_2 C_x^2 \frac{\partial^4 \theta}{\partial x^4} - I_2 C_x^2 \frac{\partial^4 \theta}{\partial y^2 \partial x^2} - I_2 C_y^2 \frac{\partial^4 \theta}{\partial y^4} \tag{14c}$$
$$-2I_2 C_x \frac{\partial^4 \theta}{\partial y^2 \partial x \partial t} - 2I_2 C_y \frac{\partial^4 \theta}{\partial y^3 \partial t} - 2I_2 C_y \frac{\partial^4 \theta}{\partial y \partial x^2 \partial t} - 2I_2 C_x \frac{\partial^4 \theta}{\partial x^3 \partial t}$$
$$-I_2 C_y^2 \frac{\partial^4 \theta}{\partial y^2 \partial x^2} + I_1 C_x^2 \frac{\partial^3 v}{\partial y \partial x^2} + I_1 C_y^2 \frac{\partial^3 v}{\partial y^3} + 2I_1 C_x \frac{\partial^3 v}{\partial y \partial x \partial t}$$
$$+2I_1 C_y \frac{\partial^3 v}{\partial y^2 \partial t} + I_1 C_y^2 \frac{\partial^3 u}{\partial y^2 \partial x} + I_1 C_x^2 \frac{\partial^3 u}{\partial x^3} + 2I_1 C_x \frac{\partial^3 u}{\partial x^2 \partial t}$$
$$+2I_1 C_y \frac{\partial^3 u}{\partial y \partial x \partial t} - I_2 \frac{\partial^4 \theta}{\partial y^2 \partial t^2} - I_2 \frac{\partial^4 \theta}{\partial x^2 \partial t^2} + I_1 \frac{\partial^3 u}{\partial x \partial t^2} + I_1 \frac{\partial^3 v}{\partial y \partial t^2} = 0,$$

δw:

$$-K_s A_{550} \frac{\partial^2 w}{\partial x^2} - K_s A_{440} \frac{\partial^2 w}{\partial y^2} + K_s A_{550} \frac{\partial^2 \theta}{\partial x^2} + K_s A_{440} \frac{\partial^2 \theta}{\partial y^2}$$
$$+2I_0 C_x C_y \frac{\partial^2 w}{\partial y \partial x} + 2I_0 C_x \frac{\partial^2 w}{\partial x \partial t} + 2I_0 C_y \frac{\partial^2 w}{\partial y \partial t} + I_0 C_x^2 \frac{\partial^2 w}{\partial x^2} \tag{14d}$$
$$+I_0 C_y^2 \frac{\partial^2 w}{\partial y^2} + I_0 \frac{\partial^2 w}{\partial t^2} - K_w W - C_d \frac{\partial W}{\partial t} + K_{gx} \frac{\partial^2 W}{\partial x^2} + K_{gy} \frac{\partial^2 W}{\partial y^2} = 0,$$

Also, by setting the coefficients of δu, δv, δw, $\delta\phi_x$ and $\delta\phi_y$ to zero, separately, the governing equations of motion based on HSDT can be determined as follows

δu:

$$B_{121} \frac{\partial^2 \phi_y}{\partial y \partial x} + B_{661} \frac{\partial^2 \phi_y}{\partial y \partial x} + B_{661} \frac{\partial^2 \phi_x}{\partial y^2} + B_{121} \frac{\partial^3 w}{\partial y^2 \partial x}$$
$$-A_{120} \frac{\partial^2 v}{\partial y \partial x} - A_{661} \frac{\partial^2 \phi_y}{\partial y \partial x} - A_{661} \frac{\partial^2 \phi_x}{\partial y^2} - A_{111} \frac{\partial^2 \phi_x}{\partial x^2}$$
$$-A_{121} \frac{\partial^2 \phi_y}{\partial y \partial x} + B_{111} \frac{\partial^2 \phi_x}{\partial x^2} - A_{660} \frac{\partial^2 v}{\partial y \partial x} + B_{111} \frac{\partial^3 w}{\partial x^3}$$
$$+2B_{661} \frac{\partial^3 w}{\partial y^2 \partial x} - A_{660} \frac{\partial^2 u}{\partial y^2} - A_{110} \frac{\partial^2 u}{\partial x^2} + 2I_1 C_y \frac{\partial^2 \phi_x}{\partial y \partial t}$$
$$-I_3 C_x^2 \frac{\partial^2 \phi_x}{\partial x^2} + 2I_1 C_x \frac{\partial^2 \phi_x}{\partial x \partial t} - I_3 C_x^2 \frac{\partial^3 w}{\partial x^3}$$
$$-2I_3 C_x \frac{\partial^2 \phi_x}{\partial x \partial t} - 2I_3 C_y \frac{\partial^2 \phi_x}{\partial y \partial t} - I_3 C_y^2 \frac{\partial^2 \phi_x}{\partial y^2} + I_1 C_y^2 \frac{\partial^2 \phi_x}{\partial y^2} \tag{15a}$$
$$-I_3 C_y^2 \frac{\partial^3 w}{\partial y^2 \partial x} + I_1 C_x^2 \frac{\partial^2 \phi_x}{\partial x^2} + I_1 \frac{\partial^2 \phi_x}{\partial t^2} - I_3 \frac{\partial^2 \phi_x}{\partial t^2}$$
$$-I_3 \frac{\partial^3 w}{\partial x \partial t^2} + I_0 \frac{\partial^2 u}{\partial t^2} + 2I_1 C_x C_y \frac{\partial^2 \phi_x}{\partial y \partial x}$$
$$-2I_3 C_x C_y \frac{\partial^2 \phi_x}{\partial y \partial x} - 2I_3 C_x C_y \frac{\partial^3 w}{\partial y \partial x^2} + I_0 C_x^2 \frac{\partial^2 u}{\partial x^2}$$
$$+I_0 C_y^2 \frac{\partial^2 u}{\partial y^2} + 2I_0 C_x \frac{\partial^2 u}{\partial x \partial t} + 2I_0 C_y \frac{\partial^2 u}{\partial y \partial t}$$
$$-2I_3 C_x \frac{\partial^3 w}{\partial x^2 \partial t} - 2I_3 C_y \frac{\partial^3 w}{\partial y \partial x \partial t} + 2I_0 C_x C_y \frac{\partial^2 u}{\partial y \partial x} = 0,$$

