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Buckling behaviours of functionally graded polymeric thin-walled hemispherical shells

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Abstract. This paper investigates the static buckling behaviours of Functionally Gradient Polymeric Material (FGPM) shells in the form of hemispherical segment. A new FGPM model based on experimental was considered to investigate the buckling problem of thin-walled spherical shells loaded by the external pressure. The spherical shells were formed by FGPM which was produced adding the two types of graphite powders into epoxy resin. The graphite powders were added to the epoxy resin as volume of 3, 6, 9, and 12%. Halpin-Tsai and Paul models were used to determine the elastic moduli of the parts of FGPM. The detailed static buckling analyses were performed by using finite element method. The influences of the types and volume of graphite powders on the buckling behaviour of the FGPM structures were investigated. The buckling loads of hemispherical FGPM shells based on Halpin-Tsai and Paul models were compared with those determined from the analytical solution of non-graphite condition existing for homogeneous material model. The comparisons between these material models showed that Paul model was overestimated. Besides, the critical buckling loads were predicted. The higher critical buckling loads were estimated for the PV60/65 graphite powder due to the compatible of the PV60/65 graphite powder with resin.

Keywords: hemispherical thin shells; functionally graded polymeric materials (FGPMs); external pressure; static buckling; finite element analysis (FEA)

1. Introduction

In automotive, aerospace, power, and other engineering industries, the demand of advanced materials increased and researches focused to maintain their capabilities even in the extreme environmental conditions. Layered structures have been preferred in various applications which are required weight sensitivity. However, delamination between layers may cause failure due to residual stresses at the interfaces (Kar and Panda 2015). In functionally graded materials (FGMs), smooth variation of materials from one surface to another surface results elimination of delamination phenomenon. The volume fraction of these materials changes uniformly along a certain direction. Thus, FGMs have a nonuniform microstructure and a continuously variable macrostructure (Foroutan *et al.* 2011). Normally, mechanical properties of polymers can be improved by the inclusion of fillers to form polymer matrix composites (McGarry 1994, Jordan *et al.* 2005, Ochelski and Gotowicki 2009, Stabik *et al.* 2009, Kaci *et al.* 2012).

It is also well known that the structures of FGMs have more preferable performance than that of fiber reinforced composite materials (FRCMs), when the material properties of the FGMs can

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be designed to vary continuously and smoothly through the thickness rather than FRCMs (Wu *et al.* 2013). Several researches of the relevant theoretical and numerical modeling of FGMs and FRCMs structures can be found in literature (Carrera and Brischetto 2009, Yas and Garmsiri 2010, Foroutan *et al.* 2011, Hadji *et al.* 2014, Bouguenina *et al.* 2015). In this work, a literature survey carried out by focusing on articles dealing with the buckling behaviour of thin spherical/cylindrical shells.

Thin spherical/cylindrical shells are widely used in many engineering fields. Their failure is mainly due to buckling behaviour of the shells being the determining factor and the buckling load being closely associated with the established of its load carrying capacity. Since the long time, the static buckling behavior of spherical shaped structures made of different materials, i.e., isotropic, orthotropic, laminated, attracted special attention of many researchers. Tillman (1970) analyzed the static buckling of isotropic shallow spherical shells under external pressure and Huang (1964) studied unsymmetrical buckling of isotropic spherical shells. The problems of buckling of orthotropic shallow spherical shells were investigated by Ganapathi and Varadan (1982). Chao and Lin (1990) studied buckling of orthotropic spherical caps based on the classical thin shell theory. Laminated spherical caps subjected to uniform external pressure and their buckling and post buckling behaviors were analyzed by Xu (1991) and Muc (1992). Chattopadhyay and Ferreira (1993) carried out a research to examine the maximum buckling load of a cylinder subject to ply stress constraints using material and geometric design variables. Ganapathi and Varadan (1995) analyzed the buckling of laminated anisotropic spherical caps using the finite element method.

