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Effect of stiffeners on steel plate shear wall systems

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Abstract. Stiffeners have widely been used in lateral load resisting systems to improve the buckling stability of shear panels in steel frames. However, due to major differences between plate girders and steel plate shear walls (SPSWs), use of plate girder equations often leads to uneconomical and, in some cases, incorrect design of stiffeners. Hence, this paper uses finite element analysis (FEA) to describe the effect of the rigidity and arrangement of stiffeners on the buckling behavior of plates. The procedures consider transverse and/or longitudinal stiffeners in various practical configurations. Subsequently, curves and formulas for the design of stiffeners are presented. In addition, the influence of stiffeners on the inward forces subjected to the boundary elements and the tension field angle is investigated as well. The results indicate that the effective application of stiffeners in SPSW systems not only improves the structural behavior, such as stiffness, overall strength and energy absorption, but also leads to a reduction of the forces that are exerted on the boundary elements.

Keywords: steel plate shear wall; stiffener; buckling; optimization; shear capacity; tension field angle

1. Introduction

A steel plate shear wall (SPSW) is made of an infill steel plate surrounded by horizontal boundary elements (beams) and vertical boundary elements (columns). It has been used as a reliable lateral load resisting system over the past few decades in the areas of high seismicity, although different countries have incorporated different design strategies. Some of the features of the SPSW system are its high initial stiffness, excellent ductility, robust resistance to cyclic degradation and significant energy dissipation.

SPSWs were first used, along with stiffeners, in the 1970's, since out-of-plane buckling of infill panel was considered as design limitation. Laboratory tests, conducted by Takahashi *et al*. (1973), on plates with various thicknesses and different stiffener dimensions, indicated that by effectively reinforcing the shear panel using stiffeners, hysteresis loops of an SPSW can be transformed from s-shaped in the thin SPSW to spindle-shaped in the stiffened SPSW as shown in Fig. 1. This transformation increases the area under the hysteresis loops, which increases the energy dissipation of the wall and simultaneously improves its performance.

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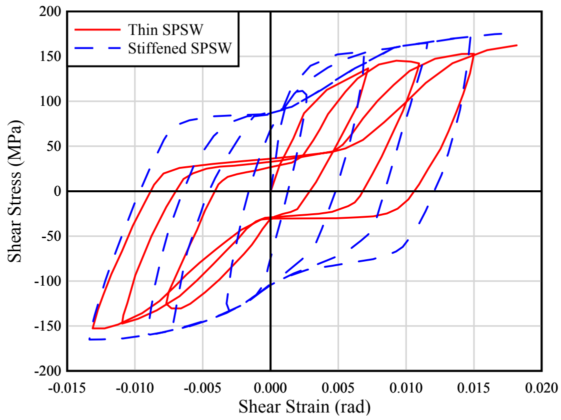


Fig. 1 Hysteresis curves in thin and stiffened SPSWs (Takahashi *et al*. 1973)

Some distinguished practical uses of this system are as follows: a 20-story office building in Tokyo, Japan; a 53-story high rise in Tokyo, Japan; a 30-story hotel in Dallas, Texas; a hospital retrofit in Charleston, South Carolina; a 6-story hospital in Los Angeles, California; and a 35-story office building in Kobe, Japan. After the construction of the last two buildings was completed, they were exposed to the 1994 Northridge and the 1995 Kobe earthquakes, respectively. Both buildings performed well during the earthquakes, and only experienced minor structural damages (Astaneh-Asl 2001).

Key researches in the 1980's showed that the post-buckling strength and ductility of thin SPSWs can be substantial because of the formation of a diagonal tension field (Bruneau *et al*. 2011). These findings led the researchers and designers, especially in the United States and Canada, to turn their attention to thin SPSWs because they were more economical than the stiffened SPSWs. Yet, from another perspective, in order to form diagonal tensions and to yield the web plate across the entire panel, boundary elements have to experience significant inward forces. Therefore, if these components do not have sufficient strength and stiffness according to capacity design principles, the web plate will not be able to reach its ultimate capacity. Because of this issue, the design of boundary elements of an SPSW faces some problems.

Current design specifications (AISC 341-10 2010, Sabelli and Bruneau 2007) have not properly addressed the issue of stiffened SPSW. Seismic Provisions for Structural Steel Buildings (AISC 341-10 2010) only gives requirements for the design of thin SPSW and the American Institute of Steel Construction (AISC) Design Guide 20 provides design specifications for stiffened SPSW based on plate girder design. However, because the boundary elements in an SPSW are different from those in a plate girder, the equations used for the plate girders cannot be assigned to SPSWs (Berman and Bruneau 2004). In this paper, the effects of stiffeners on the out-of-plane buckling behavior of the infill plate of SPSWs, as well as on the tension field orientation and the inward forces subjected to the boundary elements, are investigated using analytical study and finite element method (FEM).

2. Theoretical study on buckling stress

Since one of the principal stresses in the infill plate of SPSWs is compressive, the possibility of the occurrence of an unstable state of equilibrium plate must be considered. By the proper use of stiffeners, buckling stability could be substantially improved and consequently early elastic buckling could be prevented. In the following sections, stiffener requirements are presented in a form which can be used for design specifications.

2.1 Elastic buckling of rectangular plates in shear

At the onset of the buckling of a plate, when the flat form of equilibrium becomes unstable, the energy method can be used to calculate the critical values of forces applied in the middle plane of the plate. The energy method used in this case is because it has proven to be an excellent tool in solving a problem that cannot be solved directly as a characteristic value problem, such as stiffened plates (Timoshenko 1936). For a given plate of length L, height h and thickness t, subjected to uniformly distributed shear stress τ along the edges (Fig. 2), by using the principle of stationary potential energy, one gets

$$V + T = \text{Constant} \tag{1}$$

where, V is the strain energy of bending of the plate and T is the change of potential energy of the external forces when the plate passes from its plane form to the deflected shape (which is equal to the negative value of the work performed by the uniformly distributed shear stress).

V and T are expressed by the following

$$V = \frac{D}{2}\int_0^L\int_0^h \left\{ \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y} \right)^2 \right] \right\} dxdy \tag{2}$$

$$T = -\tau t \int_0^L\int_0^h \frac{\partial w}{\partial x}\frac{\partial w}{\partial y} dxdy \tag{3}$$

where, $D = Et^3/12(1-\nu^2)$ is the flexural rigidity of the plate, E is the Young modulus, ν is the Poisson ratio and w is the deflection of the plate in the state of buckling. In the case of reinforcing the plate by stiffeners, internal energy of bending of stiffeners should be added into Eq. (1).

The following double trigonometric series, also known as Navier solution, was widely used as the expression for the plate deflection (Timoshenko 1936)

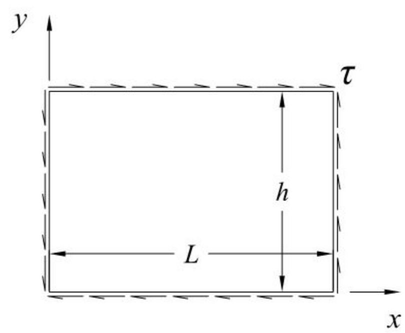


Fig. 2 A rectangular plate subjected to uniformly distributed shear stress in its middle plane

$$w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} a_{mn} \sin \frac{m\pi x}{L} \sin \frac{n\pi y}{h} \tag{4}$$

By substituting the above expression for w, which satisfies boundary conditions, into Eqs. (1)-(3), one arrives at the typical form of the expression for the critical stress of rectangular plates, i.e.

$$\tau_{cr} = k_s \frac{\pi^2 E}{12(1-\nu^2)} \left(\frac{t}{h} \right)^2 \tag{5}$$

where k_s is critical shear stress coefficient which is a function of boundary conditions and the plate aspect ratio. It should be noticed that the value of k_s is approximate, and its accuracy depends on the assumed expression of w. As the plate's resistance against rotation at its edges increases, the value of k_s for constant aspect ratio of plate will increase as well. The boundary conditions of the SPSW's infill plate and plate girder's web fall between the two extreme cases of simply supported (low rotational stiffness at edges) and clamped edge (high rotational stiffness at edges), and it can conservatively be assumed as simply supported. The following formula has been used to produce a parabolic curve, for simply supported edges, to approximate values of k_s for different properties of plates, which is used in section 3.2.2 of the AISC Design Guide 20 (Timoshenko 1936, Sabelli and Bruneau 2007) (see Appendix A).

$$k_s = 5.34 + 4\left(\frac{h}{L} \right)^2 \qquad \text{for } h \le L \tag{6}$$

2.2 Elastic buckling of plates reinforced by transverse stiffeners in shear

Eq. (5) indicates that the ratio of the plate's thickness to its smaller dimension has a remarkable influence on the critical stress of the plate in shear. If a plate is subdivided by sufficiently rigid stiffeners, smaller panels are formed, which may be considered as simply supported. Therefore, the decisive thickness-to-width ratio can be noticeably increased, and, consequently, the critical stress can be increased as well, due to its being proportional to the square of this ratio.

