Steel and Composite Structures, *Vol. 20, No. 5 (2016) 1119-1131*
DOI: http://dx.doi.org/10.12989/scs.2016.20.5.1119

A hybrid inverse method for small scale parameter estimation of FG nanobeams

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(*Received September 20, 2015, Revised January 01, 2016, Accepted January 12, 2016*)

Abstract. As a first attempt, an inverse hybrid numerical method for small scale parameter estimation of functionally graded (FG) nanobeams using measured frequencies is presented. The governing equations are obtained with the Eringen's nonlocal elasticity assumptions and the first-order shear deformation theory (FSDT). The equations are discretized by using the differential quadrature method (DQM). The discretized equations are transferred from temporal domain to frequency domain and frequencies of the nanobeam are obtained. By applying random error to these frequencies, measured frequencies are generated. The measured frequencies are considered as input data and inversely, the small scale parameter of the beam is obtained by minimizing a defined functional. The functional is defined as root mean square error between the measured frequencies and calculated frequencies by the DQM. Then, the conjugate gradient (CG) optimization method is employed to minimize the functional and the small scale parameter is obtained. Efficiency, convergence and accuracy of the presented hybrid method for small scale parameter estimation of the beams for different applied random error, boundary conditions, length-to-thickness ratio and volume fraction coefficients are demonstrated.

Keywords: small scale parameter estimation; nanobeams; hybrid numerical method

1. Introduction

The Eringen's nonlocal elasticity (Eringen 1983) has been widely used in continuum mechanics to analyze nanostructures (Malekzadeh and Shojaee 2013, Ansari *et al*. 2014, Zenkour and Abouelregal 2014, 2015, Hosseini-Hashemi *et al*. 2015). An important parameter in this nonlocal elasticity is the small scale parameter. Identification of the small scale parameter has not been understood completely yet. So, finding this parameter is of interest of researchers. Eringen (1983) obtained the small scale parameter after matching the dispersion curves via the nonlocal elasticity theory and the Rayleigh surface wave analysis results. Zhang *et al*. (2005) found the small scale parameter of single-walled carbon nanotubes (SWCNTs) using the Donnell shell theory and the molecular mechanics simulations in conjunction with buckling analysis. Wang and Hu (2005) used the nonlocal Timoshenko beam model and the molecular dynamic simulations to obtain the small scale parameter of SWCNTs. Wang (2005) compared wave propagation of carbon nanotubes (CNTs) and the nonlocal continuum mechanics model and found the small scale parameter. Zhang *et al*. (2006) gained the small scale parameter of carbon graphene sheets using elastic interactions

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http://www.techno-press.org/?journal=scs&subpage=6 ISSN: 1229-9367 (Print), 1598-6233 (Online)

analysis between the Stone-Wales and the divacancy defects. Wang *et al*. (2008) estimated the small scale parameter of carbon nanotubes via the molecular dynamic simulations (MDSs) and comparing the results with the nonlocal elasticity theory. Shen and Zhang (2010) achieved the small scale parameter of CNTs by matching the obtained buckling torque from the molecular dynamic simulations (MDSs) with the numerical results from the nonlocal shear deformable shell model. Chan and Zhao (2011) achieved the small scale parameter by considering the nonlocal elasticity in the spinning CNTs for the nonlocal first-order deformation beam model. Khademolhosseini *et al*. (2012) found the small scale parameter of nanotubes considering torsional wave propagation obtained from the MDSs by comparing with the nonlocal dispersion relations. Huang *et al*. (2012) obtained small scale parameter of single-layered graphene sheets for bending problem. They found the parameter through comparison of the displacement response under a specific load from the MDSs and the nonlocal elasticity theory.

Besides, Duan *et al*. (2007) obtained the small scale parameter of the SWCNTs using measured frequencies from the MDSs. They used the Timoshenko beam theory and solved the governing equations analytically. They found that the obtained small scale parameters from their analysis are different from those obtained using their mentioned methods via various conditions. So, introducing a hybrid inverse method for determining the small scale parameter of nanostructures may yield to find better behavior of these structures.

On the other hand, sometimes for analysis of nanostructures conservative values of the small scale parameter are used. For example, Nazemnezhad and Hosseini-Hashemi (2014) used the conservative small scale parameter for vibration analysis of functionally graded nanobeams.

Based on the above mentioned review and to our best knowledge, there is no publication on small scale parameter estimation of naobeams using inverse optimization method. So, here as a first attempt, a hybrid inverse optimization numerical method is used to estimate the small scale parameter of the FG nanobeams. In the presented inverse method the DQM and the CG method are combined. The DQM is used to discretize the governing equations and for a specific small scale parameter, by applying random error measured frequencies are onbtained. An objective function as a root mean square error between measured frequencies and the calculated frequencies from mathematical model by the DQM is defined. Then, the CG method is used to minimize the function via finding the small scale parameter.

