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Failure mechanisms of externally prestressed composite beams with partial shear connection

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Abstract. This paper proposes a model for analysing the non-linear behaviour of steel concrete composite beams prestressed by external slipping cables, taking into account the deformability of the interface shear connection. By assuming a suitable admissible displacement field for the composite beam, the balance condition is obtained by the virtual work principle. The solution is numerically achieved by approximating the unknown displacement functions as series of shape functions according to the Ritz method. The model is applied to real cases by showing the consequences of different connection levels between the concrete slab and the steel beam. Particular attention is focused on the limited ductility of the shear connection that may be the cause of premature failure of the composite girder.

Key words: external prestressing; composite beams; flexible shear connection; shear connection ductility; non-linear analysis; steel-concrete composite bridges.

1. Introduction

The use of external prestressing cables in steel-concrete composite structures is of great interest in the rehabilitation of existing under-strength bridges in that it is extremely effective, easy to implement and relatively inexpensive (Dunker *et al.* 1986, Dunker *et al.* 1990). Furthermore, such a technique finds significant application in the construction of new bridges with continuous girders where prestressing is advantageously adopted both to control cracking over the interior supports, and to achieve high global carrying capacity by limiting the steel beam dimensions (Troitsky 1990).

In the simplest case, prestressing is performed by means of a number of rectilinear cables running above the bottom flange at the sagging regions and under the top flange at the hogging regions. This makes it possible to induce constant bending moments opposing those produced by the external loads. More effective prestressing of the whole girder can be attained by shaping the cable path so as to induce variable bending moments by placing deviators (saddle points) along the beam axis (Li *et al.* 1995). In

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this last case, in order to avoid prestressing losses during cable tensioning, the friction between cable and deviators is minimised by suitable devices.

The analysis of such structures raises some difficulty with the coupling that arises between the local cable strain and the structure global deformation. The usual sectional analysis cannot be used to determine the cable elongation that must thus be calculated considering the deformation of the entire structure. As a consequence, local formulations are not effective and global formulations are more natural for the problem in question.

In the last forty years, the work of many researchers has been dedicated to developing analytical models in order to describe the behaviour of girders prestressed by external cables. The early theoretical models reported in literature date back to the 60s and deal with simply supported beams prestressed by rectilinear cables (Szilard 1959, Hoadley 1963, Reagan and Krahl 1967). Troitsky *et al.* (1989) extended the analysis to continuous beams, under the assumption of linear elastic behaviour of the materials, by considering the cable traction and the redundant reactions of the external restraints as unknowns according to the deformability method. Again, with reference to simply supported beams prestressed by rectilinear cables, an experimental and theoretical study on the non-linear behaviour of composite beams subjected both to positive and negative bending moments was presented by Saadatmanesh *et al.* (1989a-c). In these works, the differences obtained between the experimental and the theoretical results were attributed to the flexibility of the shear connection. Virlogeux (1990) continued the analysis of simply supported beams, prestressed by cables with a generic path, by proposing a non-linear compatibility condition between the cable and the girder. Later, Tong and Saadatmanesh (1992) employed the same formulation to analyse the linear behaviour of continuous composite beams. Ayyub *et al*. (1992a, b) also contributed to understanding the behaviour of the hogging regions of prestressed continuous composite beams. Once again, they observed that the analytical results were in good agreement with the experimental results only in the linear range; in the non-linear range the proposed analytical model, based on the stiff shear connection assumption, underestimates the real deformability of the structure. More recently, Dall'Asta and Dezi (1998) proposed a unitary formulation valid for the non-linear analysis of continuous girders prestressed by cables with generic paths, accounting for different construction sequences.

Despite the fact that many researchers have studied these structures and that some have pointed out the importance of taking into account the deformability of the shear connection, no paper available in the literature investigates the shear connection behaviour in externally prestressed composite beams.

In the neighbourhood of the cable anchorage, the connectors are subjected to shear force peaks inducing a beam-slab interface slip which can be more important than that induced by dead and service loads. As a consequence, the shear connectors may be involved in the collapse mechanism of the beam and, because of their limited ductility, this may lead to a significant reduction of the load carrying capacity. This aspect may be of particular importance when prestressing is applied to existing bridges because the shear connection is generally formed to be of inadequet strength. The concept of full and partial composite action, well understood for non-prestressed beams, is still an open question for prestressed composite beams. Thus, the design procedures, suggested for routine analysis by technical codes such as ENV 1994-2 (1997), are effective only for non-prestressed beams but they cannot be used in the case of prestressed beams without validation.

The aim of this paper is to overcome the limitations of previous formulations by modelling shear connection deformability (both in the linear and non-linear range) in prestressed beams in order to study the collapse modalities related to the failure of the shear connectors. The analytical formulation is derived by assuming the vertical displacement of the composite cross section and the longitudinal displacements of the steel beam and of the concrete slab, as unknowns. The equilibrium condition is enforced by the application of the virtual work principle considering generic non-linear constitutive relationships for the reinforced concrete slab, the steel beam, the cables and the shear connectors. The model makes it possible to account for different construction sequences by defining suitable residual strains for the concrete slab and the shear connection. This last aspect is of a certain importance since knowing the real strains of the components with limited ductility is fundamental to define the failure mechanisms of the composite beam, especially when prestressing is used to strengthen existing structures. The problem is numerically solved by the Ritz method by approximating the unknown functions with suitable shape functions fulfilling the kinematical boundary conditions.

In order to show the capability of the model, some applications to simply supported beams with different shear connector strength levels are reported. The results show the influence of the shear connection on the load carrying capacity of the girders by focusing attention on its limited ductility.

2. Mathematical formulation

2.1. Kinematical description

The prismatic composite beam of Fig. 1, consists of a steel beam shear-connected to a concrete upper slab. In its undeformed state, the beam has a rectilinear axis, which is assumed to be parallel to the Z axis of the orthonormal reference frame {0; X, Y, Z}. The co-ordinate plane YZ is a symmetry plane of the problem. The location of a generic point $\boldsymbol{S}$ of the beam is given by

$$\boldsymbol{S}(x, y, y)=x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k} \quad \forall(x, y) \in \{\bar{A}_c \cup \bar{A}_s\} \text{ and } z \in [0, L] \tag{1}$$

where $\boldsymbol{i}$, $\boldsymbol{j}$, $\boldsymbol{k}$ are the unit vectors of the reference axes (see Fig. 1a); $\bar{A}_c$, $\bar{A}_s$ are the closures of the domains in plane XY, representing the cross-sections of the concrete slab and of the steel beam, respectively.

