Steel and Composite Structures, *Vol. 21, No. 6 (2016) 1287-1306*
DOI: http://dx.doi.org/10.12989/scs.2016.21.6.1287

Buckling analysis of isotropic and orthotropic plates using a novel four variable refined plate theory

Fouad Bourada [*1], Khaled Amara [2,3] and Abdelouahed Tounsi [2]

[1] *Laboratoire des Structures Intelligentes, Département de Génie Civil, Institut des Sciences et de la Technologie, Centre Universitaire de Ain Témouchent, Algeria*
[2] *Material and Hydrology Laboratory, University of Sidi Bel Abbes, Faculty of Technology, Civil Engineering Department, Algeria*
[3] *Département de Génie Civil, Institut des Sciences et de la Technologie, Centre Universitaire de Ain Témouchent, Algeria*

(*Received April 04, 2016, Revised August 08, 2016, Accepted August 10, 2016*)

Abstract. The current research presents a buckling analysis of isotropic and orthotropic plates by proposing a new four variable refined plate theory. Contrary to the existing higher order shear deformation theories (HSDT) and the first shear deformation theory (FSDT), the proposed model uses a new displacement field which incorporates undetermined integral terms and involves only four variables. The governing equations for buckling analysis are deduced by utilizing the principle of virtual works. The analytical solution of a simply supported rectangular plate under the axial loading has been determined via the Navier method. Numerical investigations are performed by using the proposed model and the obtained results are compared with CPT solutions, FSDT solutions, and the existing exact solutions in the literature. It can be concluded that the developed four variable refined plate theory, which does not use shear correction coefficient, is not only simple but also comparable to the FSDT.

Keywords: refined plate theory; buckling analysis; isotropic plate; orthotropic plate

1. Introduction

The stability of rectangular plates has been a topic of investigation in engineering structures for more than a century (Matsunaga 2009, Bachir Bouiadjra *et al*. 2013, Altunsaray and Bayer 2014, Afsharmanesh *et al*. 2014, Bakora and Tounsi 2015, Bouguenina *et al*. 2015, Nguyen *et al*. 2015, Tagrara *et al*. 2015, Tebboune *et al*. 2015, Larbi Chaht *et al*. 2015, Musa 2016, Rajanna *et al*. 2016, Bouderba *et al*. 2016, Yousefitabar and Matapouri 2016, Chikh *et al*. 2016, Eltaher *et al*. 2016). A great number of exact solutions for isotropic and orthotropic plates have been proposed, and the most known are documented in Timoshenko and Woinowsky-Krieger (1959), Timoshenko and Gere (1961), Bank and Yin (1996), Kang and Leissa (2005), Hwang and Lee (2006) and Musa (2016). In company with investigations of stability response of plate, a great number of plate models have been proposed. The simplest one is the classical plate theory (CPT) which ignores the transverse shear influences. This model is not suitable for the thick and orthotropic plate with

*Corresponding author, Professor, E-mail: bouradafouad@yahoo.fr

http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

important degree of modulus ratio. To avoid this limitation, the shear deformation theory which considers the transverse shear influences is introduced by several researchers. Indeed, the FSDT (first shear deformation theory) and HSDT (high shear deformation theory) were proposed as improvement of the CPT. The FSDT is based on Reissner (1945) and Mindlin (1951) and consider the transverse shear influences by the way of linear distribution of the displacements across the thickness. Many investigations have been presented in different scientific articles by employing FSDT for the free vibration behavior of composite plates (Yan *et al*. 1966, Whitney 1969, Whitney and Pagano 1970, Ambartsumyan 1970, Sun and Whitney 1973, Bert and Chen 1978, Reddy 1979, Noor and Burton 1989a, b, Kant and Swaminathan 2001). Kant and Swaminathan (2001) extended the FSDT presented by Whitney and Pagano (1970) for the dynamic response of laminated composite and sandwich plates. Sadoune *et al*. (2014) developed a novel FSDT for mechanical behavior of laminated plates. Meksi *et al*. (2015) proposed a new simple FSDT with only four variables for static and vibration analysis of functionally graded plates. Bellifa *et al*. (2016) used a new FSDT for bending and dynamic analysis of functionally graded plates. Hadji *et al*. (2016) analyzed a FG beam using a new first-order shear deformation theory. Yaghoobi *et al*. (2014) proposed an analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loadings using VIM. Bourada *et al*. (2012) presented a new four-variable refined plate theory for thermal buckling analysis of FG sandwich plates. It is noted that various HSDTs are also used (Mahapatra *et al*. 2016a, b, Katariya and Panda 2016, Mehar and Panda 2016, Panda and Katariya 2015, Kar and Panda 2014, Panda and Singh 2013a, b, c, 2010a, b, 2011) to investigate the geometrical deformation of composite structures under the large geometrical distortion. Tounsi and his co-workers (Benachour *et al*. 2011, Houari *et al*. 2013, Tounsi *et al*. 2013, Ould Larbi *et al*. 2013, Saidi *et al*. 2013, Bousahla *et al*. 2014, Draiche *et al*. 2014, Fekrar *et al*. 2014, Hadji *et al*. 2014, Khalfi *et al*. 2014, Zidi *et al*. 2014, Klouche Djedid *et al*. 2014, Ait Yahia *et al*. 2015, Zemri *et al*. 2015, Sallai *et al*. 2015, Bennai *et al*. 2015, Bouchafa *et al*. 2015, Meradjah *et al*. 2015, Merazi *et al*. 2015, Al-Basyouni *et al*. 2015, Attia *et al*. 2015, Ait Atmane *et al*. 2015, Belkorissat *et al*. 2015, Boukhari *et al*. 2016, Bounouara *et al*. 2016, Mouaici *et al*. 2016, Beldjelili *et al*. 2016) proposed a new HSDTs to investigate the mechanical behavior of composite structures. Recently, Mantari and Granados (2015) proposed a new FSDT with four variables in which integral terms in the plate kinematics are utilized. Since FSDTs do not respect equilibrium conditions at the top and bottom surfaces of the plate, shear correction coefficients are needed to correct the unrealistic distribution of the shear strain/stress within the thickness. For these reasons, many HSDTs have been proposed to improve the limitations of FSDT such as Levinson (1980), Bhimaraddi and Stevens (1984), Reddy (1984), Ren (1986), Kant and Pandya (1988), and Mohan *et al*. (1994). A good review of these models for the investigation of laminated plates is found in (Noor and Burton 1989a, b, Reddy 1990 and 1993, Mallikarjuna and Kant 1993, Dahsin and Xiaoyu 1996). Reddy (1984) proposed a HSDT with cubic distributions for axial displacements. Based on Reddy's theory, Xiang *et al*. (2011) developed a n-order shear deformation theory. Kant and Pandya (1988), Mallikarjuna and Kant (1989) and Kant and Khare (1997) employed also HSDTs with cubic distributions for axial displacements as in the article by Reddy (1984). Recently, a new class of HSDTs is proposed by many researchers such as Shahrjerdi *et al*. (2011), Bouderba *et al*. (2013), Viswanathan *et al*. (2013), Ait Amar Meziane *et al*. (2014), Belabed *et al*. (2014), Ahmed (2014), Swaminathan and Naveenkumar (2014), Nedri *et al*. (2014), Bourada *et al*. (2015), Hamidi *et al*. (2015), Kar *et al*. (2015), Hebali *et al*. (2014), Mahi *et al*. (2015), Saidi *et al*. (2016), Bennoun *et al*. (2016) and Tounsi *et al*. (2016).

In the present research, the buckling analysis of isotropic and orthotropic plates under the in-plane loading is investigated by utilizing a novel HSDT with four variables in which instead of derivative terms in the displacement field, integral terms are employed. The displacement field presented by Mantari and Granados (2015) is improved by considering higher-order distributions of in-plane displacements within the plate thickness. By employing the Navier procedure, the closed-form solutions have been determined. Numerical results considering side-to-thickness ratio and modulus ratio are carried out to illustrate the validity of the present model in computing the critical buckling load of isotropic and orthotropic plates. It can be concluded that the present model, which does not use shear correction coefficient, is not only simple but also comparable to the FSDT.