2. The governing equations and discretization procedure

Consider a nanobeam with length L and total thickness h (Fig. 1) with continuous change of material properties through h with power law distribution.

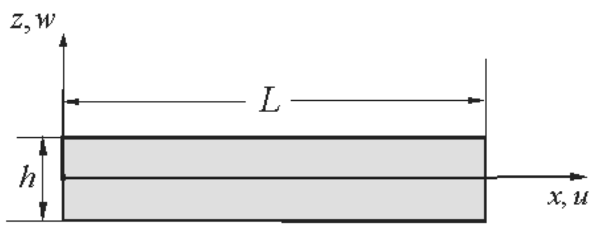


Fig. 1 The geometry of the nanobeam

Using the Eringen's nonlocal elasticity and the first-order shear deformation theories and neglecting the axial inertia, the free vibration governing equations of the functionally graded nanbeams can be obtained as (Vosoughi 2016)

$$A_{11}\frac{\partial^2 u}{\partial x^2}+B_{11}\frac{\partial^2 \varphi^x}{\partial x^2}=0 \tag{1}$$

$$A_{55}\left(\frac{\partial \varphi^x}{\partial x}+\frac{\partial^2 w}{\partial x^2}\right)=I_o\left(\ddot{w}-\mu^2\frac{\partial^2 \ddot{w}}{\partial x^2}\right) \tag{2}$$

$$B_{11}\frac{\partial^2 u}{\partial x^2}+D_{11}\frac{\partial^2 \varphi^x}{\partial x^2}-A_{55}\left(\varphi^x+\frac{\partial w}{\partial x}\right)=I_2\left(\ddot{\varphi}^x-\mu^2\frac{\partial^2 \ddot{\varphi}^x}{\partial x^2}\right) \tag{3}$$

where u, w and φ^x are the horizontal, vertical displacement component along the x and z-direction and the rotation of cross section along the y-direction, respectively. $\mu = e_0 l$ is the small scale parameter with a material constant e_0, the internal characteristic length l and

$$\left(A_{11},B_{11},D_{11}\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\frac{E(z)}{1-\upsilon(z)^2}\left(1,z,z^2\right)dz, \qquad A_{55}=\int_{-\frac{h}{2}}^{\frac{h}{2}}\kappa Q(z)\,dz,$$

$$\left(I_o,I_2\right)=\int_{-\frac{h}{2}}^{\frac{h}{2}}\rho(z)\left(1,z^2\right)dz, \qquad \ddot{X}=\frac{\partial^2 X}{\partial t^2},\left(X=w,\varphi^x\right)$$

where

$$E(z)=E_m+\left(E_c-E_m\right)\left(\frac{2z+h}{2h}\right)^n, \qquad \upsilon(z)=\upsilon_m+\left(\upsilon_c-\upsilon_m\right)\left(\frac{2z+h}{2h}\right)^n,$$

$$\rho(z)=\rho_m+\left(\rho_c-\rho_m\right)\left(\frac{2z+h}{2h}\right)^n$$

where E_i $(i = m, c)$, υ_i $(i = m, c)$ and ρ_i $(i = m, c)$ are the Young's modulus, the Poisson ratio and the density of the beam at metal and ceramic surfaces, respectively (Vosoughi 2014). Also, n is the volume fraction coefficient.

The boundary conditions for the vibration analysis of the beam are considered as

(a) Simply supported edge case

$$u=0, \quad w=0, \quad M_{xx}^{nl}=0 \tag{4a-c}$$

(b) Clamped edge case

$$u=0, \quad w=0, \quad \varphi^x=0 \tag{5a-c}$$

To discretize the above governing equations and the related boundary conditions the differential quadrature method is used. Using DQ rules (Vosoughi 2014, 2016, Vosoughi and Nikoo 2015) the discretized form of the governing equations can be obtained as

Eq. (1)

$$A_{11}\sum_{m=2}^{N_x^{'}} B_{im}^{x} u_m + B_{11}\sum_{m=1}^{N_x} B_{im}^{x}\varphi_m^x = 0 \tag{6}$$

Eq. (2)

$$A_{55}\left(\sum_{m=2}^{N_x^{'}} A_{im}^{x}\phi_m^x + \sum_{m=2}^{N_x^{'}} B_{im}^{x} w_m\right) = I_0\left(\ddot{w}_i - \sum_{m=2}^{N_x'} B_{im}^{x}\ddot{w}_m\right) \tag{7}$$