In recent years, the incorporation of material grading in which the physical and mechanical specifications change locationally can play an important role in the design of the structural systems (Librescu and Maalawi 2007). Therefore, many authors have focused on the mechanical behavior of functionally graded spherical panels and shells and their industrial applications. A common application is the design of cylindrical/spherical shells subjected to the external hydrostatic pressure which may cause collapse by buckling instability (Simitses 1996, Sridharan and Kasagi 1997, Davies and Chauchot 1999). The underground/underwater pipelines and boiler tubes under the external steam pressure and reinforced submarine structures can be given as examples. Previous numerical and experimental researches have demonstrated that failure owing to structure buckling is main risk factor for thin-walled FGMs cylindrical/spherical shells. Maalawi (2011) presented a model for enhancing the buckling stability of composite, thin-walled rings/long cylinders subjected to external pressure using radial material grading concept. The objective of the study was to maximize the critical buckling pressure while preserving the total structural mass at a constant value. Depending on the study, material grading could have important contribution to the optimization process in achieving the required structural designs with enhanced stability limits. Arefi (2014) studied the analysis of functionally graded cylindrical shell under external loads and developed a formulation which can be utilized for materials or structures with numerical distribution of material properties. Najafov et al. (2014) investigated the stability of exponentially graded cylindrical shells subjected to the hydrostatic pressure. Additionally, Ohga et al. (2005) examined the buckling of sandwich cylindrical shells under axial loading and estimated safe buckling loads by using finite element program developed for this purpose. In the study, a sandwich cylindrical structure was modeled by using shell elements allowing layered analyses and different material properties were identified through the shell thickness. Fekrar et al. (2012) considered the mechanical buckling of hybrid functionally graded materials and investigated the effect of various variables, such as FGMs volume fraction, thickness, and thickness ratio. In general, the value of the buckling load depends on shell geometry, type of boundary condition,

850

material properties of shell, the location of reinforcing material, and the type of load. These structures are very effective in resisting external loading. Prakash *et al.* (2007) studied on the nonlinear axisymmetric buckling behaviour of clamped FGMs and Ganapathi (2007) used finite element model for stability characteristics of FGMs shallow spherical shells. Indeed, by using the concept of FGMs, Kaci *et al.* (2012) studied the cylindrical bending behavior of reinforced composite under uniform pressure loading and examined the effect of material specifications on the stress state and deflections. Shen (2009) offered that the bonding strength could be improved through the usage of graded distribution in the matrix and investigated the nonlinear bending behavior of simply supported FGMs plates under a transverse uniform load.

Differently from the internal stress in cylindrical shells, the stress of in spherical shells is distributed homogeneously due to central symmetry of the geometries and loadings. When considered the same wall thickness, spherical shells have superior bearing capacity. Recently, spherical shells have been mainly applied in the storage of all kinds of gas and liquefied gas for the gas supply systems of cities and in the industries such as petrolic, chemical, and metallurgical (Huang 2002, Jiammeepreecha et al. 2012, Li et al. 2014). On the other hand, spherical shells are important structural components frequently encountered in industrial applications such as underground structures, shipbuilding, building constructions, biomechanics, and missile (Jianping and Harik 1992, Grigolyuk and Lopanitsyn 2002, Lay 1993, Hou et al. 2006, Yeh et al. 2000, Chien et al. 2006, Chen et al. 2014). They are used as elements of roof coverings and concrete arch domes (Grigolyuk and Lopanitsyn, 2002) subjected to the external pressure and nonlinear long-term buckling behavior of spherical shallow thin walled shells investigated by Hamed et al. (2010). In the field of biomechanics, spherical shells are commonly used in the cornea and crystalline lens of a human eye model (Yeh et al. 2000, Chien et al. 2006). The behavior of an underwater FGMs spherical shell has received much attention (Vo et al. 2006, Wang et al. 2006, Huang 2002, Jiammeepreecha et al. 2012) and FGMs spherical shell has been used as submerged domes (Vo et al. 2006, Wang et al. 2006).

Due to increasing usage of spherical shells made of functionally graded polymeric materials (FGPMs), the understanding of their stability characteristics is essential for designers and engineers. However, to find explicit solution for the buckling loads of spherical segments are very difficult due to their special geometrical shapes. This paper considers a new FGPMs model based on experimental in the study of Uslu (2010) developed to analysis the buckling behaviour of hemispherical segments subjected to external pressure. The critical buckling loads by using finite element method were determined. In this model, thin-walled shells were built polymeric gradient materials based on epoxy resin filled with graphite (PAM96/98 and PV60/65). The graphite powders were added to the epoxy resin as 3, 6, 9, and 12% volume. Light microscope was used for observation of graphite area. Then, Halpin-Tsai and Paul models were used to determine the elastic moduli of the parts of FGPMs. The main objective of this study is to determine a rational graphite powder type and volume ratio to increase the critical buckling load capacity. The analyses show that the use of PV60/65 graphite powder increases more the strength of stability of the hemispherical shell.