δv:

$$-A_{221} \frac{\partial^2 \phi_y}{\partial y^2} + B_{221} \frac{\partial^2 \phi_y}{\partial y^2} + B_{121} \frac{\partial^2 \phi_x}{\partial y \partial x} + B_{221} \frac{\partial^3 w}{\partial y^3}$$
$$+B_{661} \frac{\partial^2 \phi_y}{\partial x^2} - A_{120} \frac{\partial^2 u}{\partial y \partial x} - A_{661} \frac{\partial^2 \phi_x}{\partial y \partial x} - A_{661} \frac{\partial^2 \phi_y}{\partial x^2}$$
$$-A_{121} \frac{\partial^2 \phi_x}{\partial y \partial x} + B_{661} \frac{\partial^2 \phi_x}{\partial y \partial x} - A_{660} \frac{\partial^2 u}{\partial y \partial x} + B_{121} \frac{\partial^3 w}{\partial y \partial x^2}$$
$$+2B_{661} \frac{\partial^3 w}{\partial y \partial x^2} - A_{660} \frac{\partial^2 v}{\partial x^2} - A_{220} \frac{\partial^2 v}{\partial y^2} - I_3 C_y^2 \frac{\partial^2 \phi_y}{\partial y^2}$$
$$+I_1 C_y^2 \frac{\partial^2 \phi_y}{\partial y^2} - I_3 C_x^2 \frac{\partial^2 \phi_y}{\partial x^2} + I_1 C_x^2 \frac{\partial^2 \phi_y}{\partial x^2} - I_3 C_x^2 \frac{\partial^3 w}{\partial y \partial x^2} \tag{15b}$$
$$-2I_3 C_y \frac{\partial^2 \phi_y}{\partial y \partial t} - 2I_3 C_x \frac{\partial^2 \phi_y}{\partial x \partial t} + 2I_1 C_x \frac{\partial^2 \phi_y}{\partial x \partial t} + 2I_1 C_y \frac{\partial^2 \phi_y}{\partial y \partial t}$$
$$-I_3 C_y^2 \frac{\partial^3 w}{\partial y^3} - 2I_3 C_y \frac{\partial^3 w}{\partial y^2 \partial t} - 2I_3 C_x \frac{\partial^3 w}{\partial y \partial x \partial t} - I_3 \frac{\partial^3 w}{\partial y \partial t^2}$$
$$+I_1 \frac{\partial^2 \phi_y}{\partial t^2} - I_3 \frac{\partial^2 \phi_y}{\partial t^2} + I_0 \frac{\partial^2 v}{\partial t^2} + 2I_1 C_x C_y \frac{\partial^2 \phi_y}{\partial y \partial x} - 2I_3 C_x C_y \frac{\partial^2 \phi_y}{\partial y \partial x}$$
$$-2I_3 C_x C_y \frac{\partial^3 w}{\partial y^2 \partial x} + I_0 C_x^2 \frac{\partial^2 v}{\partial x^2} + I_0 C_y^2 \frac{\partial^2 v}{\partial y^2}$$
$$+2I_0 C_x \frac{\partial^2 v}{\partial x \partial t} + 2I_0 C_y \frac{\partial^2 v}{\partial y \partial t} + 2I_0 C_x C_y \frac{\partial^2 v}{\partial y \partial x} = 0,$$

δw:

$$-K_s C_{442} \frac{\partial^2 w}{\partial y^2} + 2K_s C_{441} \frac{\partial^2 w}{\partial y^2} - K_s C_{552} \frac{\partial^2 w}{\partial x^2} + 2K_s C_{551} \frac{\partial^2 w}{\partial x^2} \tag{15c}$$

$$
\begin{aligned}
&-K_s A_{550}\frac{\partial^2 w}{\partial x^2}-K_s A_{440}\frac{\partial^2 w}{\partial y^2}-K_s A_{440}\frac{\partial \phi_y}{\partial y}-K_s A_{550}\frac{\partial \phi_x}{\partial x}\\
&-K_s C_{552}\frac{\partial \phi_x}{\partial x}+2K_s C_{551}\frac{\partial \phi_x}{\partial x}-K_s C_{442}\frac{\partial \phi_y}{\partial y}+2K_s C_{441}\frac{\partial \phi_y}{\partial y}\\
&+B_{122}\frac{\partial^3 \phi_x}{\partial y^2\partial x}-2D_{661}\frac{\partial^3 \phi_y}{\partial y\partial x^2}-D_{111}\frac{\partial^3 \phi_x}{\partial x^3}-2B_{661}\frac{\partial^3 v}{\partial y\partial x^2}\\
&+2B_{122}\frac{\partial^4 w}{\partial y^2\partial x^2}+B_{222}\frac{\partial^3 \phi_y}{\partial y^3}-D_{121}\frac{\partial^3 \phi_x}{\partial y^2\partial x}+2B_{662}\frac{\partial^3 \phi_y}{\partial y\partial x^2}\\
&-2D_{661}\frac{\partial^3 \phi_x}{\partial y^2\partial x}-B_{111}\frac{\partial^3 u}{\partial x^3}+B_{122}\frac{\partial^3 \phi_y}{\partial y\partial x^2}+B_{112}\frac{\partial^3 \phi_x}{\partial x^3}\\
&-D_{121}\frac{\partial^3 \phi_y}{\partial y\partial x^2}-B_{221}\frac{\partial^3 v}{\partial y^3}+B_{222}\frac{\partial^4 w}{\partial y^4}-2B_{661}\frac{\partial^3 u}{\partial y^2\partial x}\\
&-D_{221}\frac{\partial^3 \phi_y}{\partial y^3}+2B_{662}\frac{\partial^3 \phi_x}{\partial y^2\partial x}+4B_{662}\frac{\partial^4 w}{\partial y^2\partial x^2}-B_{121}\frac{\partial^3 v}{\partial y\partial x^2}\\
&-B_{121}\frac{\partial^3 u}{\partial y^2\partial x}+B_{112}\frac{\partial^4 w}{\partial x^4}+2I_4 C_x C_y\frac{\partial^3 \phi_y}{\partial y^2\partial x}-I_5\frac{\partial^4 w}{\partial x^2\partial t^2}\\
&+2I_4 C_x C_y\frac{\partial^3 \phi_x}{\partial y\partial x^2}-2I_5 C_x C_y\frac{\partial^3 \phi_y}{\partial y^2\partial x}+2I_3 C_x C_y\frac{\partial^3 v}{\partial y^2\partial x}\\
&-2I_5 C_x C_y\frac{\partial^3 \phi_x}{\partial y\partial x^2}+2I_3 C_x C_y\frac{\partial^3 u}{\partial y\partial x^2}-2I_5 C_x C_y\frac{\partial^4 w}{\partial y^3\partial x}\\
&-2I_5 C_x C_y\frac{\partial^4 w}{\partial y\partial x^3}+2I_0 C_x C_y\frac{\partial^2 w}{\partial y\partial x}+I_3\frac{\partial^3 u}{\partial x\partial t^2}+I_3\frac{\partial^3 v}{\partial y\partial t^2}\\
&+I_4\frac{\partial^3 \phi_y}{\partial y\partial t^2}-I_5\frac{\partial^3 \phi_y}{\partial y\partial t^2}-I_5\frac{\partial^4 w}{\partial y^2\partial t^2}+I_0\frac{\partial^2 w}{\partial t^2}-I_5 C_x^2\frac{\partial^3 \phi_y}{\partial y\partial x^2}\\
&-I_5 C_y^2\frac{\partial^3 \phi_y}{\partial y^3}+I_3 C_y^2\frac{\partial^3 v}{\partial y^3}-2I_5 C_x\frac{\partial^3 \phi_x}{\partial x^2\partial t}+2I_3 C_y\frac{\partial^3 v}{\partial y^2\partial t}\\
&-2I_5 C_x\frac{\partial^3 \phi_y}{\partial y\partial x\partial t}+I_3 C_y^2\frac{\partial^3 u}{\partial y^2\partial x}-I_5 C_x^2\frac{\partial^3 \phi_x}{\partial x^3}-I_5 C_y^2\frac{\partial^3 \phi_x}{\partial y^2\partial x}\\
&+2I_3 C_x\frac{\partial^3 v}{\partial y\partial x\partial t}-2I_5 C_y\frac{\partial^4 w}{\partial y^3\partial t}-2I_5 C_x\frac{\partial^4 w}{\partial x^3\partial t}+I_0 C_x^2\frac{\partial^2 w}{\partial x^2}\\
&+I_0 C_y^2\frac{\partial^2 w}{\partial y^2}-2I_5 C_y\frac{\partial^4 w}{\partial y\partial x^2\partial t}+2I_0 C_x\frac{\partial^2 w}{\partial x\partial t}-I_5 C_x^2\frac{\partial^4 w}{\partial y^2\partial x^2}\\
&-I_5 C_y^2\frac{\partial^4 w}{\partial y^4}-I_5 C_x^2\frac{\partial^4 w}{\partial x^4}-I_5 C_y^2\frac{\partial^4 w}{\partial y^2\partial x^2}-2I_5 C_x\frac{\partial^4 w}{\partial y^2\partial x\partial t}\\
&-2I_5 C_y\frac{\partial^3 \phi_y}{\partial y^2\partial t}+2I_3 C_x\frac{\partial^3 u}{\partial x^2\partial t}+2I_3 C_y\frac{\partial^3 u}{\partial y\partial x\partial t}-2I_5 C_y\frac{\partial^3 \phi_x}{\partial y\partial x\partial t}\\
&+I_4 C_y^2\frac{\partial^3 \phi_x}{\partial y^2\partial x}+2I_4 C_x\frac{\partial^3 \phi_y}{\partial y\partial x\partial t}+2I_4 C_x\frac{\partial^3 \phi_x}{\partial x^2\partial t}+I_4 C_x^2\frac{\partial^3 \phi_x}{\partial x^3}\\
&+I_4 C_y^2\frac{\partial^3 \phi_y}{\partial y^3}+2I_4 C_y\frac{\partial^3 \phi_x}{\partial y\partial x\partial t}+I_4 C_x^2\frac{\partial^3 \phi_y}{\partial y\partial x^2}+I_3 C_x^2\frac{\partial^3 u}{\partial x^3}\\
&+2I_4 C_y\frac{\partial^3 \phi_y}{\partial y^2\partial t}+I_3 C_x^2\frac{\partial^3 v}{\partial y\partial x^2}+2I_0 C_y\frac{\partial^2 w}{\partial y\partial t}\\
&-K_w W-C_d\frac{\partial W}{\partial t}+K_{gx}\frac{\partial^2 W}{\partial x^2}+K_{gy}\frac{\partial^2 W}{\partial y^2}=0
\end{aligned}
\tag{15c}
$$