Eq. (3)

$$B_{11}\sum_{m=1}^{N_x} B_{im}^{x} u_m + D_{11}\sum_{m=1}^{N_x^{'}} B_{im}^{x}\varphi_m^x - A_{55}\left(\varphi_i^x + \sum_{m=1}^{N_x^{'}} A_{im}^{x} w_m\right) = I_2\left(\ddot{\varphi}_i^x - \mu^2\sum_{m=1}^{N_x} B_{im}^{x}\ddot{\varphi}_m^x\right) \tag{8}$$

In Eqs. (6) to (8) A_{ij}, B_{ij} and D_{ij} are the first, second and fourth-order derivatives weighting coefficients, respectively. The Chebyshev-Gauss-Lobatto quadrature points are considered and in the above equations is shown with N_x and $N'_x = N_x - 1$. Details for calculation of the weighting coefficients can be found in Refs. (Vosoughi 2014, Vosoughi and Nikoo 2015).

And the discretized form of the boundary conditions can be stated as

Eq. (4a-c)

$$u_i = 0,\ \ w_i = 0,\ \ \mathrm{D}_{11}\sum_{m=1}^{N_x} A_{im}^{x}\phi_m^x + \mu I_2\sum_{m=1}^{N_x} A_{im}^{x}\frac{d^2\phi_m^x}{dt^2} = 0, \qquad i = 1 \quad \text{and} \quad i = N_x \tag{9a-c}$$

Eq. (5a-c)

$$u_i = 0,\ \ w_i = 0,\ \ \varphi_i^x = 0, \qquad i = 1 \quad \text{and} \quad i = N_x \tag{10a-c}$$

where $i = 1$ at $x = 0$ and $i = N_x$ at $x = L$.

To transfer the discretized equations from temporal domain to frequency domain the following equations are used (Vosoughi and Nikoo 2015).

$$u(x,t) = U(x)\sin(\omega t), w(x,t) = W(x)\sin(\omega t), \varphi^x(x,t) = \phi^x(x)\sin(\omega t) \tag{11}$$

At last, the discretized form of the governing equations and the related boundary conditions in matrix form can be stated as

$$\left(\begin{bmatrix}[K_{WW}] & [K_{W\chi}]\\ [K_{\chi W}] & [K_{\chi\chi}]\end{bmatrix} - \omega^2\begin{bmatrix}[M_{WW}] & [0]\\ [0] & [M_{\chi\chi}]\end{bmatrix}\right)\begin{Bmatrix}\{W\}\\ \{\chi\}\end{Bmatrix} = \{0\} \tag{12}$$

where $\{\chi\} = \begin{Bmatrix}\{U\}\\ \{\phi^x\}\end{Bmatrix}$; $\left[K_{ij}\right](i, j = W, \chi)$ and $\left[M_{ij}\right](i, j = W, \chi)$ are the stiffness and the mass matrices, respectively.

3. Small scale parameter estimation

For small scale parameter estimation of the functionally graded nanobeams the following functional is used.

$$J\left[\omega(\mu)\right]=\sum_{k=1}^{M}\frac{\sqrt{\left[\omega_k(\mu)-\omega_k^*(\mu)\right]^2}}{M} \tag{13}$$

where M denotes the number of used frequencies in the estimation problem; ω_k and ω_k^* are the measured and the estimated (computed) frequencies, respectively.

To obtain the small scale parameter of the functionally graded nanobeam the conjugate gradient (CG) method as an optimization method is employed to minimize the functional by obtaining the optimum small scale parameter. In the conjugate gradient optimization method, gradient of the functional with respect to design variable (small scale parameter in this study) should be calculated. Here, the central difference method is used to find sensitivity coefficient of the functional according to the small scale parameter.

$$\{g\}=[\frac{\partial J}{\partial\mu}]\approx\frac{[J]_{\mu+\Delta\mu}-[J]_{\mu-\Delta\mu}}{2\Delta\mu} \tag{14}$$

In the conjugate gradient optimization method the optimum solution will be started by selecting an initial point. Then, the second point will be obtained by using the search direction and step length. This procedure will be continued, till the convergence criteria achieve.

For solving the presented problem, the following equation is used to obtain the small scale parameter in $s+1^{\text{th}}$ iteration from the achieved result from s^{th} iteration.

$$\mu^{s+1}=\mu^s+\gamma^s D^s \tag{15}$$

where D^s and γ^s are the search direction and the step length at s^{th} iteration, respectively.

Also, to solve the problem the first search direction is considered as

$$D^0=-g^0 \tag{16}$$

And $\{D\}^s$ for the other iterations is obtained as

$$D^s=-g^s+\beta D^{s-1} \tag{17}$$

where $\beta=\left(\dfrac{g^s}{g^{s-1}}\right)^2$.

Then, the golden section search algorithm is used to obtain optimum step length.