Two prestressing cables are disposed symmetrically with respect to the YZ plane. Given the problem symmetry, the cables can be replaced by a single equivalent cable having the cross section area of the two and lying in the plane of symmetry. The cable path is defined by D+1 saddle points (including the end anchorages), and it can thus be defined by the piecewise linear function

$$\boldsymbol{H}(z)=\boldsymbol{S}_{d-1}+\frac{z-z_{d-1}}{z_d-z_{d-1}}(\boldsymbol{S}_d-\boldsymbol{S}_{d-1}) \quad z \in (z_{d-1}, z_d); \quad d=1, \ldots, D \tag{2}$$

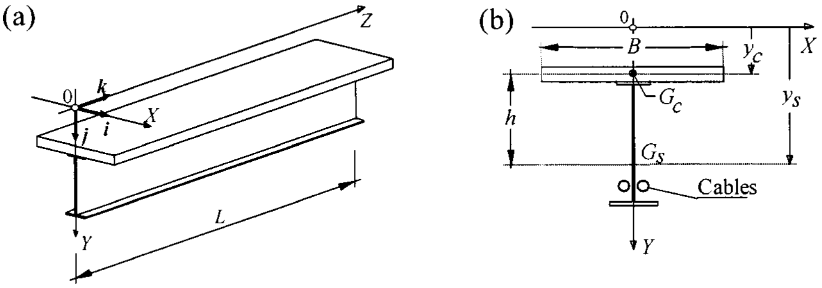

Fig. 1 Geometry of the beam: (a) prismatic composite beam; (b) cross-section

where $\boldsymbol{S}_d=y_d\boldsymbol{j}+z_d\boldsymbol{k}$ is the position vector of the dth saddle. From relation (2) it follows that the total length of the cable path is

$$\Lambda=\sum_{d=1}^{D}|\boldsymbol{S}_d-\boldsymbol{S}_{d-1}|=\sum_{d=1}^{D}\sqrt{(z_d-z_{d-1})^2+(y_d-y_{d-1})^2} \tag{3}$$

Obviously, if the beam is prestressed by more than two cables, additional expressions similar to (2) and (3) should be included.

The generic deformed configuration of the beam is characterised by the slab-beam interface slip due to the flexibility of the shear connectors. This implies that the longitudinal displacement field shows a discontinuity at the interface layer, while the vertical displacements are assumed to be continuous (in other words, the connectors prevent separation between beam and slab). According to Newmark *et al.* (1951), the preservation of plane cross section for the steel beam and the concrete slab is considered separately. Therefore, denoting the derivatives with respect to the variable z by *primes*, the final position $\boldsymbol{s}$ and the displacement $\boldsymbol{u}$ of a point in concrete and steel are expressed respectively by

$$\boldsymbol{s}(x,y,z)=\boldsymbol{S}+\boldsymbol{u}(x,y,z)=[x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}]+\{v(z)\boldsymbol{j}+[w_c(z)-(y-y_c)v'(z)]\boldsymbol{k}\}$$
$$\forall(x,y)\in\bar{A}_c,\ z\in[0,L] \tag{4}$$

$$\boldsymbol{s}(x,y,z)=\boldsymbol{S}+\boldsymbol{u}(x,y,z)=[x\boldsymbol{i}+y\boldsymbol{j}+z\boldsymbol{k}]+\{v(z)\boldsymbol{j}+[w_s(z)-(y-y_s)v'(z)]\boldsymbol{k}\}$$
$$\forall(x,y)\in\bar{A}_s,\ z\in[0,L] \tag{5}$$

where v is the scalar component of displacement in Y direction of both the concrete slab and the steel beam (Fig. 2); w_c, w_s are the scalar components of displacement in Z direction of the concrete slab and the steel beam, respectively (Fig. 2), and y_c, y_s are co-ordinates of the centroids of the concrete slab and the steel beam, respectively (Fig. 1b). Subsequently, by denoting by h the distance between the centroids of the steel beam and the concrete slab (Fig. 1b), the following expression of the slab-beam interface slip can be obtained:

$$\Gamma(z)=w_s(z)-w_c(z)+v'(z)h \tag{6}$$

From Eqs. (4) and (5), the following non-vanishing strain components can be computed:

$$\varepsilon_c(x,y,z)=w_c'-(y-y_c)v'' \quad \forall(x,y)\in\bar{A}_c,\quad z\in[0,L] \tag{7}$$

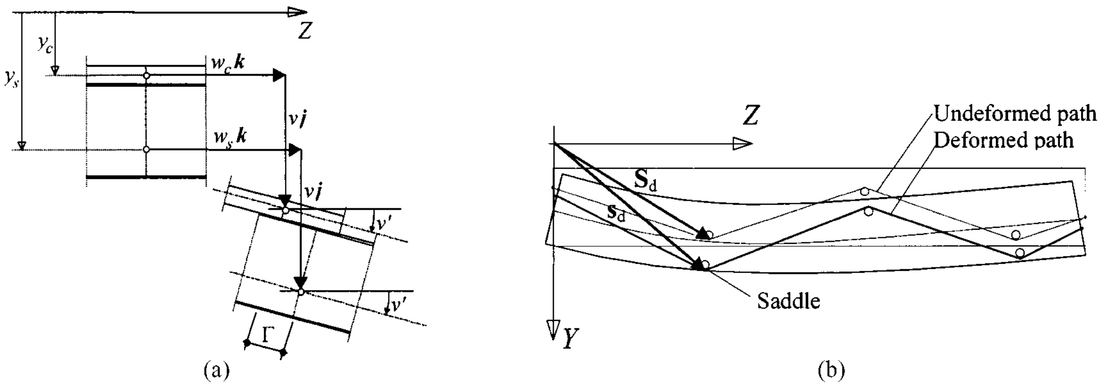

Fig. 2 Displacements: (a) composite section; (b) cable path

$$\varepsilon_s(x,y,z)=w_s{}'-(y-y_s)v'' \quad \forall(x,y)\in\bar{A}_s, \quad z\in[0,L] \tag{8}$$

With reference to the cable, since the saddle points are fixed to the steel beam, the geometry of the cable path can be obtained by combining Eqs. (4) and (5) with Eq. (2) so that

$$\boldsymbol{h}(z)=\boldsymbol{s}_{d-1}+\frac{z-z_{d-1}}{z_d-z_{d-1}}(\boldsymbol{s}_d-\boldsymbol{s}_{d-1}) \qquad z\in(z_{d-1},z_d); \quad d=1,\ldots,D \tag{9}$$