$\delta\phi_x$:

$$
\begin{aligned}
&K_s C_{552}\phi_x-2K_s C_{551}\phi_x+K_s A_{550}\phi_x+K_s A_{550}\frac{\partial w}{\partial x}\\
&+K_s C_{552}\frac{\partial w}{\partial x}-2K_s C_{551}\frac{\partial w}{\partial x}+D_{121}\frac{\partial^3 w}{\partial y^2\partial x}\\
&+B_{121}\frac{\partial^2 v}{\partial y\partial x}+2D_{121}\frac{\partial^2 \phi_y}{\partial y\partial x}+B_{661}\frac{\partial^2 v}{\partial y\partial x}-A_{662}\frac{\partial^2 \phi_y}{\partial y\partial x}\\
&-B_{112}\frac{\partial^2 \phi_x}{\partial x^2}-B_{112}\frac{\partial^3 w}{\partial x^3}-B_{662}\frac{\partial^2 \phi_y}{\partial y\partial x}-A_{662}\frac{\partial^2 \phi_x}{\partial y^2}\\
&+2D_{661}\frac{\partial^3 w}{\partial y^2\partial x}-B_{122}\frac{\partial^2 \phi_y}{\partial y\partial x}+2D_{661}\frac{\partial^2 \phi_y}{\partial y\partial x}-B_{662}\frac{\partial^2 \phi_x}{\partial y^2}\\
&+B_{661}\frac{\partial^2 u}{\partial y^2}+2D_{661}\frac{\partial^2 \phi_x}{\partial y^2}-A_{661}\frac{\partial^2 v}{\partial y\partial x}-B_{122}\frac{\partial^3 w}{\partial y^2\partial x}\\
&-2B_{662}\frac{\partial^3 w}{\partial y^2\partial x}-A_{111}\frac{\partial^2 u}{\partial x^2}+B_{111}\frac{\partial^2 u}{\partial x^2}-A_{122}\frac{\partial^2 \phi_y}{\partial y\partial x}\\
&+D_{111}\frac{\partial^3 w}{\partial x^3}-A_{112}\frac{\partial^2 \phi_x}{\partial x^2}-A_{121}\frac{\partial^2 v}{\partial y\partial x}+2D_{111}\frac{\partial^2 \phi_x}{\partial x^2}\\
&-A_{661}\frac{\partial^2 u}{\partial y^2}+I_2\frac{\partial^2 \phi_x}{\partial t^2}+I_1\frac{\partial^2 u}{\partial t^2}-I_3\frac{\partial^2 u}{\partial t^2}+I_5\frac{\partial^2 \phi_x}{\partial t^2}\\
&-2I_4\frac{\partial^2 \phi_x}{\partial t^2}-I_3 C_x^2\frac{\partial^2 u}{\partial x^2}+2I_1 C_x\frac{\partial^2 u}{\partial x\partial t}-2I_3 C_y\frac{\partial^2 u}{\partial y\partial t}\\
&+2I_5 C_x\frac{\partial^3 w}{\partial x^2\partial t}+I_1 C_y^2\frac{\partial^2 u}{\partial y^2}+I_5 C_x^2\frac{\partial^3 w}{\partial x^3}+I_5 C_y^2\frac{\partial^3 w}{\partial y^2\partial x}\\
&+I_1 C_x^2\frac{\partial^2 u}{\partial x^2}-2I_4 C_y^2\frac{\partial^2 \phi_x}{\partial y^2}-2I_4 C_x^2\frac{\partial^2 \phi_x}{\partial x^2}+2I_5 C_y\frac{\partial^2 \phi_x}{\partial y\partial t}\\
&+2I_5 C_x\frac{\partial^2 \phi_x}{\partial x\partial t}+2I_2 C_x\frac{\partial^2 \phi_x}{\partial x\partial t}+2I_2 C_y\frac{\partial^2 \phi_x}{\partial y\partial t}+I_5 C_x^2\frac{\partial^2 \phi_x}{\partial x^2}\\
&+I_2 C_x^2\frac{\partial^2 \phi_x}{\partial x^2}+I_5 C_y^2\frac{\partial^2 \phi_x}{\partial y^2}+I_2 C_y^2\frac{\partial^2 \phi_x}{\partial y^2}-4I_4 C_x\frac{\partial^2 \phi_x}{\partial x\partial t}\\
&-4I_4 C_y\frac{\partial^2 \phi_x}{\partial y\partial t}+2I_5 C_y\frac{\partial^3 w}{\partial y\partial x\partial t}-I_4 C_y^2\frac{\partial^3 w}{\partial y^2\partial x}\\
&-2I_4 C_x\frac{\partial^3 w}{\partial x^2\partial t}-I_4 C_x^2\frac{\partial^3 w}{\partial x^3}-2I_4 C_y\frac{\partial^3 w}{\partial y\partial x\partial t}\\
&+2I_1 C_y\frac{\partial^2 u}{\partial y\partial t}+2I_1 C_x C_y\frac{\partial^2 u}{\partial y\partial x}-2I_3 C_x C_y\frac{\partial^2 u}{\partial y\partial x}\\
&+2I_5 C_x C_y\frac{\partial^3 w}{\partial y\partial x^2}-2I_3 C_x\frac{\partial^2 u}{\partial x\partial t}-I_3 C_y^2\frac{\partial^2 u}{\partial y^2}\\
&+2I_2 C_x C_y\frac{\partial^2 \phi_x}{\partial y\partial x}+2I_5 C_x C_y\frac{\partial^2 \phi_x}{\partial y\partial x}-4I_4 C_x C_y\frac{\partial^2 \phi_x}{\partial y\partial x}\\
&-2I_4 C_x C_y\frac{\partial^3 w}{\partial y\partial x^2}=0,
\end{aligned}
\tag{15d}
$$