The above procedure is continued till the following convergence criterion is satisfied.

$$\left|\omega^{s+1}-\omega^s\right|/\omega^s\le\varepsilon \tag{18}$$

where ω^s and ω^{s+1} are the frequencies in ‘s’ and ‘s+1’ iterations and ε is the convergence error criterion.

4. Numerical results

In this section, first, the differential quadrature solution procedure is verified by comparing the results with those of available in the published articles. Then, applicability of the presented method for obtaining the small scale parameter estimation of the functionally graded nanobeams is demonstrated via solving some examples.

The convergence and accuracy of the presented differential quadrature solution procedure for simply supported (SS), simple-clamped (SC) and fully clamped (CC) boundary conditions are investigated in Table 1. The nonlocal to local (with $\mu = 0$) frequency ratio results for two different small scale parameters are compared with analytical solution of the Euler-Bernoulli beam theory presented by Nazemnezhad and Hosseini-Hashemi (2014). The following material properties are used to generate the numerical results.

Silicon material properties (Zhu *et al*. 2006)

$$E_c = 210GPa, \upsilon_c = 0.24, \rho_c = 2370\, kg/m^3$$

Aluminum material properties (Ogata *et al*. 2002)

$$E_m = 70GPa, \upsilon_m = 0.3, \rho_m = 2700\, kg/m^3$$

Table 1 Comparison between the obtained results using the DQM and analytical method for the nonlocal to local frequency ratio of FG nanobeams for different boundary conditions and small scale parameters ($n = 3$, $L/h = 1000$)

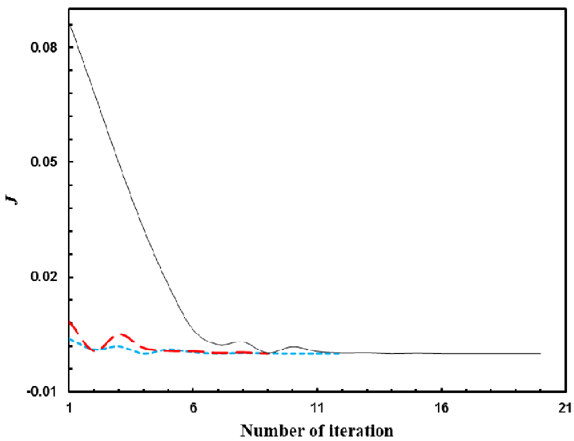
N_x	μ^2	Boundary condition		
		SS	SC	CC
7	2	0.9139	0.9014	0.8954
11		0.9139	0.9014	0.8954
17		0.9139	0.9014	0.8954
Nazemnezhad and Hosseini-Hashemi (2014)		0.9139	0.9013	--------
7	4	0.8468	0.8269	0.8175
11		0.8467	0.8269	0.8175
17		0.8467	0.8269	0.8175
Nazemnezhad and Hosseini-Hashemi (2014)		0.8467	0.8267	--------

Table 2 The convergence and accuracy of the presented hybrid inverse method for small scale parameter estimation of functionally graded nanobeams with different boundary conditions using the first measured frequency ($n = 3$, $L/h = 20$, $\mu^2_{exact} = 0$)

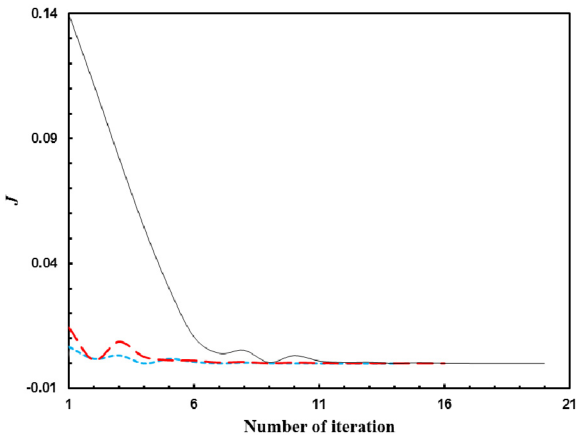
E	$(\mu^{-2})_{exact}$	Boundary condition		
		SS	SC	CC
10^{-3}	2	1.5450	1.5450	1.5450
10^{-6}		1.9990	2.0001	2.0001
10^{-3}	4	3.4550	3.4550	3.4550
10^{-6}		3.9996	3.9996	3.9996

Table 3 The average (AVE) and standard deviation (SD) of the random error for generating the measured frequencies

