

A study on natural frequencies and damping ratios of composite beams with holes

Ersin Demir*

Department of Mechatronics Engineering, Pamukkale University, Kinikli Campus, 20070 Denizli, Turkey

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Abstract. In this study, free vibration and damping characteristics of composite beams with holes are investigated, experimentally and numerically. Two types of samples with different fabrics are used: unidirectional and woven. The effects of diameter, number and location of circular holes on the vibration characteristics of composite beams are examined. The effects of rotation angle and minor to major diameter ratio of the elliptical hole are also investigated numerically. Moreover, the mode shapes of all types of beams are obtained numerically. According to the results, the natural frequency decreases with increasing hole diameter but increases very little with increasing the distance between the hole center and the clamped end. Damping ratio decreases by increasing the diameter of hole. But it fluctuates by increasing the diameters of holes of beam having three holes. Furthermore it decreases by increasing the distance between hole center and clamped end except for the range 50 mm to 100 mm.

Keywords: free vibration; damping; experimental analysis; finite element analysis; glass fibers

1. Introduction

Composite materials are well known materials in the industry and they have been widely used in many industrial fields such as in the aircraft and space industries because of their properties such as, high strength and light weight. Due to properties mentioned, dynamic and mechanical analyses of fiber reinforced and/or metal matrices composite materials have attracted the attention of many scientists during the last decade e.g.m (James *et al.* 2009, Sayer 2014, Kapuria *et al.* 2008, Callioglu *et al.* 2014, Ergun and Alkan 2014, Atlihan *et al.* 2009). Although the composites bring advantages to the industry, they can be damaged due to various reasons. So, some of these studies have focused on damage of composites. Kiral *et al.* (2012) investigated the free vibration response and damping characteristic of a cantilever composite beam having a single impact failure on it. They concluded from their experimental study that damping ratio was more sensitive to a failure on the beam than the natural frequency and, severity and location of the failure had considerable effects on the damping ratio. Baba and Thoppul (2010) examined the curvature and face/core debond influence on the vibration behavior of curved composite sandwich beams made up of carbon/epoxy laminate skins over a polyurethane foam core. The experimental results are showed that face/core debond caused reduction of the natural frequencies whereas damping loss factor values increased with presence of debond.

*Corresponding author, Ph.D., E-mail: edemir@pau.edu.tr

The irregularities, which seem like damage, can be sometimes deliberately arranged in composites. For example, cut-outs in composite structures are quite common in aircrafts. Because, the materials used in aircraft should be lightweight and internal connection lines such as electrical cables should not be interrupt. Hence, static and dynamic behaviors of composite structures with elliptical or circular holes have been the subject of curiosity. Nevertheless, the studies on this subject are relatively limited and many of them are theoretical studies (O'Boy and Krylov 2011, Lee and Chen 2011, Ozturk 2015).

Some of the few experimental studies are given as follows. Naghipour and Mehrzadi (2007) investigated dynamic properties of extra lightweight concrete sandwich beams analytically and numerically. They used three different methods to measure the damping capacity of sandwich beams. Berthelot *et al.* (2008) investigated damping characters of laminated materials, laminates with interleaved viscoelastic layers and sandwich materials. They obtained the results from experiments and finite element analysis. Sen and Sayman (2009) performed a failure analysis of two-serial bolted glass-fiber-reinforced epoxy composite plates. Their experimental results indicated that the failure responses of the two-serial-bolted joints were strictly affected by the material parameters, geometrical parameters and values of the applied preload moments. Georgiev *et al.* (2011) established reliable numerical and experimental approaches for designing, modelling and manufacturing an effective passive vibration damper using the acoustic black holes effect. Erklig *et al.* (2013) investigated the effects of cutouts, cutout position and fiber orientations of laminated composite plates on natural frequencies. The results of this work showed that fiber orientation angle and cutout location were the most important parameters on the natural frequency. Abdilrazzaq (2013) analyzed the dynamical behavior of a cantilever steel beam, with different size of a hole, at different distances from the clamped end, for each case. Stanescu *et al.* (2010) showed that a mechanical model with one degree of freedom can be used to calculate the vibrations in a certain point of a bar. Apart from these, Chandra *et al.* (1999) reviewed the researches on damping in fiber reinforced composite materials and structures.

Experimental studies on vibration and damping behaviors of composite beams with holes are very few in open literatures. As distinct from these studies, in the present study, free vibration and damping behaviors of a fiber-reinforced laminated composite beam with circular hole are investigated experimentally and numerically. In addition, the influence of elliptical hole on the natural frequency of the beam is also studied numerically. The effects of diameter of circular hole, number of circular hole, location of circular hole on the natural frequencies, damping ratios and mode shapes are discussed according to the results obtained from experiments. Moreover, the results obtained from experiments are compared with those from SOLIDWORKS® Simulation Module (SolidWorks Corp., USA) for natural frequencies. The effects of rotation angle and minor to major diameters ratio of elliptical hole on the natural frequencies are also discussed according to the results obtained from SOLIDWORKS® Simulation Module (SolidWorks Corp., USA). The results obtained from experiments are found to be consistent with the results obtained from SOLIDWORKS® Simulation Module (SolidWorks Corp., USA) (see Figs. 8, 9, 11 and 12).

