Steel and Composite Structures, Vol. 2, No. 6 (2002) 429-440
DOI: http://dx.doi.org/10.12989/scs.2002.2.6.429

Finite strip method in probabilistic fatigue analysis of steel bridges

W.C. Li† and M.S. Cheung‡

Department of Civil & Environmental Engineering, Carleton University, Ottawa, Ontario, Canada K1S 5B6

(Received January 28, 2002, Accepted November 6, 2002)

Abstract. A finite strip method is developed for fatigue reliability analysis of steel highway bridges. Flat shell strips are employed to model concrete slab and steel girders, while a connection strip is formed using penalty function method to take into account eccentricity of girder top flange. At each sampling point with given slab thickness and modulus ratio, a finite strip analysis of the bridge under fatigue truck is performed to calculate stress ranges at fatigue-prone detail, and fatigue failure probability is evaluated following the AASHTO approach or the LEFM approach. After the failure probability is integrated over all sampling points, fatigue reliability of the bridge is determined.

Key words: finite strip method; reliability; fatigue; fracture; AASHTO approach; LEFM approach.

1. Introduction

It has been reported that 80 to 90 percent of failures in steel structures are related to fatigue and fracture (Committee, 1982). Therefore, fatigue and fracture reliability evaluation is a very important task in design and management of steel bridges.

In order to accurately calculate stress ranges at fatigue-prone structural details, refined stress analysis is preferable for complex bridge structures. In the last decade, the finite strip method has proved to be one of the most efficient methods of bridge analysis and has received wide acceptance and recognition by the engineering community (Cheung *et al*. 1996). The technique has matured to such an extent that a considerable number of design codes in the world now specifically recognize the finite strip method as one of the acceptable tools for bridge analysis. For straight or circularly curved highway bridges with a constant cross section, the finite strip method uses a series of beam eigenfunctions to describe the longitudinal profile of displacement components. Therefore, the dimensions of the analysis are reduced at least by one. Consequently, the cost of analysis is reduced significantly not only due to the reduction in the required computer time and storage space, but also due to the substantial simplification of input data preparation and results interpretation. Combined with the flexibility approach, the finite strip method is also applicable to the bridges with intermediate supports and transverse stiffeners. For more complicated bridge geometry, such as arbitrarily curved slab bridges, haunched continuous slab-on-

†Research Associate
‡Adjunct Professor

girder or box-girder bridges, and cable-stayed bridges, the spline finite strip method (Cheung *et al.* 1996) has been developed, and considerable computational saving is also achieved due to a significant reduction in the number of required degrees of freedom. The advantage of the finite strip method becomes more remarkable for reliability analysis since hundreds, thousands or even millions similar stress analyses may be required to reach a satisfactory accuracy for reliability assessment of a complex structure.

In the present study, a finite strip method is developed for the fatigue reliability analysis of steel highway bridges. The flat shell strips are employed to model the concrete slab and steel girders, while a connection strip is formed by means of the penalty function method in order to take into account the eccentricity of the top flange of girder. The effects of loading are modeled by the fatigue truck developed by Laman and Nowak (1996). The slab thickness, the ratio between elasticity moduli of steel and concrete, the fatigue or fracture parameters of fatigue-sensitive detail, and the impact factor are taken as the basic variables. At each sampling point with given values of slab thickness and modulus ratio, a finite strip analysis of the bridge under fatigue truck is carried out, and the stress ranges at the structural detail are calculated. Based on these stress ranges and intended service life, the fatigue failure probability can be evaluated by following the AASHTO approach or the Linear-Elastic Fracture Mechanics (LEFM) approach. After the failure probability is integrated over all sampling points, the fatigue reliability of the bridge is determined.

Details are described in the following sections. Numerical examples are presented to illustrate the proposed methodology.

2. Finite strip method

In the present study, the flat shell strip (Fig. 1) is employed to model slab and girders of a highway bridge. For a straight bridge simply supported at both ends, the displacement parameters of the strip for the m-th series term are taken as:

$$\{\delta\}_m = [u_{im}\ v_{im}\ w_{im}\ \theta_{im}\ u_{jm}\ v_{jm}\ w_{jm}\ \theta_{jm}]^T$$

where u, v, w and θ are defined in Fig. 1.