2. Theoretical formulation

2.1 Kinematics and strains

In this research work, further simplifying supposition are considered to the conventional HSDT so that the number of variables is diminished. The kinematic of the conventional HSDT is expressed by

$$u(x,y,z) = u_0(x,y) - z\frac{\partial w_0}{\partial x} + f(z)\varphi_x(x,y) \tag{1a}$$

$$v(x,y,z) = v_0(x,y) - z\frac{\partial w_0}{\partial y} + f(z)\varphi_y(x,y) \tag{1b}$$

$$w(x,y,z) = w_0(x,y) \tag{1c}$$

where u_0; v_0; w_0, φ_x, φ_y are five unknown displacements of the mid-plane of the plate, $f(z)$ denotes shape function defining the distribution of the transverse shear strains and stresses within the thickness. By supposing that $\varphi_x = \int \theta(x,y)dx$ and $\varphi_y = \int \theta(x,y)dy$, the displacement field of the current theory model can be found in a simpler form as

$$u(x,y,z,t) = u_0(x,y,t) - z\frac{\partial w_0}{\partial x} + k_1 f(z)\int \theta(x,y,t)\,dx \tag{2a}$$

$$v(x,y,z,t) = v_0(x,y,t) - z\frac{\partial w_0}{\partial y} + k_2 f(z)\int \theta(x,y,t)\,dy \tag{2b}$$

$$w(x,y,z,t) = w_0(x,y,t) \tag{2c}$$

In this investigation, the proposed higher-order shear deformation plate theory is determined by considering

$$f(z) = z\left(\frac{5}{4} - \frac{5z^2}{3h^2}\right) \tag{3}$$

It can be observed that the kinematic in Eq. (2) incorporates only four variables (u_0, v_0, w_0 and θ). The nonzero strains associated with the kinematic in Eq. (2) are

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix}, \tag{4}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_0}{\partial x^2} \\ -\dfrac{\partial^2 w_0}{\partial y^2} \\ -2\dfrac{\partial^2 w_0}{\partial x \partial y} \end{Bmatrix}, \tag{5a}$$

$$\begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} k_1\, \theta \\ k_2\, \theta \\ k_1 \dfrac{\partial}{\partial y} \int \theta \, dx + k_2 \dfrac{\partial}{\partial x} \int \theta \, dy \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^0 \\ \gamma_{xz}^0 \end{Bmatrix} = \begin{Bmatrix} k_2 \int \theta \, dy \\ k_1 \int \theta \, dx \end{Bmatrix},$$

and

$$g(z) = \frac{df(z)}{dz} \tag{5b}$$

The integrals used in the above expressions shall be resolved by a Navier type method and can be expressed as follows

$$\frac{\partial}{\partial y} \int \theta \, dx = A' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \frac{\partial}{\partial x} \int \theta \, dy = B' \frac{\partial^2 \theta}{\partial x \partial y}, \quad \int \theta \, dx = A' \frac{\partial \theta}{\partial x}, \quad \int \theta \, dy = B' \frac{\partial \theta}{\partial y} \tag{6}$$

where the coefficients A' and B' are adopted according to the type of solution employed, in this case via Navier. Hence, A' and B' are defined as follows

$$A' = -\frac{1}{\alpha^2}, \quad B' = -\frac{1}{\beta^2}, \quad k_1 = \alpha^2, \quad k_2 = \beta^2 \tag{7}$$

where α and β are defined in expression (20).

2.2 Constitutive equations

For elastic and orthotropic plate, the constitutive relations can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \\ \tau_{yz} \\ \tau_{xz} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 & 0 & 0 \\ Q_{12} & Q_{22} & 0 & 0 & 0 \\ 0 & 0 & Q_{66} & 0 & 0 \\ 0 & 0 & 0 & Q_{44} & 0 \\ 0 & 0 & 0 & 0 & Q_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \\ \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} \tag{8}$$

where Q_{ij} present the plane stress reduced elastic constants in the material axes of the plate, and are expressed as

$$\begin{gathered} Q_{11} = \frac{E_1}{1-\nu_{12}\nu_{21}}, \quad Q_{12} = \frac{\nu_{12} E_2}{1-\nu_{12}\nu_{21}}, \quad Q_{22} = \frac{E_2}{1-\nu_{12}\nu_{21}}, \\ Q_{66} = G_{12}, \quad Q_{44} = G_{23}, \quad Q_{55} = G_{13} \end{gathered} \tag{9}$$

where E_1, E_2 are Young's modulus, G_{12}, G_{23}, G_{13} are shear modulus, and ν_{12}, ν_{21} are Poisson's ratios. For the isotropic plate, these above material characteristics reduce to $E_1 = E_2 = E$, $G_{12} = G_{23} = G_{13} = G$, $\nu_{12} = \nu_{21} = \nu$. The subscripts 1, 2, 3 correspond to x, y, z directions of Cartesian coordinate system, respectively.

2.3 Governing equations

The principle of virtual works is employed herein to derive the governing equations. The principle can be expressed in analytical form as

$$\delta U + \delta V = 0 \tag{10}$$

where δU is the variation of strain energy; and δV is the variation of the external work done by external load applied to the plate.

The variation of strain energy of the plate is given by

$$\begin{aligned} \delta U &= \int_V \left[\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \tau_{xy} \delta \gamma_{xy} + \tau_{yz} \delta \gamma_{yz} + \tau_{xz} \delta \gamma_{xz} \right] dV \\ &= \int_A \left[N_x \delta \varepsilon_x^0 + N_y \delta \varepsilon_y^0 + N_{xy} \delta \gamma_{xy}^0 + M_x^b \delta k_x^b + M_y^b \delta k_y^b + M_{xy}^b \delta k_{xy}^b \right. \\ &\quad \left. + M_x^s \delta k_x^s + M_y^s \delta k_y^s + M_{xy}^s \delta k_{xy}^s + S_{yz}^s \delta \gamma_{yz}^s + S_{xz}^s \delta \gamma_{xz}^0 \right] dA = 0 \end{aligned} \tag{11}$$

where A is the top surface and the stress resultants N, M, and S are expressed by

$$\left(N_i, M_i^b, M_i^s\right) = \int_{-h/2}^{h/2} (1, z, f)\sigma_i dz\,, \quad (i = x, y, xy) \text{ and } \left(S_{xz}^s, S_{yz}^s\right) = \int_{-h/2}^{h/2} g\left(\tau_{xz}, \tau_{yz}\right) dz \tag{12}$$

The variation of the external work can be expressed as

$$\delta V = -\int_A \left(N_x^0 \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial x} + 2N_{xy}^0 \frac{\partial w_0}{\partial x}\frac{\partial \delta w_0}{\partial y} + N_y^0 \frac{\partial w_0}{\partial y}\frac{\partial \delta w_0}{\partial y} \right) dA \tag{13}$$

where (N_x^0, N_y^0, N_{xy}^0) are transverse and in-plane applied loads, respectively.

By substituting Eqs. (11) and (13) into Eq. (10), the following governing equations can be derived

$$\begin{aligned}
&\delta u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
&\delta v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
&\delta w_0 : \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + N_x^0 \frac{\partial^2 w_0}{\partial x^2} + 2 N_{xy}^0 \frac{\partial^2 w_0}{\partial x \partial y} + N_y^0 \frac{\partial^2 w_0}{\partial y^2} = 0 \\
&\delta \theta : \ - M_x^s - M_y^s - (A' + B')\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + A' \frac{\partial S_{xz}^s}{\partial x} + B' \frac{\partial S_{yz}^s}{\partial y} = 0
\end{aligned} \tag{14}$$

By substituting Eq. (4) into Eq. (8) and the subsequent results into Eq. (12), the stress resultants can be written as below

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \\ M_x^b \\ M_y^b \\ M_{xy}^b \\ M_x^s \\ M_y^s \\ M_{xy}^s \end{Bmatrix} \begin{bmatrix} A_{11} & A_{12} & 0 & B_{11} & B_{12} & 0 & B_{11}^s & B_{12}^s & 0 \\ A_{12} & A_{22} & 0 & B_{12} & B_{22} & 0 & B_{12}^s & B_{22}^s & 0 \\ 0 & 0 & A_{66} & 0 & 0 & B_{66} & 0 & 0 & B_{66}^s \\ B_{11} & B_{12} & 0 & D_{11} & D_{12} & 0 & D_{11}^s & D_{12}^s & 0 \\ B_{12} & B_{22} & 0 & D_{12} & D_{22} & 0 & D_{12}^s & D_{22}^s & 0 \\ 0 & 0 & B_{66} & 0 & 0 & D_{66} & 0 & 0 & D_{66}^s \\ B_{11}^s & B_{12}^s & 0 & D_{11}^s & D_{12}^s & 0 & H_{11}^s & H_{12}^s & 0 \\ B_{12}^s & B_{22}^s & 0 & D_{12}^s & D_{22}^s & 0 & H_{12}^s & H_{22}^s & 0 \\ 0 & 0 & B_{66}^s & 0 & 0 & D_{66}^s & 0 & 0 & H_{66}^s \end{bmatrix} \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \\ k_x^b \\ k_y^b \\ k_{xy}^b \\ k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} \tag{15a}$$

$$\begin{Bmatrix} S_{xz}^s \\ S_{yz}^s \end{Bmatrix} = \begin{bmatrix} A_{55}^s & 0 \\ 0 & A_{44}^s \end{bmatrix} \begin{Bmatrix} \gamma_{xz}^0 \\ \gamma_{yz}^0 \end{Bmatrix} \tag{15b}$$

and stiffness components are expressed as

$$\left\{A_{ij}, B_{ij}, D_{ij}, B_{ij}^{s}, D_{ij}^{s}, H_{ij}^{s}\right\} = \int_{-h/2}^{h/2} Q_{ij} \left\{1, z, z^2, f(z), z\, f(z), f^2(z)\right\} dz, \quad i, j = 1,2,6 \tag{16a}$$

$$A_{ij}^{s} = \int_{-h/2}^{h/2} Q_{ij} \left[g(z)\right]^2 dz, \quad i, j = 4,5 \tag{16b}$$