Since the displacements $\boldsymbol{u}$ are small, as per the linear formulation of the problem, the final length of the cable can be expressed as

$$\lambda=\Lambda+\sum_{d=1}^{D}\frac{\boldsymbol{P}_d-\boldsymbol{P}_{d-1}}{|\boldsymbol{P}_d-\boldsymbol{P}_{d-1}|}\cdot(\boldsymbol{u}_d-\boldsymbol{u}_{d-1})=\Lambda+\sum_{d=1}^{D}\boldsymbol{a}_d\cdot(\boldsymbol{u}_d-\boldsymbol{u}_{d-1}) \tag{10}$$

Under the assumption of no-friction between cable and saddle points and by assuming a negligible weight for the cable, the strain of the cable ε_{ca} can now be obtained by the ratio between the global stretching of the cable and its initial length (Dall'Asta and Dezi 1998)

$$\varepsilon_{ca}=\frac{\lambda-\Lambda}{\Lambda}=\frac{1}{\Lambda}\sum_{d=1}^{D}\{a_{d_y}(v_d-v_{d-1})+a_{d_z}[(w_{s_d}-w_{s_{d-1}})-(y_d-y_s)v_d{}'+(y_{d-1}-y_s)v_{d-1}{}']\} \tag{11}$$

where a_{dy} and a_{dz} are the components of the unit vectors $\boldsymbol{a}_d$ defining the direction of the d-th section of the cable path in the undeformed configuration.

2.2. Constitutive relationships

It is assumed that the behaviour of the materials composing the beam is described by generic non-linear single-valued stress strain laws. In particular, G_c, G_s and G_{ca} will denote the functions furnishing the stresses in the upper slab, in the lower beam and in the cable, respectively, as functions of the strains measured by starting from the natural state of the material (i.e., where no stress is present). For the problem in question, a state of the structure in which all the components are in their natural state does not exist and it is necessary to express the constitutive relationships in the form:

$$\sigma_c=G_c(x,y;\varepsilon_c-\varepsilon_{0c}); \quad \sigma_s=G_s(x,y;\varepsilon_s-\varepsilon_{0s}); \quad \sigma_{ca}=G_{ca}(\varepsilon_{ca}-\varepsilon_{0ca}) \tag{12}$$

where ε_c, ε_s and ε_{ca} are the strains in the actual state and ε_{0c}, ε_{0s} and ε_{0ca} are the strains in the reference state. By choosing the reference configuration such that the steel beam is in the natural state, ε_{0s} appearing in the second of Eqs. (12) vanishes. Notice that ε_{0ca} is necessary to impose the cable pretension while the introduction of a generic residual strain field for the concrete slab permits analysing the different construction sequences as will be shown in the sequel. No restrictive assumption is made on the distribution of different materials in the cross section, and the dependence on x and y simply means that different points can be defined in different materials. This makes it possible to consider the presence of reinforcing rebars or internal prestressing cables in the slab under the assumption of full bond simply by defining different stress-strain laws for the reinforcing steel and for the concrete.

The shear connection is assumed to be a continuous elastic device having only shear deformability and the following constitutive law is considered between the longitudinal shear force per unit length q

and the interface slip Γ

$$q = G_{conn}(\Gamma - \Gamma_0) \tag{13}$$

where Γ_0 is the beam-slab residual slip.

2.3. Equilibrium condition

By assuming that the beam is subjected to vertical forces $p(z)$, the equilibrium conditions can be written in the variational form equalising the virtual work of internal stresses to that of external forces for each variation (δv, δw_c, δw_s) of the admissible displacements. With expressions (4) and (5) for the displacements, (6) for the interface slip and (7), (8) and (11) for the strains, the application of virtual work principle (VWP) gives

$$\int_0^L (N_c \delta w_c' - M_c \delta v'')dz + \int_0^L (N_s \delta w_s' - M_s \delta v'')dz + \int_0^L q\delta\Gamma dz + \Lambda \tau_{ca}\delta\varepsilon_{ca} = \int_0^L p\delta v dz$$
$$\forall \delta v,\ \delta w_c,\ \delta w_s \tag{14}$$

The first and second integrals group the contributions to the internal work due to axial and flexural deformations in the slab and beam, respectively. In particular

$$N_c = \int_{A_c} G_c[x, y;\ w_c' - v''(y - y_c) - \varepsilon_{0c}]dA_c \tag{15a}$$

$$M_c = \int_{A_c} G_c[x, y;\ w_c' - v''(y - y_c) - \varepsilon_{0c}](y - y_c)dA_c \tag{15b}$$

$$N_s = \int_{A_s} G_s[x, y;\ w_s' - v''(y - y_s) - \varepsilon_{0s}]dA_s \tag{15c}$$

$$M_s = \int_{A_s} G_s[x, y;\ w_s' - v''(y - y_s) - \varepsilon_{0s}](y - y_s)dA_s \tag{15d}$$

are the generalised stress resultants for concrete slab and steel beam. The third integral is the contribution of the shear connection where

$$q = G_{conn}(w_s - w_c + v'h - \Gamma_0) \tag{16}$$

Finally, the term outside the integral is related to the prestressing cable. In fact, since this can slip with negligible friction on the deviators, the stresses are constant along the cable and the relevant integral reduces to the product between the cable length, the virtual cable strain $\delta\varepsilon_{ca}$, expressed according to (11), and the constant resultant traction

$$\tau_{ca} = -A_{ca}G_{ca}\varepsilon_{0ca} + A_{ca}G_{ca}\sum_{d=1}^{D}\{a_{d_y}(v_d - v_{d-1})$$
$$+ a_{d_z}[(w_{s_d} - w_{s_{d-1}}) - (y_d - y_s)v_d' + (y_{d-1} - y_s)v_{d-1}']\} \tag{17}$$

The variational equilibrium condition (14) provided by the VWP is the most natural for the problem formulation. Firstly it is no more complicated than that of non-prestressed beams in which the term outside the integrals, related to the external prestressing cable, does not appear. Secondly, when more cables are present, the formulation can be promptly extended by considering as many terms as the number of cables. Furthermore, as demonstrated in the sequel, the splitting of the virtual work into different contributions, each related to a different component of the structure (steel beam, concrete slab, shear connection, prestressing cables), is particularly convenient for analysing the various construction sequences. Vice versa, local formulation does not lead to simply differential equations as in the case of non prestressed beams since the cable traction (17) is a functional of the beam displacements. Furthermore, in the case in question, where the cable follows a path defined by saddles, the local equations must be completed by a number of continuity conditions for the cross sections at saddle locations.