2. Preparation of composite plates

In order to see the effect of different stacking sequences, two different types of composite fabric are used. Unidirectional E-glass fabric with 600 warp tex and 200 weft tex (METYX Composites, Turkey) and Woven E-glass fabric with 600 warp tex and 600 weft tex (FIBROTEKS

Table 1 Mechanical properties of composite plates

Type of stacking sequences	E_1 [MPa]	E_2 [MPa]	G_{12} [MPa]	$G_{13} = G_{23}$ [MPa]	ν_{12}	$\nu_{13} = \nu_{23}$	ρ [g/mm ³]
Unidirectional	20443	5184	1856	1113	0.36	0.09	0.001526
Woven	21651	21651	1646	988	0.35	0.35	0.001578

Weaving Industry, Turkey) are used as fiber. The weights of the Unidirectional and Woven E-glass fabrics are 301 g/m² and 300 g/m², respectively. Epo Kem 1000 liquid epoxy resin and Keh 2000 Polyamide epoxy hardener are used as epoxy material in the production. These materials are supplied from PAG Chemical Industry, Turkey.

Glass-epoxy composite plates with eight layers are fabricated by ATARD Defence and Aerospace Inc., Turkey. Dimensions of the plates are produced as 500 mm × 1000 mm and vacuum assisted resin infusion method are used in the production. The average thicknesses of the plates are measured as 2.1 mm.

Engineering constants (Elasticity moduli, in-plane shear modulus, mass density and Poisson’s ratio) of unidirectional and woven composite plates are calculated from data sheets of materials purchased companies by using Classical Mixed Rules (Gibson 1994) and given in Table 1.

3. Geometries and designs of samples

In this study, glass fiber-epoxy composite plates manufactured are cut in the size of 250 mm × 20 mm for vibration tests. The beam is clamped at 50 mm from the one end and the other end of the beam was released. In order to see the effect of holes on the natural frequencies and damping responses of composite beams, three different cases are handled. In the first case, four different samples are drilled at 100 mm distances from the clamped end to see the effect of diameter of hole. The diameter of hole is taken as 4 mm, 8 mm, 12 mm and 16 mm. These samples are called as D-type samples in this study. In the second case, three holes are drilled in each sample. These samples are called as N-type samples to examine the effect of number of holes. The diameters of holes are changed in the samples like D-type. In the last case, a 8 mm diameter hole is drilled at different distances from the clamped end of the beam. The effect of the location of hole is investigated in the last case. The distance between the center of the hole and the clamped end of the beam sample (L) are changed as 25 mm, 50 mm, 75 mm, 100 mm, 125 mm, 150 mm, 175 mm. These samples are called as L-type samples. Intact and the three different type samples mentioned above are used in the vibration tests. To summarize the above issues, the schematic representation of D-type, N-type and L-type samples are shown in Figs. 1(a)-(b)-(c), respectively.

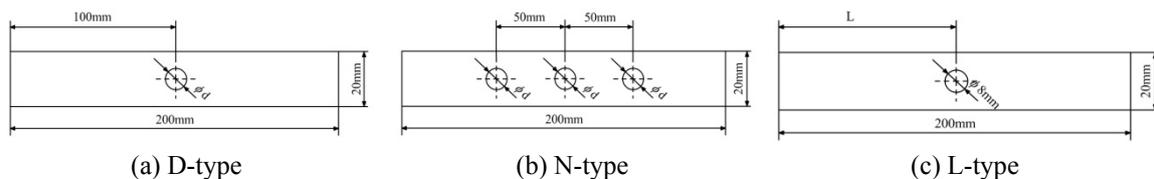


Fig. 1 Schematic representation of samples

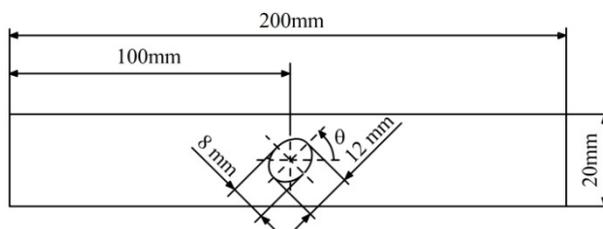


Fig. 2 Variation of the rotation angle of the elliptical hole

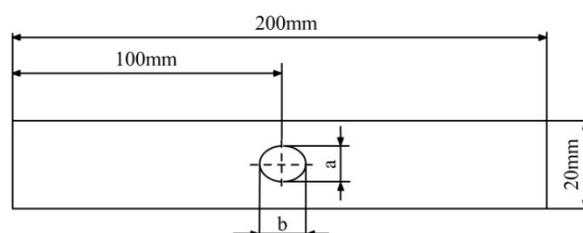


Fig. 3 Variation of the size of the elliptical hole

In addition, the effect of elliptical hole on the natural frequency is also investigated numerically in the study. To achieve this goal, two different cases are handled. In the first case, the rotation angle (θ) of the elliptical hole is changed from 0° to 180° by an increment of 15° , as seen in Fig. 2.

In the second, it can be seen from Fig. 3 that the minor to major diameter ratios of the elliptical hole (a/b) is chosen as $12/4$, $12/6$, $12/8$, $12/10$, $12/12$, $10/12$, $8/12$, $6/12$ and $4/12$.

4. Experimental setup and free vibration tests

Measurements are carried out for different types of the composite beam samples in order to demonstrate the effects of the holes on the natural frequencies and damping ratios. Boundary conditions of the beam samples are taken to be clamped-free. The free vibration is initiated by giving an initial displacement. In order to keep constant value of the initial displacement for all cases, a reference rigid (steel) beam is bolted on the clamp, as seen in Figs. 4(a)-(b). Free end of the cantilever beam is pulled until it touches the reference beam, then it is released for the free vibration. Schematic representation and real image of the experimental setup are shown in Figs. 4(a)-(b), respectively.