In the local coordinate system, the displacement components at any point within the middle plane of the strip are expressed in terms of the displacement parameters as follows:

$$u = \sum_{m=1}^{r} [(1-X)u_{im} + Xu_{jm}]\sin k_m y$$

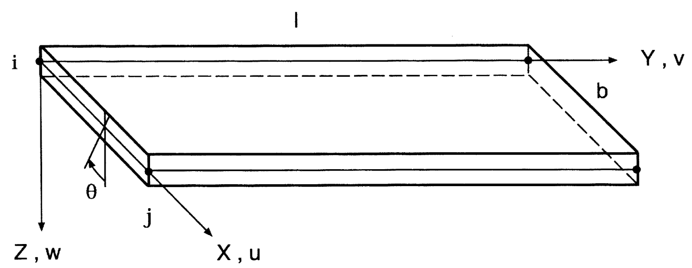

Fig. 1 Flat shell strip

$$v = \sum_{m=1}^{r} [(1-X)v_{im} + Xv_{jm}]\cos k_m y$$

$$w = \sum_{m=1}^{r} [(1-3X^2+2X^3)w_{im} + x(1-2X+X^2)\theta_{im} + (3X^2-2X^3)w_{jm} + x(X^2-X)\theta_{jm}]\sin k_m y \quad (1)$$

where $X = x/b$ and b is the width of the strip, r is the total number of series terms considered in the analysis, $k_m = m\pi/l$ and l is the length of the strip.

Based on these shape functions and the principle of minimum total potential energy, the stiffness matrix and load vector of the strip are formed and have been listed in the book written by Cheung *et al.* (1996).

In order to take into account the eccentricity of top flange of girder, a connection strip is developed, which connects nodal line i in the middle plane of slab and nodal line j in the middle plane of top flange, as shown in Fig. 2.

At any cross-section with coordinate y, the displacements of nodal lines i and j have the following relationships:

$$u_j = u_i - e\theta_i$$

$$v_j = v_i - e\frac{dw_i}{dy}$$

$$w_j = w_i$$

$$\theta_j = \theta_i$$

where e is the eccentricity.

By substitution of Eq. (1) into above relationships, the following matrix equation can be obtained:

$$[B]_m\{\delta\}_m = 0$$

where

$$[B]_m = \begin{bmatrix} 1 & 0 & 0 & -e & -1 & 0 & 0 & 0 \\ 0 & 1 & -ek_m & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix}$$

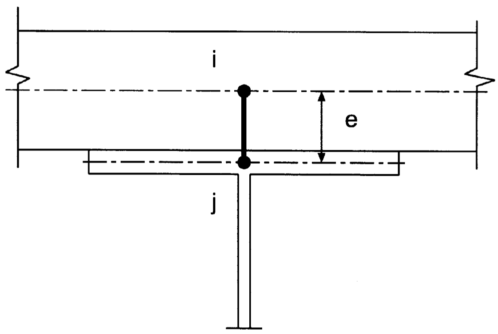

Fig. 2 Connection strip

Thus, the stiffness matrix of the connection strip can be derived using the penalty function method (Cheung *et al.* 1996) as:

$$[K]_m = \alpha[B]_m^T[B]_m$$

in which α is a large stiffness coefficient. In the present study, α is taken as 1000 times the maximum diagonal item of structural stiffness matrix for series term m.

For applications of the finite strip method to bridges with more complicated geometry, it is recommended to consult the book written by Cheung *et al.* (1996).

3. Failure probability under given stress ranges

Two approaches are available for evaluation of fatigue failure probability of a fatigue-prone detail under given stress ranges in the intended service life, namely the AASHTO approach and the Linear-Elastic Fracture Mechanics (LEFM) approach (Zhao *et al.* 1994). The AASHTO approach is based on *S-N* design curves included in design code, and is easy to use for structural engineers. Using this approach, the fatigue reliability of an aging steel bridge can be evaluated analytically, as long as the loading history is available in terms of stress ranges in the critical fatigue-sensitive detail. On the other hand, the LEFM approach is based on the crack propagation theory in the linear-elastic fracture mechanics. If the initial crack sizes can be measured with reasonable accuracy, this approach leads to a more accurate evaluation for the fatigue reliability of a given structural detail. In addition, the LEFM approach is also capable of updating the reliability assessment of an aging structure according to crack sizes measured during inspections.