2.4. Construction sequences

Under service load, the stress state of composite structures is strongly influenced by the construction sequence. This aspect is fundamental for calibrating the external prestressing level to strengthen existing decks in order to evaluate the effectiveness of the operation. As already stated, the proposed model makes it easy to analyse the main construction techniques of practical interest by suitably defining the residual strains ε_{0c}, and Γ_0 which appear in the constitutive relationships. Under the assumption of preservation of the plane cross section for concrete slab and steel beam considered separately, the residual strains are evaluated by

$$\varepsilon_{0c}=w_{0c}'-v_0''(y-y_c) \qquad \Gamma_0=w_{0s}-w_{0c}+v_0'h \tag{18}$$

where w_{0c}, w_{0s} and v_0 are the solutions of preventive structural analyses. The modalities for performing the preventive structural analyses, for the main construction techniques, will be described.

2.4.1. Propped beam

This technique is used only for short span decks given the high cost of the temporary supports and the longer construction times. It permits full exploitation of beam-slab composite action since the self-weight is sustained by the composite beam as a whole. While pouring the concrete slab, the steel beam is supported by a number of intermediate temporary supports. At this stage, both the steel beam and the concrete slab are in their natural state so that ε_{0c} is zero and no preventive analysis is required. After removing the intermediate bearings, the cable is pretensioned by assigning a suitable non zero value to ε_{0ca}.

2.4.2. Unpropped beam

In order to cut down deck construction times and when the use of provisional piers is not economically convenient, the slab is poured on the unpropped steel beams which sustain the weight of the whole deck. In this case, the residual stresses must be evaluated by preventive analysis by eliminating the terms related to the concrete slab and to the prestressing cable in equation (14); the solution is of the following kind:

$$v_0(z)\neq 0 \qquad w_{0s}(z)=0 \qquad w_{0c}(z)=v_0'(z)h \tag{19}$$

In the following analysis steps, the entire expression of (14) must be considered and the pretension of the external cables may be controlled as usual by assigning a non zero value to ε_{0ca}.

2.4.3. Beams with prestressed cast in situ slabs

In some cases post tensioned cables are placed in the interior of the concrete slab in the sections over the inner supports of continuous beams in order to control the concrete cracking. This technique is generally not combined with the external prestressing, this case may be of interest however when existing bridges with prestressed slabs require rehabilitation.

The concrete slab is prestressed when it is shear-connected to the steel beam and the analysis should be performed according to the following steps:

– evaluation of the residual strains according to the technique of slab pouring (see the previous sections regarding propped or unpropped beams);
– tensioning of the internal cables by suitably imposing additional $\bar{\varepsilon}_{0c}$ in Eq. (12a) and by considering Eq. (14) without the external cable term;
– tensioning of the external cables by assigning a suitable value to ε_{0ca} and by considering the entire expression (14).

2.4.4. Beams with prestressed precast concrete slabs

Sometimes the slab is constructed by using precast segments that are placed over the unpropped steel beams. Initially, the shear connection is rendered ineffective by means of suitable devices which allow beam-slab slip. Thus, cables are placed at the interior of the concrete slab and tensioned in order to make the segmental slab monolithic. Finally the slab is connected to the steel beam by sealing the shear connectors. In this case, the analysis must be performed according to the following steps:

– evaluation of the residual strains as in the case of unpropped beams by eliminating the terms related to the concrete slab and the external cables in (14); the following solution is thus obtained

$$v_0^{(1)}(z) \neq 0 \quad w_{0s}^{(1)}(z)=0 \quad w_{0c}^{(1)}(z)=v_0'(z)h \tag{20}$$

– evaluation of the residual strains by eliminating the terms related to the shear connection to the concrete slab and by imposing the strain $\bar{\varepsilon}_{0c}$ to the internal cables; the new solution is

$$v_0^{(2)}(z)=0 \quad w_{0s}^{(2)}(z)=0 \quad w_{0c}^{(2)}(z) \neq 0 \tag{21}$$

– tensioning of the external cables by assigning a suitable value to ε_{0ca} and by considering the entire expression (14) in which the residual strains are given by the superposition of (20) and (21).

2.5. Numerical solution

A numerical solution of the problem can be undertaken by the Ritz method after approximating the unknown displacement functions in the form

$$w_c(z)=\boldsymbol{w}_c \cdot \boldsymbol{\varphi}_c(z) \quad w_s(z)=\boldsymbol{w}_s \cdot \boldsymbol{\varphi}_s(z) \quad w_s(z)=\boldsymbol{v} \cdot \boldsymbol{\psi}(z) \tag{22a, b, c}$$

In other words, the unknown displacement functions are expressed as linear combinations of known shape functions, $\boldsymbol{\varphi}_c$, $\boldsymbol{\varphi}_s$, $\boldsymbol{\psi}$, and the unknown coefficients, $\boldsymbol{w}_c$, $\boldsymbol{w}_s$, $\boldsymbol{v}$. By substituting Eqs. (22) into Eqs. (6), (7), (8) and (11), the following expressions are obtained for the interface slip and for the strain components of the concrete slab, the steel beam and the cable, respectively:

$$\Gamma=\boldsymbol{w}_s \cdot \boldsymbol{\varphi}_s - \boldsymbol{w}_c \cdot \boldsymbol{\varphi}_c + h\, \boldsymbol{v} \cdot \boldsymbol{\psi}' \tag{23}$$

$$\varepsilon_c=\boldsymbol{w}_c \cdot \boldsymbol{\varphi}_c' - (y-y_c)\boldsymbol{v} \cdot \boldsymbol{\psi}'' \tag{24}$$

$$\varepsilon_s=\boldsymbol{w}_s \cdot \boldsymbol{\varphi}_s' - (y-y_s)\boldsymbol{v} \cdot \boldsymbol{\psi}'' \tag{25}$$

$$\varepsilon_{ca}=\frac{1}{\Lambda}\sum_{d=1}^{D}\{[a_{d_y}(\psi_d-\psi_{d-1})+a_{d_z}(-(y_d-y_s)\psi_d{}'+(y_{d-1}-y_s)\psi_{d-1}{}')]\cdot\boldsymbol{v}$$
$$+a_{d_z}(\varphi_{s_d}-\varphi_{sd-1})\cdot\boldsymbol{w}_s\} \tag{26}$$