$\delta\phi_y$:

$$
\begin{aligned}
&K_s C_{442}\phi_y-2K_s C_{441}\phi_y+K_s A_{440}\phi_y+K_s A_{440}\frac{\partial w}{\partial y}\\
&+K_s C_{442}\frac{\partial w}{\partial y}-2K_s C_{441}\frac{\partial w}{\partial y}-A_{122}\frac{\partial^2 \phi_x}{\partial y\partial x}-B_{122}\frac{\partial^2 \phi_x}{\partial y\partial x}\\
&-2B_{662}\frac{\partial^3 w}{\partial y\partial x^2}-B_{222}\frac{\partial^2 \phi_y}{\partial y^2}-A_{662}\frac{\partial^2 \phi_y}{\partial x^2}+B_{221}\frac{\partial^2 v}{\partial y^2}\\
&+2D_{661}\frac{\partial^2 \phi_y}{\partial x^2}-A_{221}\frac{\partial^2 v}{\partial y^2}-B_{662}\frac{\partial^2 \phi_x}{\partial y\partial x}-A_{661}\frac{\partial^2 u}{\partial y\partial x}\\
&-A_{121}\frac{\partial^2 u}{\partial y\partial x}+2D_{661}\frac{\partial^3 w}{\partial y\partial x^2}-B_{222}\frac{\partial^3 w}{\partial y^3}+2D_{661}\frac{\partial^2 \phi_x}{\partial y\partial x}
\end{aligned}
\tag{15e}
$$

$$
\begin{aligned}
&+B_{661}\frac{\partial^2 u}{\partial y\partial x}-B_{122}\frac{\partial^3 w}{\partial y\partial x^2}-B_{662}\frac{\partial^2 \phi_y}{\partial x^2}-A_{222}\frac{\partial^2 \phi_y}{\partial y^2}\\
&+B_{661}\frac{\partial^2 v}{\partial x^2}-A_{662}\frac{\partial^2 \phi_x}{\partial y\partial x}+D_{121}\frac{\partial^3 w}{\partial y\partial x^2}+B_{121}\frac{\partial^2 u}{\partial y\partial x}\\
&+2D_{221}\frac{\partial^2 \phi_y}{\partial y^2}-A_{661}\frac{\partial^2 v}{\partial x^2}+2D_{121}\frac{\partial^2 \phi_x}{\partial y\partial x}+D_{221}\frac{\partial^3 w}{\partial y^3}\\
&-I_3C_y^{\,2}\frac{\partial^2 v}{\partial y^2}+I_1C_y^{\,2}\frac{\partial^2 v}{\partial y^2}+I_5C_x^{\,2}\frac{\partial^3 w}{\partial y\partial x^2}+I_1C_x^{\,2}\frac{\partial^2 v}{\partial x^2}\\
&-I_3C_x^{\,2}\frac{\partial^2 v}{\partial x^2}+I_5C_y^{\,2}\frac{\partial^3 w}{\partial y^3}-2I_3C_y\frac{\partial^2 v}{\partial y\partial t}-2I_3C_x\frac{\partial^2 v}{\partial x\partial t}\\
&+2I_1C_x\frac{\partial^2 v}{\partial x\partial t}+2I_1C_y\frac{\partial^2 v}{\partial y\partial t}+2I_5C_x\frac{\partial^3 w}{\partial y\partial x\partial t}\\
&+2I_5C_x\frac{\partial^2 \phi_y}{\partial x\partial t}+2I_5C_y\frac{\partial^2 \phi_y}{\partial y\partial t}+2I_2C_x\frac{\partial^2 \phi_y}{\partial x\partial t}\\
&+2I_2C_y\frac{\partial^2 \phi_y}{\partial y\partial t}-2I_4C_x^{\,2}\frac{\partial^2 \phi_y}{\partial x^2}-2I_4C_y^{\,2}\frac{\partial^2 \phi_y}{\partial y^2}\\
&+I_5C_x^{\,2}\frac{\partial^2 \phi_y}{\partial x^2}+I_2C_x^{\,2}\frac{\partial^2 \phi_y}{\partial x^2}+I_5C_y^{\,2}\frac{\partial^2 \phi_y}{\partial y^2}\\
&+I_2C_y^{\,2}\frac{\partial^2 \phi_y}{\partial y^2}-4I_4C_x\frac{\partial^2 \phi_y}{\partial x\partial t}-4I_4C_y\frac{\partial^2 \phi_y}{\partial y\partial t}\\
&+2I_5C_y\frac{\partial^3 w}{\partial y^2\partial t}-2I_4C_x\frac{\partial^3 w}{\partial y\partial x\partial t}-2I_4C_y\frac{\partial^3 w}{\partial y^2\partial t}\\
&-I_4C_y^{\,2}\frac{\partial^3 w}{\partial y^3}-I_4C_x^{\,2}\frac{\partial^3 w}{\partial y\partial x^2}-2I_4\frac{\partial^2 \phi_y}{\partial t^2}-I_3\frac{\partial^2 v}{\partial t^2}\\
&+I_5\frac{\partial^3 w}{\partial y\partial t^2}+I_5\frac{\partial^2 \phi_y}{\partial t^2}+I_2\frac{\partial^2 \phi_y}{\partial t^2}+I_1\frac{\partial^2 v}{\partial t^2}-I_4\frac{\partial^3 w}{\partial y\partial t^2}\\
&-2I_4C_xC_y\frac{\partial^3 w}{\partial y^2\partial x}+2I_1C_xC_y\frac{\partial^2 v}{\partial y\partial x}-2I_3C_xC_y\frac{\partial^2 v}{\partial y\partial x}\\
&+2I_5C_xC_y\frac{\partial^3 w}{\partial y^2\partial x}+2I_5C_xC_y\frac{\partial^2 \phi_y}{\partial y\partial x}+2I_2C_xC_y\frac{\partial^2 \phi_y}{\partial y\partial x}\\
&-4I_4C_xC_y\frac{\partial^2 \phi_y}{\partial y\partial x}=0,
\end{aligned}
\tag{15e}
$$

It should be noted that all the parameters which are expressed in Eqs. (13)-(15) are defined in Appendix A.