Vibration signals are recorded via an accelerometer (3035A1G, Dytran Instruments, USA). In order to reduce the influence of gravity, the samples are clamped perpendicularly to the surface and, although the accelerometer is quite light (2.3 g) it is located very close to the clamped end of the beam. The signal of the accelerometer is conditioned and amplified using a current source power unit (4102C, Dytran Instruments, USA). Then the signal is sent to the computer through a data acquisition card (NI USB 6008, National Instruments, USA). The signal is converted from the time domain to the frequency domain using Fast Fourier Transform Method. Therefore, a MATLAB code (MathWorks, USA) is developed to obtain the vibration frequencies in this study. As an example, the frequency spectrum of the unidirectional intact composite beam is shown in Fig. 5.

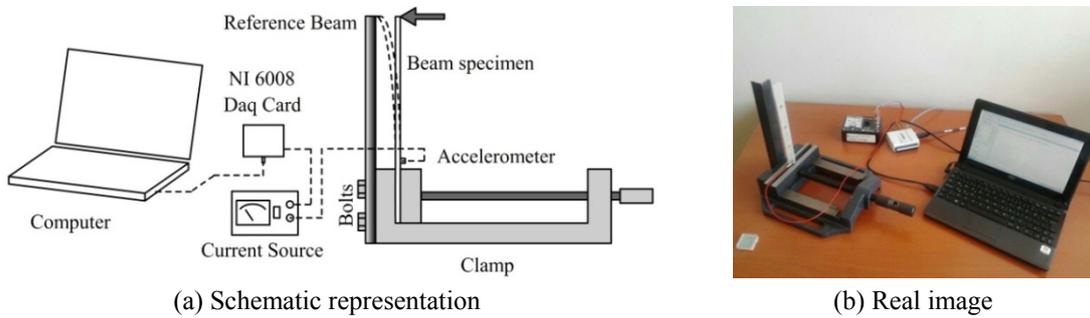


Fig. 4 The experimental setup

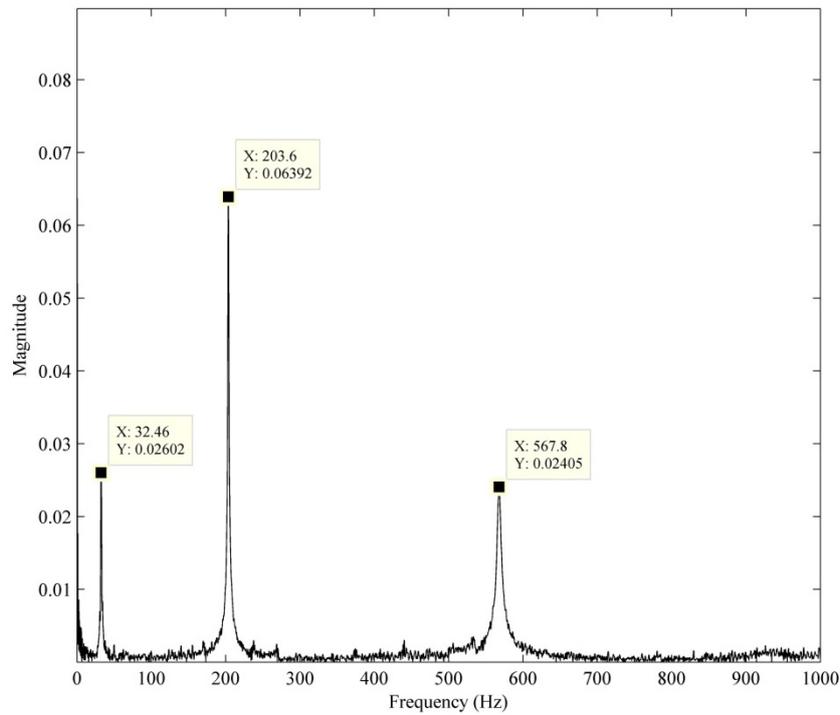


Fig. 5 Frequency spectrum of the intact composite beam

5. Obtaining the damping ratios from time history

When the free end of the beam is released from an initial position, free damped vibrations occurred. The well-known time history graph of underdamped vibration is given in Fig. 6.

It can be seen from the figure that the displacement is described by the following equation (Krodkiewski 2008).

$$x(t) = Ce^{-\xi\omega_n t} \sin(\omega_d t + \phi) \tag{1}$$

where C is the amplitude, ξ is the damping ratio, t is the time. ω_d is called as damped natural

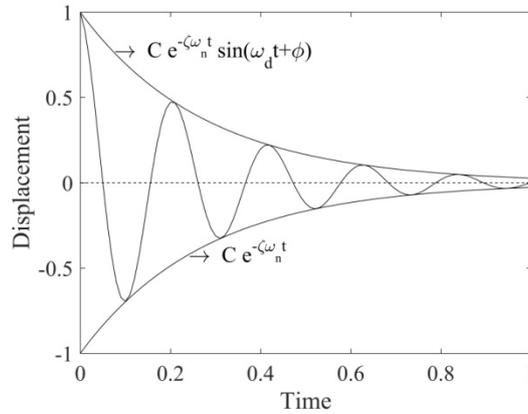


Fig. 6 Time history graph of underdamped vibration

frequency and it is equal to $\omega_n \sqrt{1-\xi^2}$ where ω_n is undamped natural frequency. ϕ is the phase shift, which can be calculated from the initial conditions.