By expressing the displacement and strain variations according to Eqs. (22-26), the equilibrium condition (14) provides (I+J+K) non-linear algebraic equations having the form

$$\int_0^L[\int_{A_c}G_c(x,y;\varepsilon_c-\varepsilon_{0c})dA_c\varphi_{ci}{}'-G_{conn}(\Gamma-\Gamma_0)\varphi_{ci}]dz=0 \quad i=1,\ldots,I \tag{27}$$

$$\int_0^L[\int_{A_s}G_s(x,y;\varepsilon_s-\varepsilon_{0s})dA_s\varphi_{si}{}'+G_{conn}(\Gamma-\Gamma_0)\varphi_{si}]dz$$
$$+A_{ca}G_{ca}(\varepsilon_{ca}-\varepsilon_{0ca})\sum_{d=1}^{D}a_{dz}(\varphi_{sd_i}-\varphi_{sd-1_i})=0 \quad i=1,\ldots,J \tag{28}$$

$$-\int_0^L(\int_{A_c}G_c(x,y;\varepsilon_c-\varepsilon_{0c})(y-y_c)dA_c+\int_{A_s}G_s(x,y;\varepsilon_s-\varepsilon_{0s})(y-y_s)dA_s)\psi_i{}''dz$$
$$+A_{ca}G_{ca}(\varepsilon_{ca}-\varepsilon_{0ca})\sum_{d=1}^{D}a_{dy}(\psi_{d_i}-\psi_{d-1_i})+a_{dz}[-(y_d-y_s)\psi_{d_i}{}'$$
$$+(y_{d-1}-y_s)\psi_{d-1_i}{}']\}+\int_0^L[hG_{conn}(\Gamma-\Gamma_0)\psi_i{}']dz=\int_0^L p\psi_i dz \quad i=1,\ldots,K \tag{29}$$

The equations obtained constitute a coupled system of non-linear equations of the kind

$$\boldsymbol{a}(\boldsymbol{x})=\boldsymbol{b} \tag{30}$$

in which $\boldsymbol{x}[w_{c1},\ldots,w_{cI},w_{s1},\ldots,w_{sJ},v_1,\ldots,v_K]$ is the vector of the unknown coefficients defined in (22); $\boldsymbol{a}$ is a non-linear operator involving the integrals of the stresses in the composite beam and in the shear connection. The terms with summation account for the prestressing cable. Finally, $\boldsymbol{b}$ is the vector which groups the terms obtained by the integration of the forces applied to the beam.

The non-linear system obtained can be numerically solved by an iterative procedure. The choice of the iterative method depends strongly on the form of the non-linear operator $\boldsymbol{a}$ that is related mainly to the material constitutive laws adopted in the analysis. In the applications reported in this paper the Newton-Raphson method, based on the recursive formula

$$\boldsymbol{x}^{(k+1)}=\boldsymbol{x}^{(k)}+(\nabla\boldsymbol{a}(\boldsymbol{x}^{(k)}))^{-1}(\boldsymbol{b}-\boldsymbol{a}(\boldsymbol{x}^{(k)})) \tag{31}$$

is adopted. This method allows quick convergence. Also it is particularly simple to apply since, by adopting the usual constitutive laws, the gradient of the function $\boldsymbol{a}$ can be expressed in analytical form (see Appendix). The ultimate load can be obtained by using the incremental method by controlling the load level with an auto-stepping procedure.

3. Applications

The influence of the deformation of the shear connection on the failure modes of a simply supported

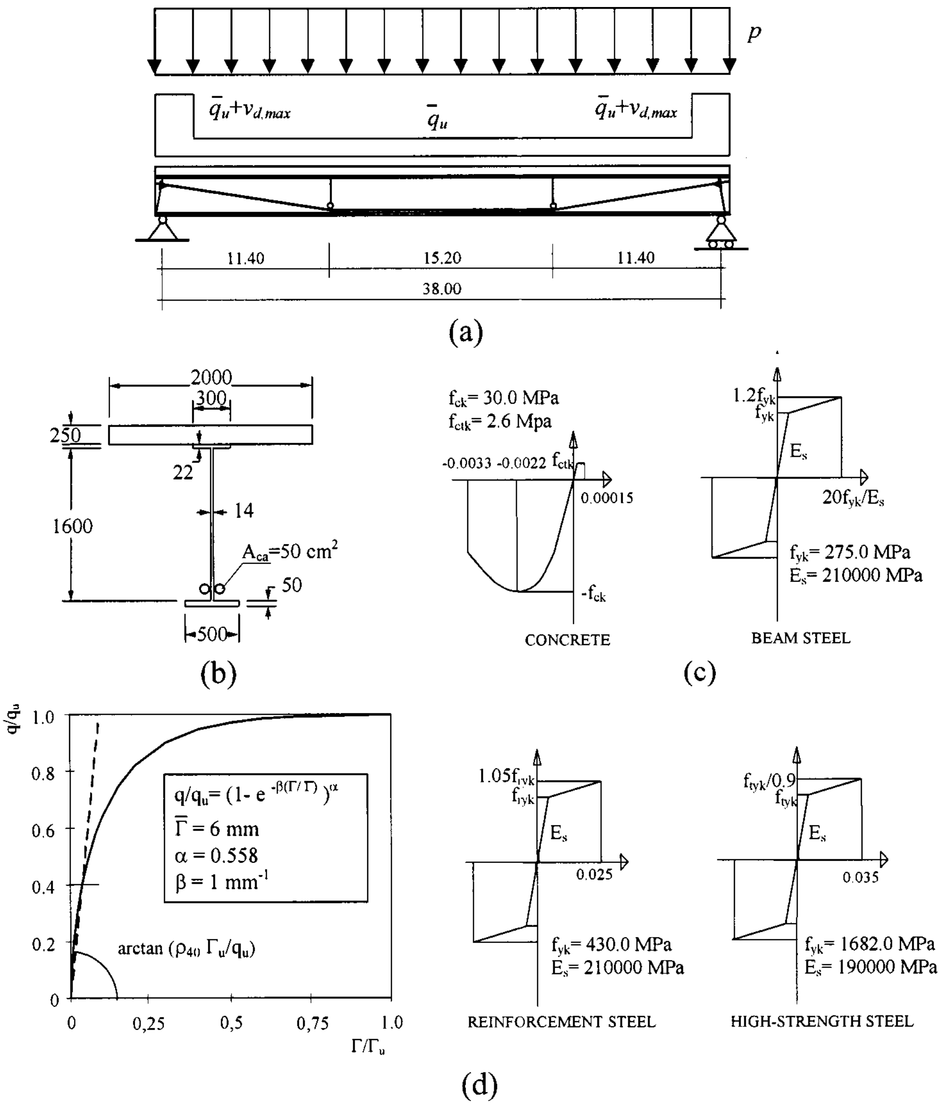

Fig. 3 Data of applications: (a) static scheme; (b) cross section details; (c) non-linear constitutive relationship of the materials; (d) non-linear constitutive relationship of the shear connection

beam is studied. For the sake of simplicity, a prismatic beam is used (Fig. 3b). The prestressing cables, having a total area of 5000 mm^2 and an initial tension of 5000 kN, are anchored at the end cross-sections at the level of the centroid of the composite beam and their path is defined by the two saddle points shown in Fig. 3a. The prestressing force applied is the maximum value that can be applied to the beam without causing cracking of the slab.