2. Description of the present FGPMs

The present FGPMs models consist of the epoxy resin and the graphite powders. The graphite powders PAM96/98 and PV60/65 produced by Koh-I-Noor (2010) were used as filler. The

Mine U. Uysal



Fig. 1 Graphite powder graphite type PV60/65, 12% vol.

FGPMs models show that the possibility of the graphite powders PAM96/98 and PV60/65 applications as innovatory filler of polymers as described in the articles: (Uslu 2010, Koh-I-Noor 2010, Stabik *et al.* 2009, 2010a, Uslu and Kremzer 2011).

The epoxy resin (Epidian 6) which was cured with Z1 material and produced by "Organika Sarzyna" Chemical Plant S.A. (Poland), was used as polymeric matrix components (Stabik *et al.* 2010b). Epidian 6 is a thermosetting matrix and the density of epoxy resin is 1.17 g/cm^3 .

The graphite ratios of 3, 6, 9, and 12% volume were added into the epoxy resin and the centrifugal casting method was chosen to produce the FGPMs. The rotational speed was adjusted as 535.3 rpm for two hours. The samples wet grinded, polished, then etched in ethyl acetate pure (Poch Co. Gliwice, Poland). All samples were observed by the light microscope LECIA (MEF4A) in magnification 200X equipped with Axiovision software (Uslu 2010). The PV60/65 ratio of 12% vol. is presented in Fig. 1.

The graphite particles in FGPMs structure were detected and the area percentages of the detected elements were calculated by using image analyzer Leica QWin (Uslu 2010). In Table 1, I, II, III, IV, and V represent the five regions from outside to inside and the percentages of graphite area density in these regions were given.

In this model, the predictions of the Young's moduli for each five regions are necessary. Several empirical models for predicting the Young's module of polymer composites with nonspherical fillers have been proposed such as Guth model, Brodnyan model, Halpin-Tsai model, Lewis-Nielsen model, Verbeek-Focke model, and Paul model.

Table 1 The percentages of graphite density (% area)					
Material (%)	Ι	II	III	IV	V
3% PAM	2,1	1,8	1,6	1,1	1,0
3% PV	12,0	4,2	1,2	0,2	0,0
6% PAM	2,5	2,2	1,9	0,8	0,0
6% PV	7,8	5,6	3,6	2,0	0,3
9% PAM	1,7	0,95	0,69	0,3	0,0
9% PV	10,9	6,7	6,2	2,3	0,2
12% PAM	2,6	3,3	3,0	1,8	0,3
12% PV	22,1	19,2	17,0	10,0	5,0

Table 1 The percentages of graphite density (% area)

The Halpin-Tsai model and the Paul model were chosen for predicting equivalent moduli of elasticity of the material in straightforward way. Then, the buckling loads of hemispherical FGPM shells based on Halpin-Tsai and Paul models were compared. These expressions of models are given below.

Thus, model proposed by Paul (1960) as seen in Eq. (1).

$$E_{composite} = \frac{E^2_{polymer} + \left(E_{polymer}E_{filler} - E^2_{polymer}\right)V_f^{2/3}}{E_{polymer} + \left(E_{filler} - E_{polymer}\right)V_f^{2/3}\left(1 - V_f^{2/3}\right)}$$
(1)

where V_f is the % volume of filler, E_{filler} and $E_{polymer}$ are the modulus of elasticity of the PAM96/98 or PV60/65 graphite powder particle and the epoxy resin (Epidian 6), respectively.

The Halpin-Tsai equations (Halpin and Kardos 1976) are widely used to predict the modulus of unidirectional composites (Tucher III and Liang 1999, Fornes and Paul 2003). The Halpin-Tsai equations are a fact that general form of the Kerner equation and many other equations (Nielsen and Landel 1994). In result, the modulus of elasticity of FGPMs can be determined by Eq. (2).

$$\frac{E_{composite}}{E_{polymer}} = \frac{1 + \xi \eta v_f}{1 - \eta v_f}$$
(2)

where ξ is a shape factor that depends on the geometry of the filler particle and v_f is the % volume of filler, the shape factor was assumed as 5,1 due to the fact that graphite shape was flake (Nielsen and Landel 1994), and the parameter η is determined in Eq. (3).

$$\eta = \frac{E_{filler} / E_{polymer} - 1}{E_{filler} / E_{polymer} + \xi}$$
(3)

In this study, the moduli of elasticity were calculated by using Eqs. (1)-(3) and Table 1. In calculations, E_{PAM} , E_{PV} , and E_{epoxy} were taken as 1000, 1020, and 3,24 GPa, respectively and Poisson's ratios v_{PAM} , v_{PV} , and v_{epoxy} were taken as 0,261, 0,272, and 0,25, respectively.

