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Behavior and modeling of single bolt lap-plate connections

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Abstract. A research investigation of single bolt lap-plate connection load-deformation behavior is presented. Each important characteristic of this behavior is evaluated and two methods for analytically approximating the behavior are developed and presented. The first of these methods is a component method in which the behavior of the connection is modeled as a combination of the behavior of the parts. The second method utilizes a number of parametric relationships that relate the connection parameters to coefficients of two non-linear continuous analytical curves. The test results from four independent experimental programs that investigated the behavior of single bolt lap-plate connections are used in the development and verification of these methods.

Key words: partially restrained connections; load vs slip behavior of bolt connect.

1. Background

The research reported in this paper was developed as part of a larger research program that investigated the behavior of partially-restrained (PR) composite connections. The primary hypothesis of the overall research program is that a PR connection can be modeled as a combination of connection components. One of the fundamental connection components is a high strength bolt in single shear. It has been assumed that the behavior of this component can be represented by the behavior of a single bolt lap-plate connection. This assumption has been shown to be generally valid in a separate research investigation (Rex and Easterling 1996c). A schematic of a PR composite beam-girder connection and the associated model components are illustrated in Fig. 1.

The research presented in this paper is a summary of a detailed study of single bolt lap-plate connection behavior. Rex and Easterling (1996b) report the details of the full study. The report contains an appendix of all the experimental data considered in the study.

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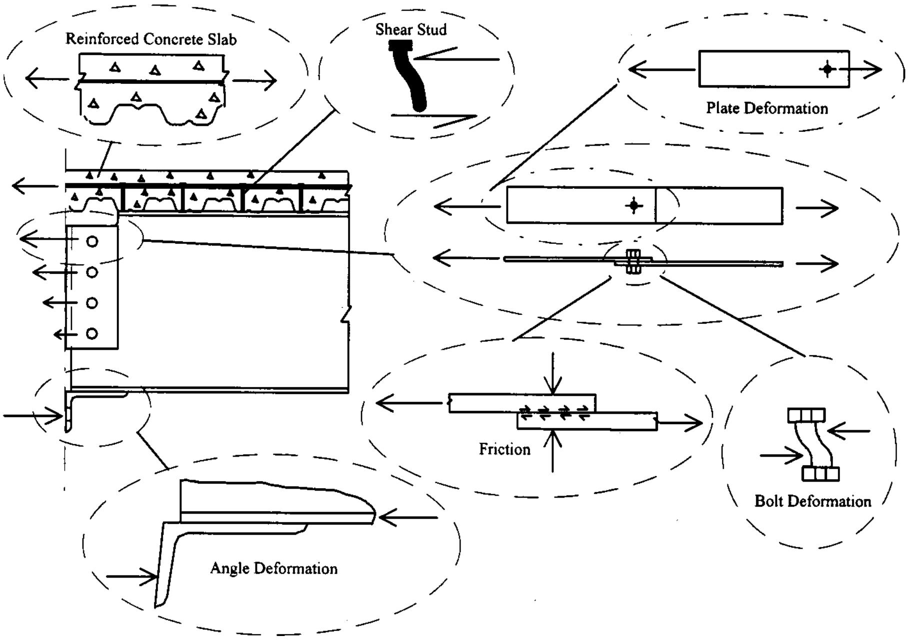

Fig. 1 Primary components of proposed beam-girder connection

2. Introduction

2.1. Component model

A study of the load-deformation behavior of single bolt lap-plate connections is presented in this paper. The hypothesis of this study is that this load-deformation behavior can be modeled as a combination of three, more fundamental, component load-deformation behaviors: plate friction, plate bearing, and bolt shear. This concept is shown schematically in Fig. 2. Once these three component behaviors can be modeled, then the overall lap-plate connection behavior can be modeled using springs to represent the components. These springs are combined in series and in parallel as shown in Fig. 3. This method of modeling connection behavior is known as a component model.

2.2. Objectives and methods

The objective of the research on single bolt lap-plate connections is to be able to model the load-deformation behavior of these connections. Two methods of modeling this behavior are developed. These methods are the Component Model (discussed above) and a Parametric Model. To develop these methods, the following research was conducted:

1. All readily available experimental data for single bolt lap-plate connections was collected and evaluated.
2. A plate bearing load-deformation behavior model was developed.
3. A bolt shear load-deformation model was developed.
4. A plate friction load-deformation model was developed.