With regard to the shear connection device, a uniform distribution of stud connectors with ultimate strength per unit length $q_u=\bar{q}_u$ is adopted along the whole length of the beam, excepting that at the end regions where, according to ENV 1994-2 (1997), a stronger connection system with ultimate strength $q_u=\bar{q}_u+v_{d,\max}$ is provided on a segment having a length of 1.0 m. For the case under consideration $v_{d,\max}$=3000 kN/m.

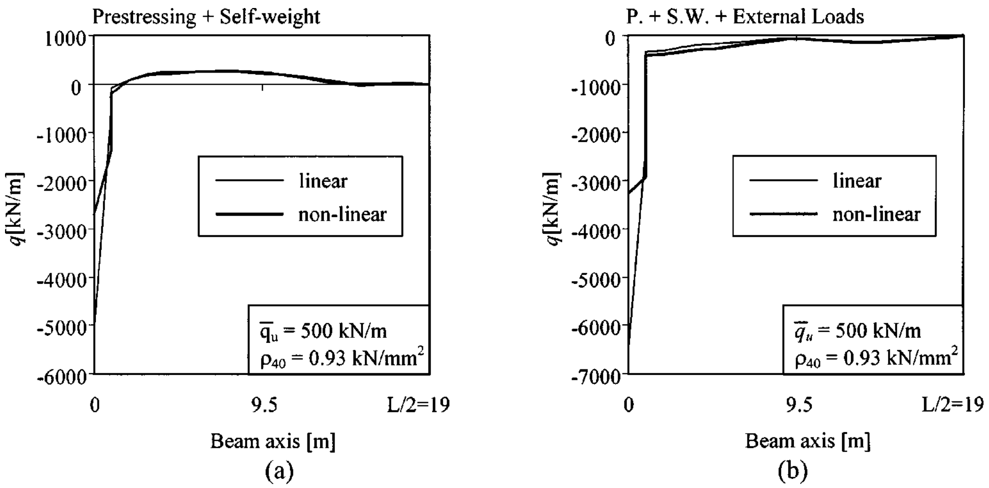

Fig. 4 Longitudinal force on the shear connection with linear and non-linear behaviour of the connectors: (a) beam subjected to prestress and self-weight; (b) beam subjected to prestress, self-weight and service loads

The non-linear constitutive laws adopted for steel and concrete are shown in Fig. 3c, while the classic Ollgaard constitutive law (Ollgaard *et al.* 1971, Johnson and Molenstra 1991) is considered for the shear connectors (Fig. 3d). Tschebichef's polynomial sequences are adopted as shape functions for the displacements.

The first diagrams of Fig. 4 show the differences between the results obtained with the model presented by considering a connection with the reference ultimate strength $\bar{q}_u$ =500 kN/m and those given by an elastic analysis in which the shear-connection stiffness per unit length (ρ_{40}) is fixed as equal to the secant stiffness at 40% of the ultimate strength q_u. The figure shows the longitudinal shear force at the interface connection for two different external load levels: in Fig. 4a only the self-weight of the composite beam is considered while the results in Fig. 4b are obtained by considering an external load

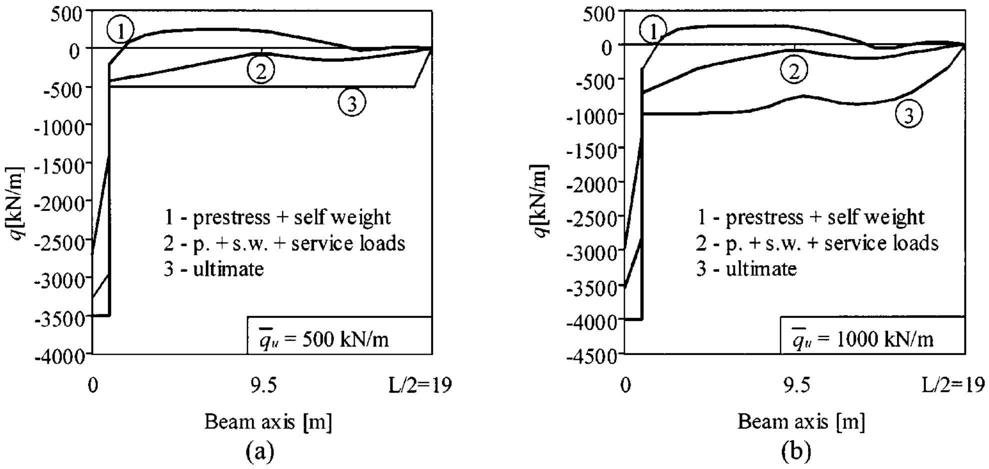

Fig. 5 Development of the longitudinal force on the shear connectors: (a) beam with weak connection; (b) beam with strong connection

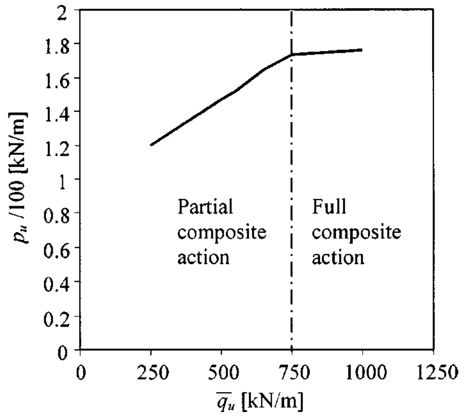

Fig. 6 External ultimate load versus shear connection strength

simulating the service condition (8 kN/m). Since the problem is symmetric, the diagrams refer only to half the length of the beam. As it is well known, the shear connection is subjected to peak forces near the regions of the prestressing cable anchorage (Dezi *et al*. 1995): the prestressing force applied entirely to the steel beam is partially transmitted to the concrete slab by the shear connectors. Such peaks, in the linear case, have an exponential shape and reach very high values, as compared to those obtained in the non-linear case. This means that in the case of prestressed beams the assumption of linear behaviour of the shear connection is not very realistic.