In this study, the damping ratio is obtained in two ways. In the first way, the time histories of the samples are obtained from tests. Then, the 11 peak values obtained are transferred to commercial program, EXCEL (Microsoft, USA). Because damped and undamped natural frequencies are almost the same for small damping ratios, the exponentially decay term of Eq. (1) ($Ce^{-\xi\omega_n t}$) is obtained easily from the fitted exponential curve. In the second way, logarithmic decrement of damping is used. The logarithmic decrement of damping is the natural logarithm of the peak amplitudes ratio of two successive cycles in the response. The logarithmic decrement of damping is defined by (Krodkiewski 2008)

$$\delta = \ln \frac{x(t)}{x(t+T_d)} = \ln \frac{C e^{-\xi\omega_n t} \sin(\omega_d t + \phi)}{C e^{-\xi\omega_n(t+T_d)} \sin(\omega_d(t+T_d) + \phi)} = \frac{2\pi\xi}{\sqrt{1-\xi^2}} \quad (2)$$

and damping ratio ξ is given by Krodkiewski (2008).

$$\xi = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}} \quad (3)$$

In this study, in order to calculate the damping ratio, 11 successive peak values are considered. 10 damping ratios are obtained by substituting the 11 peak values into above equations. Then, the damping ratio of the beam is calculated by taking the arithmetic mean of these 10 values. As the damping ratios are calculated by using both methods for Mode 1, they are calculated only by using Envelope Curve method for Mode 2.

6. Numerical analysis

In order to support the results obtained from the experiments for natural frequencies, a finite element based commercial program SOLIDWORKS® (SolidWorks Corp., USA) is utilized in the

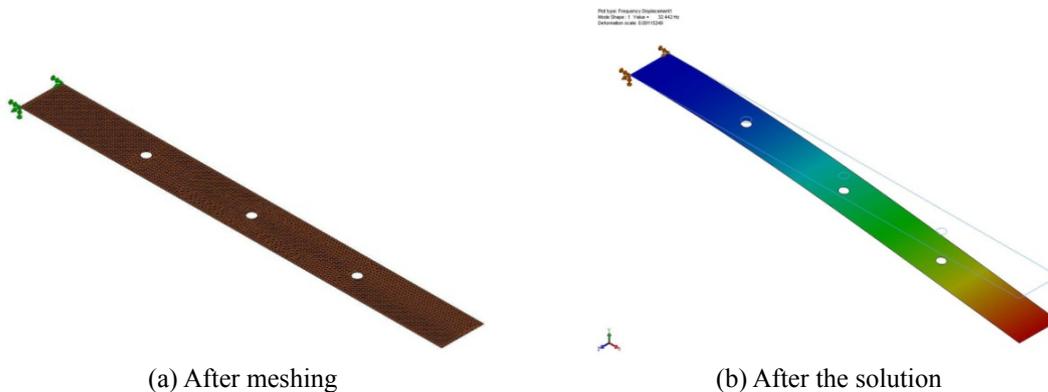


Fig. 7 Finite element model of a composite beam with holes

numerical solution. Initially, the beam with hole is modeled in the drawing module of the program, and then the simulation module of the program is run. Numerical vibration analysis of each composite sample is performed in this module.

Two different material types are defined in the program as unidirectional and woven according to their material properties given in Table 1. Clamped-free boundary conditions are applied and the model is meshed, as shown in Fig. 7(a). The solution is completed in order to find the natural frequencies for the first two modes. An example is given in Fig. 7(b) for N-type sample of 4 mm diameter. Numerical and experimental natural frequencies for the first two modes are compared in Section 7. The results obtained by both methods are in good agreement (see Figs. 8, 9, 11 and 12).

Mode shapes of all type of beams can be obtained by SOLIDWORKS®. In order to obtain the mode shape of beam, all nodes along the edge length of the beam are considered. The effects of the size, number and hole position on mode shape are also discussed in Section 7 (see Figs. 18-20).

7. Results and discussion

In this work, the effects of holes on the natural frequencies and damping responses of the composite beams are investigated experimentally and numerically. In the first three subsections below, the effects of diameter, number and location of holes on the natural frequency and the damping ratio are discussed according to the results obtained experimentally and numerically. In the next two subsections, the effects of the rotation angle and the minor to major diameters ratio of the elliptical hole are also discussed according to the results obtained numerically. In the last two subsections below, the effects on the damping ratios and mode shapes are discussed.

7.1 The effect of diameter of circular holes

In order to reveal the effect of diameter of the circular hole on the natural frequency of the composite beam, a hole is drilled at the middle of the sample as shown in Fig. 1(a). The diameter of the hole is increased in 4 steps until 16 mm. The variations of the natural frequencies of unidirectional and woven composite beam having different hole diameters are depicted in Figs. 8(a)-(b), respectively.

It can be seen from the figures that natural frequencies decrease with increasing diameter of the

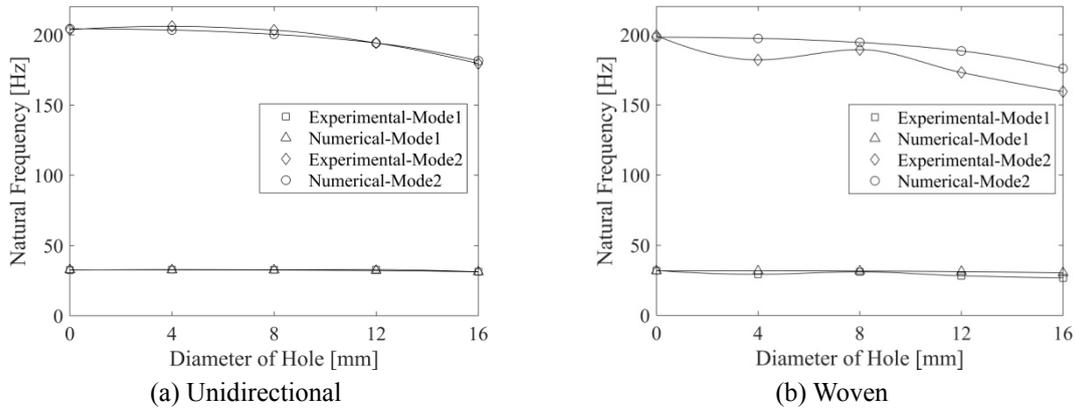


Fig. 8 Variation of the natural frequency of D-type composite beam with diameter of hole