3.1. AASHTO approach

The AASHTO approach employs the fatigue design curves in AASHTO Specifications (1994) to evaluate the fatigue failure probability. The relationships between allowable stress ranges and number of cycles in these curves are obtained from test results and used for fatigue design of highway bridges. Based on results of fatigue experiments, Wirsching (1995) suggested that the fatigue failure occurs if the following limit state is reached:

$$\sum_{i=1}^{k} \frac{S_i^m N_i}{A} = \Delta \tag{2}$$

where N_i is the number of loading cycles at stress range level S_i during intended service life, A is the fatigue strength coefficient of the structural detail, whilst Δ is the damage index at failure.

Extensive studies by Wirsching have shown that the cycles-to-failure data can be best fit if A and Δ are both modeled by lognormal variables and m is assumed as a constant.

The AASHTO Specifications have given design values of A and design curves in $\log N$ - $\log S$ scales for a variety of structural details. Typically, the design curve is two standard deviations below the median curve for a particular structural detail. Therefore, the median value $\tilde{A}$ can be calculated from the design value A_0 as below:

$$\ln\tilde{A} = \ln A_0 + 2\sigma_{lnA}$$

where $\sigma_{\ln A}$ is the standard deviation of $\ln A$ and equal to $\sqrt{\ln(1+V_A^2)}$ with V_A denoting the coefficient of variation of A. Thus

$$\tilde{A} = A_0 \exp[2\sqrt{\ln(1+V_A^2)}]$$

Moreover, if the stress range S_i is calculated using a static analysis, the impact factor I should be included to take into account the dynamic effects of moving vehicles as below:

$$S_i = IS_{oi}$$

where S_{oi} is the stress range obtained from the static analysis. The impact factor I in turn can be modeled by a lognormal variable (Wirsching 1995).

Since all the random variables in Eq. (2) are lognormally distributed, it is ready to obtain the fatigue failure probability P_{fs} of a structural detail for a given stress ranges as:

$$P_{fs} = \Phi(-\beta_s) \tag{3}$$

where $\Phi(\ )$ is the standard normal cumulative distribution function, and

$$\beta_s = \frac{\ln\left[(\tilde{\Delta}\tilde{A})\Big/\left(\sum_{i=1}^{k}\tilde{I}^m S_{oi}^m N_i\right)\right]}{\sqrt{\ln[(1+V_\Delta^2)(1+V_A^2)(1+V_I^2)^{m^2}]}} \tag{4}$$

with V denoting the coefficient of variation and the tilde representing the median for each variable.

3.2. LEFM approach

The Linear-Elastic Fracture Mechanics (LEFM) approach is based on a crack propagation theory (Harris 1995). Since the effect of crack size is taken into consideration, this approach yields more accurate results for fatigue reliability assessment if the current crack size can be measured.

In welded bridge details, the welding process inherently results in initial flaws from which crack growth may occur under cyclic loading. The most commonly used crack growth model is the well-known Paris relation:

$$\frac{da}{dN} = C(\Delta K)^m \tag{5}$$

where a is crack size, which refers to the crack depth for the surface crack, or the half of crack length for through crack, N is the number of stress cycles, C and m are crack growth parameters, which are material properties, and ΔK is the stress intensity range, which is evaluated as

$$\Delta K = K_{\max} - K_{\min} = F(a, Y)S\sqrt{\pi a}$$

in which S is the stress range, $F(a, Y)$ is the geometry function to account for the possible stress concentration, Y is a vector of geometrical parameters, such as the dimensions of crack and specimen under consideration (Murakami 1987).

Due to scatter in test data, crack growth parameters C and m, as material properties, should be considered as random variables. It is generally accepted that, for steel, the crack growth parameter C approximately follows a lognormal distribution while the crack growth exponent m is assumed to be a constant (Harris 1995).