Introducing relations (15) into Eq. (14), the governing equations can be expressed in terms of displacements (δu_0, δv_0, δw_0, $\delta\theta$) and the appropriate equations take the form

$$\begin{aligned} &A_{11} d_{11} u_0 + A_{66}\, d_{22} u_0 + \left(A_{12} + A_{66}\right) d_{12} v_0 - \left(B_{11} d_{111} w_0 + \left(B_{12} + 2B_{66}\right) d_{122} w_0\right) \\ &+ \left(k_1 A' + k_2 B'\right) B_{66}^{s}\, d_{122}\theta + \left(k_1 B_{11}^{s} + k_2 B_{12}^{s}\right) d_1\theta = 0, \end{aligned} \tag{17a}$$

$$\begin{aligned} &\left(A_{12} + A_{66}\right) d_{12} u_0 + A_{66}\, d_{22} v_0 + A_{22}\, d_{22} v_0 - \left(\left(B_{12} + 2B_{66}\right) d_{112} w_0 + B_{22} d_{222} w_0\right) \\ &+ \left(k_1 A' + k_2 B'\right) B_{66}^{s}\, d_{112}\theta + \left(k_2 B_{22}^{s} + k_1 B_{12}^{s}\right) d_2\theta = 0, \end{aligned} \tag{17b}$$

$$\begin{aligned} &\left(B_{11} d_{111} u_0 + \left(B_{12} + 2B_{66}\right) d_{122} u_0\right) + \left(\left(B_{12} + 2B_{66}\right) d_{112} v_0 + B_{22} d_{222} v_0\right) \\ &- D_{11} d_{1111} w_0 - 2\left(D_{12} + 2D_{66}\right) d_{1122} w_0 - D_{22}\, d_{2222} w_0 \\ &+ \left(k_1 D_{11}^{s} + k_2 D_{12}^{s}\right) d_{11}\theta + 2\left(k_1 A' + k_2 B'\right) D_{66}^{s}\, d_{1122}\theta + \left(k_1 D_{12}^{s} + k_2 D_{22}^{s}\right) d_{22}\theta \\ &+ N_x^0\, d_{11} w_0 + 2N_{xy}^0\, d_{12} w_0 + N_y^0\, d_{22} w_0 = 0 \end{aligned} \tag{17c}$$

$$\begin{aligned} &-\left(\left(k_1 A' + k_2 B'\right) B_{66}^{s}\, d_{122} u_0 + \left(k_1 B_{11}^{s} + k_2 B_{12}^{s}\right) d_1 u_0\right) - \left(\left(k_1 A' + k_2 B'\right) B_{66}^{s}\, d_{112} v_0 + \left(k_2 B_{22}^{s} + k_1 B_{12}^{s}\right) d_2 v_0\right) \\ &+ \left(D_{11}^{s} + D_{12}^{s}\right) d_{11} w_0 + 2\left(k_1 A' + k_2 B'\right) D_{66}^{s}\, d_{1122} w_0 + \left(D_{12}^{s} + D_{22}^{s}\right) d_{22} w_0 \\ &- k_1^{\,2} H_{11}^{s}\theta - k_2^{\,2} H_{22}^{s}\theta - 2k_1 k_2 H_{12}^{s}\theta - \left(k_1 A' + k_2 B'\right)^2 H_{66}^{s}\, d_{1122}\theta \\ &+ A_{44}^{s} \left(k_2 B'\right)^2 d_{22}\theta + A_{55}^{s} \left(k_1 A'\right)^2 d_{11}\theta = 0 \end{aligned} \tag{17d}$$

where d_{ij}, d_{ijl} and d_{ijlm} are the following differential operators

$$d_{ij} = \frac{\partial^2}{\partial x_i \partial x_j}, \quad d_{ijl} = \frac{\partial^3}{\partial x_i \partial x_j \partial x_l}, \quad d_{ijlm} = \frac{\partial^4}{\partial x_i \partial x_j \partial x_l \partial x_m}, \quad d_i = \frac{\partial}{\partial x_i}, \quad (i, j, l, m = 1,2). \tag{18}$$

2.4 Closed-form solution

The critical buckling loads of simply supported, orthotropic, rectangular plate will be computed in this work by employing the Navier method. Considering that the plate is subjected to in-plane compressive loads of form: $N_x^0 = \gamma_1 N_{cr}$, $N_y^0 = \gamma_2 N_{cr}$, $N_{xy}^0 = 0$ (here γ_1 and γ_2 are non-dimensional load parameters). The following displacement functions are chosen to automatically satisfy the boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_0 \\ \theta \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn}\cos(\alpha\, x)\sin(\beta\, y) \\ V_{mn}\sin(\alpha\, x)\cos(\beta\, y) \\ W_{mn}\sin(\alpha\, x)\sin(\beta\, y) \\ X_{mn}\sin(\alpha\, x)\sin(\beta\, y) \end{Bmatrix} \tag{19}$$

with

$$\alpha = m\pi / a \qquad \text{and} \qquad \beta = n\pi / b \tag{20}$$

and U_{mn}, V_{mn}, W_{mn}, X_{mn} are arbitrary parameters to be determined. Substituting Eq. (19) into Eq. (17), the closed-form solution of buckling load can be determined from

$$\begin{bmatrix} S_{11} & S_{12} & S_{13} & S_{14} \\ S_{12} & S_{22} & S_{23} & S_{24} \\ S_{13} & S_{23} & S_{33}+k & S_{34} \\ S_{14} & S_{24} & S_{34} & S_{44} \end{bmatrix} \begin{Bmatrix} U_{mn} \\ V_{mn} \\ W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{21}$$

where

$$\begin{gathered}
S_{11} = -\left(A_{11}\alpha^2 + A_{66}\beta^2\right), \quad S_{12} = -\alpha\beta\left(A_{12} + A_{66}\right), \quad S_{13} = \alpha\left(B_{11}\alpha^2 + B_{12}\beta^2 + 2\,B_{66}\,\beta^2\right), \\
S_{14} = \alpha\left(k_1 B_{11}^s + k_2 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s\,\beta^2\right), \\
S_{22} = -\left(A_{66}\,\alpha^2 + A_{22}\,\beta^2\right), \quad S_{23} = \beta\left(B_{22}\beta^2 + B_{12}\alpha^2 + 2B_{66}\alpha^2\right), \\
S_{24} = \beta\left(k_2 B_{22}^s + k_1 B_{12}^s - \left(k_1 A' + k_2 B'\right) B_{66}^s \alpha^2\right), \\
S_{33} = -\left(D_{11}\alpha^4 + 2(D_{12} + 2D_{66})\alpha^2\beta^2 + D_{22}\beta^4\right), \\
S_{34} = -\left(k_1 D_{11}^s + k_2 D_{12}^s\right)\alpha^2 + 2(k_1 A' + k_2 B')\, D_{66}^s \alpha^2\beta^2 - \left(k_2 D_{22}^s + k_1 D_{12}^s\right)\beta^2, \\
S_{44} = -k_1\left(k_1 H_{11}^s + k_2 H_{12}^s\right) - (k_1 A' + k_2 B')^2\left(H_{66}^s \alpha^2\beta^2\right) - k_2\left(k_1 H_{12}^s + k_2 H_{22}^s\right) \\
-\left(k_1 A'\right)^2 A_{55}^s\,\alpha^2 - \left(k_2 B'\right)^2 A_{44}^s\,\beta^2 \\
k = N_{cr}\left(\gamma_1\alpha^2 + \gamma_2\beta^2\right)
\end{gathered} \tag{22}$$

The geometrical instability has been taken care in the present analysis by considering the determinant of the coefficient matrix in Eq. (21) to be equal zero. Indeed, for nontrivial solution, the determinant of the coefficient matrix in Eq. (21) must be zero. For each choice of m and n, there is a corresponding unique value of N_{cr}. The critical buckling load is the smallest value of N_{cr} (m, n).