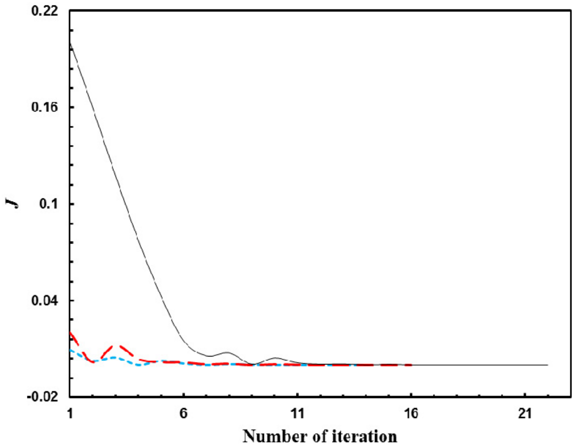
Random error (%)		Number of used measured frequency		
		One	Two	Three
+5	AVE	4.42	4.48	3.09
	SD	0	0.04	1.14
+10	AVE	6.78	8.18	7.41
	SD	0	0.99	0.91



(a) SS

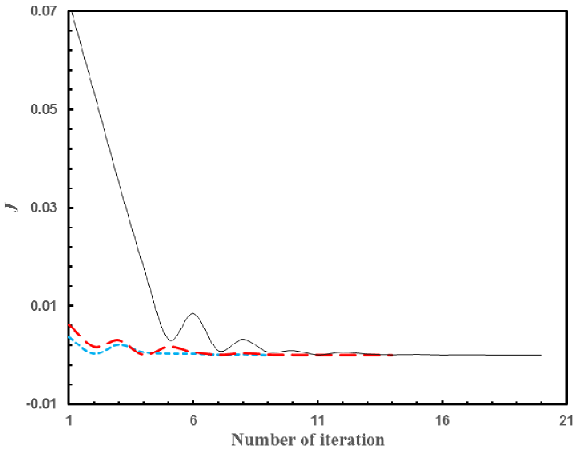


(b) CS

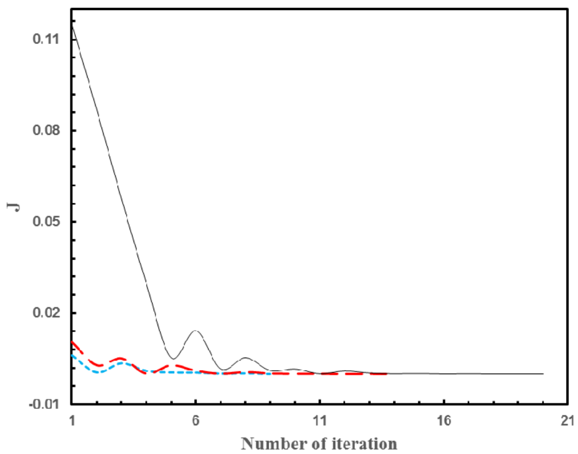


(c) CC

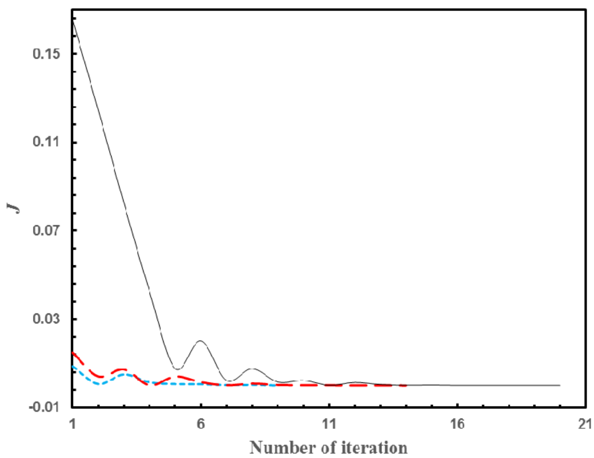
Fig. 2 Influence of start point on convergence and robustness of the conjugate gradient method for small scale parameter estimation of functionally graded nanobeams with different boundary conditions ($\mu^2_{start} = 2$, $n = 3$, $L/h = 20$, $\varepsilon = 10^{-6}$) $\mu^2_{start} = 0$: ; $\mu^2_{start} = 10$: ; $\mu^2_{start} = 100$:



(a) SS



(b) CS



(c) CC

Fig. 3 Influence of start point on convergence and robustness of the conjugate gradient method for small scale parameter estimation of functionally graded nanobeams with different boundary conditions ($\mu^2_{start} = 4$, $n = 3$, $L/h = 20$, $\varepsilon = 10^{-6}$) $\mu^2_{start} = 0$: ; $\mu^2_{start} = 10$: ; $\mu^2_{start} = 100$:

According to Table 1 the fast rate of convergence and accuracy of the differential quadrature method for free vibration analysis of the FG nanobeam are clarified. So, hereafter $N_x = 11$ is selected to solve the small scale effect parameter estimation problem.