3. FGPMs hemispherical shell under external pressure

The static buckling problems of FGPMs hemispheres subjected the external pressure were analyzed. The FGPMs hemispheres were made of five layers. Total thickness of layers (t) was 1 mm and the elastic moduli of the five layers obtained based on both Halpin-Tsai model and Paul model. The geometrical dimensions of the spherical shells are shown in Fig. 2.

The values of critical pressures and buckling modes were determined by using finite element analysis. In literature, the buckling problem of the homogeneous isotropic thin walled spherical shells subjected to the external pressure was solved firstly by Zoelly (1915). Besides, the same solution was found independently by Leibenson. The formula for the critical pressure for the sphere is called Zoelly-Leibenson formula (Marcinowski 2007). They used the classical theory of small deflections and the solutions of linear differential equations. Based upon the work, the critical elastic buckling pressure P_{cr} for thin spherical shells found as in Eq. (4).



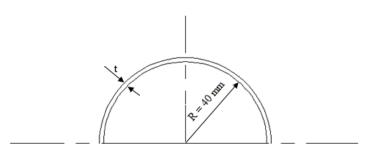


Fig. 2 Geometrical dimensions of the spherical shells

$$p_{cr}^{0} = \frac{2E}{\sqrt{3(1-\nu^{2})}} \left(\frac{t}{R}\right)^{2}$$
(4)

where t is the shell thickness, R is the radius, E is Young's modulus, and v is Poisson's ratio.

4. Finite element model

The numerical model of joint supported FGPMs hemispherical shell was developed by using the commercial software package ANSYS[®]. This finite element programme enables the prediction of buckling load and global behavior of FGPMs hemispherical shell subjected to external pressure. The finite element model of hemispherical shell can be shown in Fig. 3.

The numerical model of FGPMs hemispherical shell was divided into finite number of elements satisfying the equilibrium and compatibility at each node and along the boundaries between the elements as seen in Fig. 3. In FEM models, it is important to define a sufficiently accurate mesh for the elements, since the convergence of simulations as well as the accuracy of numerical results may be seriously compromised. Because of this reason, mesh convergence analyses were done for reducing element size in the model and solve the problem again. And then the desired quantities was not change by more than 2%, so it can be said the selecting mesh is fine

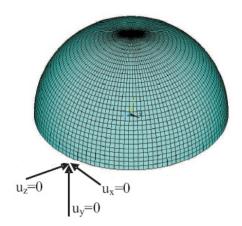


Fig. 3 Finite element model of hemispherical shell, joint support along the lower edges of the hemisphere

enough and the results do not depend on the mesh size. The FEM model was built with SHELL181 elements. SHELL181 may be used for layered applications for modeling composite shells. The accuracy in modeling composite shells is governed by the first-order shear-deformation theory (Ansys[®] 13.0 2010).

In practical engineering applications, graphite powders used as filler in composite makes it possible to gain new and cheaper polymeric materials with many possible implications. The influences of this filler (PAM96/98 and PV60/65) on properties indicate that processing of these new materials may be accompanied with some problems as example electrical (Szczepanik et al. 2009), magnetic (Stabik et al. 2010c, 2012), and wear resistant (Stabik and Chomiak 2013) problems. Thus, PAM PAM96/98 and PV60/65 hemispherical shells were created to investigate material performance under external pressure. Two types of FGPMs hemispherical shells were modeled and the effects of graphite powder type/volume ratios on the buckling loads were investigated by finite element analysis. The important step of the work is that graphite distribution of FGPMs samples were investigated by light microscope and the Young moduli of each region were predicted based on the image processing program and with the help Halpin-Tsai and Paul equations. Then, the critical buckling pressures of hemispherical FGPM shells based on Halpin-Tsai model, in which graphite shape factor is involved, and Paul model, which doesn't have any shape factor, were compared. The main originality of this study is to investigate the buckling behavior thin walled hemispherical shells by using a new FGPMs model based on experimental developed recently.