By applying the condensation approach to eliminate the in-plane displacements U_{mn} and V_{mn}, Eq. (21) can be rewritten as

$$\begin{bmatrix} \bar{S}_{33}+k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{bmatrix} \begin{Bmatrix} W_{mn} \\ X_{mn} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \tag{23}$$

where

$$\bar{S}_{33} = S_{33} - \frac{S_{13}\left(S_{13}S_{22} - S_{12}S_{23}\right) - S_{23}\left(S_{11}S_{23} - S_{12}S_{13}\right)}{S_{11}S_{22} - S_{12}^2}$$
$$\bar{S}_{34} = S_{34} - \frac{S_{14}\left(S_{13}S_{22} - S_{12}S_{23}\right) - S_{24}\left(S_{11}S_{23} - S_{12}S_{13}\right)}{S_{11}S_{22} - S_{12}^2} \tag{24}$$

$$\bar{S}_{43} = S_{34} - \frac{S_{13}\left(S_{14}S_{22} - S_{12}S_{24}\right) - S_{23}\left(S_{11}S_{24} - S_{12}S_{14}\right)}{S_{11}S_{22} - S_{12}^2}$$
$$\bar{S}_{44} = S_{44} - \frac{S_{14}\left(S_{14}S_{22} - S_{12}S_{24}\right) - S_{24}\left(S_{11}S_{24} - S_{12}S_{14}\right)}{S_{11}S_{22} - S_{12}^2} \tag{24}$$

The system of homogeneous Eq. (23) has a nontrivial solution only for discrete values of the buckling load. For a nontrivial solution, the determinant of the coefficients (W_{mn}, X_{mn}) must equal zero

$$\begin{vmatrix} \bar{S}_{33}+k & \bar{S}_{34} \\ \bar{S}_{43} & \bar{S}_{44} \end{vmatrix} = 0 \tag{25}$$

The resulting equation may be solved for the buckling load. This gives the following expression for buckling load

$$k = \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{26}$$

By employing the Eq. (25), the following expression for critical buckling load is determined

$$N_{cr} = \frac{1}{\left(\gamma_1\alpha^2 + \gamma_2\beta^2\right)} \frac{\bar{S}_{34}\bar{S}_{43} - \bar{S}_{33}\bar{S}_{44}}{\bar{S}_{44}} \tag{27}$$

3. Numerical results and discussions

In this section the accuracy of the proposed HSDT which uses a kinematic with only four variables, is assessed. For this end, a simply supported rectangular plate under loading conditions, as presented in Fig. 1, is examined to demonstrate the accuracy of the current formulation in investigating the buckling response of the plate. For proposed examples, the following engineering constants are employed (Reddy 1997)

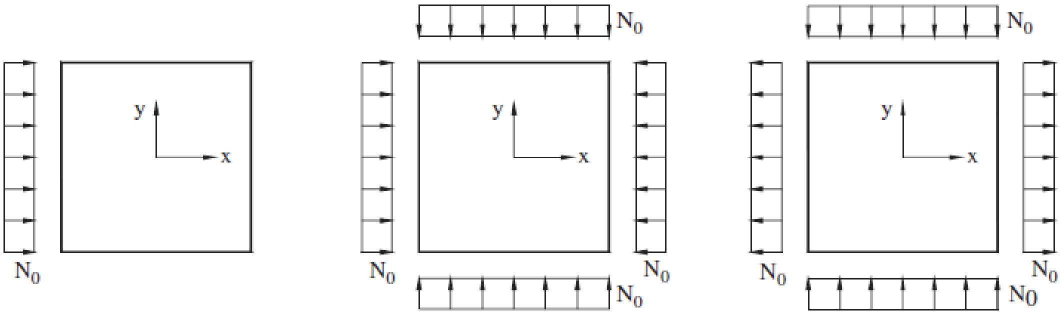


Fig. 1 The loading conditions of square plate for (a) uniaxial compression; (b) biaxial compression; and (c) tension in the *x* direction and compression in the *y* direction

$$E_1/E_2 \text{ varied} \quad G_{12}/E_2 = G_{13}/E_2 = 0.5, \quad G_{23}/E_2 = 0.2, \quad \nu_{12} = 0.25 \tag{28}$$

For convenience, the following non-dimensional buckling load is utilized

$$\overline{N} = \frac{N_{cr}a^2}{E_2h^3} \tag{29}$$

where a is the length of the square plate and h is the thickness of the plate.

Table 1 Comparison of non-dimensional critical buckling load of square plates under uniaxial compression

a/h	Theories	Isotropic $\nu = 0.3$	Orthotropic $E_1/E_2 = 10$	$E_1/E_2 = 25$	$E_1/E_2 = 40$
5	Present	2.9512	6.3478	9.1039	10.5785
	RPT(*)	2.9512	6.3478	9.1039	10.5785
	FSDT(*) ($k_s = 2/3$)	2.8200	5.5679	7.1122	7.7411
	FSDT(*) ($k_s = 5/6$)	2.9498	6.1804	8.2199	9.1085
	FSDT(*) ($k_s = 1$)	3.0432	6.6715	9.1841	10.3463
10	Present	3.4224	9.3732	16.7719	22.2581
	RPT(*)	3.4224	9.3732	16.7719	22.2581
	FSDT(*) ($k_s = 2/3$)	3.3772	8.8988	14.7011	18.3575
	FSDT(*) ($k_s = 5/6$)	3.4222	9.2733	15.8736	20.3044
	FSDT(*) ($k_s = 1$)	3.4530	9.5415	16.7699	21.8602
20	Present	3.5650	10.6534	21.3479	31.0685
	RPT(*)	3.5650	10.6534	21.3479	31.0685
	FSDT(*) ($k_s = 2/3$)	3.5526	10.4926	20.4034	28.85
	FSDT(*) ($k_s = 5/6$)	3.5650	10.6199	20.9528	30.0139
	FSDT(*) ($k_s = 1$)	3.5733	10.7066	21.3363	30.8451

50	Present	3.6071	11.0780	23.1225	34.9717
	RPT(*)	3.6071	11.0780	23.1225	34.9717
	FSDT(*) ($k_s = 2/3$)	3.6051	11.0497	22.9366	34.4886
	FSDT(*) ($k_s = 5/6$)	3.6071	11.0721	23.0461	34.7487
	FSDT(*) ($k_s = 1$)	3.6085	11.0871	23.1197	34.9244
100	Present	3.6132	11.1415	23.4007	35.6120
	RPT(*)	3.6132	11.1415	23.4007	35.6120
	FSDT(*) ($k_s = 2/3$)	3.6127	11.1343	23.3527	35.4852
	FSDT(*) ($k_s = 5/6$)	3.6132	11.1400	23.3810	35.5538
	FSDT(*) ($k_s = 1$)	3.6135	11.1438	23.3999	35.5996

* Taken from Ref (Kim *et al*. 2009)

Table 2 Comparison of non-dimensional critical buckling load of square plates under biaxial compression

a/h	Theories	Isotropic $\nu = 0.3$	Orthotropic		
			$E_1/E_2 = 10$	$E_1/E_2 = 25$	$E_1/E_2 = 40$
5	Present	1.4756	2.8549[a]	3.3309[a]	3.3800[a]
	RPT(*)	1.4756	2.8549[a]	3.3309[a]	3.3800[a]
	FSDT(*) ($k_s = 2/3$)	1.4100	2.5042[a]	2.7332[a]	2.8303[a]
	FSDT(*) ($k_s = 5/6$)	1.4749	2.8319[a]	3.1422[a]	3.2822[a]
	FSDT(*) ($k_s = 1$)	1.5216	3.1027[a]	3.4933[a]	3.6793[a]
10	Present	1.7112	4.6718[a]	6.0646[a]	7.2536[a]
	RPT(*)	1.7112	4.6718[a]	6.0646[a]	7.2536[a]
	FSDT(*) ($k_s = 2/3$)	1.6886	4.4259	5.4351[a]	6.0797[a]
	FSDT(*) ($k_s = 5/6$)	1.7111	4.6367	5.8370[a]	6.6325[a]
	FSDT(*) ($k_s = 1$)	1.7265	4.7708	6.1425[a]	7.0690[a]
20	Present	1.7825	5.3267	7.6643[a]	9.6614[a]
	RPT(*)	1.7825	5.3267	7.6643[a]	9.6614[a]
	FSDT(*) ($k_s = 2/3$)	1.7763	5.2463	7.3701[a]	8.9895[a]
	FSDT(*) ($k_s = 5/6$)	1.7825	5.3100	7.5546[a]	9.3049[a]
	FSDT(*) ($k_s = 1$)	1.7866	5.3533	7.6834[a]	9.5297[a]
50	Present	1.8036	5.5390	8.2784[a]	10.6576[a]
	RPT(*)	1.8036	5.5390	8.2784[a]	10.6576[a]
	FSDT(*) ($k_s = 2/3$)	1.8025	5.5249	8.2199[a]	10.5111[a]
	FSDT(*) ($k_s = 5/6$)	1.8036	5.5361	8.2566[a]	10.5810[a]
	FSDT(*) ($k_s = 1$)	1.8042	5.5436	8.2812[a]	10.6282[a]
100	Present	1.8066	5.5707	8.3744[a]	10.8172[a]
	RPT(*)	1.8066	5.5707	8.3744[a]	10.8172[a]
	FSDT(*) ($k_s = 2/3$)	1.8063	5.5672	8.3593[a]	10.7788[a]
	FSDT(*) ($k_s = 5/6$)	1.8066	5.5700	8.3687[a]	10.7972[a]
	FSDT(*) ($k_s = 1$)	1.8068	5.5719	8.3751[a]	10.8095[a]

* Taken from Ref (Kim *et al*. 2009)

[a] Mode for plate is $(m, n) = (1, 2)$.