hole. It can be also seen from the figures that the reduction in the range of 8 mm - 16 mm is greater than those of 0 mm - 8 mm. Moreover, it can be seen from Fig. 8(b) that some minor differences are observed between the numerical and test results obtained for woven composite beam. It is thought that the differences are based on the local irregularities taking place during manufacture. But the discrepancy between the experimental and numerical natural frequency values is generally acceptable.

As a result, as expected, the increase in the diameter of hole causes decrease in the values of the natural frequency. Nevertheless, the reduction is more significant after the diameter of hole reach approximately the quarter of the beam width.

7.2 The effect of number of circular holes

In order to expose the effect of number of circular holes on the natural frequency of the composite beam, three holes are drilled at the middle of the sample, as shown in Fig. 1(b). The diameters of the holes are increased in 4 steps from 0 mm to 16 mm. Figs. 9(a)-(b) illustrates the

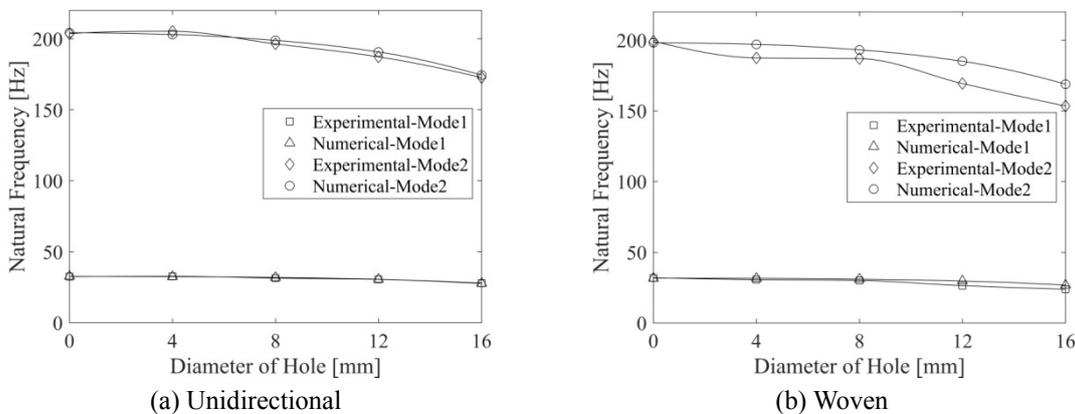


Fig. 9 Variation of the natural frequency of N-type composite beam with diameter of hole

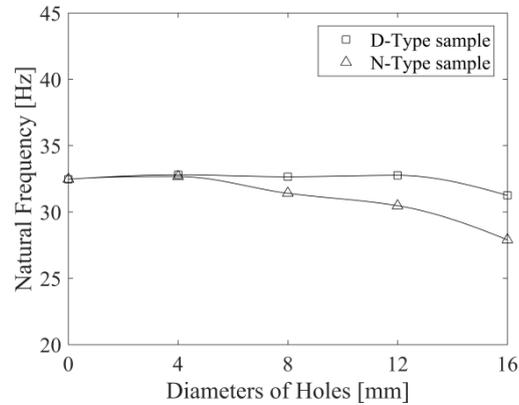


Fig. 10 Comparison of the experimental fundamental frequencies of D-type and N-type samples with diameter of holes

variations of the natural frequencies of unidirectional and woven composite beams versus diameter of hole, respectively.

As expected, increase in the diameter of the hole causes decrease in the natural frequencies. As a result of this, in order to achieve desired natural frequencies, the diameter of hole can be arranged. In addition, it can be seen from Fig. 10 that the reduction in the natural frequencies for mode 1 of beam having three holes is greater than those of one hole.

Consequently, a reduction in stiffness leads to a reduction in the natural frequencies. Moreover, as seen from the figures, the results obtained numerically and experimentally are in acceptable agreement.

7.3 The effect of location of circular holes

To see the effect of the location of holes on the natural frequency of the composite beam, a hole with a diameter of 8 mm is drilled at the middle of the sample as shown in Fig. 1(c). The distance between the center of the hole and the clamped end of the beam sample (L) are changed as 25 mm, 50 mm, 75 mm, 100 mm, 125 mm, 150 mm and 175 mm for each sample. In contrast to the above effects, the graphics of first and second natural frequencies show different tendency from each other. The variations of the first and second natural frequencies of unidirectional composite beam with different hole distance are shown in Figs. 11(a)-(b), respectively.

According to Fig. 11(a), the values of fundamental natural frequencies increase gradually by increasing the distance between the center of the hole and the clamped end of the beam sample. As for the values of second natural frequency, it is not much affected by changing the location of the hole. But, interestingly, it shows a tendency to rise when the hole is located at one quarter of the length of the beam from the clamped end. It can be seen from Fig. 12 that similar tendency is obtained for woven composite beam samples. Consequently, in order to achieve desired natural frequencies, the location of hole can also be arranged.