When the crack grows from its initial dimension a_0 to its critical size a_c, fatigue failure occurs (Fisher 1984). The limit state equation can be obtained by rearranging items in Eq. (5) and then integrating both sides from a_0 to a_c:

$$\int_{a_0}^{a_c} \frac{da}{[F(a, Y)\sqrt{\pi a}]^m} = C\sum_{i=1}^{k} S_i^m N_i \tag{6}$$

Due to considerable uncertainties in material properties and welding process, also due to limited crack detectability of current inspection techniques, crack sizes a_0 and a_c are both random valuables. As indicated by Harris (1995), the uncertainty in initial crack dimension a_0 has a dominant effect on the results of fatigue reliability assessment, and it is suggested that as a reasonable approximation, a_0 can be modeled by a lognormal variable, while a_c can be considered as a constant. For very important structures, the distribution parameters can be determined through measurements, or can be backed out from fatigue test data.

Similarly to AASHTO approach, a lognormal variable, impact factor I, should be introduced if the stress range S_i is determined by a static analysis.

For each sampling value of a_0, the integration on the left-hand side of Eq. (6) can be performed numerically, and the fatigue failure probability for the given stress ranges can be evaluated in the way similar to AASHTO approach:

$$P_{fs} = \Phi(-\beta_s) \tag{7}$$

in which

$$\beta_s = \frac{\ln\left[\int_{a_0}^{a_c} \frac{da}{[F(a, Y)\sqrt{\pi a}]^m} / \tilde{C}\sum_{i=1}^{k} \tilde{I}^m S_{oi}^m N_i\right]}{\sqrt{\ln[(1+V_C^2)(1+V_I^2)^{m^2}]}} \tag{8}$$

where V denotes the coefficient of variation, while the tilde represents the median for each variable.

4. Solution procedure

In addition to the aforementioned random variables, which determine the fatigue failure probability of a structural detail under given stress ranges, the structural geometry, material properties and loading etc. also vary from case to case. The variability in these structural and loading parameters affects the results of stress calculation, and must be considered before the stress analysis is performed. In the present study, the thickness of concrete slab, t_c, the elasticity modulus of concrete, E_c, and the elasticity modulus of steel, E_s, are treated as random variables. The uncertainties in other parameters are neglected because of their secondary importance. However, their randomness can be easily incorporated in the same manner if any concern is raised. In general, fatigue is caused by repeatedly applied stresses, under which only elastic deformation occurs in a properly designed bridge. For elastic stress analysis, it is the modulus ratio $\rho = E_s / E_c$ instead of E_s and E_c themselves that affects the resulting stresses. Therefore, only two basic variables are considered here, namely the slab thickness t_c and the modulus ratio ρ. If E_s and E_c both have lognormal distribution, the modulus ratio ρ also follows the lognormal distribution with the following probabilistic parameters:

$$\tilde{\rho} = \tilde{E}_s/\tilde{E}_c$$

$$\sigma_{\ln\rho} = \sqrt{\ln[(1+V_{E_s}^2)(1+V_{E_c}^2)]}$$

in which V denotes the coefficient of variation, the tilde represents the median, and σ stands for the standard deviation.

A simulation technique (Ayyub *et al.* 1995) is used for the final solution. At each sampling point with given values of t_c and ρ, the finite strip analysis is carried out for the considered bridge under fatigue truck developed by Laman and Nowak (1996). Thus, the stress ranges at the fatigue-prone detail are calculated. If AASHTO approach is employed, the fatigue failure probability, P_{fs}, under these given stress ranges is determined using Eq. (3) and Eq. (4) for intended service life. Integrating numerically over all sampling points yields the fatigue failure probability of the structural detail as:

$$P_F = \iint P_{fs} f_{t_c}(t_c) f_\rho(\rho) dt_c d\rho \tag{9}$$

where f_{t_c} and f_ρ are the probability density functions of t_c and ρ, respectively.