Example 1**:** Table 1 gives the values of the non-dimensional buckling loads $\overline{N}$ of isotropic and orthotropic square plates subjected to uniaxial compression for various values of thickness ratio a / h and modulus ratio E_1 / E_2. The obtained results are compared with the data reported by Kim *et al*. (2009) based on FSDT and refined plate theory (RPT). Many shear correction coefficients ($k_s = 2 / 3$, $k_s = 5 / 6$ and $k_s = 1$) are employed for the FSDT (Kim *et al*. 2009) in comparison with the present model. An excellent agreement is proved for all types ranging from thin to very thick plates.

Example 2**:** In Table 2 we found the values of the non-dimensional buckling loads $\overline{N}$ of isotropic and orthotropic square plates under biaxial compression for various values of thickness ratio a / h and modulus ratio E_1 / E_2. Again, the computed values are compared with those given by Kim *et al*. (2009) based on FSDT and RPT. It is remarked that there is an excellent agreement between the values computed by the present model and RPT (Kim *et al*. 2009) for all values of thickness ratio a / h and modulus ratio E_1 / E_2.

Example 3**:** This example is performed for isotropic and orthotropic square plates subjected to tension in the x direction and compression in the y direction. In Table 3 the comparison of non-dimensional buckling loads $\overline{N}$ computed via present model with those provided by Kim *et al.*

Table 3 Comparison of non-dimensional critical buckling load of square plates subjected to tension in the x direction and compression in the y direction

a / h	Theories	Isotropic $\nu = 0.3$	Orthotropic		
			$E_1 / E_2 = 10$	$E_1 / E_2 = 25$	$E_1 / E_2 = 40$
5	Present	4.8274[a]	4.0258[b]	4.1044[c]	4.1525[c]
	RPT(*)	4.8274[a]	4.0258[b]	4.1044[c]	4.1525[c]
	FSDT(*) ($k_s = 2 / 3$)	4.4175[a]	3.2849[d]	3.3001[e]	3.3053[e]
	FSDT(*) ($k_s = 5 / 6$)	4.8158[a]	3.9241[c]	3.9794[c]	4.0075[d]
	FSDT(*) ($k_s = 1$)	5.1237[a]	4.4488[b]	4.5691[c]	4.6073[c]
10	Present	6.6024[a]	7.7863	8.5471[b]	9.1638[b]
	RPT(*)	6.6024[a]	7.7863	8.5471[b]	9.1638[b]
	FSDT(*) ($k_s = 2 / 3$)	6.4032[a]	7.2656	7.7820[b]	8.1208[b]
	FSDT(*) ($k_s = 5 / 6$)	6.6010[a]	7.7748	8.4774[b]	8.9039[b]
	FSDT(*) ($k_s = 1$)	6.7398[a]	8.0651	9.0153[b]	9.5197[b]
20	Present	7.2754[a]	9.2811	11.6347[b]	12.8031[b]
	RPT(*)	7.2754[a]	9.2811	11.6347[b]	12.8031[b]
	FSDT(*) ($k_s = 2 / 3$)	7.2139[a]	9.1310	11.2544[b]	12.1990[b]
	FSDT(*) ($k_s = 5 / 6$)	7.2753[a]	9.2782	11.6015[b]	12.6339[b]
	FSDT(*) ($k_s = 1$)	7.3168[a]	9.3790	11.8453[b]	12.9428[b]
50	Present	7.4895[a]	9.8101	12.9531[b]	14.4177[b]
	RPT(*)	7.4895[a]	9.8101	12.9531[b]	14.4177[b]

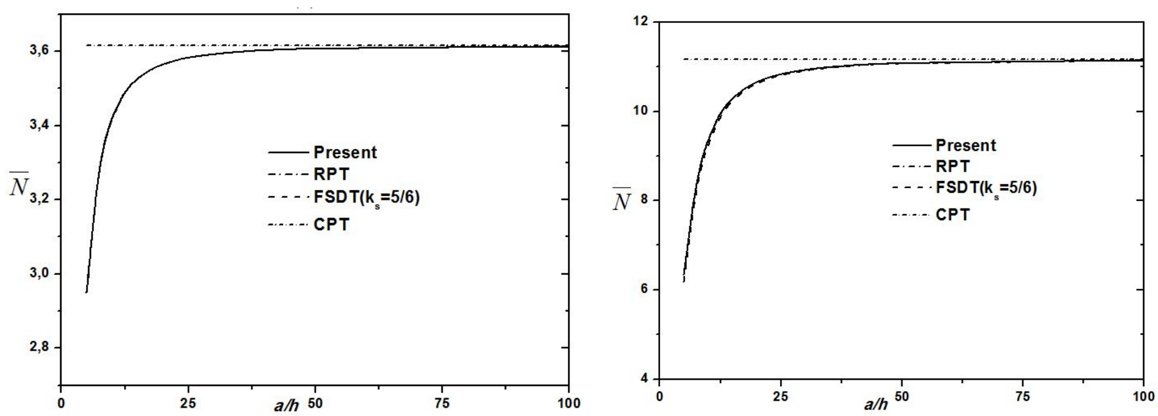
	FSDT[(*)] ($k_s = 2/3$)	7.4790[a]	9.7830	12.8751[b]	14.2839[b]
	FSDT[(*)] ($k_s = 5/6$)	7.4895[a]	9.8097	12.9463[b]	14.3789[b]
	FSDT[(*)] ($k_s = 1$)	7.4965[a]	9.8275	12.9942[b]	14.4430[b]
	Present	7.5211[a]	9.8907[a]	13.1666[b]	14.6827[b]
	RPT[(*)]	7.5211[a]	9.8907[a]	13.1666[b]	14.6827[b]
100	FSDT[(*)] ($k_s = 2/3$)	7.5185[a]	9.8838[a]	16.1463[b]	14.6474[b]
	FSDT[(*)] ($k_s = 5/6$)	7.5211[a]	9.8906[a]	13.1648[b]	14.6724[b]
	FSDT[(*)] ($k_s = 1$)	7.5229[a]	9.8951[a]	13.1772[b]	14.6891[b]

* Taken from Ref (Kim *et al*. 2009)
[a] Mode for plate is (m, n) = (1, 2); [b] Mode for plate is (m, n) = (1, 3); [c] Mode for plate is (m, n) = (1, 4)
[d] Mode for plate is (m, n) = (1, 5); [e] Mode for plate is (m, n) = (1, 6).

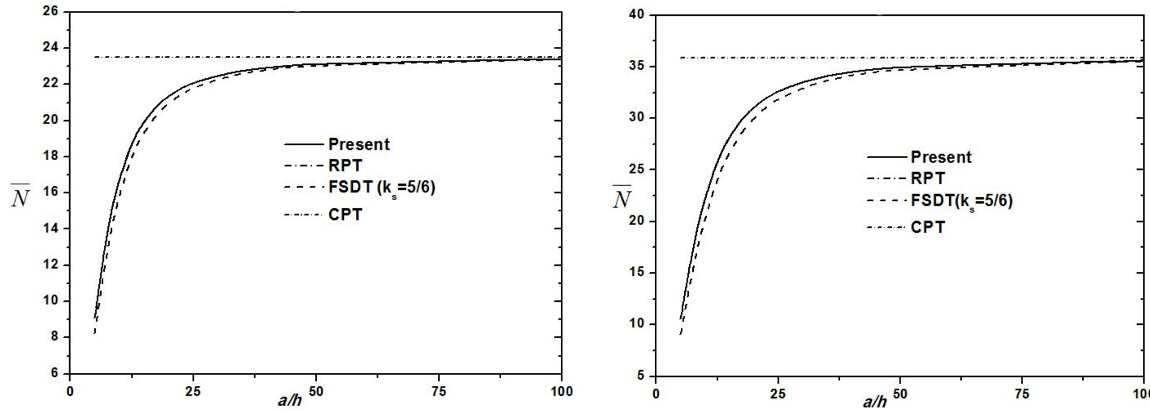
(2009) based on FSDT and RPT is presented. It can be seen that there is an excellent agreement for a wide range of values of thickness ratio and modulus ratio.

Fig. 2 demonstrates the effect of thickness and modulus ratios on the non-dimensional critical buckling load $\overline{N}$ of square plate subjected to uniaxial compression. It is observed that the present novel four variable refined plate theory and RPT predict almost the same data, and CPT over-estimates the buckling loads of plate due to neglecting transverse shear deformation influences. The difference between CPT and shear deformation models diminishes when the side-to-thickness ratio a / h increases. As well as the plate becomes more orthotropic, the difference between the present theory and FSDT will increase with respect to the increase of thickness ratio.