Fig. 5 shows a comparison between the results obtained by considering the beam with $\bar{q}_u$ =500 kN/m and a second beam having double the ultimate strength. The shear connection is assumed to be perfectly ductile. As is evident, the behaviour of the two beams is very different: in the case of the weaker connection, the shear connection becomes fully plastic at the ultimate load level while in the case of the stronger connection the ultimate capacity of the shear connection is reached only over a

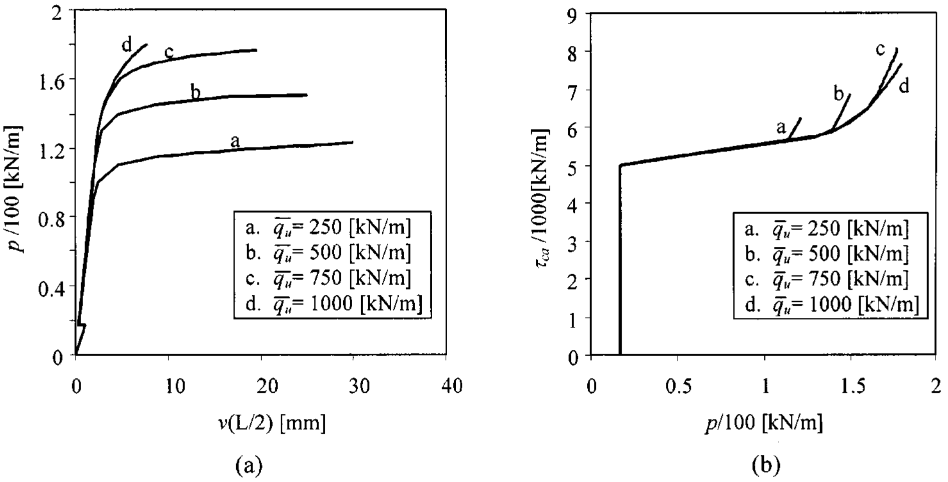

Fig. 7. Comparison of behaviour of beams with different strength of shear connections: (a) load versus deflection at midspan; (b) cable tension versus external load

portion of the beam.

Under the assumption of perfectly ductile connection, the influence of the shear connection strength $\bar{q}_u$ on the load capacity of the beam is shown in Fig. 6. It is evident that the ultimate load p_u grows almost linearly until $\bar{q}_u$ is about equal to 750 kN/m. Once this value is reached the ultimate load for the beam remains constant. In other words for values of $\bar{q}_u$ higher than 750 kN/m, the shear connection permits the development of the fully plastic bending moment at the midspan cross section (full composite action) so that the maximum load capacity of the beam is reached. On the other hand, for lower values of the shear connection strength failure occurs under sensibly lower loads without the formation of a classic plastic hinge at midspan (partial composite action).

This affects the overall behaviour of the beams as shown in Fig. 7a. After an initial common load-deflection path, characterised by discontinuity due to the application of prestressing force, the curves relative to weaker shear connections branch out from that of the strongest connection and after mild strain-hardening reach the state of collapse. Conversely, the beam with the strongest connection reaches its ultimate load, due to crushing of concrete, after a gradual reduction in stiffness. With $\bar{q}_u \geq 750$ kN/m, although the full plastic moment capacity is realised at the midspan section, the beam deflection and failure mechanisms at collapse are different. In the case of $\bar{q}_u = 750$ kN/m, the shear connectors are at their limit state along the whole beam. With $\bar{q}_u = 1000$ kN/m, the shear connectors reach the limit state in the cable anchorage region only. Fig. 7b shows the variation of the cable force with the external loads. As for the deflections, this parameter is also representative of the overall behaviour of the beam since, as already stated, under the assumption of no friction between the cables and the saddle points, the cable tension is constant along its length and depends on the deformation of the whole beam. The higher deflections of the composite beam due to the plasticisation of the shear connection produces a large increment of traction on the cable even if it does not reach the yield level in any of the cases considered. The prestressing force grows gradually showing linear behaviour and undergoes a sudden increment as the ultimate load is approached. At failure, the increment of the cable tension is important and, in the cases examined, it was found to reach about 60% of the initial tension.

The preceding results were obtained under the assumption of perfectly ductile shear connection in

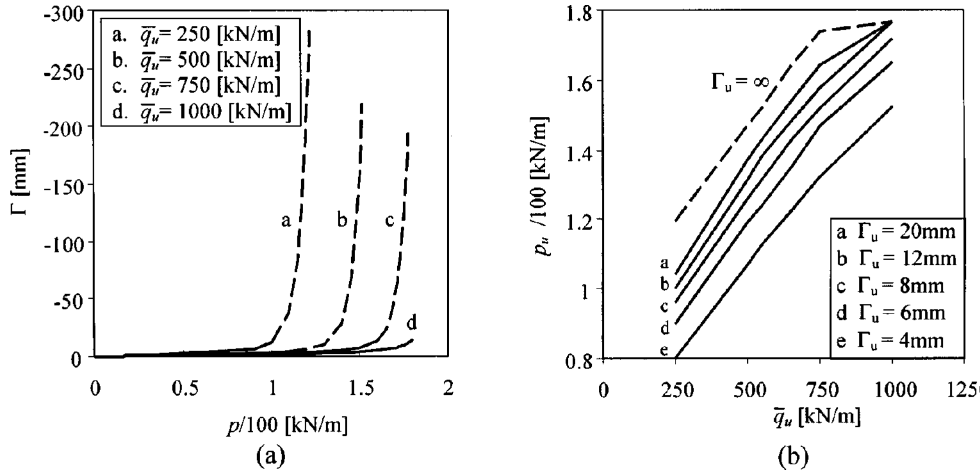

Fig. 8 Influence of shear connection with limited ductility: (a) interface slipping at beam end versus external load; (b) external ultimate load versus shear connection strength

order to permit the complete development of the failure mechanisms which can be used in a classic plastic analysis. Push-out tests demonstrate however that welded stud connections may undergo maximum slips no higher than 10 mm (Newmark *et al*. 1951, Ollgaard *et al*. 1971). From Fig. 8a, in which the evolution of the interface slipping at the beam ends versus the load level are plotted, it is evident that the slippage predicted by the analysis is so high that the results previously shown for shear connectors of lower strength seem almost meaningless. In other words, the ultimate loads previously shown are not true since they are limited by reaching of the ultimate slip of the shear connection which leads to non-ductile failure of the whole beam. Fig. 8b groups curves showing the importance of the shear connection ductility in the development of the failure mechanism. It is evident how, only for the strongest shear connection, it is possible to obtain the ultimate load level estimated under the assumption of perfectly ductile connection. For the minimum value of the shear connection strength that ensures the full connection (750 kN/m), the real value of the ultimate load can be dangerously overestimated. This problem is of particular importance in the case considered where the application of concentrated forces due to the prestressing cables induces large interface slips around the cable anchorages. In these cases, controlling interface slipping may become crucial in the design of the connectors, so that the classic plastic analysis, based on equilibrium conditions only, cannot be applied.