Using the non-dimensional parameter $\gamma = EI_s/Dd$, in which I_s is the effective moment of inertia of the stiffener, d is the stiffener spacing, and the energy method as before, it can be proved that the inclined waves of the buckled plate run across the stiffener if the rigidity of the stiffener is not sufficient. This case is also known as global buckling mode, in which buckling of the plate is associated with bending of stiffener. By subsequently increasing γ, the buckling pattern of the stiffened plate changes, and we reach a limiting value of γ_0, which ensures that the stiffeners remain straight. This is known as local buckling mode. A further increase of γ, practically, does not add to the buckling strength of the reinforced plate (Fig. 3). When reinforced by stiffeners having a ratio of γ_0, optimal dimensions of stiffeners are obtained and each plate panel can be considered as a simply supported plate in shear.

Stein and Fralich (1949) proved that Eq. (4) cannot be used in stiffened plates to estimate the limiting value of γ_0. They used two different deflection functions w:

For the low rigidity of stiffeners

$$w = \sin \frac{\pi x}{\lambda} \sum_{n=2,4,...}^{\infty} a_n \sin \frac{n\pi y}{h} + \cos \frac{\pi x}{\lambda} \sum_{n=1,3,...}^{\infty} b_n \sin \frac{n\pi y}{h} \tag{7}$$

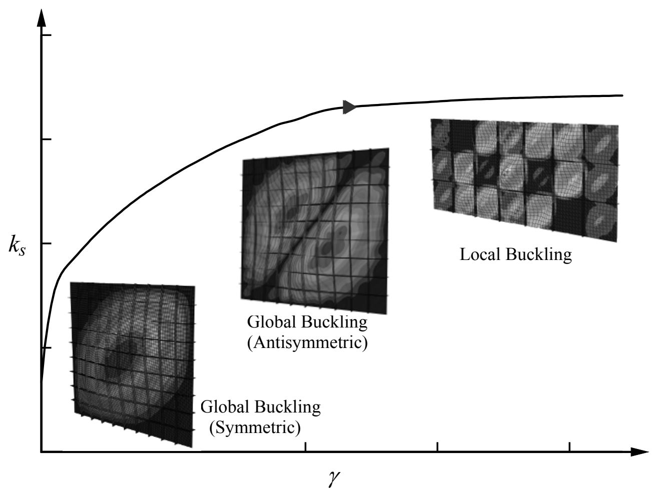


Fig. 3 Change in buckling pattern due to the increase of stiffeners' rigidity

in which λ is the half wave length of the buckled plate between two subsequent stiffeners.

For the high rigidity of stiffeners

$$w = \sum_{m=2,4,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} a_{mn} \sin\frac{m\pi x}{d} \sin\frac{n\pi y}{h} + \sum_{m=0,2,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} b_{mn} \cos\frac{m\pi x}{d} \sin\frac{n\pi y}{h} \tag{8}$$

and

$$w = \sum_{m=2,4,\ldots}^{\infty} \sum_{n=1,3,\ldots}^{\infty} a_{mn} \sin\frac{m\pi x}{d} \sin\frac{n\pi y}{h} + \sum_{m=0,2,\ldots}^{\infty} \sum_{n=2,4,\ldots}^{\infty} b_{mn} \cos\frac{m\pi x}{d} \sin\frac{n\pi y}{h} \tag{9}$$

Having used the above buckling configurations, Stein and Fralich (1949) improved Timoshenko's method for plates reinforced by equidistant vertical stiffeners of equal flexural rigidity (Fig. 4(a)) and presented curves to obtain the value of k_s for practical subpanel aspect ratios of 1, 2 and 5 (Fig. 5). The assumption of simply supported panels for high rigidities of

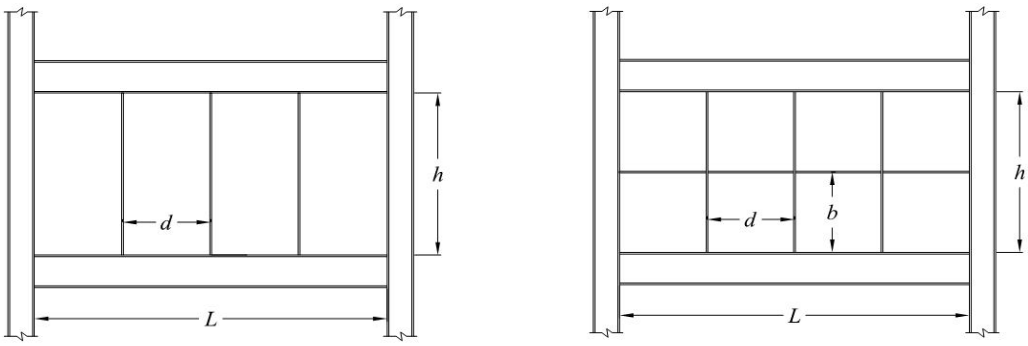


Fig. 4 (a) Steel plate shear wall with vertical stiffeners; (b) Steel plate shear wall with vertical and horizontal stiffeners

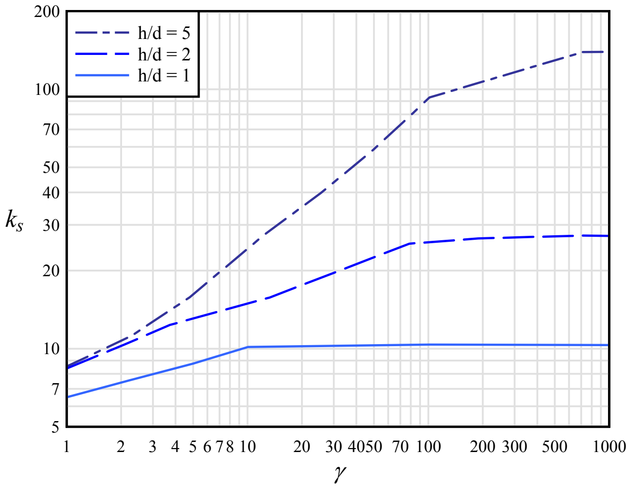


Fig. 5 Critical shear stress coefficient for vertically stiffened plates and subpanel aspect ratio of 1, 2, 5 (Stein and Fralich 1949)

stiffeners seems to be fairly conservative, due to overlooking the continuity of plate across the stiffeners (Fig. 6). On the other hand, it is only when rigidity of the stiffeners is low and they are placed with little distance from each other that the plate can be assumed orthotropic (Stein and Fralich 1949). Thus, although orthotropic plate solution can be used to determine the minimum required moment of inertia of stiffeners that shifts the buckling mode from global to local, conservatively (Sabouri-Ghomi *et al*. 2008), it is not appropriate to estimate critical buckling stress of a stiffened plate.

Based on the curves in Fig. 5, Bleich (1952) suggested the following equation for calculating the limiting value of γ_0

$$\gamma_0 = 4\left\{7\left(\frac{h}{d}\right)^2 - 5\right\} \quad \text{for } d \le h \tag{10}$$

By substituting $\gamma_0 = 12(1-v^2)I_{s,0}/(t^3 d)$ and $v = 0.3$ into Eq. (10), a formula for the required minimum effective moment of inertia of stiffeners ($I_{s,0}$) can be obtained, which is used in section G2 of AISC 360 with a slight modification (Galambos 1998, AISC 360-05 2005) (see Appendix B).

In a recent work, using the commercial finite element (FE) software package ABAQUS (2005), Alinia and Sarraf Shirazi (2009) developed the subsequent expression for the optimal dimensions of vertical stiffeners placed on one side of a square infill plate

$$t_s h_s^2 \ge \begin{cases} 0.7t^2 h & n = 1 \\ 0.7\left(1 + \dfrac{2n}{10}\right)t^2 h & n > 1 \end{cases} \tag{11}$$

in which, t_s and h_s are the thickness and the height of stiffener and n is the number of vertical stiffeners.