The variations of non-dimensional critical buckling load $\overline{N}$ of square plate as a function of the modulus ratio is illustrated in Fig. 3. The considered plate is subjected to biaxial compression. The curves plotted by employing the present model are compared with the curves plotted by utilizing the CPT, FSDT and the RPT. From these results can be concluded that the resulting curves are very close to the curves plotted by using RPT and the CPT over-estimates the results when the

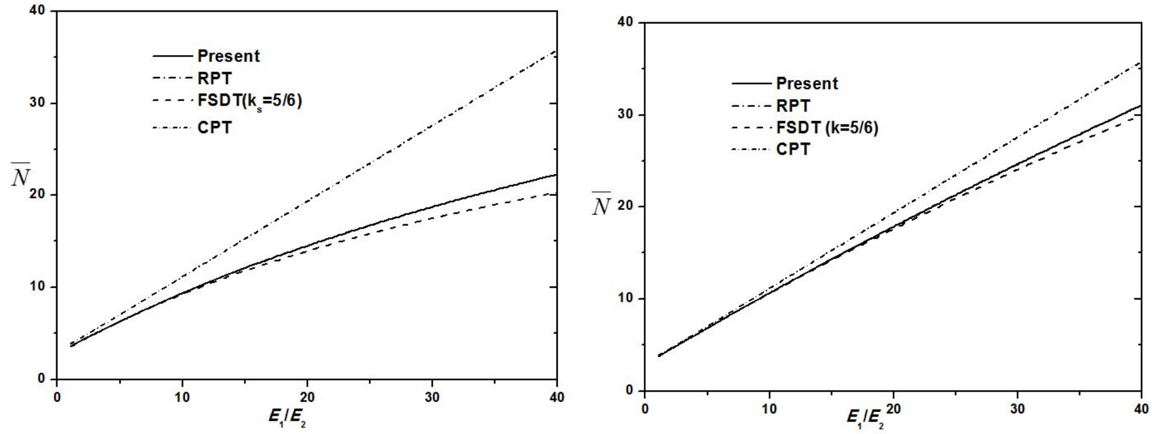


(a) (b)



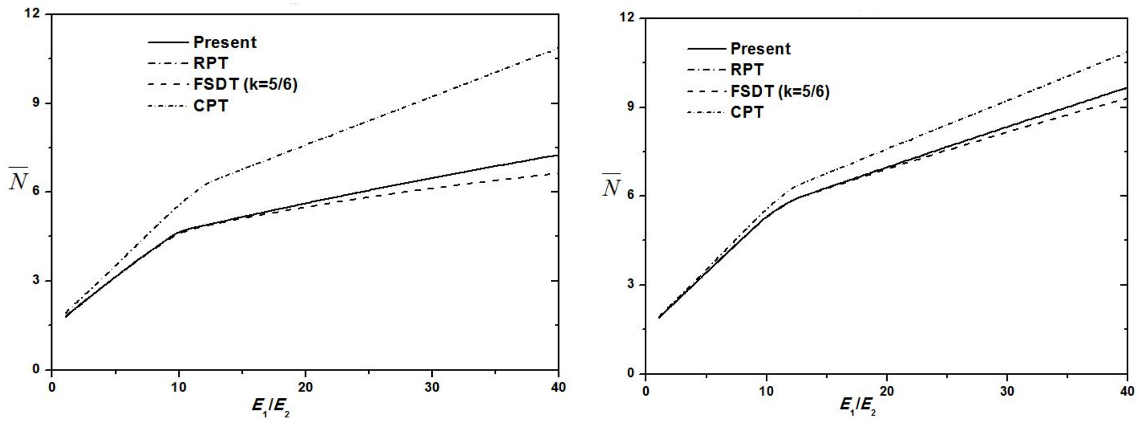
(c) (d)

Fig. 2 The effectofside-to thickness and modulus ratios on the critical buckling load of square plate subjected to uniaxial compression: (a) isotropic; (b) E_1 / E_2 = 10; (c) E_1 / E_2 = 25; and (d) E_1 / E_2 = 40



(a) (b)

Fig. 3 The effect of modulus ratio on the critical buckling load of square plate subjected to uniaxial compression: (a) a = 10 h; and (b) a = 20 h



(a) (b)

Fig. 4 The effect of modulus ratio on the critical buckling load of square plate subjected to biaxial compression: (a) a = 10 h ; and (b) a = 20 h

modulus ratio increases. It can be also seen that the difference between the present theory and SDT will increase with increasing the modulus ratio.

The effect of modulus ratio on the critical buckling load of square plate subjected to tension in the x direction and compression in the y direction is shown in Fig. 4. Again from Fig. 4 can be observed that the resulting curves are very close to the curves plotted by using RPT and the CPT over-estimates the results when the modulus ratio increases.

It can be observed from Tables 1-3 and Figs. 2-4 that the difference of critical buckling load between the present theory and FSDT depends on not only the thickness and modulus ratios, but also the in-plane loading conditions (Fig. 1).

4. Conclusions

A simplified HSDT with only four unknowns was developed for buckling analysis of isotropic and orthotropic plates. Governing equations are obtained from the principle of virtual works. Closed-form solutions are obtained for simply supported orthotropic plates. The accuracy of the developed model has been checked for the stability analysis of isotropic and orthotropic plates.

References

Afsharmanesh, B., Ghaheri, A. and Taheri-Behrooz, F. (2014), "Buckling and vibration of laminated composite circular plate on Winkler-type foundation", *Steel Compos. Struct., Int. J.*, **17**(1), 1-19.

Ahmed, A. (2014), "Post buckling analysis of sandwich beams with functionally graded faces using a consistent higher order theory", *Int. J. Civil Struct. Environ.*, **4**(2), 59-64.

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Atmane, H., Tounsi, A., Bernard, F. and Mahmoud, S.R. (2015), "A computational shear displacement model for vibrational analysis of functionally graded beams with porosities", *Steel Compos. Struct., Int. J.*,

19(2), 369-384.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), “Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories”, *Struct. Eng. Mech., Int. J.*, **53**(6), 1143-1165.

Al-Basyouni, K.S., Tounsi, A. and Mahmoud, S.R. (2015), “Size dependent bending and vibration analysis of functionally graded micro beams based on modified couple stress theory and neutral surface position”, *Compos. Struct.*, **125**, 621-630.

Altunsaray, E. and Bayer, I. (2014), “Buckling of symmetrically laminated quasi-isotropic thin rectangular plates”, *Steel Compos. Struct., Int. J.*, **17**(3), 305-320.

Ambartsumyan, S.A. (1970), *Theory of Anisotropic Plates*, Technomic Publishing Company, Westport, CT, USA.

Attia, A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), “Free vibration analysis of functionally graded plates with temperature-dependent properties using various four variable refined plate theories”, *Steel Compos. Struct., Int. J.*, **18**(1), 187-212.

Bachir Bouiadjra, R., Adda Bedia, E.A., Tounsi, A. (2013), “Nonlinear thermal buckling behavior of functionally graded plates using an efficient sinusoidal shear deformation theory”, *Struct. Eng. Mech., Int. J.*, **48**(4), 547-567.

Bakora, A. and Tounsi, A. (2015),” Thermo-mechanical post-buckling behavior of thick functionally graded plates resting on elastic foundations”, *Struct. Eng. Mech., Int. J.*, **56**(1), 85-106.

Bank, L. and Yin, J. (1996), “Buckling of orthotropic plates with free and rotationally restrained unloaded edges”, *Thin-Wall. Struct.*, **24**(1), 83-96.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), “An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates”, *Compos.: Part B*, **60**, 274-283.

Beldjelili, Y., Tounsi, A. and Mahmoud, S.R. (2016), “Hygro-thermo-mechanical bending of S-FGM plates resting on variable elastic foundations using a four-variable trigonometric plate theory”, *Smart Struct. Syst., Int. J.*, **18**(4). [In Press]

Belkorissat, I., Houari, M.S.A., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2015), “On vibration properties of functionally graded nano-plate using a new nonlocal refined four variable model”, *Steel Compos. Struct., Int. J.*, **18**(4), 1063-1081.

Bellifa, H., Benrahou, K.H., Hadji, L., Houari, M.S.A. and Tounsi, A. (2016), “Bending and free vibration analysis of functionally graded plates using a simple shear deformation theory and the concept the neutral surface position”, *J. Braz. Soc. Mech. Sci. Eng.*, **38**(1), 265-275.

Benachour, A., Daouadji, H.T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), “A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient”, *Compos. Part B*, **42**(6), 1386-1394.

Bennai, R., Ait Atmane, H. and Tounsi, A. (2015), “A new higher-order shear and normal deformation theory for functionally graded sandwich beams”, *Steel Compos. Struct., Int. J.*, **19**(3), 521-546.

Bennoun, M., Houari, M.S.A. and Tounsi, A. (2016), “A novel five variable refined plate theory for vibration analysis of functionally graded sandwich plates”, *Mech. Adv. Mater. Struct.*, **23**(4), 423-431.

Bert, C.W. and Chen, T.L.C. (1978), “Effect of shear deformation on vibration of antisymmetric angle ply laminated rectangular plates”, *Int. J. Solid. Struct.*, **14**(6), 465-473.

Bhimaraddi, A. and Stevens, L.K. (1984), “A higher order theory for free vibration of orthotropic, homogeneous and laminated rectangular plates”, *J. Appl. Mech.-T ASME*, **51**(1), 195-198.

Bouchafa, A., Bachir Bouiadjra, M., Houari, M.S.A. and Tounsi, A. (2015), “Thermal stresses and deflections of functionally graded sandwich plates using a new refined hyperbolic shear deformation theory”, *Steel Compos. Struct., Int. J.*, **18**(6), 1493-1515.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), “Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations”, *Steel Compos. Struct., Int. J.*, **14**(1), 85-104.