5. Numerical results and discussions

The elastic buckling analysis was applied to predict the critical buckling pressure of the FGPMs hemispherical shells. The values of critical pressure and their corresponding buckling modes were calculated as well. First four modes of the 3% PAM-Paul Model hemisphere were presented in Fig. 4. The mode shapes were similar for the other graphite type and volume ratios.

The influences of material models, i.e, Halpin-Tsai and Paul models, graphite powder types, and graphite volume ratios on the critical buckling pressures are presented in Figs. 5-7.

In Fig. 5, the critical buckling pressures of the FGPMs hemispherical shells were examined and they were compared with according to volume ratios and material models for the first mode. The critical buckling pressure was obtained as 2397,9 Pa for the sample added neither PAM nor PV graphite powders by using finite element analysis. This value was calculated 2414,9 Pa for non graphite hemispherical shell by using Zoelly-Leibenson analytical formula (Marcinowski 2007). The critical buckling pressures for 3% vol. of PAM were determined as 2540,8 Pa by using Halpin-Tsai model and as 2993,1 Pa by using Paul model. The Fig. 5 shows that the critical buckling pressures increase with addition the small amount of graphite such as 3% volume to the resin matrix. This results show that the critical buckling pressure determined by Halpin-Tsai model more converged to the analytical result (Marcinowski 2007) than that determined by Paul model. Since Halpin-Tsai model considers the graphite shape factor differently from Paul model, leads to this result, possibly. In general, the critical buckling pressures increase with increasing the volume ratio of graphite powder. As seen in Fig. 5, the critical buckling pressure increased 19,44% when the volume ratio of PAM graphite increased from 3% to 12% for Halpin-Tsai model. This increment was calculated as 9,49% for PV graphite. The highest critical buckling pressure was observed for PV hemispherical shell and this result presented the PV graphite powder was

Mine U. Uysal

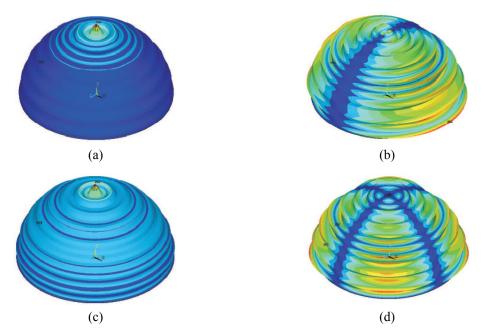


Fig. 4 Mode shapes of the 3% PAM-Paul Model hemisphere: (a) mode 1; (b) mode 2; (c) mode 3; (d) mode 4

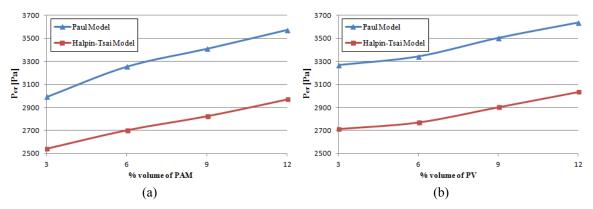


Fig. 5 Effects of the graphite volume ratios on the critical buckling pressure: (a) PAM; (b) PV

compatible with resin and the PV hemispherical shells responded much better against the static buckling.

The buckling pressure values that correspond to the subsequent eight modes of buckling were also calculated in Fig. 6. These values were calculated for 3% vol. of PAM, 3% vol. of PV, and non-graphite hemispherical shells. It should be remarked that the subsequent values of critical pressures were situated very close to one another. A difference in values of critical pressures for the first eight eigenmodes did not exceed 6% for all graphite types and all material models. The critical buckling pressures of hemispherical FGPMs shells based on Halpin-Tsai model and Paul model were compared with the numerical solution of non-graphite condition. Paul model results

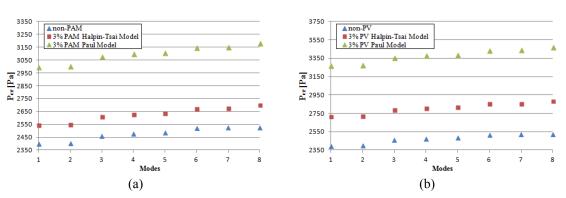


Fig. 6 Buckling pressures according to the modes (3% vol.): (a) PAM; (b) PV

are found to overestimate according to the comparisons between these material models.