3. Solution approach

The Navier's type solution is selected to solve the equations of motion of simply-supported composite plate.Based on this procedure, the displacement variables are assumed as functions which satisfy at least the different geometric boundary conditions. It should be noted that the simply supported boundary condition is selected at x = {0, a} and y = {0, b}. According to above explanations, the functions of displacement variables can be considered as follows (Wang *et al*. 2000)

$$
\begin{aligned}
u(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}U_{mn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},\\
v(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}V_{mn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},\\
w(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}W_{mn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},\\
\phi_x(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}\phi_{xmn}\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},\\
\phi_y(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}\phi_{ymn}\sin\left(\frac{m\pi x}{a}\right)\cos\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},\\
\theta(x,y,t)&=\sum_{n=1}^{N}\sum_{m=1}^{M}\phi_{ymn}\sin\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)e^{i\omega_{mn}t},
\end{aligned}
\tag{16}
$$

where m and n present half axial and transverse wave numbers, respectively and ω_{mn} is the natural frequency of composite plate.

Therefore, final relations can be obtained as a following matrices form

$$
[P]_{5\times5}\{U_{mn}\quad V_{mn}\quad W_{mn}\quad \phi_{xmn}\quad \phi_{ymn}\}^T=\{0\},
\tag{17}
$$

for CPT, FSDT, TSDT, SSDT and HSDT

$$
[R]_{4\times4}\{U_{mn}\quad V_{mn}\quad W_{mn}\quad \theta_{mn}\}^T=\{0\},\quad \text{for NFSDT}
\tag{18}
$$

The elements of matrices P and R in above relations can be determined by substituting Eq. (16) into Eqs. (13)-(15) for six theories, separately.

4. Numerical results and discussion

In this section, effects of various parameters such as distribution and volume fraction of CNTs, moving speed in x and y directions, orthotropic pattern of viscoelastic foundation, aspect ratio and thickness on the vibration characteristics of moving CNTRC plate are discussed in details. Poly methyl methacrylate, referred to as PMMA, is selected for the matrix of CNTRC plate inside CNTs fibers. The effective material properties of PMMA and CNTs are presented in Table 1.

It should be noted that η_1 = 0.149, η_2 = 0.934 for the case of V^*_{CNT} = 0.11, η_1 = 0.150, η_2 = 0.941 for the case of V^*_{CNT} = 0.14, and η_1 = 0.149, η_2 = 1.381 for the case of V^*_{CNT} = 0.17. Moreover, it's assumed that $\eta_2 = \eta_3$ and G_{12}, G_{13} = G_{23} according to Zhu *et al*. (2012). It should be mentioned that following dimensionless parameters are defined to obtain dimensionless results

Table 1 Mechanical properties of PMMA and SWCNT with 10 (Wang and Shen 2012)

ρ^{CNT} (Kg/m^3)	E_{11}^{CNT} (GPa)	E_{22}^{CNT} (GPa)	υ^{CNT}	G_{12}^{CNT} (GPa)	ρ_m (Kg/m^3)	υ_m	E_m (GPa)
1400	600	10	0.19	17.2	1190	0.3	2.5

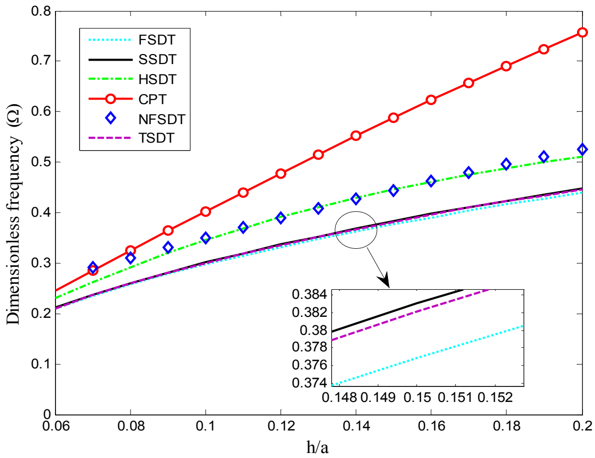

Fig. 2 The real part of dimensionless frequency versus length to thickness ratio in different plate theories

$$(\zeta,\eta)=\left(\frac{x}{a},\frac{y}{b}\right),\quad (U,V,W)=\left(\frac{u}{a},\frac{v}{b},\frac{w}{h}\right),$$
$$(\alpha,\beta,\gamma)=\left(\frac{h}{a},\frac{h}{b},\frac{a}{b}\right), \tag{19}$$
$$C^*=C\sqrt{\frac{Q_{11}}{\rho}}\ P^*=\frac{N_x}{Q_{11}a}\ \Omega=\omega a\sqrt{\frac{\rho}{Q_{11}}}.$$

The comparison between six plate theories is performed in Fig. 2. This figure shows the real part of dimensionless frequency versus length to thickness ratio. It can be seen that dimensionless frequencies decrease with increasing length to thickness ratio. As can be observed, the difference between the results of various theories is more prominent in low length to thickness ratio. In addition, it can be concluded that for low a/h, the results of CPT, NFSDT and HSDT is incredible. Moreover, it is evident that the results of TSDT are close to SSDT and these theories are more reliable than others. Based on the results of this figure, TSDT is selected to obtain the effect of various parameters on vibration characteristics of moving composite plate.

In another attempt, the comparison between six shear deformation plate theories is performed in Tables 2-3. Table 2 presents dimensionless natural frequencies of CNTRC plate (with zero moving speed) for different types of distribution and volume fraction of CNTs which are obtained by six shear deformation plate theories. It can be found that the in different types of distribution and volume fraction of CNTs, the results of HSDT and SSDT is more reliable than other plate theories. Note that we assume that the UD-CNTRC and FG-CNTRC plates have the same overall mass fraction w_{CNT} of the carbon nanotube for the purpose of comparisons. It can be observed that the volume fraction of the CNT has so much influence on the real part of dimensionless frequencies of the plates. It is noticeable that the dimensionless frequencies of FG-X CNTRC plates are larger than deflections of UD-CNTRC plates while those of FG-O CNTRC plate are smaller through these four types of plates with the same mass fraction of the CNT. This is because that the form of distribution of reinforcements can affect the stiffness of plates and it is thus expected that the desired stiffness can be achieved by adjusting the distribution of CNTs along the thickness direction of plates. It is concluded that reinforcements distributed close to top and bottom are more efficient than those distributed nearby the mid-plane for increasing the stiffness of plates.

Table 3 shows the influences of moving speed in both x and y directions on the real part of dimensionless frequencies of nanocomposite plate. This table approves that with increasing C_x^* and C_y^*, the frequencies of composite plate decreases, while the reduction percent is lower for the moving plate in x direction. This is due to the fact that adding CNTs along x direction leads to increase stability of moving plate and consequently the frequencies increase.