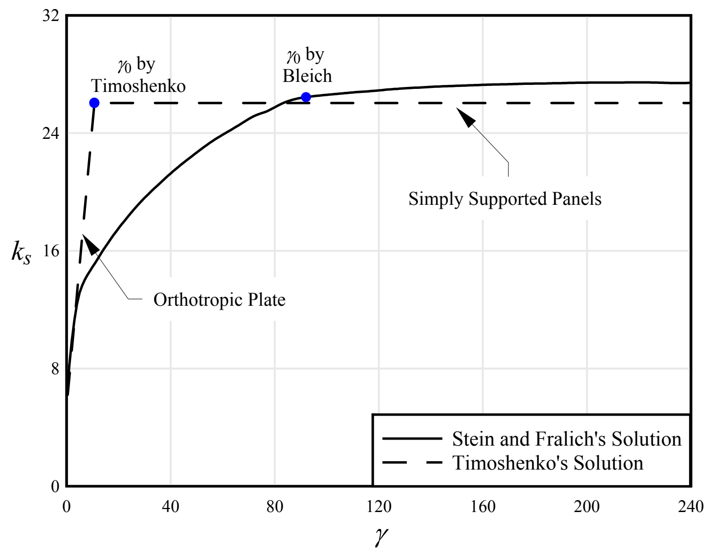


Fig. 6 Critical shear stress coefficient for plates with vertical stiffeners and subpanel aspect ratio of 2 (Stein and Fralich 1949)

To improve the previous research, the linear analysis of ABAQUS (2005) was used to compute the elastic shear buckling of the stiffened panels. Plates and stiffeners were modelled by the sufficient number of general purpose quadrilateral shell elements with reduced integration (ABAQUS element S4R). S4R has 4 nodes, with all 6 active degrees of freedom per node and therefore, the out-of-plane behavior of the stiffened plates was included in the models. The results are given in Fig. 7 for square or rectangular stiffened plates with subpanel aspect ratios of 1 to 5.

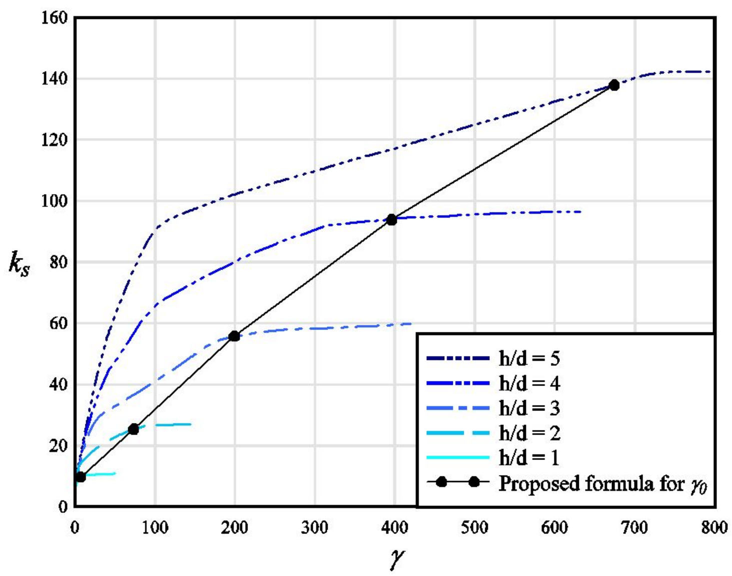


Fig. 7 Critical shear stress coefficient for plates with vertical stiffeners

Based on these curves, the succeeding formulas can be used for any subpanel aspect ratio:

- To estimate the limiting value of γ_0

$$\gamma_0 = 2\left(\frac{h}{d}\right)^3 + 17.5\left(\frac{h}{d}\right)^2 - 12 \quad \text{for } d \le h \tag{12}$$

or, in term of the effective moment of inertia of stiffeners.

$$I \ge \frac{t^3 d}{12\left(1-\nu^2\right)}\left\{2\left(\frac{h}{d}\right)^3 + 17.5\left(\frac{h}{d}\right)^2 - 12\right\} \quad \text{for } d \le h \tag{13}$$

To estimate the critical shear stress coefficient, k_s

$$k_s = \begin{cases} \left\{\left(\frac{h}{d}\right)^2\left(5.34 - \frac{4}{n^2}\right) - 1.34\right\}\frac{\gamma}{\gamma_0} + \frac{4}{n^2}\left(\frac{h}{d}\right)^2 + 5.34 & \gamma \le \gamma_0 \\ 5.34\left(\frac{h}{d}\right)^2 + 4 & \gamma \ge \gamma_0 \end{cases} \tag{14}$$

The difference between the FE results and the proposed formulas is shown in Fig. 8 for subpanel aspect ratio of 3. AISC Design Guide 20 (Sabelli and Bruneau 2007) recommends using the requirements of Chapter G of AISC 360 to evaluate required stiffness of vertical stiffeners, i.e.

$$I_{s,0} \ge dt^3 j \tag{15}$$

where, $j = 2.5(h/d)^2 - 2 \ge 0.5$.

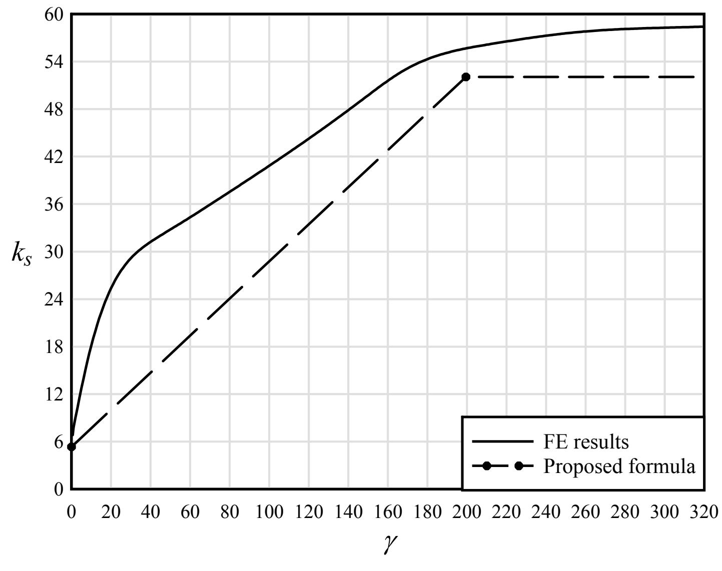
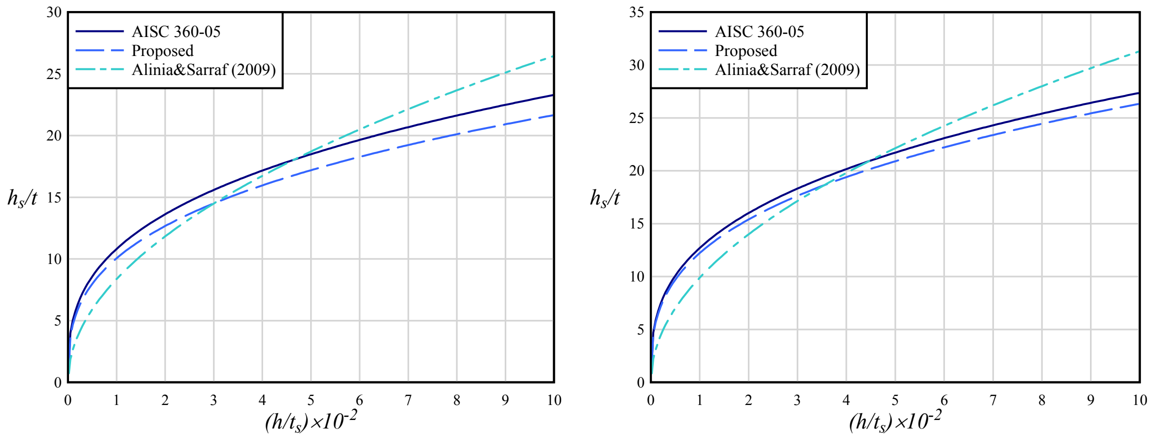


Fig. 8 Comparison of FE results and proposed formula for vertically stiffened plates with subpanel aspect ratio of 3



(a) Square plate with one-sided vertical stiffeners and subpanel aspect ratio of 2

(b) Square plate with one-sided vertical stiffeners and subpanel aspect ratio of 3

Fig. 9 Comparison of optimal stiffener dimensions

Comparison curves between the AISC 360 equation, the proposed formula by Alinia and Sarraf (2009), and Eq. (13) are given in Fig. 9 for subpanel aspect ratios of 2 and 3. The results indicate that Alinia and Sarraf's formula underestimates required stiffener height for thicker and overestimates it for thinner stiffeners. Being prone to fatigue and breathing of web plates were claimed to be causes of the difference between the proposed formula by Alinia and Sarraf and the AISC 360 equation for thicker stiffeners, while derivation of the AISC 360 equation was completely mathematical (Stein and Fralich 1949, Bleich 1952, Galambos 1998). It seems that selection of different parameters than γ, has resulted in such differences. However, comparison between the AISC 360 equation and Eq. (13) shows that the AISC 360 equation is, to some extent, conservative. Also, these two approaches are in good agreement, because of utilizing appropriate parameter, γ.