7.4 The effect of rotation angle of the elliptical hole

It can be seen from Fig. 2 that the rotation angle of elliptical hole is changed from 0° to 180° by

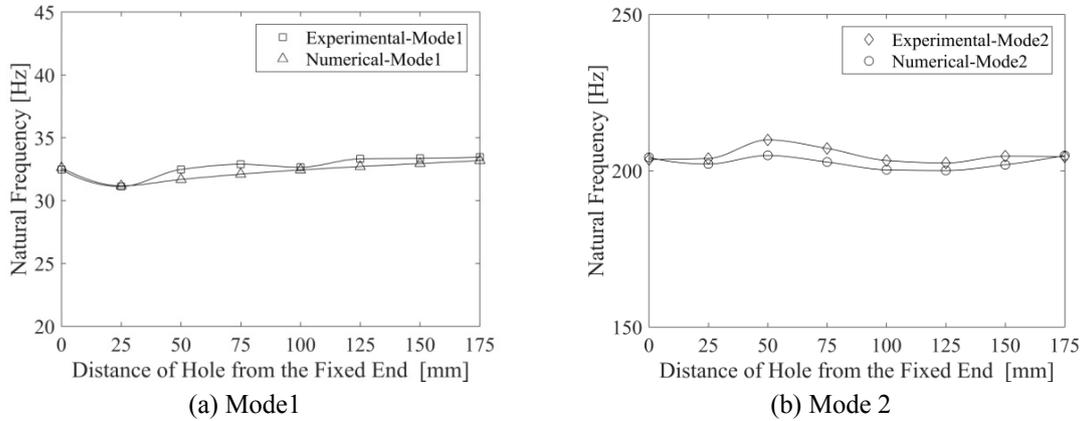


Fig. 11 Variation of the natural frequencies of unidirectional composite beam with distance of hole from the clamped end

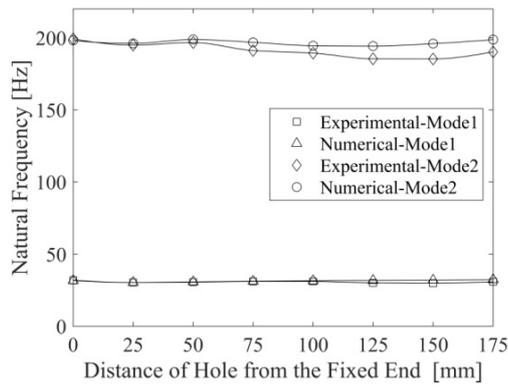


Fig. 12 Variation of the natural frequency of woven composite beam with distance of hole from the clamped end

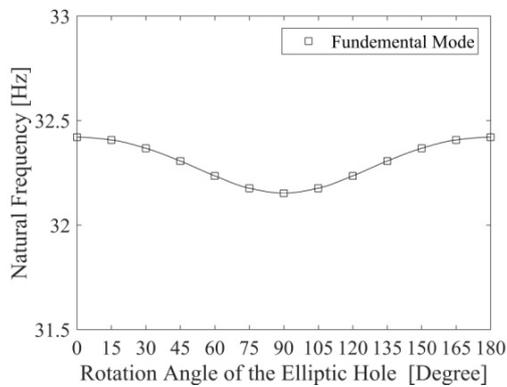


Fig. 13 Variation of the fundamental frequency of unidirectional composite beam with rotation angle of elliptical hole

an increment of 15° in order to see the effect of the rotation angle. The rotation angle is taken as 0° when the major axis of the elliptical hole is oriented horizontally. The major and minor diameters of the elliptical hole are taken as 12 mm and 8 mm, respectively. The natural frequencies, obtained from numerical analysis, versus the rotation angle of the elliptical hole are presented in Fig. 13.

As can be seen from the figure, the maximum natural frequency value is obtained when the elliptical hole is oriented horizontally. However, it decreases gradually with increasing rotation angle up to 90° . After 90° , the natural frequency value increases gradually. In other words, the natural frequencies obtained between 0° and 90° vary symmetrically with those obtained between 90° and 180° . Consequently, the natural frequency obtained for beam sample with vertical elliptical hole is lower than those with horizontal elliptical hole so that the bending stiffness of the beam with vertical elliptical hole is lower than those with horizontal elliptical hole.

7.5 The effect of minor to major diameter ratio of the elliptical hole

Fig. 14 shows the variations of the natural frequencies obtained from numerical analysis versus the ratio of minor to major diameter. In this subsection, a unidirectional beam specimen with circular hole is taken as reference specimen. The diameter of the circular hole (when the ratio of minor to major diameter is equal to 12/12) is taken as 12 mm. In order to see the effect of changes in minor and major diameter separately, the minor and major diameter of the hole is decreased gradually from 12 to 4. It can be seen from Fig. 14 that the natural frequency decreases with increasing major diameter of the elliptical hole.

Similarly, decrease in the minor diameter of the elliptical hole causes increase in the values of the natural frequency. However, the rate of increase in the latter case is approximately four and a half times greater than that in the previous case. It is concluded that the effect on the natural frequency is more considerable when the major axis of the elliptical hole is parallel to the longitudinal axis of the beam.