Bouderba, B., Houari, M.S.A. and Tounsi, A. and Mahmoud, S.R. (2016), “Thermal stability of functionally graded sandwich plates using a simple shear deformation theory”, *Struct. Eng. Mech., Int. J.*, **58**(3), 397-

422.
Bouguenina, O., Belakhdar, K., Tounsi, A. and Adda Bedia, E.A. (2015), "Numerical analysis of FGM plates with variable thickness subjected to thermal buckling", *Steel Compos. Struct., Int. J.*, **19**(3), 679-695.
Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.
Boukhari, A., Ait Atmane, H., Tounsi, A., Adda Bedia, E.A. and Mahmoud, S.R. (2016), "An efficient shear deformation theory for wave propagation of functionally graded material plates", *Struct. Eng. Mech., Int. J.*, **57**(5), 837-859.
Bounouara, F., Benrahou, K.H., Belkorissat, I. and Tounsi, A. (2016), "A nonlocal zeroth-order shear deformation theory for free vibration of functionally graded nanoscale plates resting on elastic foundation", *Steel Compos. Struct., Int. J.*, **20**(2), 227-249.
Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct., Int. J.*, **18**(2), 409-423.
Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Computat. Method.*, **11**(6), 1350082.
Chikh, A., Bakora, A., Heireche, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2016), "Thermo-mechanical postbuckling of symmetric S-FGM plates resting on Pasternak elastic foundations using hyperbolic shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(4), 617-639.
Dahsin, L. and Xiaoyu, L. (1996), "An overall view of laminate theories based on displacement hypothesis", *J. Compos. Mater.*, **30**(14), 1539-1561.
Draiche, K., Tounsi, A. and Khalfi, Y. (2014), "A trigonometric four variable plate theory for free vibration of rectangular composite plates with patch mass", *Steel Compos. Struct., Int. J.*, **17**(1), 69-81.
Eltaher, M.A., Khater, M.E., Park, S., Abdel-Rahman, E. and Yavuz, M. (2016), "On the static stability of nonlocal nanobeams using higher-order beam theories", *Adv. Nano Res., Int. J.*, **4**(1), 51-64.
Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**(4), 795-810.
Hadji, L., Daouadji, T.H., Tounsi, A. and Bedia, E.A. (2014), "A higher order shear deformation theory for static and free vibration of FGM beam", *Steel Compos. Struct., Int. J.*, **16**(5), 507-519.
Hadji, L., Hassaine Daouadji, T., Ait Amar Meziane, M., Tlidji, Y. and Adda Bedia, E.A. (2016), "Analysis of functionally graded beam using a new first-order shear deformation theory", *Struct. Eng. Mech., Int. J.*, **57**(2), 315-325.
Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 235-253.
Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "A new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *ASCE J. Eng. Mech.*, **140**(2), 374-383.
Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int. J. Mech. Sci.*, **76**, 102-111.
Hwang, I. and Lee, J.S. (2006), "Buckling of orthotropic plates under various in-plane loads", *KSCE J. Civil Eng.*, **10**(5), 349-356.
Khalfi, Y., Houari, M.S.A. and Tounsi, A. (2014), "A refined and simple shear deformation theory for thermal buckling of solar functionally graded plates on elastic foundation", *Int. J. Comput. Meth.*, **11**(5), 135007.
Kang, J.H. and Leissa, A.W. (2005), "Exact solutions for the buckling of rectangular plates having linearly varying in-plane loading on two opposite simply supported edges", *Int. J. Solid. Struct.*, **42**(14), 4220-

4238.
Kant, T. (1989), “A higher-order theory for free vibration of unsymmetrically laminated composite and sandwich plates-finite element evaluations”, *Comput. Struct.*, **32**(5), 1125-1132.
Kant, T. and Khare, R.K. (1997), “A higher-order facet quadrilateral composite shell element”, *Int. J. Numer. Meth. Eng.*, **40**(24), 4477-4499.
Kant, T. and Pandya, B.N. (1988), “A simple finite element formulation of a higher-order theory for unsymmetrically laminated composite plates”, *Compos. Struct.*, **9**(3), 215-264.
Kant, T. and Swaminathan, K. (2001), “Analytical solutions for free vibration of laminated composite and sandwich plates based on a higher-order refined theory”, *Compos. Struct.*, **53**(1), 73-85.
Kar, V.R. and Panda, S.K. (2014), “Large deformation bending analysis of functionally graded spherical shell using FEM”, *Struct. Eng. Mech., Int. J.*, **53**(4), 661-679.
Kar, V.R., Mahapatra, T.R. and Panda, S.K. (2015), “Nonlinear flexural analysis of laminated composite flat panel under hygro-thermo-mechanical loading”, *Steel Compos. Struct., Int. J.*, **19**(4), 1011-1033.
Katariya, P.V. and Panda, S.K. (2016), “Thermal buckling and vibration analysis of laminated composite curved shell panel”, *Aircraft Eng. Aerosp. Technol.*, **88**(1), 97-107.
Kim, S.E., Thai, H.T. and Lee, J. (2009), “Buckling analysis of plates using the two variable refined plate theory”, *Thin-Wall. Struct.*, **47**(4), 455-462.
Klouche Djedid, I., Benachour, A., Houari, M.S.A., Tounsi, A. and Ameur, M. (2014), “A n-order four variable refined theory for bending and free vibration of functionally graded plates”, *Steel Compos. Struct., Int. J.*, **17**(1), 21-46.
Larbi Chaht, F., Kaci, A., Houari, M.S.A., Tounsi, A., Anwar Bég, O. and Mahmoud, S.R. (2015), “Bending and buckling analyses of functionally graded material (FGM) size-dependent nanoscale beams including the thickness stretching effect”, *Steel Compos. Struct., Int. J.*, **18**(2), 425-442.
Levinson, M. (1980), “An accurate simple theory of the statics and dynamics of elastic plates”, *Mech. Res. Commun.*, **7**(6), 343-350.
Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016a), “Geometrically nonlinear flexural analysis of hygro-thermo-elastic laminated composite doubly curved shell panel”, *Int. J. Mech. Mater. Des.*, **12**(2), 153-171.
Mahapatra, T.R., Panda, S.K. and Kar, V.R. (2016b), “Nonlinear hygro-thermo-elastic vibration analysis of doubly curved composite shell panel using finite element micromechanical model”, *Mech. Adv. Mater. Struct.*, **23**(11), 1343-1359.
Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), “A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates”, *Appl. Math. Model.*, **39**(9), 2489-2508.
Mallikarjuna, M. and Kant, T. (1993), “A critical review and some results of recently developed refined theories of fiber-reinforced laminated composites and sandwiches”, *Compos. Struct.*, **23**(4), 293-312.
Mantari, J.L. and Granados, E.V. (2015), “A refined FSDT for the static analysis of functionally graded sandwich plates”, *Thin-Wall. Struct.*, **90**, 150-158.
Matsunaga, H. (2009), “Thermal buckling of functionally graded plates according to a 2D higher-order deformation theory”, *Compos. Struct.*, **90**(1), 76-86.
Mehar, K. and Panda, S.K. (2016), “Geometrical nonlinear free vibration analysis of FG-CNT reinforced composite flat panel under uniform thermal field”, *Compos. Struct.*, **143**, 336-346.
Meksi, A., Benyoucef, S., Houari, M.S.A. and Tounsi, A. (2015), “A simple shear deformation theory based on neutral surface position for functionally graded plates resting on Pasternak elastic foundations”, *Struct. Eng. Mech., Int. J.*, **53**(6), 1215-1240.
Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), “A new higher order shear and normal deformation theory for functionally graded beams”, *Steel Compos. Struct., Int. J.*, **18**(3), 793-809.
Merazi, M., Hadji, L., Daouadji, T.H., Tounsi, A. and Adda Bedia, E.A. (2015), “A new hyperbolic shear deformation plate theory for static analysis of FGM plate based on neutral surface position”, *Geomech. Eng., Int. J.*, **8**(3), 305-321.
Mindlin, R.D. (1951), “Influence of rotary inertia and shear on flexural motions of isotropic elastic plates”, *J.*