2.3 Buckling of plates reinforced by longitudinal and transverse stiffeners in shear

Fig. 4(b) depicts the general configuration of plates reinforced by longitudinal and transverse stiffeners in shear. By defining α_p as the panel aspect ratio and β as the subpanel aspect ratio, it can be found that when buckling occurs in subpanels, for a constant value of β, α_p does not have any influence on k_s (Fig. 10). On the other hand, it can be concluded that when subpanels are square, i.e., $b = d$, τ_{cr} reaches its maximum value from Eqs. (5)-(6).

By using the linear analysis of ABAQUS (2005) same as previous case, the following curves are obtained, and presented in Fig. 11, to evaluate k_s for practical values of h/d for plates reinforced by vertical and horizontal stiffeners of equal flexural rigidity with square subpanels. It should be noted that for the global buckling range of each curve, average values of k_s have been used. Based on these curves, the succeeding formulas can be used for any h/d ratio:

- To estimate the limiting value of γ_0,

$$\gamma_0 = -2\left(\frac{h}{d}\right)^2 + 48\left(\frac{h}{d}\right) - 40 \quad \text{for } d \le h \tag{16}$$

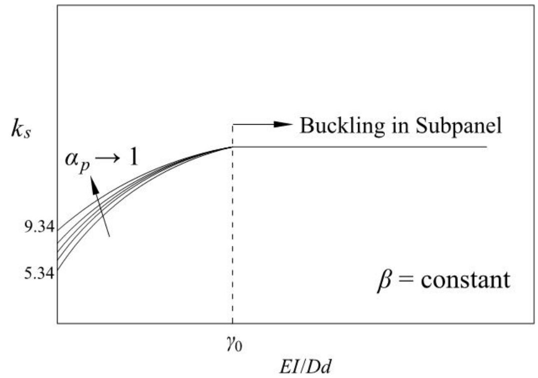


Fig. 10 Influence of changing panel aspect ratio on the critical shear stress coefficient for constant subpanel aspect ratio

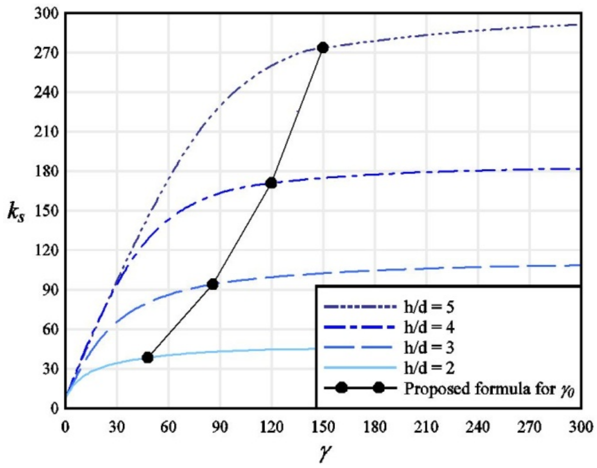


Fig. 11 Critical shear stress coefficient for plates with vertical/horizontal stiffeners and square subpanels

or, in term of the effective moment of inertia of stiffeners

$$I \geq \frac{t^3 d}{6\left(1-\nu^2\right)}\left\{-\left(\frac{h}{d}\right)^2+24\left(\frac{h}{d}\right)-20\right\} \quad \text{for } d \leq h \tag{17}$$

- To estimate the critical shear stress coefficient, k_s

$$k_s = \begin{cases} \left\{\left(\frac{h}{d}\right)^2\left(9.34-\frac{4}{n^2}\right)-5.34\right\}\frac{\gamma}{\gamma_0}+\frac{4}{n^2}\left(\frac{h}{d}\right)^2+5.34 & \gamma \leq \gamma_0 \\ 9.34\left(\frac{h}{d}\right)^2 & \gamma \geq \gamma_0 \end{cases} \tag{18}$$

The difference between the FE results and the proposed formulas is shown in Fig. 12 for h/d ratio of 4. Alinia and Sarraf Shirazi (2009) proposed the below formula to calculate the optimal

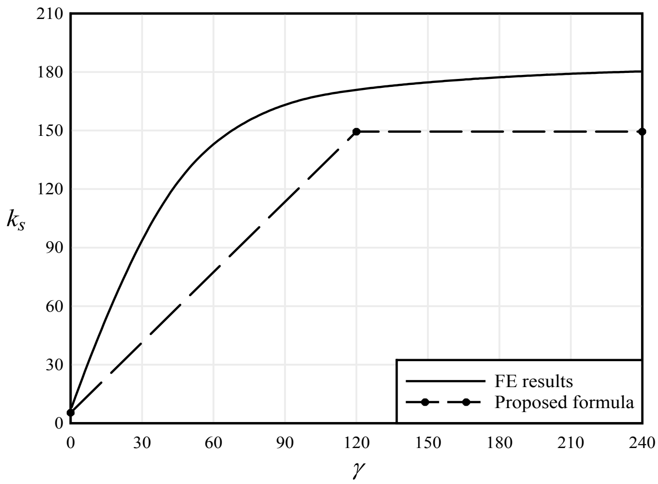


Fig. 12 Comparison of FE results and proposed formula for vertically and horizontally stiffened plate with square subpanel and *h*/*d* ratio of 4

dimensions of vertical and horizontal stiffeners placed on one side of a square infill plate

$$t_s h_s^{2.5} \geq \begin{cases} 1.8 t^{2.5} h & n = 1 \\ 1.8\left(1 - \dfrac{n}{10}\right) t^{2.5} h & n > 1 \end{cases} \tag{19}$$

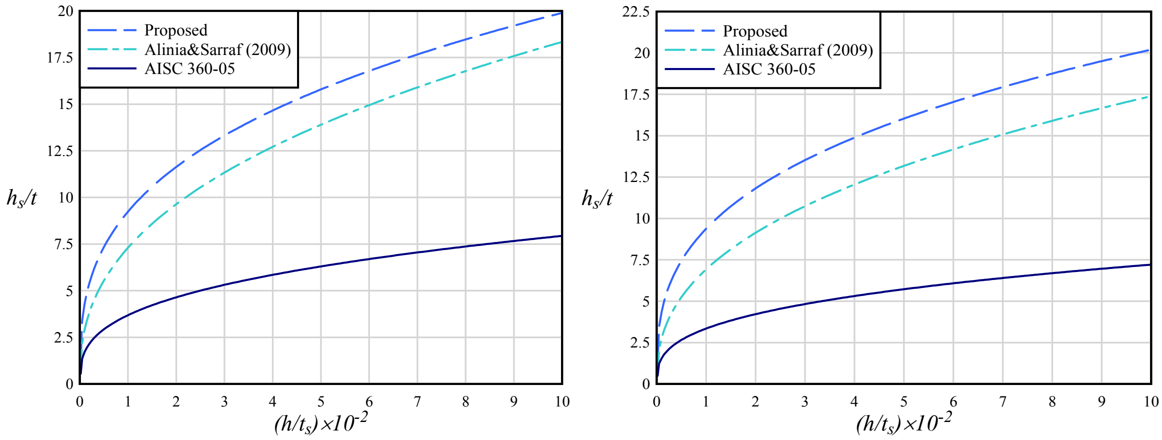
AISC Design Guide 20 (Sabelli and Bruneau 2007) suggests using subpanel aspect ratio, i.e., *b*/*d*, in the AISC 360 equation to evaluate required stiffness of transverse and longitudinal stiffeners. In other words

$$I \geq t^3 d \left\{ \frac{2.5}{\left(\dfrac{b}{d}\right)^2} - 2 \right\} \tag{20}$$

The differences between three approaches are shown in Fig. 13, for cases of square panels and subpanels with *h*/*d* ratios of 3 and 4. The curves imply that using subpanel aspect ratio in the AISC 360 equation leads to incorrect design of stiffeners. On the other hand, same as vertically stiffened case, Alinia and Sarraf's formula underestimate required stiffener height because of utilizing inappropriate parameters.

Unlike plate girders, the only goal for a stiffened SPSW is for the infill plate to have a shear yielding. Due to the existence of strong boundary elements, there is no persistence on stiffeners to remain straight up to the ultimate capacity of the infill plate. However, when a designer requires stiffener to remain straight up to the ultimate capacity, minimum stiffness of stiffener can be evaluated from Eqs. (13) and (17). For the infill plate to yield in shear, the following requirement should be met

$$\tau_{cr} = k_s \frac{\pi^2 E}{12(1-\upsilon^2)}\left(\frac{t}{h}\right)^2 \geq \tau_y \;\rightarrow\; k_s t^2 \geq 0.6388 \frac{h^2 \sigma_y}{E} \tag{21}$$



(a) Square plate reinforced by vertical and horizontal stiffeners with h/d ratio of 3

(b) Square plate reinforced by vertical and horizontal stiffeners with h/d ratio of 4

Fig. 13 Comparison of optimal stiffener dimensions

in which $\tau_y = \sigma_y/\sqrt{3}$ is the shear yield stress, σ_y the specified minimum yield stress, and k_s can be obtained from Fig. 7 and 11 or Eqs. (14) and (18), for different configurations of the stiffened SPSWs. It should be noted that the aforementioned curves and formulas for evaluating k_s are only valid for stiffeners with negligible torsional rigidity.