If the LEFM approach is applied, an additional sampling over the initial crack dimension a_0 is required after the stress ranges are determined. Thus, Eq. (9) becomes:

$$P_F = \iiint [P_{fs} f_{a_0}(a_0) da_0] f_{t_c}(t_c) f_\rho(\rho) dt_c d\rho \tag{10}$$

in which P_{fs} is evaluated by means of Eq. (7) and Eq. (8), while $f_{a_0}(a_0)$ denotes the probability density function of a_0.

Then, the reliability index can be calculated as:

$$\beta = -\Phi^{-1}(P_F)$$

where $\Phi^{-1}(\,)$ is the inverse of standard normal cumulative distribution.

It can be noticed that this solution procedure is a combination of analytical solution and simulation technique. In addition, the probability density functions are explicitly calculated at each sampling point, so that the required sampling points are reduced significantly in comparison with the Monte-Carlo method, leading to an enhanced efficiency.

5. Numerical examples

5.1. Bridge girders of weathering steel without painting, AASHTO approach

Fatigue reliability of a typical composite concrete-slab on steel-girder bridge is evaluated following

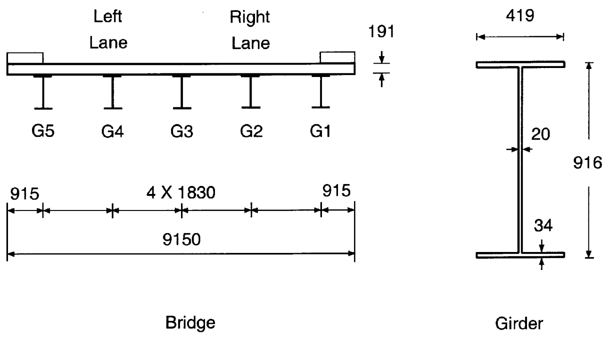

Fig. 3 Bridge with weathering steel-girders (mm)

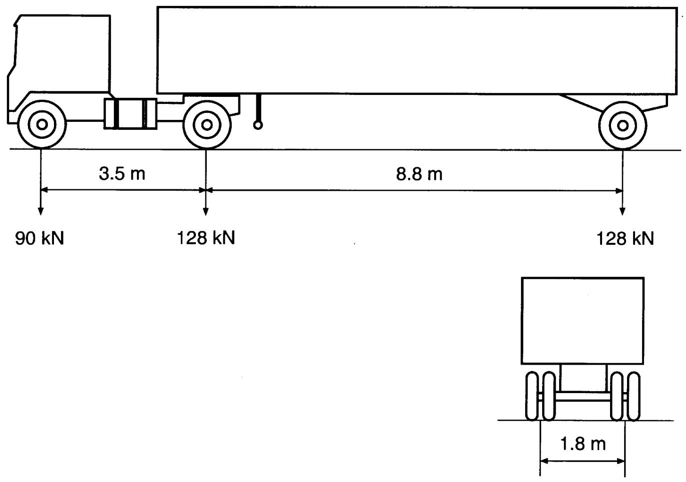

Fig. 4 Fatigue truck

the AASHTO approach. The bridge has a simple span of 24.0 m and two lanes with traffic in the same direction. The cross section is shown in Fig. 3. Five steel girders are made of weathering steel without painting. The loading effects are modeled by the fatigue truck, developed by Laman and Nowak as shown in Fig. 4, with gross vehicle weight (GVW) of 346 kN. The average daily truck traffic (ADTT) is 5000 with 66% in the right lane, 33% in the left lane and 1% passing side by side in both lanes. According to the data of Laman and Nowak, each truck passage causes one stress cycle. Under repeated tensile stresses, fatigue cracks may be developed in the bottom flange of steel girder. According to AASHTO specifications, the girders made of unpainted weathering steel belong to category B, if there is no other source of stress concentration.