The influences of moving speed in both x and y directions are demonstrated in Fig. 3. The left figure is related to real part of dimensionless frequency versus dimensionless moving speed in x direction when the

Table 2 Dimensionless frequencies of composite plate reinforced by CNTs which are obtained by various shear deformation theories ($a = 10\,h$, $T = 300$ K, $C_x^* = 0$, $C_y^* = 0$)

		CPT	FSDT	TSDT	HSDT	SSDT	NFSDT
$V^*_{CNT} = 0.11$	UD	0.3482	0.2855	0.2790	0.3043	0.2792	0.3075
	FG-X	0.3986	0.2989	0.2931	0.3001	0.2937	0.3406
	FG-O	0.2892	0.2621	0.2554	0.3109	0.2551	0.2645
	FG-V	0.3112	0.2722	0.2674	0.3100	0.2675	0.2812
$V^*_{CNT} = 0.14$	UD	0.3618	0.3076	0.3014	0.3392	0.3015	0.3237
	FG-X	0.4104	0.3229	0.3172	0.3345	0.3176	0.3571
	FG-O	0.3060	0.2824	0.2764	0.3466	0.2762	0.2819
	FG-V	0.3272	0.2933	0.2888	0.3454	0.2889	0.2982
$V^*_{CNT} = 0.17$	UD	0.3642	0.3114	0.3053	0.3457	0.3054	0.3265
	FG-X	0.4126	0.3270	0.3213	0.3407	0.3218	0.3601
	FG-O	0.3085	0.2859	0.2799	0.3535	0.2797	0.2846
	FG-V	0.3292	0.2966	0.2922	0.3522	0.2923	0.3007

Table 3 The influence of moving speed on dimensionless frequencies of composite plate reinforced by CNTs which are obtained by various shear deformation theories (a = 10 h, T = 300 K, V^*_{CNT} = 0.17, FG-X distribution)

		CPT	FSDT	TSDT	HSDT	SSDT	NFSDT
$C^*_x = 0$	$C^*_y = 0$	0.4022	0.2971	0.3005	0.3104	0.3010	0.3457
	$C^*_y = 0.01$	0.3972	0.2904	0.2938	0.3040	0.2944	0.3400
	$C^*_y = 0.02$	0.3820	0.2692	0.2729	0.2838	0.2735	0.3221
	$C^*_y = 0.03$	0.3553	0.2297	0.2340	0.2466	0.2347	0.2898
$C^*_x = 0.02$	$C^*_y = 0$	0.3972	0.2904	0.2938	0.3040	0.2944	0.3400
	$C^*_y = 0.01$	0.3922	0.2835	0.2870	0.2974	0.2876	0.3341
	$C^*_y = 0.02$	0.3768	0.2618	0.2656	0.2768	0.2662	0.3159
	$C^*_y = 0.03$	0.3497	0.2209	0.2254	0.2385	0.2261	0.2829
$C^*_x = 0.04$	$C^*_y = 0$	0.3820	0.2692	0.2729	0.2838	0.2735	0.3221
	$C^*_y = 0.01$	0.3768	0.2618	0.2656	0.2768	0.2662	0.3159
	$C^*_y = 0.02$	0.3608	0.2381	0.2423	0.2545	0.2429	0.2966
	$C^*_y = 0.03$	0.3323	0.1923	0.1974	0.2123	0.1982	0.2612

velocity in y direction is zero. Also, in the absence of C^*_x, the effect of moving speed in y direction on vibration frequencies of moving plate is illustrated in the right side of Fig. 3. As can be seen, the critical speed of composite plate moving along the x direction is higher than when it moves along the y direction. It's due to the fact that mechanical properties of CNTRC plates are much higher in the longitudinal direction (in x axis) than in the transverse direction (in y axis) since CNTs only align in x direction. Therefore, the stability and consequently the critical speed increase when the moving plate reinforced by CNT along the x direction. Moreover, Fig. 3 demonstrates the effect of distribution of CNTs on vibration frequencies of composite plate. It can be concluded that the highest and lowest frequencies are related to FG-X and FG-O distributions, respectively, due to reinforcements distributed close to top and bottom are more efficient than those distributed near the mid-plane for increasing the stiffness of CNTRC plates.

The influences of CNT distribution on dimensionless frequencies versus aspect ratio of CNTRC plate are demonstrated in Fig. 4. This figure approved that increasing aspect ratio of composite plate leads to increase frequencies of moving system. In addition, the effect of CNTs distribution is more significant at square plate. Also, it can be found that the frequencies of composite plate which is reinforced by CNTs in FG-X distribution are more than others.

The influences of volume fractions of composite plate on dimensionless frequencies versus dimensionless moving speed ratio are demonstrated in Fig. 5. This figure approved that increasing moving speed ratio leads to decrease frequencies of moving system. In addition, the effect of CNTs reinforcement is more significant at in high moving speed ratio. Also, it can be found that the frequencies of composite plate reinforced by 0.17 volume fractions of CNTs are close to 0.14. So, in this study V^*_{CNT} = 0.17 is selected for the composite plate. It should be noted that designers could meet their purposes by selecting the suitable percent of fiber in composite structures.

As mentioned ago, SWCNTs is selected as a reinforcement of face sheets of sandwich plate. The mechanical properties of CNTs at different temperatures are adopted

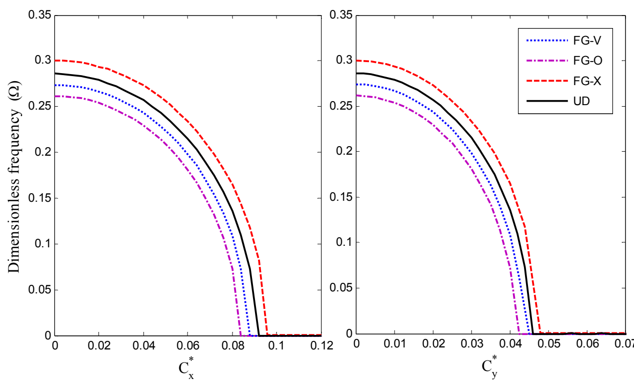

Fig. 3 The influences of CNT distribution on the real part of dimensionless frequency versus dimensionless moving speed

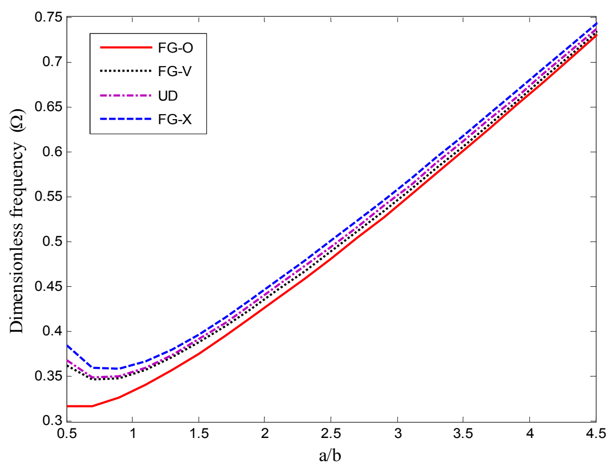

Fig. 4 The influences of CNT distribution on the real part of dimensionless frequency versus aspect ratio of composite plate