3. Demands on boundary elements

Based on the theory of incomplete diagonal tension, most practical web plates are acting, even near failure, in a state of stress, which is in between of the state of pure diagonal tension and the state of stress that exists before the web plate buckles (Kuhn *et al.* 1952). So, the applied shear stress τ can be divided into two parts: a pure shear part τ_s and a diagonal tension part τ_{DT}. On the other hand, if the applied shear stress τ is larger than τ_{cr}, the diagonal tension effects are expected to be produced merely by the excess stress $(\tau - \tau_{cr})$; therefore, the following equations are obtained

$$\tau = \tau_{DT} + \tau_s \tag{22}$$

$$\tau_{DT} = \tau - \tau_{cr} = \tau\left(1 - \frac{\tau_{cr}}{\tau}\right) = k\tau \tag{23}$$

$$k = 1 - \frac{\tau_{cr}}{\tau} \quad \text{for } \tau_{cr} \le \tau \tag{24}$$

$$\tau_s = (1-k)\tau \tag{25}$$

where k is diagonal tension factor which represents the degree to which the diagonal tension is developed. Provided $k = 0$, an unbuckled web plate with no diagonal tension is represented, while a value of $k = 1$ describes a web panel in pure diagonal tension.

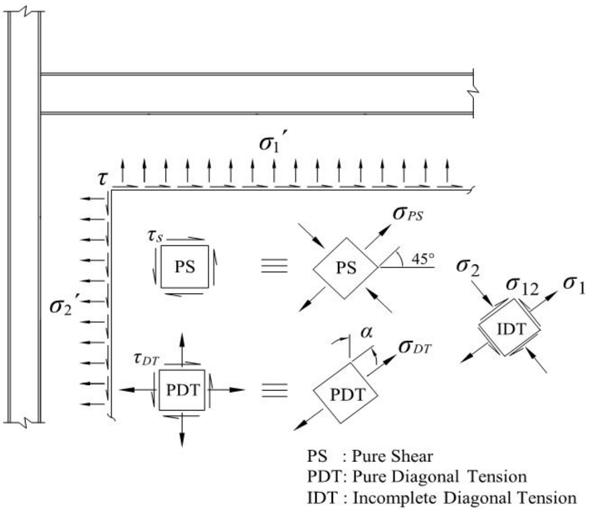


Fig. 14 Stress systems in infill plate and imposed stresses on boundary elements of a stiffened SPSW

The general case is obtained by superposing the two stress systems shown in Fig. 14, which gives for the stress σ_1 along the direction of diagonal tension α, the stress σ_2 perpendicular to this direction and shear stress σ_{12}, respectively.

$$\sigma_1 = \frac{2k\tau}{\sin 2\alpha} + \tau(1-k)\sin 2\alpha \tag{26}$$

$$\sigma_2 = -\tau(1-k)\sin 2\alpha \tag{27}$$

$$\sigma_{12} = -\tau(1-k)\cos 2\alpha \tag{28}$$

The transverse stresses applied to horizontal and vertical boundary elements can be presented as the following, respectively

$$\sigma_1' = k\tau \cot\alpha \tag{29}$$

$$\sigma_2' = k\tau \tan\alpha \tag{30}$$

Eqs. (29)-(30) are obtained by assuming that the boundary elements are strong enough not to produce significant non-uniformity of stress. With regard to the theory of incomplete diagonal tension, it can be understood that τ_{DT} and consequently the inward forces can be reduced by using stiffeners and increasing critical shear stress. Eqs. (29)-(30) state this fact as well.

It will be shown later that $\sin^2 2\alpha$ can be taken as unity. By using the plasticity hypothesis of Huber, von Mises and Hencky (so-called distortion energy), and Eqs. (26)-(28), the following relationship is obtained

$$\tau_u = \frac{R_y \sigma_y}{\sqrt{k_u^2 + 3}} \tag{31}$$

in which, τ_u is the ultimate shear capacity of stiffened panel, k_u is the diagonal tension factor in ultimate state and R_y is the ratio of the expected yield stress to the specified minimum yield stress, σ_y. For two extreme cases of the pure shear and the pure diagonal tension, τ_u can be obtained by the following, respectively

$$k_u = 0 \rightarrow \tau_u = \frac{R_y \sigma_y}{\sqrt{3}} \tag{32}$$

$$k_u = 1 \rightarrow \tau_u = \frac{R_y \sigma_y}{2} \tag{33}$$

The diagonal tension factor in ultimate state, k_u, can be expressed by

$$k_u = 1 - \frac{4}{1 + \sqrt{4\left(\frac{R_y \sigma_y}{\tau_{cr}}\right)^2 - 3}} \tag{34}$$

Finally, by considering strain hardening of the infill plate's material for capacity design of boundary elements, Eqs (29)-(30) become

$$\sigma'_{1,u} = \frac{c_{sh}\tau_{cr}}{4}\left(\sqrt{4\left(\frac{R_y \sigma_y}{\tau_{cr}}\right)^2 - 3} - 3\right)\cot\alpha \tag{35}$$

$$\sigma'_{2,u} = \frac{c_{sh}\tau_{cr}}{4}\left(\sqrt{4\left(\frac{R_y \sigma_y}{\tau_{cr}}\right)^2 - 3} - 3\right)\tan\alpha \tag{36}$$

in which $\sigma'_{1,u}$ and $\sigma'_{2,u}$ are exerted stresses on horizontal and vertical boundary elements in ultimate state, respectively, and c_{sh} stands for the strain hardening factor of the infill plate material.

4. The angle of diagonal tension field in the stiffened SPSWs

A derivation for the angle of diagonal tension field was first presented by Wagner (1931) for the limiting case of pure diagonal tension state in the web of plate girders. Due to the fact that pure diagonal tension state is only possible for very thin webs, the angle of tension field was improved by Kuhn *et al*. (1952) for the practical case of incomplete diagonal tension state. In the SPSW, the inclination of the tension field, α, is defined as the angle between the column and the line of action of the inclined tensile forces (Fig. 14). Using least work principle, the tension field angle in elastic region was derived by Thorburn *et al*. (1983) and improved later by Timler and Kulak (1983). The goal of the subsequent investigation is to develop a formula for the tension field angle in buckled stiffened SPSWs, i.e., when $\tau_{cr} < \tau_y$.

For a single story stiffened SPSW sub-assemblage shown in Fig. 4(b), the energy within the frame consist of contributions from the web, the stiffeners, one beam, and two columns. The work components of each will be evaluated separately and then summed to give the total internal work done by the sub-assemblage when subjected to the shear stress. Thus

$$W_{total} = W_w + \underset{Axial}{W_{s,v}} + \underset{Axial}{W_{s,h}} + \underset{Bending}{W_{s,v}} + \underset{Bending}{W_{s,h}} + W_{b,Axial} + W_{c,Axial} + W_{c,Bending} \tag{37}$$

The work done by the web can be evaluated from general expression for internal work, i.e.

$$W_w = Lht\int_0^{\varepsilon_i} \sigma_i d\varepsilon_i \tag{38}$$

in which, σ_i and ε_i are, respectively, stress and strain intensity. The stress intensity of the stress system shown in Fig. 14, is defined by

$$\sigma_i^2 = \sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 + 3\tau^2 = \frac{4k^2\tau^2}{\sin^2 2\alpha} + 3\tau^2\left(1-k^2\right) \tag{39}$$

The axial stress in each vertical and horizontal stiffener are expressed by, respectively

$$\sigma_{s,v} = \frac{\sigma_{DT} td\cos^2\alpha}{A_{s,v}} \tag{40}$$

$$\sigma_{s,h} = \frac{\sigma_{DT} tb\sin^2\alpha}{A_{s,h}} \tag{41}$$

where $\sigma_{DT} = 2k\tau / \sin 2\alpha$ is the stress along angle α due to the diagonal tension part of the applied stress, $A_{s,v}$ is cross-sectional area of vertical stiffener, and $A_{s,h}$ is cross-sectional area of horizontal stiffener. Substituting these values into the general expression for the internal work, results in the following equations for the work done by axial stresses in the vertical and horizontal stiffeners, respectively.