In order to calculate these tensile stresses, the bridge is modeled by 45 flat sell strips and 15 connection strips as shown in Fig. 5, whilst 15 series terms are taken. Only little improvement can be achieved if more strips and terms are employed. A series of static analyses is carried out to evaluate stresses within each girder under the fatigue truck located at different positions. It is found that the maximum tensile stress due to the truck in the right lane appears in the first girder G_1 at cross section 13.25 meters away from bridge entrance when the middle axle of the truck just acts at this section. The maximum tensile stresses in girders G_2 and G_3 also occur at this cross section with the same truck

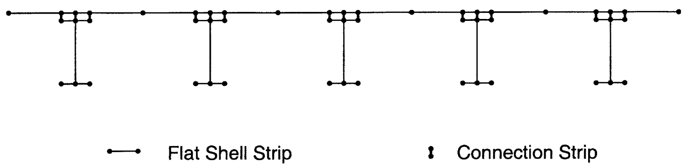

Fig. 5 Finite strip model of slab-on-girder bridge

Table 1 Basic variables for girders of weathering steel (AASHTO approach)

Variables		Distribution	Median		COV
Thickness of concrete slab	t_c	Normal	191	mm	0.067
Elasticity modulus of concrete	E_c	Lognormal	30000	MPa	0.2
Elasticity modulus of steel	E_s	Lognormal	210000	MPa	0.06
Fatigue strength coefficient	A	Lognormal	9.28×10^{12}	MPa^3	0.45
Damage index at failure	Δ	Lognormal	1.0		0.3
Impact factor	I	Lognormal	1.15		0.1

location. However, in girders G_4 and G_5, the maximum tensile stresses appear at midspan with the middle axle acting at y = 14.5 m. These results are used to locate the fatigue truck and to identify the critical cross section in each girder during the subsequent simulation process.

The thickness of concrete slab, elasticity moduli of concrete and steel, fatigue strength coefficient of steel girder, damage index at failure, and impact factor are considered as random variables in the probabilistic analysis. The distributions, medians and coefficients of variation are listed in Table 1. In this table, the median of fatigue strength coefficient is calculated from the design value $A_o = 3.93\times10^{12}$ MPa^3, which is given in AASHTO Specifications (1994) for detail category B. In addition, the constant m is equal to 3.0, as listed in AASHTO Specifications.

Based on these probabilistic distributions, the fatigue failure probability of each steel girder is evaluated by following the proposed solution procedures for AASHTO approach. The resulting reliability index of each girder is depicted in Fig. 6 as a function of service life. It can be noticed that the critical girder is G_1 with the reliability index $\beta = 2.86$ after 70 years in service, whilst girder G_3 has the highest reliability index $\beta = 4.15$ within the same period. In addition, girder G_2 has the safety level approximately the same as girder G_1, while girders G_4 and G_5 have the safety level similar to girder G_3. If the target reliability index is 2.0, as suggested by Moses *et al.* (1987), and the intended service life is 70 years, all the girders have adequate fatigue reliability.

5.2. Steel girders with cover-plates, LEFM approach

The span length of the bridge in the previous example is extended to 28 meters. A cover-plate is welded to the bottom flange of each girder, as shown in Fig. 7. The distance from each end of the cover-plate to the adjacent bearing is 2.0 m. The weld leg size at the end of cover-plate is 16 mm. Under repeated tensile stresses, fatigue cracks may form at the end weld of the cover-plate and penetrate into the bottom flange, even into the web. Based on the works of Fisher (1984), the cracks can be modeled as semi-elliptical surface cracks with the depth a in the bottom flange. The geometry function $F(a, Y)$ can be calculated using equations listed in Fisher (1984).

For the same cover-plate, structural analysis shows that the end weld on the side of bridge exit