7.6 The effects on damping ratios

Figs. 15-17 show the variation of the damping ratio by changing the diameter, number and location of holes for unidirectional and woven composite beams (D-type, N-type and L-type),

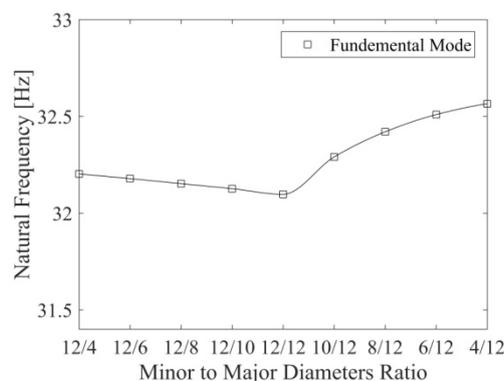


Fig. 14 Variation of the fundamental frequency of unidirectional composite beam with rotation ratio of minor to major diameters

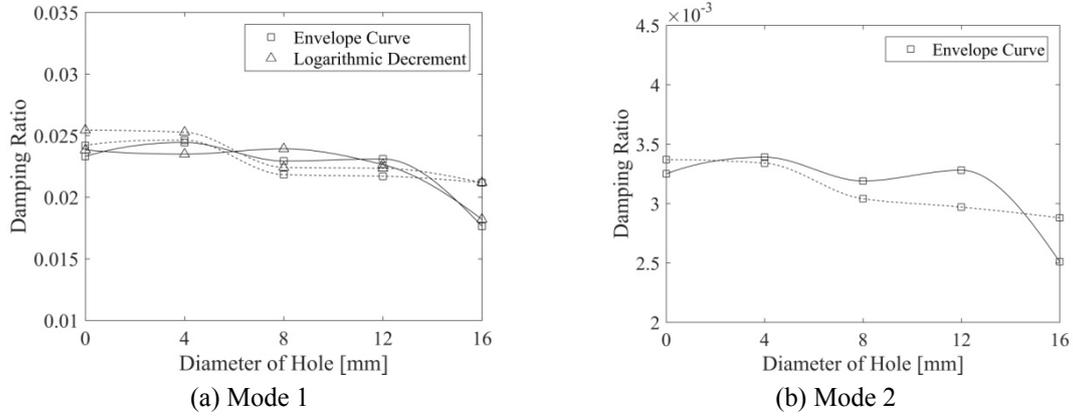


Fig. 15 Variation of the damping ratio (D-type) with diameter of hole (- unidirectional, -- woven)

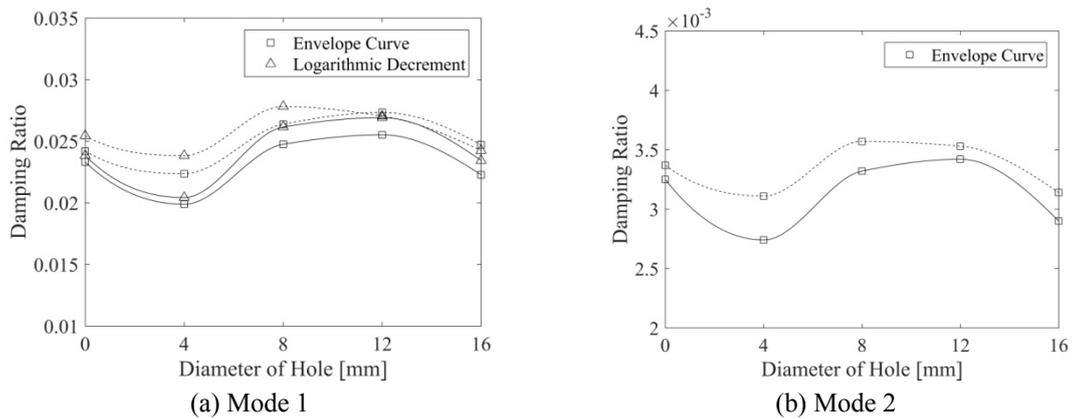


Fig. 16 Variation of the damping ratio (N-type) with diameter of hole (- unidirectional, -- woven)

respectively.

As illustrated in the Figs. 15(a)-(b), the damping ratio decreases by increasing the diameter of hole. But the decline is much steeper in the range of 12 mm and 16 mm for unidirectional composite beam. This rapid decline of damping ratio values is not observed for woven composite beam. The curve characteristics of the first and second modes are almost the same.

Figs. 16(a)-(b) show the damping ratio for first and second modes with diameter of hole for N-type composite beam, respectively. According to the figure, the curves obtained for unidirectional and woven composite beams show similar character. They decrease first and increase after and then decrease again. The curve characteristics of the first and second modes are also almost the same.

As for the effect of hole position on the damping ratio, a quite interesting graph is obtained as shown in Fig. 17. It can be seen from the figure that the curve increases very little up to 25 mm for both unidirectional and woven composite beam, and it then decreases gradually for woven composite beam. As for the unidirectional composite beam, it decreases a certain amount, after it increases up to earlier level. Finally, it decreases gradually, again. The curve characteristics of the