$$\underset{Axial}{W_{s,v}} = n\frac{\sigma_{s,v}^2}{2E}A_{s,v}h = \frac{nk^2\tau^2t^2d^2h}{2EA_{s,v}\sin^2\alpha} \tag{42}$$

$$\underset{Axial}{W_{s,h}} = m\frac{\sigma_{s,h}^2}{2E}A_{s,h}L = \frac{mk^2\tau^2t^2b^2L}{2EA_{s,h}\cos^2\alpha} \tag{43}$$

in which, n and m are, respectively, numbers of stiffeners in the longitudinal and transverse direction, and L is the bay width. The deflection of the n'th vertical stiffener and the m'th horizontal stiffener can be represented by Fourier sine series, respectively

$$\left(w_{s,v}\right)_n = \sum_{i=1}^{\infty} \Delta_{i,n}\sin\frac{i\pi y}{h} \tag{44}$$

$$\left(w_{s,h}\right)_m = \sum_{i=1}^{\infty} \Delta_{i,m}\sin\frac{i\pi x}{L} \tag{45}$$

where $\Delta_{i,n}$ and $\Delta_{i,m}$ are coefficients of deflection functions (Chen and Atsuta 2007). The expressions for these contributions to the total work are

$$W_{\substack{s,v\\Bending}} = \sum_{n=1}^{n} \int_0^h \left(\frac{M_{s,v}^2}{2EI_{s,v}} \right)_n dy \tag{46}$$

$$W_{\substack{s,h\\Bending}} = \sum_{m=1}^{m} \int_0^L \left(\frac{M_{s,h}^2}{2EI_{s,h}} \right)_m dx \tag{47}$$

in which, $M_{s,v} = \sigma_{s,v} A_{s,v} w_{s,v}$, is the moment distribution in the vertical stiffener, $M_{s,h} = \sigma_{s,h} A_{s,h} w_{s,h}$, is the moment distribution in the horizontal stiffener, $I_{s,v}$ is the moment of inertia of the vertical stiffener, $I_{s,h}$ the moment of inertia of the horizontal stiffener. Substituting Eqs. (44)-(45) into Eqs. (46)-(47), gives

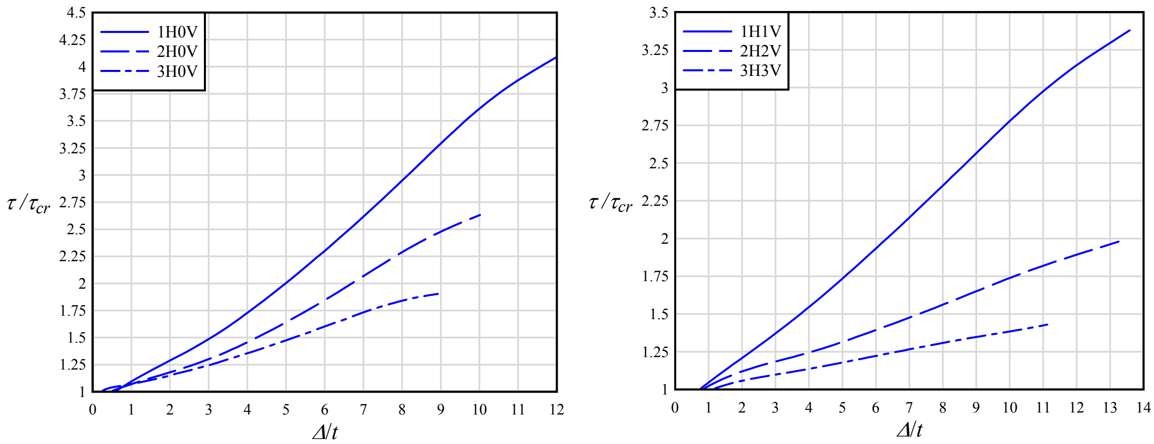
$$W_{\substack{s,v\\Bending}} = \frac{nk^2\tau^2t^2d^2h}{4EI_{s,v}} \cot^2\alpha \sum_{i=1}^{\infty} \Delta_{i,n}^2 \tag{48}$$

$$W_{\substack{s,h\\Bending}} = \frac{mk^2\tau^2t^2b^2L}{4EI_{s,h}} \tan^2\alpha \sum_{i=1}^{\infty} \Delta_{i,m}^2 \tag{49}$$

The coefficients $\Delta_{i,n}$ and $\Delta_{i,m}$ are function of τ/τ_{cr}, t and ratio of buckle half wavelength to effective compressive width of the infill plate (Rhodes 2003). The effects of τ/τ_{cr} on the out-of-plane displacement are shown in Fig. 15. It can be seen that by increasing τ/τ_{cr}, the out-of-plane displacement increases as well. Loughlan and Hussain (2014) reached the same conclusion through their detailed investigation on the out-of-plane behavior of the stiffened shear panel. However, for simplicity, based on curves in Fig. 15, the subsequent approximation will be used

$$\sum_{i=1}^{\infty} \Delta_{i,n}^2 = \sum_{i=1}^{\infty} \Delta_{i,m}^2 \approx (12t)^2 \tag{50}$$

Putting the above approximation into Eqs. (48)-(49) yields in



(a) Plate with n horizontal stiffeners (b) Plate with n horizontal and m vertical stiffeners

Fig. 15 Load-displacement curves in post-buckling range (Alinia and Sarraf Shirazi 2009)

$$W_{\substack{s,v\\Bending}} = \frac{36nk^2\tau^2t^4d^2h}{EI_{s,v}}\cot^2\alpha \tag{51}$$

$$W_{\substack{s,h\\Bending}} = \frac{36mk^2\tau^2t^4b^2L}{EI_{s,h}}\tan^2\alpha \tag{52}$$

Lastly, the contributions of the columns and the beam into the total work are considered same as Timler and Kulak (1983) by the following simplified expressions

$$W_{c,Axial} = \frac{V_{DT}^2 h}{4A_c E\tan^2\alpha} = \frac{k^2\tau^2t^2L^2h}{4A_c E\tan^2\alpha} \tag{53}$$

$$W_{c,Bending} = \frac{V_{DT}^2 h^5\tan^2\alpha}{720EI_cL^2} = \frac{k^2\tau^2t^2h^5\tan^2\alpha}{720EI_c} \tag{54}$$

$$W_{b,Axial} = \frac{V_{DT}^2 h^2\tan^2\alpha}{2A_bEL} = \frac{k^2\tau^2t^2Lh^2\tan^2\alpha}{2A_bE} \tag{55}$$

in which $V_{DT} = k\tau tL$ is the diagonal tension part of applied shearing force, I_c is the moment of inertia of the column, A_c is the cross-sectional area of the column, and A_b is the cross-sectional area of the beam. To find the critical value of α, the expression for the internal work of each component is differentiated with respect to α and the summation of them is set to zero. It is assumed that only the plate deforms plastically and the stiffeners and the boundary frame remain elastic. For the web plate

$$\frac{\partial W_w}{\partial\alpha} = \frac{\partial W_w}{\partial\sigma_i}\frac{\partial\sigma_i}{\partial\alpha} \tag{56}$$

Now, each part can be evaluated separately and then multiplied. Thus

$$\frac{\partial W_w}{\partial\sigma_i} = Lht\frac{\partial\varepsilon_i}{\partial\sigma_i}\frac{1}{\partial\varepsilon_i}\int_0^{\varepsilon_i}\sigma_i d\varepsilon_i = \frac{\sigma_i}{E_t}Lht \tag{57}$$

$$\frac{\partial\sigma_i}{\partial\alpha} = -\frac{8k^2\tau^2}{\sigma_i}\left(\frac{\cos 2\alpha}{\sin^3 2\alpha}\right) \tag{58}$$

where E_t is tangent modulus of the web plate. So

$$\frac{\partial W_w}{\partial\alpha} = -\frac{8k^2\tau^2Lht}{E_t}\left(\frac{\cos 2\alpha}{\sin^3 2\alpha}\right) \tag{59}$$

Differentiating of the other components is easy to obtain. Finally, by setting the resulted relationship for first derivative of the total work equal to zero and solving it for α, gives

$$\tan^4\alpha = \frac{1+\left(\dfrac{E_t}{E}\right)tL\left(\dfrac{1}{2A_c}+\dfrac{n}{(n+1)^2}\left\{\dfrac{1}{A_{s,v}}+\dfrac{72t^2}{I_{s,v}}\right\}\right)}{1+\left(\dfrac{E_t}{E}\right)th\left(\dfrac{h^3}{360I_cL}+\dfrac{1}{A_b}+\dfrac{m}{(m+1)^2}\left\{\dfrac{1}{A_{s,h}}+\dfrac{72t^2}{I_{s,h}}\right\}\right)} \tag{60}$$

In the case of thin SPSW, $n=m=0$, Eq. (60) yields in an equation which was originally derived by Timler and Kulak (1983) for the elastic behavior of the web plate and recently improved by Webster *et al*. (2014) for the inelastic behavior of the web plate. However, it is more general since it includes the effects of stiffeners as well. Eq. (60) implies that as the plate deforms plastically, the angle of inclination moves toward 45° because E_t/E ratio reaches zero.