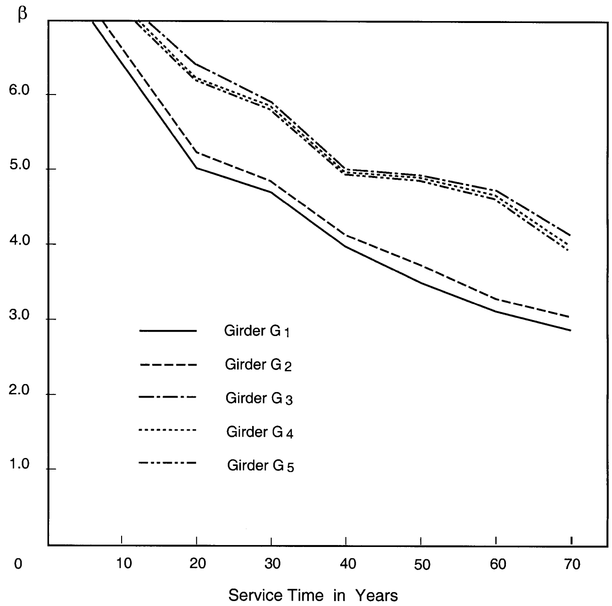

Fig. 6 Fatigue reliability indices of weathering steel girders

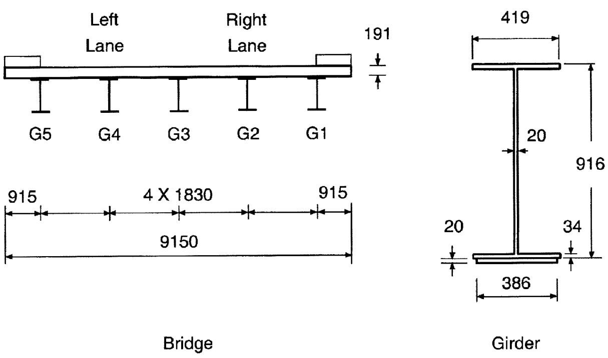

Fig. 7 Steel-girder bridge with cover-plates (mm)

experiences higher stress range than the end on the side of bridge entrance under the fatigue truck. The maximum tensile stress occurs when the front axle of the fatigue truck acts just above the exit end of cover-plate. Under given bending moment in a girder, the critical fatigue stress is located in the bottom flange at the end of cove plate. Therefore, it should be calculated based on cross section of the girder without cover-plate. However, the cover-plates stiffen the girders, and increase the bending moment shared by the girder right underneath the truck, and reduce the loads shared by other girders. In order to take into account this effect, the transverse load distribution factors are first determined using a finite strip model of the bridge with full-length cover-plates, and then evaluated again using the same model but without cover-plates. Thus, a modification coefficient for the load distribution factor of each girder is obtained as $\lambda = f_{cp} / f_o$, with f_{cp} denoting the load distribution factor of the girder from the first model and f_o representing the load distribution factor from the second model. During simulation process, the maximum stress at end of cover-plate is calculated using the finite strip model without cover-plate, and

Table 2 Basic variables for steel girders with cover-plates (LEFM approach)

Variable		Distribution	Mean		C.O.V
Thickness of Concrete Slab	t_c	Normal	191	mm	0.067
Elasticity Modulus of Concrete	E_c	Lognormal	30000	MPa	0.2
Elasticity Modulus of Steel	E_s	Lognormal	210000	MPa	0.06
Crack Growth Constant	C	Lognormal	1.26×10^{-13}	$\mathrm{MPa^{-3}mm^{-1/2}}$	0.63
Initial Crack Dimension	a_0	Lognormal	0.762	mm	0.5
Impact Factor	I	Lognormal	1.15		0.1

then multiplied by this modification coefficient λ.

In this finite strip model, 45 flat sell strips and 15 connection strips are employed as shown in Fig. 5, whilst 20 series terms are taken. Only little improvement can be achieved if more strips and terms are used.

The thickness of concrete slab, elasticity moduli of concrete and steel, crack growth constant C, initial crack dimension and impact factor are considered as random variables in the probabilistic analysis. The distributions, mean values and coefficients of variation are listed in Table 2. In addition, the constant m is taken as 3.0, while the critical crack size a_c is determined as 25.4 mm (Fisher 1984).

Based on these probabilistic distributions, the fatigue failure probability of each steel girder is evaluated by following the proposed procedures for LEFM approach. The resulting reliability index of each girder is illustrated in Fig. 8 as a function of service life. It can be noticed that the critical girder is G_2 with the reliability index $\beta = 1.56$ after 70 years in service, whilst girder G_5 has the highest reliability index $\beta = 3.83$ in the same period. If the target reliability index is 2.0, and the intended service life is 70 years, girder G_2 has inadequate fatigue strength and should be strengthened. In fact, if all the cover-plates are extended to the full length of span, the end welds with high stress concentration will be removed from the zones of tensile stress. Thus, the fatigue category of steel girder will be changed from E to B (AASHTO 1994), and corresponding fatigue reliability will be enhanced significantly at a little extra cost.