To show convergence and accuracy of the presented hybrid inverse method for the small scale parameter estimation of functionally grade nanobeams Table 2 is prepared. In this table the small scale parameters of FG nanobeams with different boundary conditions are estimated using the first measured frequency without applying any random error. In this table by increasing the convergence criteria the good estimation of the parameter is achieved and so, in the following solved examples $\varepsilon = 10^{-6}$ considered to generate numerical results.

Convergence of the presented method for the parameter estimation of the nanobeams for two sets of the small scale parameters are investigated in Figs. 2 and 3. In these figures influences of different boundary conditions and initial small scale parameters on convergence of the problem are studied. From these figure one can see that by decreasing the initial small scale parameter, more rapid convergence is obtained. So, $\mu^2_{start} = 0$ is selected to generate the other numerical results.

In Table 3 the average and standard deviation of used applied random error for generating measured frequencies are shown. In this table, the average and standard deviation for generating three sets of number of frequencies are tabulated.

In Tables 4-6 influences of number of measured frequencies with different applied random error on small scale parameter estimation of the nanobeams with different boundary conditions are investigated. From these tables, it is obvious that the estimated parameters are sensitive with the inserted random error. Also, by increasing the number of measured frequencies the better

Table 4 Influence of the number of measured frequencies (NMF) on the small scale parameter estimation of SS supported functionally graded nanobeams for different random error (n = 3, L/h = 20, $\varepsilon = 10^{-6}$)

$(\mu^2)_{exact}$	NMF	Random error (%)	Estimated parameter (μ^2)	J × 10^{-6}
2	1	0	1.9990	8.4737 (12)*
		+5	1.0973	9.3082 (12)
		+10	0.6627	7.0341 (13)
	2	0	2.0001	8.6107 (17)
		+5	1.6424	8.7264 (17)
		+10	1.2969	9.4548 (15)
	3	0	2.0001	8.6108 (17)
		+5	1.9825	9.9908 (18)
		+10	1.6790	9.1787 (16)
4	1	0	3.9996	2.9110 (9)
		+5	2.9305	6.5014 (8)
		+10	2.4197	1.8325 (14)
	2	0	4.0002	9.1755 (16)
		+5	3.4733	6.5647 (15)
		+10	2.9634	2.6880 (10)
	3	0	4.0002	9.1755 (16)
		+5	3.9710	2.2624 (17)
		+10	3.4637	5.4606 (15)

* The number in parentheses shows the number of iteration in which convergence is occurred

Table 5 Influence of the number of measured frequencies (NMF) on the small scale parameter estimation of CS functionally graded nanobeams for different random error (n = 3, L/h = 20, $\varepsilon = 10^{-6}$)

$(\mu^2)_{exact}$	NMF	Random error (%)	Estimated parameter (μ^2)	J × 10^{-6}
2	1	0	2.0020	2.7883 (13)*
		+5	1.8384	1.3598 (11)
		+10	0.7935	9.1408 (12)

Table 5 Continued

$(\mu^2)_{exact}$	NMF	Random error (%)	Estimated parameter (μ^2)	J × 10^{-6}
2	2	0	2.0002	7.2621 (14)
		+5	1.6463	5.1652 (15)
		+10	1.3045	2.5683 (17)
	3	0	2.0001	9.9872 (17)
		+5	1.9825	3.2603 (18)
		+10	1.6774	4.2281 (15)
4	1	0	3.9996	4.8775 (9)
		+5	3.0261	1.6333 (12)
		+10	2.5551	3.7471 (14)
	2	0	4.0002	6.8095 (16)
		+5	3.4722	9.5103 (14)
		+10	2.9638	7.7284 (17)
	3	0	4.0000	1.9148 (18)
		+5	3.9705	7.7460 (17)
		+10	3.4589	9.0912 (14)

* See footnote of Table 4

Table 6 Influence of the number of measured frequencies (NMF) on the small scale parameter estimation of CC functionally graded nanobeams for different random error (n = 3, L/h = 20, $\varepsilon = 10^{-6}$)

$(\mu^2)_{exact}$	NMF	Random error (%)	Estimated parameter (μ^2)	J × 10^{-6}
2	1	0	2.0001	2.3883 (19)*
		+5	1.1653	6.4752 (15)
		+10	0.7640	3.0295 (16)
	2	0	2.0001	9.0029 (14)
		+5	1.6493	9.3066 (16)
		+10	1.3093	6.6940 (18)
	3	0	2.0000	1.3992 (19)
		+5	1.9822	4.9671 (14)
		+10	1.6741	5.9567 (19)
4	1	0	3.9996	6.9482 (9)
		+5	3.0041	51.2918 (4)
		+10	2.5252	4.3422 (12)
	2	0	4.0002	8.4844 (6)
		+5	3.4813	3.0069 (16)
		+10	2.9794	3.7822 (17)
	3	0	4.0000	2.2053 (18)
		+5	3.9699	4.5331 (18)
		+10	3.4539	5.1914 (17)