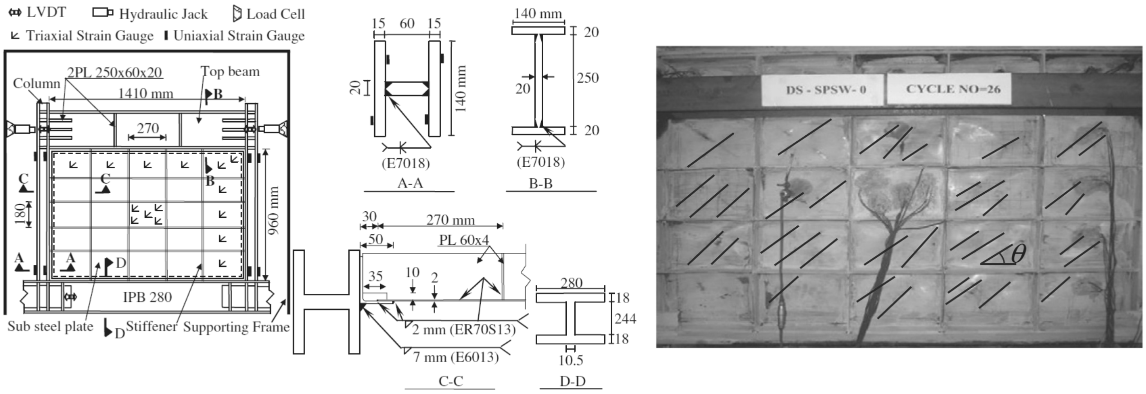
Values of the diagonal tension angle in the thin SPSWs commonly fall between 38° and 45° (Bruneau *et al*. 2011). In the stiffened SPSWs, based on Eq. (60), the diagonal tension angle is slightly larger, due to the fact that usually stiffeners are designed with the same dimensions, i.e., $A_{s,v}=A_{s,h}$ and $I_{s,v}=I_{s,h}$, and $n\geq m$. So, it is still convenient to assume $\sin^2 2\alpha \approx 1$.

To obtain angle α, AISC Design Guide 20 (Sabelli and Bruneau 2007) recommends that two angles should be considered, one of which is derived for the thin SPSWs from AISC 341-10 (2010) equation (17-2). The other one is an angle obtained based on the plate girder design. To better illustrate this procedure, the following equations define the diagonal tension angle

$$\tan\alpha = \begin{cases} \sqrt[4]{\dfrac{1+\dfrac{tL}{2A_c}}{1+th\left(\dfrac{1}{A_b}+\dfrac{h^3}{360I_cL}\right)}} & \text{Global buckling of the infill plate (a)} \\ or & \\ \dfrac{d}{b}\,\text{(Subpanel aspect ratio)} & \text{Buckling in subpanels (b)} \end{cases} \tag{61}$$

Then the angle, of which the resultant forces are greater, will be taken into consideration in the design process.

Despite many experimental investigations on the thin SPSW (Bruneau *et al*. 2011, Vatansever and Yardimci 2011, Webster *et al*. 2014), there are few tests on the stiffened SPSW (Takahashi *et al*. 1973, De Matteis *et al*. 2008, Sabouri-Ghomi and Asad Sajjadi 2012, Brando and De Matteis 2014, Sabouri-Ghomi and Mamazizi 2015). Among these, only Sabouri-Ghomi and Asad Sajjadi (2012) examined the diagonal tension angle as a part of their test. Configuration of the aforementioned specimen, called DS-SPSW-0%, is shown in Fig. 16(a). The post buckling tension field of this specimen is shown in Fig. 16(b). It was reported that the angle values of the diagonal tensions were 36° to 55° (Sabouri-Ghomi and Asad Sajjadi 2012). Eq. (60) gives values of α between 43.4° to 45° for this specimen. Eqs. (61a)-(61b) result in 39.8° and 56.3°, respectively. Comparison between test and theoretical result shows good agreement of Eq. (60) with test. Also, it can be concluded that neglecting the effect of stiffeners on the diagonal tension angle (using Eq.



(a) (b)

Fig. 16 (a) Detailed description; and (b) diagonal tension field of DS-SPSW-0% specimen (Sabouri-Ghomi and Asad Sajjadi 2012)

(61a)) may be small for ultimate shear capacity, but, based on Eqs. (35)-(36), it results in decrease and increase in flexural demand of the vertical and horizontal boundary elements, respectively. Inversely, utilizing Eq. (61b), leads to increase and decrease in the aforesaid demands, respectively.

5. Finite element analysis

In the last two decades, FE analysis comprising both material and geometrical nonlinearities has been extensively used to investigate behavior of SPSWs. Utilizing FE method has several advantages, one of which is that out-of-plane buckling of the plate can be modeled explicitly.

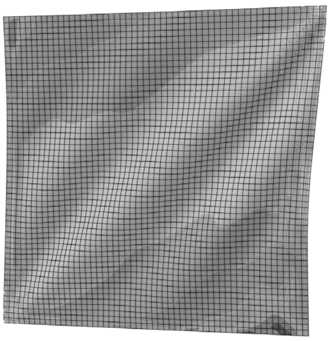
To illustrate the influence of stiffeners on the behavior of a SPSW, a lab specimen of SPSW2 used in Lubell (1997) was studied as a benchmark specimen. The specimen was a 30% scale model of an inner residential building core, with the floor-to-floor and column-to-column distance equal to 900 mm and with the infill panel comprised of a plate with 1.5 mm thickness. Its frame elements were constructed of Canadian S75×8 sections, while a double section was used for the upper beam. The test started by cyclic loading of the specimen according to ATC-24 (1992), and then stopped after three cycles because a progressive fracture in the sample column was observed.

The role of boundary elements in the SPSW is to anchor the produced diagonal tensions. Thus if the boundary elements are not strong enough, the deformation they experience will be significant, which leads to the release of tensile stresses in the infill plate. As a result, the infill plate will not be able to reach its ultimate capacity. In such case, deformation of the boundary element is called the "hourglass shape" (Lubell 1997). The previously mentioned specimen was analyzed by using ABAQUS (2005). The shear wall infill plate was modeled using 4-noded doubly curved shell elements with reduced integration (ABAQUS element S4R5), and boundary elements were modeled using 3-noded quadratic beam elements (ABAQUS element B32). The material stress-strain behavior determined from the coupon tests was used with bilinear representation in the FE model. The entire model was divided into a sufficient number of elements with 15×15 mm mesh size to adequately represent the deformations and stress gradients. The FE model was analyzed and calibrated by introducing an initial imperfection, which was obtained from a linear buckling mode, and using the arc-length method (ABAQUS Riks method). Despite the load control and displacement control methods, the arc-length method is useful in bifurcation buckling problem whose secondary branches in the equilibrium path must be traced. In the load control method, the load is kept constant during a load step and in the displacement control method, displacement is kept constant during an increment. Unlike these methods, the arc-length method uses an additional unknown in equilibrium equations, known as the load-factor, which at each iteration is modified so that the solution follows some specified path until convergence is achieved.

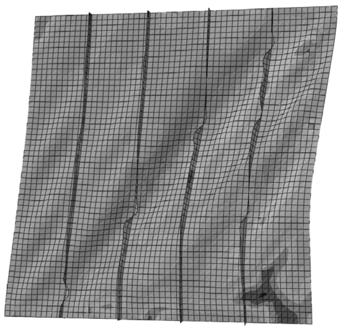
Afterwards, the FE model was stiffened by adding eight vertical stiffeners with a 25×1 mm^2 cross section, which were placed in each side of the panel, with a 165 mm distance between them. The same elements which were used for the infill plate were utilized for the stiffeners as shown in Fig. 17.

As is illustrated in Fig. 18, by adding stiffeners, the initial stiffness is increased. On the other hand, as the diagonal tension factor is reduced and the inward forces are decreased, the displacement capacity and energy absorption of the system are increased as well.

In order to verify the accuracy of the obtained equations to estimate demands on the boundary elements, three different cases of Takahashi *et al*. (1973) were modeled and analyzed using ABAQUS (Fig. 19). The results are in good agreement with the theoretical values, as shown in Fig. 19.