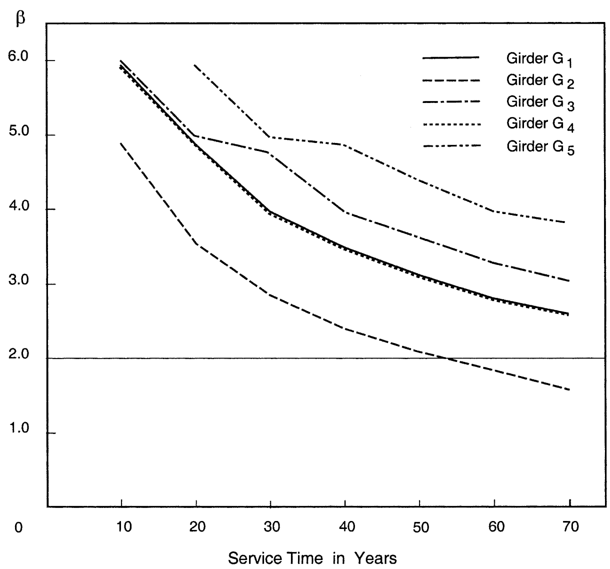

Fig. 8 Fatigue reliability indices of girders with cover-plates

6. Conclusions

In the present study, a finite strip method for fatigue reliability analysis of highway bridges has been developed. The flat shell strips are used to model the concrete slab and steel girders, while a connection strip is formed to take into account the eccentricity of girder top flanges. The loading effects are modeled by the fatigue truck developed by Laman and Nowak. At each sampling point with given values of slab thickness and modulus ratio of materials, a finite strip analysis is carried out to determine stress ranges at fatigue prone details, while the AASHTO approach or LEFM approach is used to calculate the corresponding fatigue failure probability of the structural details. After the failure probability is integrated over all the sampling points, the fatigue reliability of the bridge is obtained. Numerical examples have shown the capability of the proposed method.

Acknowledgments

The financial supports from the Natural Sciences and Engineering Research Council of Canada are gratefully acknowledged.

References

AASHTO (1994), AASHTO LRFD Bridge Design Specifications, American Association of State Highway and Transportation Officials, Washington, D. C.

Ayyub, B.M. and Mccuen, R.H. (1995), "Simulation-based reliability method", *Probabilistic Structural Mechanics Handbook*, Sundararajan, C. editor, Chapter 4, Chapman & Hall Ltd., New York, NY.

Cheung, M.S., Li, W. and Chidiac, S.E. (1996), Finite Strip Analysis of Bridges, E & FN SPON, London.

Committee on Fatigue and Fracture Reliability of the Committee on Structural Safety and Reliability of the Structural Division, (1982), "Fatigue reliability 1 to 4", *J. Structural Engineering*, ASCE, **108**(1), 3-88.

Fisher, J.W. (1984), *Fatigue and Fracture in Steel Bridges*, Case Studies, John Wiley & Sons, New York, NY.

Harris, D.O. (1995), "Probabilistic fracture mechanics", *Probabilistic Structural Mechanics Handbook*, Sundararajan, C. editor, Chapter 6, Chapman & Hall Ltd., New York, NY.

Laman, J.A. and Nowak, A.S. (1996), "Fatigue-load models for girder bridges", *J. Structural Engineering*, ASCE, **122**(7), 726-733.

Moses F, Schilling, C.G. and Raju, K.S. (1987), Fatigue Evaluation Procedures for Steel Bridges. NCHRP Report 299, Washington, D.C.

Murakami, Y. (1987), *Stress Intensity Factors Handbook*, Pergamon, Oxford, UK.

Zhao, Z., Haldar, A. and Breen, F.L. (1994), "Fatigue-reliability evaluation of steel bridges", *J. Structural Engineering*, ASCE, **120**(5), 1608-1623.

Wirsching, P.H. (1995), "Probabilistic fatigue analysis", *Probabilistic Structural Mechanics Handbook*, Sundararajan, C. editor, Chapter 7, Chapman & Hall Ltd., New York, NY.

CU