* See footnote of Table 4

Table 7 Influence of length-to-thickness ratio on small scale parameter estimation of functionally graded nanobeams for different boundary conditions (BC) and random error by using the first three measured frequencies (n = 3, $\varepsilon = 10^{-6}$)

BC	$(\mu^2)_{exact}$	Random error	L/h		
			100	20	10
SS	2	0	2.0001	2.0001	2.0001
		+5	1.9816	1.9825	1.9840
		+10	1.6643	1.6790	1.7085
	4	0	4.0002	4.0002	4.0002
		+5	3.9701	3.9710	3.9714
		+10	3.4489	3.4637	3.4801
SC	2	0	2.0001	2.0001	2.0001
		+5	1.9818	1.9825	1.9830
		+10	1.6692	1.6773	1.6904
	4	0	4.0002	4.0000	4.0002
		+5	3.9696	3.9705	3.9705
		+10	3.4511	3.4589	3.4598
CC	2	0	2.0001	2.0000	2.0001
		+5	1.9820	1.9822	1.9822
		+10	1.6738	1.6741	1.6745
	4	0	4.0002	4.0000	4.0002
		+5	3.9701	3.9699	3.9701
		+10	3.4530	3.4539	3.4548

* See footnote of Table 4

Table 8 Influence of fraction volume coefficients on small scale parameter estimation of functionally graded nanobeams for different boundary conditions (BC) and random error by using the first three measured frequencies (L/h = 10, $\varepsilon = 10^{-6}$)

BC	$(\mu^2)_{exact}$	Random error	L/h		
			100	20	10
SS	2	0	2.0000	2.0000	2.0001
		+5	1.9840	1.9840	1.9840
		+10	1.7085	1.7085	1.7085
	4	0	4.0000	4.0000	4.0002
		+5	3.9714	3.9714	3.9714
		+10	3.4801	3.4801	3.4801
SC	2	0	2.0000	2.0000	2.0001
		+5	1.9829	1.9830	1.9830
		+10	1.6903	1.6904	1.6904
	4	0	4.0000	4.0000	4.0002
		+5	3.9702	3.9702	3.9705
		+10	3.4598	3.4598	3.4598

Table 8 Influence of fraction volume coefficients on small scale parameter estimation of functionally graded nanobeams for different boundary conditions (BC) and random error by using the first three measured frequencies (L/h = 10, $\varepsilon = 10^{-6}$)

BC	$(\mu^2)_{exact}$	Random error	L/h		
			100	20	10
CC	2	0	2.0000	2.0000	2.0001
		+5	1.9822	1.9822	1.9822
		+10	1.6745	1.6745	1.6745
	4	0	4.0000	4.0000	4.0002
		+5	3.9701	3.9701	3.9701
		+10	3.4548	3.4548	3.4548

* See footnote of Table 4

estimation of the parameter is obtained.

In Table 7 influences of length-to-thickness ratio on the small scale parameter estimation of the functionally graded nanobeams are investigated for different boundary conditions. In this table the first three measured frequencies are considered. It is obtained that in the three cases of length-to-thickness ratio with different measured frequencies and same applied random error similar small scale parameters are obtained. In Table 8 influences of the volume fraction coefficients on the small scale parameter estimation of the functionally graded nanobeams using the first three measured frequencies are investigated. In this table different boundary conditions are considered. From this table, it is obvious that the volume fraction coefficients of the nanobeams cannot affect estimation of the small scale parameter.

5. Conclusions

As a first attempt, a hybrid inverse numerical method for small scale parameter estimation of functionally grade nanobeam is introduced. The first-order shear deformation and the Eringen's nonlocal elasticity theories are used to obtain the governing equations. The equations are discretized using the differential quadrature method (DQM). After transferring the discretized equations from temporal domain to frequency domain the frequencies of the nanobeam are calculated. By applying random error to the calculated frequencies measured frequencies are generated. Then, a functional as root mean square error between the measured and the calculated frequencies is defined. The conjugate gradient (CG) method is adopted to minimize the functional by selecting the small scale parameter as a design variable. Convergence and accuracy of the presented method for solving the problem with different length-to-thickness ratio, volume fraction coefficient and boundary conditions are demonstrated. It can be concluded that the presented hybrid inverse method can be used to estimate small scale parameter of the nanobeams using measured frequencies.

References

Ansari, R., Mohammadi, V., Faghih Shojaei, M., Gholami, R. and Rouhi, H. (2014), "Nonlinear vibration analysis of Timoshenko nanobeams based on surface stress elasticity theory", *Eur. J. Mech. A-Solid.*, **45**, 143-152.