The above research provided the behavior models needed to complete the component modeling

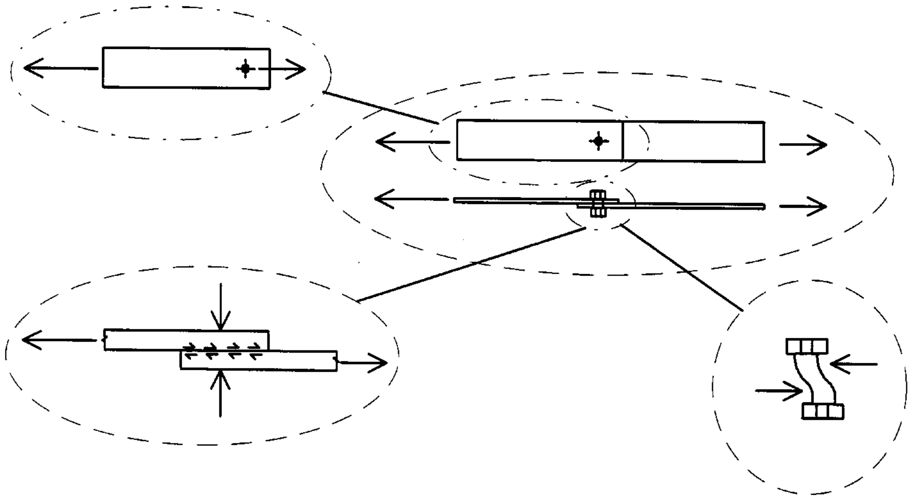

Fig. 2 Primary components of a single bolt lap-plate connection

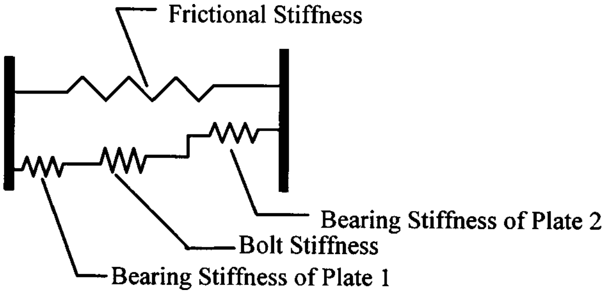

Fig. 3 Component model of single bolt lap-plate connection

method. The experimental test data was used to develop the parametric model. Finally, comparisons between the experimental results and the two methods developed here along with existing methods for modeling the single bolt lap-plate load-deformation behavior are presented.

3. Experimental data for single bolt lap-plate connections

Four experimental investigations of single bolt lap-plate connection tests were found in the literature. The experimental data, including the geometric and material parameters and the raw load-deformation data from these experimental investigations, was compiled and input into a commercial database program for analysis.

A schematic of a single bolt lap-plate connection test specimen is shown in Fig. 4. The method in which the load was applied to the plates and the method in which deformation was measured varied depending on the experimental investigator. Generally, the free ends of the connection were bolted to a testing assembly that was placed in a universal testing machine to apply the load. The deformation was measured as the change in the distance from a fixed point on one plate to a fixed point on the opposite plate.

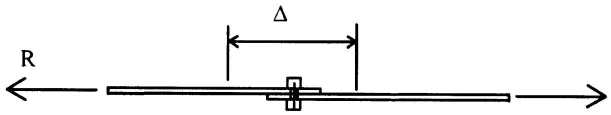

Fig. 4 Typical single bolt lap plate connection test

The following sections present a brief summary of each of the four experimental investigations. In some cases it was necessary to make assumptions about and adjustments to the data so that the tests could be included in the analysis. The assumptions and adjustments made are also discussed briefly in the following sections.

3.1. Lap-plate connection tests reported by Karsu (1995)

Karsu (1995) reported a total of 61 lap-plate connection tests. Each test actually consisted of two lap-plate connections that were pulled at the same time. The average load and deformation measurements for the two connections were used.

Parameters varied in the experimental study included bolt diameter, plate thickness of both plates in the connection, bolt end distance, and plate edge condition. All bolt holes were drilled. Washers were placed under both the nut and bolt head. Electronic potentiometers were used to measure the deformation.