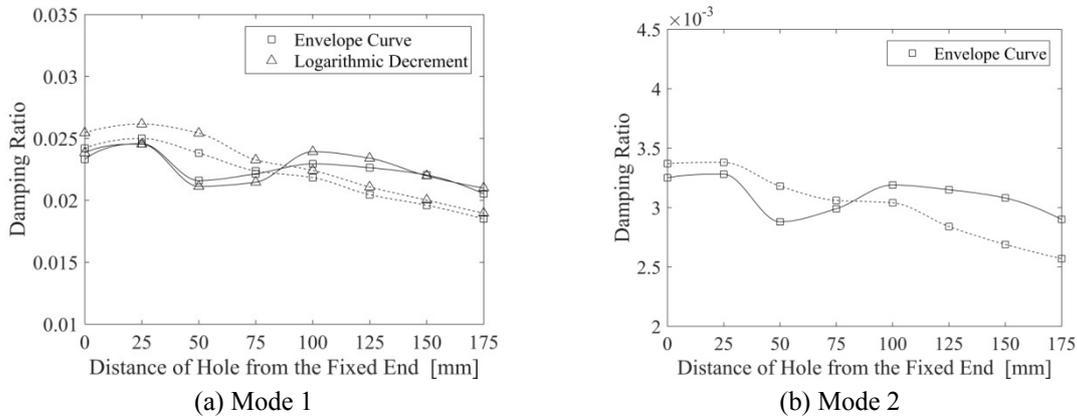


Fig. 17 Variation of the damping ratio (L-type) with distance of hole from the clamped end (- unidirectional, -- woven)

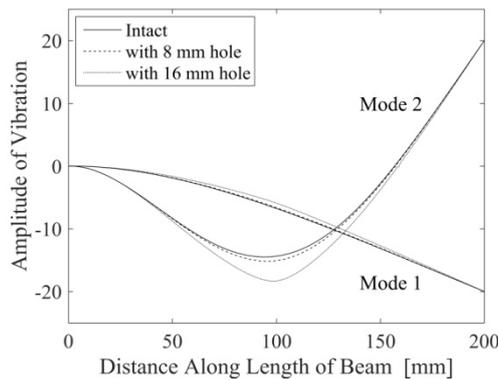


Fig. 18 Mode shape of D-type beam

first and second mode are almost the same as with the others.

7.7 The effects on mode shape

Fig. 18 represents the first and the second mode shapes of D-Type beam. It can be seen from the figure that although the increase in the hole diameter does not affect the first mode shape it affects the second mode shape. Because, the hole is drilled at the center of the beam.

Fig. 19 represents the first and the second mode shapes of N-Type beam. It can be also seen from the figure that increase in the hole diameter does affect very few the first mode shape. But the holes affect more the second mode shape. As mentioned above, N-type beam has 3 hole. As can be seen from the figure that the holes at 100 mm and 150 mm from clamped end affect the second mode shape. However the effect of hole at 50 mm from clamped end on the second mode shape is not observed.

Fig. 20 represents the first and the second mode shape of L-Type beam. It can be seen from the figure that increase in the location of the hole almost not affect the first and second mode shape.

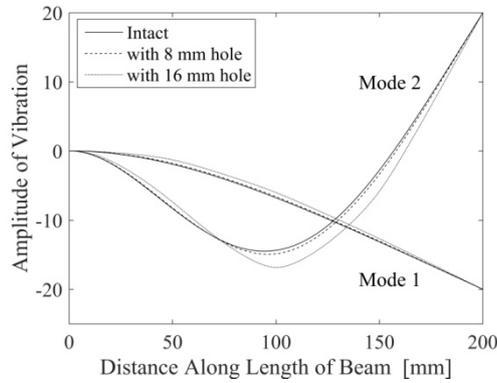


Fig. 19 Mode shape of N-type beam

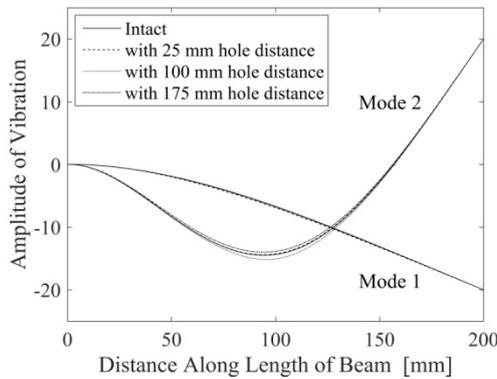


Fig. 20 Mode shape of L-type beam

8. Conclusions

In the analysis, effects of diameter, number and location of circular hole on the natural frequencies, damping ratios and mode shapes are investigated experimentally. The experimental results obtained for natural frequencies are supported with the numerical results. Moreover the effect of rotation angle and minor to major diameter ratio of the elliptical hole are discussed numerically. In the light of the above investigation, the following conclusions can be drawn:

- Increase in the diameter of hole causes decrease in the values of the natural frequency. The decrease in natural frequencies relatively apparent after the diameter of hole reaches approximately the quarter of the beam width, whereas it becomes considerably slow before it reaches quarter of the beam width.
- The natural frequency decrease with increasing the number of holes.
- The fundamental natural frequency increase gradually by increasing the distance between the center of the hole and the clamped end of the beam. As for the values of second natural frequency, it is not much affected by varying the location of the hole. But, interestingly, it shows a tendency to rise, when the hole is located at one quarter of the length of the beam from the clamped end.

- The natural frequency is taken minimum value when the major axis of the elliptical hole is perpendicular to the longitudinal axis of the beam.
- The natural frequency decrease with increasing major diameter of the elliptical hole. Similarly, decrease in the minor diameter of the elliptical hole causes increase in the values of the natural frequency. However, the rate of increase in the second case is approximately four and a half times greater than that in the first case.
- Increase in the diameter of the circular hole results in decrease in the values of the damping ratio obtained for unidirectional and woven beams. But the reduction is more noticeable in the range of 12 mm and 16 mm for unidirectional beam.
- The damping ratio fluctuates by increase in the diameters of holes of beam having three holes.
- The damping ratio gradually decreases with increasing the distance between the center of hole and clamped end of the beam except for a range between about 25 mm and 100 mm.
- Increasing in the diameter of the hole affects the second mode shape.
- A change in the location of the beam does not affect the mode shapes.

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