4. Conclusions

In this paper a model to investigate the non-linear behaviour of externally prestressed steel-concrete composite beams, taking into account the deformability of the shear connection, has been proposed. It is of general validity and can be applied to beams with various static schemes, each cable path and various construction sequences. By means of an incremental analysis, the model makes it possible to follow the stress-strain histories of all the elements of the beam under increasing loads and thus it is a powerful tool to numerically simulate failure modes of structures of this kind. Some applications to a simply supported beam permitted drawing a number of conclusions.

The evaluation of the shear flow on the beam-slab interface connection should be performed by a non-linear analysis since the elastic analysis overestimates the peaks arising around the cable anchorages. This is due to the non-linear behaviour of the connectors, particularly important even for low interface slip values, which implies a redistribution of the interface force.

The cable tension at ultimate load is very different from the initial value depending on the global ductility of the beam.

The beam failure modes are strongly influenced by the flexibility of the shear connection. Plastic failure mechanisms can develop only if the shear connection is very ductile and for usual connectors the ultimate load can be much lower than that provided by calculations under the assumption of perfectly ductile interface connection. This means that the concept of full composite action, as commonly understood, is not applicable to externally prestressed composite beams because it does not permit controlling the shear connection slippage responsible for overall beam failure.

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Appendix I

The expressions of the components of $\nabla \boldsymbol{a}$ are reported in the sequel.

- for i=1, …, I and j=1, …, I

$$[\nabla \boldsymbol{a}]_{ij}=\int_0^L \varphi_{ci}{}'\varphi_{cj}{}'\left[\int_{A_c}\left(\frac{dG_c(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_c-\varepsilon_{0c}} dA_c\right]+\varphi_{ci}\varphi_{cj}\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma-\Gamma_0} dz$$

- for i=1, …, I and j=I+1, …, I+J

$$[\nabla \boldsymbol{a}]_{ij}=[\nabla \boldsymbol{a}]_{ji}=\int_0^L -\varphi_{ci}\varphi_{sj}\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma-\Gamma_0} dz$$

- for i=1, …, I and j=I+J+1, …, I+J+K

$$[\nabla \boldsymbol{a}]_{ij}=[\nabla \boldsymbol{a}]_{ji}=\int_0^L \varphi_{ci}{}'\psi_j{}''\left[\int_{A_c} -\left(\frac{dG_c(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_c\text{-}\varepsilon_{0c}}(y-y_c)dA_c\right]-\varphi_{ci}\psi_j{}'h\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma\text{-}\Gamma_0}dz$$

- for $i=I+1, \ldots, I+J$ and $j=I+1, \ldots, I+J$

$$[\nabla \boldsymbol{a}]_{ij}=\int_0^L \varphi_{si}{}'\varphi_{sj}{}'\left[\int_{A_s}\left(\frac{dG_s(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_s\text{-}\varepsilon_{0s}}dA_s\right]+\varphi_{si}\varphi_{sj}\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma\text{-}\Gamma_0}dz$$

$$+\frac{A_{ca}}{\Lambda}\left(\frac{dG_{ca}(\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_{ca}\text{-}\varepsilon_{0ca}}\sum_{d=1}^{D}a_{dz}(\varphi_{sd_i}-\varphi_{sd-1_i})\sum_{d=1}^{D}a_{dz}(\varphi_{sd_j}-\varphi_{sd-1_j})$$

- for $i=I+1, \ldots, I+J$ and $j=I+J+1, \ldots, I+J+K$

$$[\nabla \boldsymbol{a}]_{ij}=[\nabla \boldsymbol{a}]_{ji}=\int_0^L \varphi_{si}{}'\psi_j{}''\left[\int_{A_s} -\left(\frac{dG_s(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_s\text{-}\varepsilon_{0s}}(y-y_s)dA_s\right]+\varphi_{si}\psi_j{}'h\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma\text{-}\Gamma_0}dz$$

$$+\frac{A_{ca}}{\Lambda}\left(\frac{dG_{ca}(\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_{ca}\text{-}\varepsilon_{0ca}}\sum_{d=1}^{D}a_{dz}(\varphi_{sd_i}-\varphi_{sd-1_i})$$

$$\sum_{d=1}^{D}a_{dy}(\psi_{d_j}-\psi_{d-1_j})+a_{dz}(-(y_d-y_s)\psi_{dj}{}'+(y_{d-1}-y_s)\psi_{d-1j}{}')$$

- for $i=I+J+1, \ldots, I+J+K$ and $j=I+J+1, \ldots, I+J+K$

$$[\nabla \boldsymbol{a}]_{ij}=\int_0^L \psi_i{}''\psi_j{}''\left[\int_{A_c}\left(\frac{dG_c(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_c\text{-}\varepsilon_{0c}}(y-y_c)^2dA_c+\int_{A_s}\left(\frac{dG_s(x,y;\varepsilon)}{d\varepsilon}\right)_{\varepsilon=\varepsilon_s\text{-}\varepsilon_{0s}}(y-y_s)^2dA_s\right]dz$$

$$+\int_0^L \psi_i{}'\psi_j{}'h^2\left(\frac{dG_{conn}(\gamma)}{d\gamma}\right)_{\gamma=\Gamma\text{-}\Gamma_0}dz+\frac{A_{ca}}{\Lambda}\left(\frac{dG_{ca}(\gamma)}{d\gamma}\right)_{\varepsilon=\varepsilon_{ca}\text{-}\varepsilon_{0ca}}$$

$$\sum_{d=1}^{D}a_{dy}(\psi_{d_i}-\psi_{d-1_i})+a_{dz}(-(y_d-y_s)\psi_{di}{}'+(y_{d-1}-y_s)\psi_{d-1_i}{}')$$

$$\sum_{d=1}^{D}a_{dy}(\psi_{d_j}-\psi_{d-1_j})+a_{dz}(-(y_d-y_s)\psi_{dj}{}'+(y_{d-1}-y_s)\psi_{d-1_j}{}')$$

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