Chan, K.T. and Zhao, Y. (2011), "The dispersion characteristics of the waves propagating in a spinning single-walled carbon nanotube", *Sci. China-Phys. Mech. Astron.*, **54**(10), 1854-1865.

Duan, W.H., Wang, C.M. and Zhang, Y.Y. (2007), "Calibration of nonlocal scaling effect parameter for free vibration of carbon nanotubes by molecular dynamics", *J. Appl. Phys.*, **101**(2), 024305.

Eringen, A.C. (1983), "On differential equations of nonlocal elasticity and solutions of screw dislocation and surface waves", *J. Appl. Phys.*, **54**(9), 4703-4710.

Hosseini-Hashemi, S., Nazemnezhad, R. and Rokni, H. (2015), "Nonlocal nonlinear free vibration of nanobeams with surface effect", *Eur. J. Mech. A-Solid.*, **52**, 44-53.

Huang, L.Y., Han, Q. and Liang, Y.J. (2012), "Calibration of nonlocal scale effect parameter for bending single-layered grapheme sheet under molecular dynamics", *Nano.*, **7**(5), 1250033.

Khademolhosseini, F., Phani, A.S., Nojeh, A. and Rajapakse, N. (2012), "Nonlocal continuum modeling and molecular dynamics simulation of torsional vibration of carbon nanotubes", *IEEE Trans. Nanotech.*, **11**(1), 34-43.

Malekzadeh, P. and Shojaee, M. (2013), "Surface and nonlocal effects on the nonlinear free vibration of non-uniform nanobeams", *Compos. B. Eng.*, **52**, 84-92.

Nazemnezhad, R. and Hosseini-Hashemi, S. (2014), "Nonlocal nonlinear free vibration of functionally graded nanobeams", *Compos. Struct.*, **110**, 192-199.

Ogata, S., Li, J. and Yip, S. (2002), "Ideal pure shear strength of aluminum and copper", *Sci.*, **298**(5594), 807-811.

Shen, H.S. and Zhang, C.L. (2010), "Torsional buckling and postbuckling of double-walled carbon nanotubes by nonlocal shear deformable shell model", *Compos. Struct.*, **92**(5), 1073-1084.

Vosoughi, A.R. (2014), "Thermal postbuckling analysis of functionally graded beams", *J. Therm. Stress.*, **37**(4), 532-544.

Vosoughi, A.R. (2016), "Nonlinear free vibration of functionally graded nanobeams on nonlinear elastic foundation", *Iran. J. Sci. Tech. Trans. Civil Eng.*, **45**(1), 581-586.

Vosoughi, A.R., Malekzadeh, P. and Razi, H. (2013), "Response of moderately thick laminated composite plates on elastic foundation subjected to moving load", *Compos. Struct.*, **97**, 286-295.

Vosoughi, A.R. and Nikoo, M.R. (2015), "Maximum fundamental frequency and thermal buckling temperature of laminated composite plates by a new hybrid multi-objective optimization technique", *Thin-Wall. Struct.*, **95**, 408-415.

Wang, Q. (2005), "Wave propagation in carbon nanotubes via nonlocal continuum mechanics", *J. Appl. Phys.*, **98**(12), 124301.

Wang, L.F. and Hu, H.Y. (2005), "Flexural wave propagation in single-walled carbon nanotubes", *Phys. Rev. B.*, **71**(19), 195412.

Wang, Q., Han, Q.K. and Wen, B.C. (2008), "Estimate of material properties of carbon nanotubes via nonlocal elasticity", *Adv. Theor. Appl. Mech.*, **1**(1), 1-10.

Zenkour, A.M. and Abouelregal, A.E. (2014), "The effect of two temperatures on a FG nanobeam induced by a sinusoidal pulse heating", *Struct. Eng. Mech., Int. J.*, **51**(2), 199-214.

Zenkour, A.M. and Abouelregal, A.E. (2015), "Thermoelastic interaction in functionally graded nanobeams subjected to time-dependent heat flux", *Steel Compos. Struct.*, **18**(4), 909-924.

Zhang, X., Jiao, K., Sharma, P. and Takobson, B.I. (2006), "An atomistic and non-classical continuum field theoretic perspective of elastic interactions between defects (force dipoles) of various symmetries and application to grapheme", *J. Mech. Phys. Solid.*, **54**(11), 2304-2329.

Zhang, Y.Q., Liu, G.R. and Xie, X.Y. (2005), "Free transverse vibrations of double-walled carbon nanotubes using a theory of nonlocal elasticity", *Phys. Rev. B.*, **71**(19), 195404.

Zhu, R., Pan, E., Chung, P.W., Cai, X., Liew, K.M., Buldum, A. (2006), "Atomistic calculation of elastic moduli in strained silicon", *Semiconduct. Sci. Tech.*, **21**: 906-911.

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