(a)



(b)

Fig. 17 (a) FE model of SPSW2; (b) FE model of stiffened SPSW2

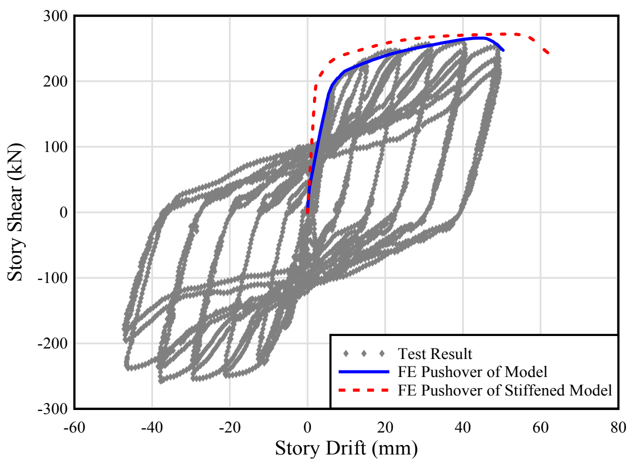
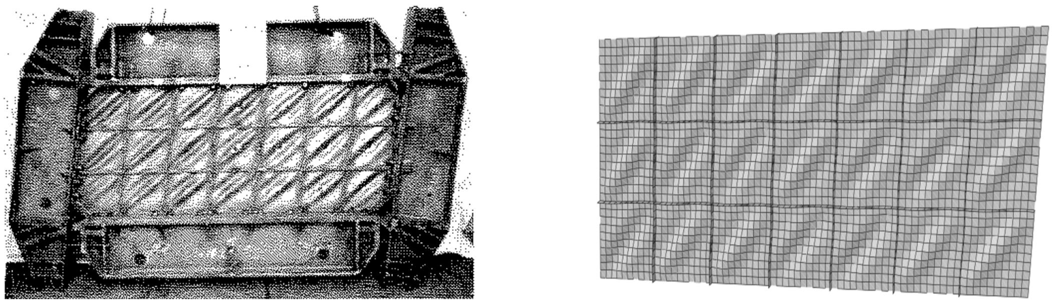


Fig. 18 Comparison of test results with FE pushover analysis results



(a) (b)

Fig. 19 (a) Specimen PR-2.3-M2-60 at the end of test (Takahashi *et al.* 1973); (b) FE model of PR-2.3-M2-60 at the end of analysis

Table 1 Specimen configurations and material specifications

Case	Plate dimensions (mm)			σ_y (MPa)	Stiffeners	b (mm)	d (mm)	I (mm^4)
	L	h	t					
Stiffened SPSW2	825	825	1.5	320	Vertical arrangement on both sides	-	165	10416.7
PR-2.3-M2-60	2100	900	2.3	310	Vertical and horizontal arrangement on both sides	300	300	81000
PR-3.2-M2-40	2100	900	3.2	280	Vertical and horizontal arrangement on both sides	300	300	17066.7
PR-3.2-M2-60	2100	900	3.2	232	Vertical and horizontal arrangement on both sides	300	300	81000

Table 2 Stresses applied to the boundary element, based on the obtained equations and FEA

Case	h/d	γ	Buckling mode	k_s	$k_{s,\,orth}$ [a]	τ_{cr} (MPa)	$(\tau_{cr})_{FE}$ (MPa)	k_u	τ_u (MPa)	α (deg)	Demands on boundary elements (MPa)			
											σ'_1	$\sigma'_{1,FE}$ [b]	σ'_2	$\sigma'_{2,FE}$ [b]
Stiffened SPSW2	5	204.3	Global	102.2	168.1	61.6	61.36	0.677	189.3	49.2	126.8	104.9	170.2	126.4
PR-2.3-M2-60	3	242.3	Local	107.1	875.9	126.44	132.73	0.345	193.1	45	108.3	93.18	108.3	98.15
PR-3.2-M2-40	3	18.96	Global	49.94	79.84	114.1	139.4	0.346	174.4	45	96.43	13.54	96.43	49.84
PR-3.2-M2-60	3	90	Local	94.58	336.7	216.1	279.6	0	140.8	-	0	3.73	0	2.77

[a] Using orthotropic plate solution.
A summary of orthotropic plate solution can be found in Ventsel and Krauthammer (2001).
[b] Average values

6. Conclusions

In this study, the characteristics of stiffeners on the steel plate shear wall are investigated. By stiffening a thin SPSW, its critical shear stress increases. The increase in the buckling capacity of the SPSW depends on the rigidity and arrangement of stiffeners. The use of orthotropic plate solution to calculate the buckling stress of stiffened plates in shear may be considered without error for only a limited number of cases. Curves and simplified formulas have been introduced to estimate the critical shear stress coefficient for various practical configurations of stiffeners. Also, a general formula for the angle at which the tension field stresses act was established. Having used the theory of incomplete diagonal tension, equations were obtained to evaluate the ultimate shear capacity of stiffened SPSWs and the inward forces exerted on the boundary elements. It has been shown that the application of stiffener results in increasing initial stiffness, ultimate strength and energy absorption of the system, and reducing the inward forces applied to the boundary elements. Such decrease in demands leads to an easier design of boundary elements and prevents the occurrence of inappropriate states such as substantial hour glass shape of system.

In contrast to plate girders, due to the existence of strong boundary elements in SPSWs, stiffeners can be stripped of the task of anchoring diagonal tensions. So, by neglecting the limiting value of γ_0, the desired buckling strength can be provided, using lighter stiffeners. Since buckling of plates is a stability problem, using low-yield-strength (LYS) steel, instead of mild steel, reduces the need for stiffeners and the diagonal tension's portion in lateral load resistance. This leads to reducing demands on the boundary elements. However, more studies should be conducted with regard to other configurations of stiffeners and the effects of using LYS steel in stiffened SPSWs.

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Appendix A

The general equation for the critical buckling stress of a rectangular plate under shear stress is

$$\tau_{cr} = k_s \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{b}\right)^2 \tag{A1}$$

For a stiffened SPSW, if buckling occurs in the subpanels, the critical shear stress coefficient can be obtained from the following Eq. (Timoshenko 1936)

$$k_s = 5.34 + 4\left(\frac{b}{d}\right)^2 \qquad \text{for } b \le d \tag{A2}$$

By substituting Eq. (A1) into Eq. (A2), one has

$$\tau_{cr} = \frac{\pi^2 E t^2}{12(1-\nu^2)}\left(\frac{5.34}{b^2} + \frac{4}{d^2}\right) \tag{A3}$$

For the infill plate to yield in shear, the below requirement should be met

$$\tau_{cr} \ge \tau_y = \frac{\sigma_y}{\sqrt{3}} \tag{A4}$$

By assuming that the infill plate buckles in subpanel, minimum required plate thickness can be obtained by putting Eq. (A3) into Eq. (A4), i.e.

$$t \ge \sqrt{\frac{12(1-\nu^2)\dfrac{\sigma_y}{\sqrt{3}}}{\pi^2 E\left(\dfrac{5.34}{b^2} + \dfrac{4}{d^2}\right)}} \tag{A5}$$

Where the spacing of stiffeners is equal in each direction, substituting $b = d$ into Eq. (A5) gives limiting web slenderness below which shear buckling is prevented, i.e.

$$\frac{b}{t} \le 3.824\sqrt{\frac{E}{\sigma_y}} \tag{A6}$$

Where stiffeners are used in one direction, putting $d \to \infty$ into Eq. (A5) gives the previous limiting slenderness by the following

$$\frac{b}{t} \le 2.89\sqrt{\frac{E}{\sigma_y}} \tag{A7}$$

Eqs. (A5)-(A7) are used in section 3.2.2 of AISC Design Guide 20 (Sabelli and Bruneau 2007).

Appendix B

The suggested equation by Bleich (1952) is

$$\gamma_0 = 4\left\{7\left(\frac{h}{d}\right)^2 - 5\right\} \quad \text{for } d \le h \tag{B1}$$

So, for stiffener to not buckle with plate the following requirement should be satisfied

$$\gamma \ge \gamma_0 = 4\left\{7\left(\frac{h}{d}\right)^2 - 5\right\} \tag{B2}$$

By substituting $\gamma_0 = 12(1-v^2)I_{s,0}/t^3 d$ and $v = 0.3$ into the previous equation, one gets

$$I_{s,0} \ge t^3 d\left\{2.564\left(\frac{h}{d}\right)^2 - 1.83\right\} \tag{B3}$$

or in slightly modified form

$$I_{s,0} \ge t^3 d\left\{\frac{2.5}{\left(\frac{d}{h}\right)^2} - 2\right\} \tag{B4}$$

and

$$d \le h \rightarrow j = \frac{2.5}{\left(\frac{d}{h}\right)^2} - 2 \ge 0.5 \tag{B5}$$

Combining Eqs. (B4)-(B5), gives

$$I_{s,0} \ge dt^3 j \tag{B6}$$

where, $j = 2.5(h/d)^2 - 2 \ge 0.5$.

This formula is used in section G.2.2 of AISC 360 (AISC 360-05 2005) to calculate the required transverse stiffness of stiffeners.