In Fig. 7, the critical buckling pressures of FGPMs hemispherical shells were investigated according to the graphite volume ratios and material models. The critical buckling pressure determined by Paul model 20,4% higher than that of Halpin-Tsai model, for 6% volume of PAM graphite. Similar, this value was calculated 20,7% higher for 6% volume of PV graphite. The analysis shows that Paul model predicts overestimate the critical buckling pressures for the all graphite types and volume ratios.

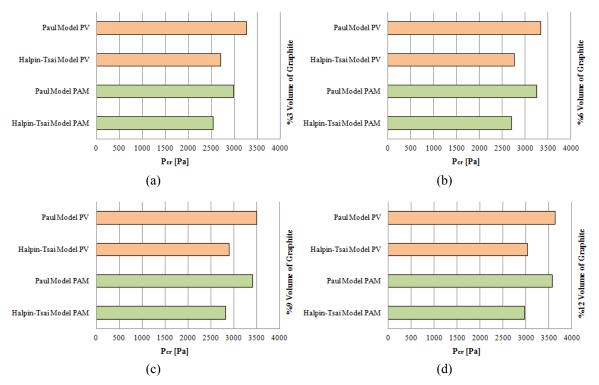


Fig. 7 Effects of the material models and graphite types on the critical buckling pressure: (a) 3% vol.; (b) 6% vol.; (c) 9% vol.; (d) 12% vol.



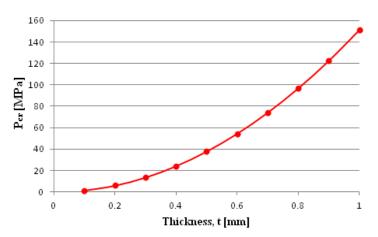


Fig. 8 The critical pressure according to the hemispherical wall thickness

6. Validation of the finite element analysis

Niezgodzinski and Swiniarski's study (2010) was taken into consideration to control the analysis, due to not achieving the previously conducted similar study. In their study, researchers presented FEM calculations of the stability of thin walled steel spherical shells for various thickness, different boundary conditions, and dilation angles. The values of critical pressure for steel material as a function of wall thickness of the hemisphere with the joint supported can be seen in Fig. 8.

The critical pressure was determined as 151,3 MPa, when the wall thickness of hemisphere was 1 mm (Marcinowski 2007).

In this study, the present FEM analysis was applied to the steel and the results were compared with the analytical solution (Marcinowski 2007) and FEM solution (Niezgodzinski and Swiniarski 2010) as shown in Table 2 and Fig. 9. The difference between present solution and the analytical solution (Marcinowski 2007) was determined as 0,86% and this pointed at the precision of the present FEM analysis.

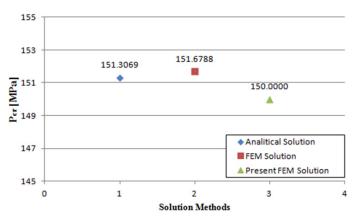


Fig. 9 The critical pressure values for each solution method

859

Analytical-numerical solution for hemisphere joint support ($t = 10 \text{ mm}, R = 40 \text{ mm}, \text{St37} E = 200 \text{ GPa}, v = 0.3$)					
Solution Comparisons	P _{cr} [MPa]	Differences [%]			
Analytical Solution (Marcinowski 2007)	151,30	0,00			
FEM Solution (Niezgodzinski and Swiniarski 2010)	151,67	0,25			
Present FEM Solution	150,00	0,86			

Table 2 Values of critical loads

7. Conclusions

In this study, the static buckling problem in hemispherical shells subjected external pressure was investigated in detail by using new developed functionally graded polymeric materials (FGPMs) model based on experimental. The obtained result reveals that the critical buckling pressure was affected differently by the graphite types, material model and volume ratios of graphite powders. The results determined by Halpin-Tsai model were more reasonable and reliable due to including the shape factor. The selecting of PV graphite was better to increase the buckling strength of hemispherical shell. The most buckling strength functionally graded polymer was obtained when the 12% volume ratio was applied. The external load type could be met at FGM spherical shells were affected by underwater when they used as submerged domes (Vo *et al.* 2006, Wang *et al.* 2006) and shell storage container (Huang 2002, Jiammeepreecha *et al.* 2012). The present interesting results can be useful for the stability problems of FGM hemispherical shells.

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