Appl. Mech.-T ASME, **18**(1), 31-38.
Mohan, P.R., Naganarayana, B.P. and Prathap, G. (1994), "Consistent and variationally correct finite elements for higher-order laminated plate theory", *Compos. Struct.*, **29**(4), 445-456.
Mouaici, F., Benyoucef, S., Ait Atmane, H. and Tounsi, A. (2016), "Effect of porosity on vibrational characteristics of non-homogeneous plates using hyperbolic shear deformation theory", *Wind Struct., Int. J.*, **22**(4), 429-454.
Musa, I.A. (2016), "Buckling of plates including effect of shear deformations: A hyperelastic formulation", *Struct. Eng. Mech., Int. J.*, **57**(6), 1107-1124.
Nedri, K., El Meiche, N. and Tounsi, A. (2014), "Free vibration analysis of laminated composite plates resting on elastic foundations by using a refined hyperbolic shear deformation theory", *Mech. Compos. Mater.*, **49**(6), 629-640.
Nguyen, K.T., Thai, T.H. and Vo, T.P. (2015), "A refined higher-order shear deformation theory for bending, vibration and buckling analysis of functionally graded sandwich plates", *Steel Compos. Struct., Int. J.*, **18**(1), 91-120.
Noor, A.K. and Burton, W.S. (1989a), "Assessment of shear deformation theories for multilayered composite plates", *Appl. Mech. Rev.*, **42**(1), 1-13.
Noor, A.K. and Burton, W.S. (1989b), "Stress and free vibration analysis of multilayer composite plates", *Compos. Struct.*, **11**, 183-204.
Ould Larbi, L., Kaci, A., Houari, M.S.A. and Tounsi, A. (2013), "An efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams", *Mech. Based Des. Struct. Mach.*, **41**(4), 421-433.
Panda, S.K. and Katariya, P.V. (2015), "Stability and free vibration behaviour of laminated composite panels under thermo-mechanical loading", *Int. J. Appl. Comput. Math.*, **1**(3), 475-490.
Panda, S.K. and Singh, B.N. (2009), "Thermal post-buckling behaviour of laminated composite cylindrical/hyperboloidal shallow shell panel using nonlinear finite element method", *Compos. Struct.*, **91**(3), 366-384.
Panda, S.K. and Singh, B.N. (2010a), "Nonlinear free vibration analysis of thermally post-buckled composite spherical shell panel", *Int. J. Mech. Mater. Des.*, **6**(2), 175-188.
Panda, S.K. and Singh, B.N. (2010b), "Thermal post-buckling analysis of a laminated composite spherical shell panel embedded with shape memory alloy fibres using non-linear finite element method", *Proceedings of IMechE Part C: Journal of Mechanical Engineering and Science*, **224**(4), 757-769.
Panda, S.K. and Singh, B.N. (2011), "Large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel using nonlinear FEM", *Finite Elem. Anal. Des.*, **47**(4), 378-386.
Panda, S.K. and Singh, B.N. (2013a), "Large amplitude free vibration analysis of thermally post-buckled composite doubly curved panel embedded with SMA fibres", *Nonlinear Dyn.*, **74**(1), 395-418.
Panda, S.K. and Singh, B.N. (2013b), "Thermal postbuckling behavior of laminated composite spherical shell panel using NFEM", *Mech. Based Des. Struct. Mach.*, **41**(4), 468-488.
Panda, S.K. and Singh, B.N. (2013c), "Nonlinear finite element analysis of thermal post-buckling vibration of laminated composite shell panel embedded with SMA fibre", *Aerosp. Sci. Technol.*, **29**(1), 47-57.
Panda, S.K. and Singh, B.N. (2013d), "Post-buckling analysis of laminated composite doubly curved panel embedded with SMA fibers subjected to thermal environment", *Mech. Adv. Mater. Struct.*, **20**(10), 842-853.
Rajanna, T., Banerjee, S., Desai, Y.M. and Prabhakara, D.L. (2016), "Vibration and buckling analyses of laminated panels with and without cutouts under compressive and tensile edge loads", *Steel Compos. Struct., Int. J.*, **21**(1), 37-55.
Reddy, J.N. (1979), "Free vibration of antisymmetric angle ply laminated plates including transverse shear deformation by the finite element method", *J. Sound Vib.*, **66**(4), 565-576.
Reddy, J.N. (1984), "A simple higher-order theory for laminated composite plates", *J. Appl. Mech.*, **51**(4), 745-752.
Reddy, J.N. (1990), "A review of refined theories of laminated composite plates", *Shock Vib. Dig.*, **22**(7), 3-17.

Reddy, J.N. (1993), "An evaluation of equivalent-single-layer and layerwise theories of composite laminates", *Compos. Struct.*, **25**(1-4), 21-35.

Reddy, J.N. (1997), *Mechanics of Laminated Composite Plate: Theory and Analysis*, CRC Press, New York, NY, USA.

Reissner, E. (1945), "The effect of transverse shear deformation on the bending of elastic plates", *J. Appl. Mech.-T ASME*, **12**(2), 69-77.

Ren, J.G. (1986), "A new theory of laminated plate", *Compos. Sci. Technol.*, **26**(3), 225-239.

Sadoune, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2014), "A novel first-order shear deformation theory for laminated composite plates", *Steel Compos. Struct., Int. J.*, **17**(3), 321-338.

Saidi, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2013), "Thermo-mechanical bending response with stretching effect of functionally graded sandwich plates using a novel shear deformation theory", *Steel Compos. Struct., Int. J.*, **15**(2), 221-245.

Saidi, H., Tounsi, A. and Bousahla, A.A. (2016), "A simple hyperbolic shear deformation theory for vibration analysis of thick functionally graded rectangular plates resting on elastic foundations", *Geomech. Eng., Int. J.*, **11**(2), 289-307.

Sallai, B., Hadji, L., Hassaine Daouadji, T. and Adda Bedia, E.A. (2015), "Analytical solution for bending analysis of functionally graded beam", *Steel Compos. Struct., Int. J.*, **19**(4), 829-841.

Shahrjerdi, A., Mustapha, F., Bayat, M. and Majid, D.L.A. (2011), "Free vibration analysis of solar functionally graded plates with temperature-dependent material properties using second order shear deformation theory", *J. Mech. Sci. Technol.*, **25**(9), 2195-2209.

Sun, C.T. and Whitney, J.M. (1973), "Theories for the dynamic response of laminated plates", *AIAA J.*, **11**(2), 178-183.

Swaminathan, K. and Naveenkumar, D.T. (2014), "Higher order refined computational models for the stability analysis of FGM plates: Analytical solutions", *Eur. J. Mech. A/Solids*, **47**, 349-361.

Tagrara, S.H., Benachour, A., Bachir Bouiadjra, M. and Tounsi, A. (2015), "On bending, buckling and vibration responses of functionally graded carbon nanotube-reinforced composite beams", *Steel Compos. Struct., Int. J.*, **19**(5), 1259-1277.

Tebboune, W., Benrahou, K.H., Houari, M.S.A. and Tounsi, A. (2015), "Thermal buckling analysis of FG plates resting on elastic foundation based on an efficient and simple trigonometric shear deformation theory", *Steel Compos. Struct., Int. J.*, **18**(2), 443-465.

Timoshenko, S.P. and Gere, J.M. (1961), *Theory of Elastic Stability*, McGraw-Hill, New York, NY, USA.

Timoshenko, S.P. and Woinowsky-Krieger, S. (1959), *Theory of Plates and Shells*, McGraw-Hill, New York, NY, USA.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), "A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates", *Aerosp. Sci. Technol.*, **24**(1), 209-220.

Tounsi, A., Houari, M.S.A. and Bessaim, A. (2016), "A new 3-unknowns non-polynomial plate theory for buckling and vibration of functionally graded sandwich plate", *Struct. Eng. Mech., Int. J.*, [In press].

Viswanathan, K.K., Javed, S. and Abdul Aziz, Z. (2013), "Free vibration of symmetric angle-ply layered conical shell frusta of variable thickness under shear deformation theory", *Struct. Eng. Mech., Int. J.*, **45**(2), 259-275.

Whitney, J.M. (1969), "The effect of transverse shear deformation on the bending of laminated plates", *J. Compos. Mater.*, **3**(3), 534-547.

Whitney, J.M. and Pagano, N.J. (1970), "Shear deformation in heterogeneous anisotropic plates", *J. Appl. Mech.-T ASME*, **37**(4), 1031-1036.

Xiang, S., Jin, Y.X., Bi, Z.Y., Jiang, S.X. and Yang, M.S. (2011), "A n-order shear deformation theory for free vibration of functionally graded and composite sandwich plates", *Compos. Struct.*, **93**(11), 2826-2832.

Yaghoobi, H., Valipour, M.S., Fereidoon, A. and Khoshnevisrad, P. (2014), "Analytical study on post-buckling and nonlinear free vibration analysis of FG beams resting on nonlinear elastic foundation under thermo-mechanical loadings using VIM", *Steel Compos. Struct., Int. J.*, **17**(5), 753-776.

Yan, P.C., Norris, C.H. and Stavsky, Y. (1966), "Elastic wave propagation in heterogeneous plates", *Int. J.*

Solid. *Struct*., **2**(4), 665-684.
Yousefitabar, M. and Matapouri, M.Kh. (2016), “Thermally induced buckling of thin annular FGM plates”, *J. Brazil*. *Soc*. *Mech*. *Sci*. *Eng*., 1-12.
Zemri, A., Houari, M.S.A., Bousahla, A.A. and Tounsi, A. (2015), “A mechanical response of functionally graded nanoscale beam: an assessment of a refined nonlocal shear deformation theory beam theory”, *Struct*. *Eng*. *Mech*., *Int*. *J*., **54**(4), 693-710.
Zidi, M., Tounsi, A., Houari M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), “Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory”, *Aerosp*. *Sci*. *Technol*., **34**, 24-34.

CC