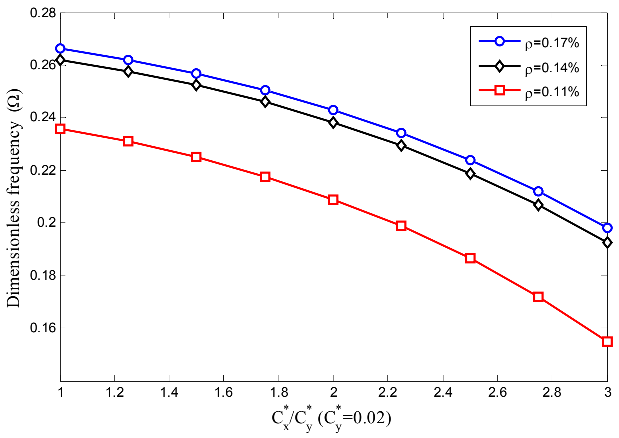

Fig. 5 The influences of volume fraction of CNTs on the dimensionless frequency versus dimensionless moving speed ratio in two direction

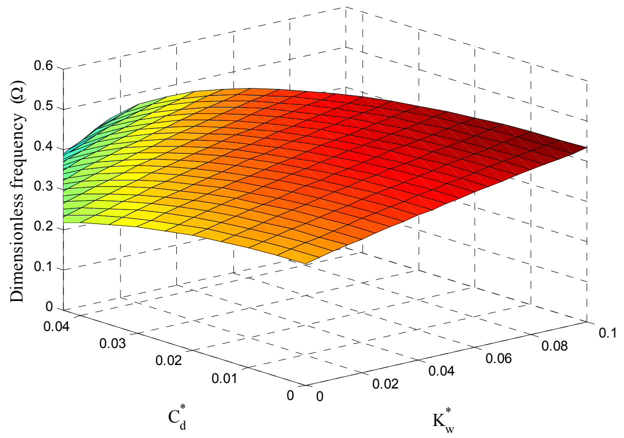

Fig. 7 Three-dimensional plot of dimensionless frequency variation versus Pasternak shear constant in ς and η directions

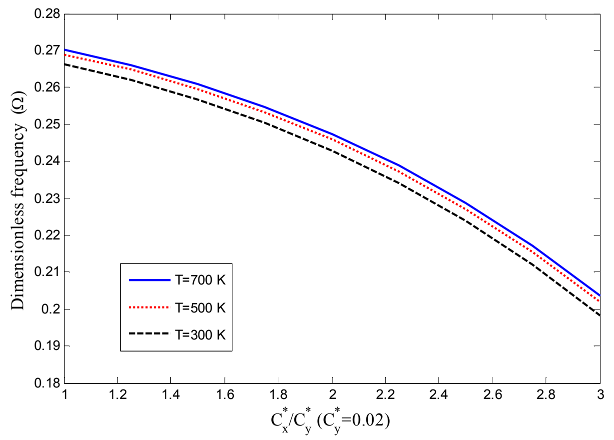

Fig. 6 The real part of dimensionless frequency versus dimensionless moving speed ratio for various temperature changes

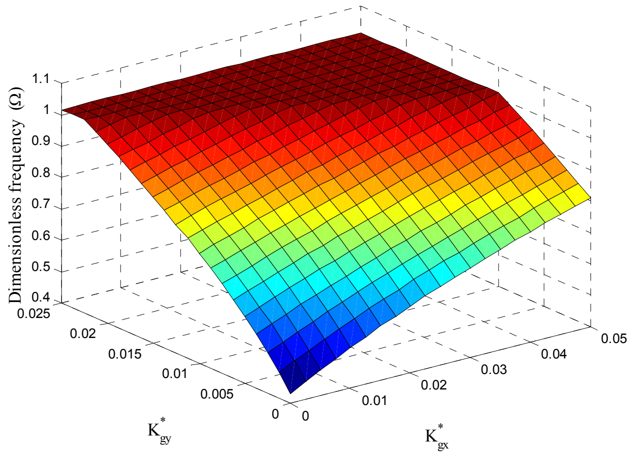

Fig. 8 Three-dimensional plot of dimensionless frequency variation versus Winkler constant and damping constant

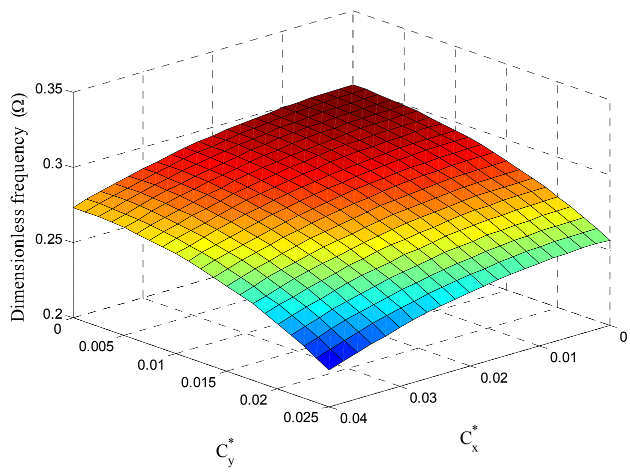

Fig. 9 Three-dimensional plot of dimensionless frequency variation versus moving speed in x and y directions

from Wang and Shen (2012). Fig. 6 presents the effect of temperature on vibration frequencies of moving sandwich plate. As can be seen, increasing temperature leads to increase the frequencies of moving composite plate. So by considering appropriate temperature, the stability of moving structures can be controlled and optimized to design and use this kind of structures.

Figs. 7 and 8 illustrate the three-dimensional plot of simultaneous effects of (K^*_w, C^*_d) and (K^*_{gx}, K^*_{gy}) on the dimensionless frequency of composite plate, respectively. Fig. 7 shows that the stability of CNTRC plate decreases with increasing damping coefficient of elastic medium and decreasing Winkler constant. Also, Fig. 8 demonstrates that increasing both K^*_{gx} and K^*_{gy} leads to increase stability of CNTRC plate, while the influence of K^*_{gy} is more effective than K^*_{gx}. So, the optimum values of foundation parameters can be selected to improve the stability of CNTRC plate by using the results of recent two figures.

The influences of moving speed in x and y directions, simultaneously, is shown in Fig. 9. As can be seen, the variation of dimensionless frequencies with increasing C^*_y is more evident than C^*_x. It can be concluded that due to existence of CNTs reinforcement align in x direction, the mechanical properties of CNTRC plates are much higher in the longitudinal direction (in x axis) than in the transverse direction (in y axis) and consequently the slop of curve associated with moving speed in x direction is lower than other case.

5. Conclusions

Based on various types of shear deformation theories, vibration analysis of composite plate moving in two directions was developed for the first time. Orthotropic visco-Pasternak foundation was developed to consider the influences of orthotropy angle, damping coefficient, normal and shear modulus. Considering simply supported boundary conditions, the motion equations were obtained using Hamilton's principle and solved by analytical solution. It was found that vibrating behavior of moving CNTRC plate was strongly dependent on moving speed, so that, with increasing moving speed, system stability decreases and became susceptible to buckling. In addition, the stability of FGCNTRC plate can be improved, considerably, by changing the distribution of CNTs. Moreover, orthotropic visco-Pasternak foundation plays an important role on the stability of axially moving SLGS, so that, varying the shear modulus of orthotropic elastic medium cause to change the intensity and the trend of orthotropy angle. The results of this study is hoped to be used in optimum design of aerospace and military equipment.

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References

An, C. and Su, J. (2014), "Dynamic analysis of axially moving orthotropic plates: Integral transforms solution", *Appl. Math. Comput.*, **228**, 489-507.
Banichuk, N., Jeronen, J., Neittaanmäki, P. and Tuovinen, T. (2010), "On the instability of an axially moving elastic plate", *Int. J. Solids Struct.*, **47**(1), 91-99.
Banichuk, N., Jeronen, J., Kurki, M., Neittaanmäki, P., Saksa, T. and Tuovinen, T. (2011), "On the limit velocity and buckling phenomena of axially moving orthotropic membranes and plates", *Int. J. Solids Struct.*, **48**(13), 2015-2025.
Ghayesh, M.H., Amabili, M. and Païdoussis, M.P. (2013), "Nonlinear dynamics of axially moving plates", *J. Sound Vib.*, **332**(2), 391-406.
Ghorbanpour Arani, A. and Haghparast, E. (2016), "Size-dependent vibration of axially moving viscoelastic microplates based on sinusoidal shear deformation theory", *Int. J. Appl. Mech.*
Ghorbanpour Arani, A., Haghparast, E. and Baba Akbar Zarei, H. (2016), "Vibration of axially moving 3-phase CNTFPC plate resting on orthotropic foundation", *Struct. Eng. Mech.*, *Int. J.*, **57**(1), 105-126.
Gibson, R.F. (1994), *Principles of Composite Material Mechanics*, McGraw-Hill, Inc, New York, USA.
Hatami, S., Azhari, M. and Saadatpour, M.M. (2007), "Free vibration of moving laminated composite plates", *Compos. Struct.*, **80**(4), 609-620.
Hatami, S., Ronagh, H.R. and Azhari, M. (2008), "Exact free vibration analysis of axially moving viscoelastic plates", *Comput. Struct.*, **86**(17), 1738-1746.
Kim, J., Cho, J., Lee, U. and Park, S. (2003), "Modal spectral element formulation for axially moving plates subjected to in-plane axial tension", *Comput. Struct.*, **81**(20), 2011-2020.
Lin, C.C. (1997), "Stability and vibration characteristics of axially moving plates", *Int. J. Solids Struct.*, **34**(24), 3179-3190.
Marynowski, K. (2010), "Free vibration analysis of the axially moving Levy-type viscoelastic plate", *Eur. J. Mech. A Solids*, **29**(5), 879-886.
Marynowski, K. and Grabski, J. (2013), "Dynamic analysis of an axially moving plate subjected to thermal loading", *Mech. Res. Commun.*, **51**, 67-71.
Mohammadimehr, M., Rousta Navi, B. and Ghorbanpour Arani, A. (2015), "Free vibration of viscoelastic double-bonded polymeric nanocomposite plates reinforced by FG-SWCNTs using MSGT, sinusoidal shear deformation theory and meshless method", *Compos. Struct.*, **131**, 654-671.
Tang, Y.Q. and Chen, L.Q. (2011), "Nonlinear free transverse vibrations of in-plane moving plates: Without and with internal resonances", *J. Sound Vib.*, **330**(1), 110-126.
Tang, Y.Q. and Chen, L.Q. (2012), "Primary resonance in forced vibrations of in-plane translating viscoelastic plates with 3:1 internal resonance", *Nonlinear Dyn.*, **69**(1), 159-172.
Wang, X. (1999), "Numerical analysis of moving orthotropic thin plates", *Comput. Struct.*, **70**(4), 467-486.
Wang, Z.X. and Shen, H.S. (2012), "Nonlinear vibration and bending of sandwich plates with nanotube-reinforced composite face sheets", *Composites Part B*, **43**(2), 411-421.
Wang, C.M., Reddy, J.N. and Lee, K.H. (2000), *Shear Deformable Beams and Plates*, Elsevier.
Yang, T. and Fang, B. (2013), "Asymptotic analysis of an axially viscoelastic string constituted by a fractional differentiation law", *Int. J. Non-Linear Mech.*, **49**, 170-174.
Yang, T., Fang, B., Chen, Y. and Zhen, Y. (2009), "Approximate solutions of axially moving viscoelastic beams subject to multi-frequency excitations", *Int. J. Non-Linear Mech.*, **44**(2), 230-238
Yang, X.D., Chen, L.Q. and Zu, J.W. (2011), "Vibrations and stability of an axially moving rectangular composite plate", *J. Appl. Mech.*, **78**(1), 11018-11028.
Yang, X.D., Zhang, W., Chen, L.Q. and Yao, M.H. (2012), "Dynamical analysis of axially moving plate by finite difference method", *Nonlinear Dyn.*, **67**(2), 997-1006.
Yang, T., Fang, B., Yang, X.D. and Li, Y. (2013), "Closed-form approximate solution for natural frequency of axially moving beams", *Int. J. Mech. Sci.*, **74**, 154-160.
Zho, Y. and Wang, Z. (2008), "Vibrations of axially moving viscoelastic plate with parabolically varying thickness", *J. Sound Vib.*, **316**(1), 198-210.
Zhu, P., Lei, Z.X. and Liew, K.M. (2012), "Static and free vibration analyses of carbon nanotube-reinforced compo-site plates using finite element method with first order shear deformation plate theory", *Compos. Struct.*, **94**(4), 1450-1460.

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Appendix A

All the parameters which are defined in Eqs. (13)-(15) can be written as follows:

Strain energy parameters:

$$A_{ijk} = \int_{-h/2}^{h/2} \left(Q_{ij} z^{k}\right) dz\,, \tag{A-1}$$

$$B_{ijk} = \int_{-h/2}^{h/2} \left(Q_{ij} \Gamma(z)^{k}\right) dz\,, \tag{A-2}$$

$$C_{ijk} = \int_{-h/2}^{h/2} \left(Q_{ij} \left(\frac{d\Gamma(z)}{dz}\right)^{k}\right) dz\,, \tag{A-3}$$

$$D_{ijk} = \int_{-h/2}^{h/2} \left(Q_{ij} \left(z\,\Gamma(z)\right)^{k}\right) dz\,, \tag{A-4}$$

Kinetic energy parameters:

$$I_0 = \int_{-h/2}^{h/2} \rho_c dz\,, \tag{A-5}$$

$$I_1 = \int_{-h/2}^{h/2} \left(\rho_c z\right) dz\,, \tag{A-6}$$

$$I_2 = \int_{-h/2}^{h/2} \left(\rho_c z^{2}\right) dz\,, \tag{A-7}$$

$$I_3 = \int_{-h/2}^{h/2} \left(\rho_c \Gamma(z)\right) dz\,, \tag{A-8}$$

$$I_4 = \int_{-h/2}^{h/2} \left(\rho_c z\,\Gamma(z)\right) dz\,, \tag{A-9}$$

$$I_5 = \int_{-h/2}^{h/2} \left(\rho_c \left(\Gamma(z)\right)^{2}\right) dz\,, \tag{A-10}$$

where $\Gamma(z) = \begin{cases} \psi(z) & \mathrm{CPT, FSDT,} T\,\mathrm{SDT, SSDT} \\ 0 & NFSDT \\ f(z) & H\,\mathrm{SDT} \end{cases}$