The test specimens were assembled and put into a testing rig. The bolts were tightened to the snug-tight condition and then a pre-load was applied to the specimen. While the pre-load was applied the bolts were fully tensioned by turn-of-nut. The pre-load was then removed and the test was started from zero load. This process was intended to eliminate any sudden slips in the connection during the test.

The deformations of interest in this research are the local bolt and plate deformations and not the overall elastic plate deformations between points of measurement. The data reported by Karsu (1995) included elastic deformations. Consequently, a method of estimating these deformations was developed and they were removed from the data.

3.2. Lap-plate connection tests reported by Gillett (1978)

Gillett (1978) reported a total of 75 lap-plate connection tests. Load-deformation data was available for only 66 of these tests. Parameters varied in the experimental study included bolt grade and diameter, steel grade, plate thickness, and end distance.

A local steel fabricator provided the fabricated test plates. The plates were sheared and the bolt holes were punched to standard sizes. Two dial gages were used to measure deformations, one in front and one in back of the specimen.

The test specimens were assembled and put into the testing rig. The bolts were tightened to the snug-tight condition and then a pre-load of 5 kips was applied to the specimen. While the pre-load was applied the bolts were fully tensioned by turn-of-nut. The pre-load was then removed and the test was started from zero load. This process eliminated any sudden slips in the connection during the test.

Three assumptions about this testing program have been made so that the tests could be included in the analysis. First, no material properties were given for the 5/8-in. thick plates used in the test program. It was assumed that the steel properties of these plates were consistent with other A36 steel properties given in the report and the average of the A36 steel properties given was used.

Second, in some cases the mode of failure was not clear. The mode of failure for a group of tests was reported rather than for the individual tests. In some cases, two modes of failure were indicated for the same group of tests. In these cases, a failure mode was assumed based on the options given for the group and a comparison of the load-deformation behaviors for the tests in the group.

Lastly, it was assumed that the bolt threads were excluded from the shearing plane. This was based on a comparison of the expected bolt shearing load to the test load reported.

3.3. Lap-plate connection tests reported by Caccavale (1975)

Caccavale (1975) reported 11 lap-plate connection tests. The plate thickness was the only parameter varied. All bolt holes were drilled and washers were placed under the nuts of the bolts.

The author did not specifically report failure modes of specimens. However, the author indicates "The test results show that under these conditions no visible mark of shear deformation occur in the bolt." This would tend to indicate some sort of plate failure. For analysis purposes it was assumed that plate bearing/tearout failures occurred.

Because of the way that deformations were measured in these tests, it is highly likely that the initial deformation readings included test setup deformations that were not intended to be measured. Because of the uncertainty of the measurement, only the strength values from this data are included in subsequent development and verification work.

3.4. Lap-plate connection tests reported by Sarkar and Wallace (1992)

Sarkar and Wallace (1992) reported 16 lap-plate connection tests. Parameters that were varied included the bolt type, plate thickness and end distance. All bolt holes were drilled. The report did not indicate how the bolts had been tightened. Based on a comparison to the test data from Karsu (1995) and Gillett (1978) it is believed that the bolts were only tightened to the snug condition and the tests are treated as such for analysis purposes in this report.

4. Plate bearing behavior model

4.1. Existing models

A previous research investigation of the load-deformation behavior of a single plate bearing on a single bolt was conducted (Rex and Easterling 1996a, 2003).

$$\frac{R}{R_n} = \frac{1.74\bar{\Delta}}{(1 + \bar{\Delta}^{0.5})^2} - 0.009\bar{\Delta} \tag{1}$$

Where:

R = Plate Load

R_n = Nominal Plate Strength = $L_e\, t_p\, F_u \leq 2.4\, d_b\, t_p\, F_u$ (1993)

$\bar{\Delta}$ = Normalized Hole Elongation = $\Delta\, \beta\, K_i / R_n$

Δ = Hole Elongation

β = Steel Correction Factor = 30%/%Elongation (for typical steels taken as 1.0)

K_i = Initial stiffness given by

$$K_i = \frac{1}{\frac{1}{K_{br}} + \frac{1}{K_b} + \frac{1}{K_v}} \tag{2}$$

Where:

K_{br} = Bearing stiffness = 120 $F_y\ t_p\ d_b^{0.8}$ (units are kips and inches)
K_b = Bending stiffness = 32 $E\ t_p\ (L_e / d_b - 0.5)^3$
K_v = Shearing stiffness = 6.67 $G\ t_p\ (L_e / d_b - 0.5)$
d_b = Bolt diameter
t_p = Plate thickness
L_e = Distance from the centerline of the bolt to the end of the plate

To develop the normalized behavior, the data from tests conducted at *VT* that failed by bearing, tearout, or splitting was normalized by the maximum load for the test. Based on this normalized data, a method of normalizing the test deformations was then developed. After the load and deformation values were normalized, non-linear regression was used to fit the Richard Equation (Richard and Elsalti 1991) to the data. The resulting relationship is given by Eq. (1). Additional background on the development is presented by Rex and Easterling (1996a, 2003).

4.2. Evaluation of plate strength

A comparison of test load to predicted load is presented in Table 1. The test load is defined as the load when the specimen failed or when the test was stopped. The predicted load is based on the AISC

Table 1 Plate test strength to predicted strength

	Average	COV	No.
All Plate Failures			
All Researchers	1.02	12.0%	85
Gillett	0.97	14.0%	36
Karsu	1.07	7.8%	36
Caccavale	1.07	8.9%	11
Sarkar and Wallace	0.88	5.1%	2
Bearing/Tearout Failures			
All Researchers	1.06	10.0%	43
Gillett	0.98	12.1%	10
Karsu	1.09	8.3%	22
Caccavale	1.07	8.9%	11
Sarkar and Wallace	-	-	-
Splitting Failures			
All Researchers	0.99	13.1%	42
Gillett	0.96	14.8%	26
Karsu	1.06	6.9%	14
Caccavale	-	-	-
Sarkar and Wallace	0.88	5.1%	2

Specification (*Load and* 1993). Only tests that failed by bearing, tearout, or splitting were considered.

The following observations are made based on review of Table 1.

- Bearing / Tearout failures had mean strengths about 7% higher than splitting failures. This is partly attributable to the very low test strengths reported by Sarkar and Wallace (1992).
- Evaluation of bearing/tearout failures shows the predicted strength is about 6% conservative. A comparison of the specification equation to bearing/tearout strength for single bolt single plate specimens showed an average ratio of 0.998 (Rex and Easterling 1996a). This may be an indication of a slight increase in plate strength associated with the single bolt lap-plate connections compared to the single plate single bolt type specimens. One possible reason for the increased strength is the confinement of the steel in front of the bolt provided by the bolt nut and head and washers if present. Another reason may be that some load was being carried by friction between the two plates which leads to calculated bearing stresses higher than the real bearing stresses (Fisher and Struik 1974). However, it should be noted that the upper limit on the bearing stress was shown by Perry (1981) to be unaffected by bolt tension.
- Considering all the researchers data, there is good correlation between the test load and the predicted load.
- Most of the variation results from the tests conducted by Gillett (1978).
- Splitting failures are physically very different than tearout failures. The current expression given in the AISC Specification (*Load and* 1993) is based on the physical behavior associated with tearout. Despite this, the expression appears to correlate very well with the test strengths associated with splitting failures as well.

The only general conclusion that can be made based on this evaluation is that the current expression given in the AISC Specification (*Load and* 1993) for determining tearout strength correlates well with all the test data considered.

4.3. Failure deformation

The previous study (Rex and Easterling 1996a) had not developed a method for predicting the plate deformation at failure. Using the normalized load-deformation behavior given above and the experimental test data, an approximate plate failure deformation was determined. In normalized form, this deformation is given as:

$$\bar{\Delta}_f = 22.87 \tag{3}$$

Where:

$\bar{\Delta}_f$ = Normalized Hole Elongation at Plate Failure = $\Delta_f \, \beta \, K_i / R_n$

5. Bolt shearing behavior model

5.1. Bolt shear strength

There are basically five bolt shear strength models that have been recommended over the last 30 years. These models were primarily developed from bolt shear tests where the bolt was in double shear. The basic differences in these models lie in the value of the ultimate shearing stress and the value of the

Table 2 Evaluation of bolt shear strength models

Model	F_{vb}/F_{ub}	A_{bv}/A_b	Average Test/Predict	COV
Fisher and Struik (1974)	0.62	0.75	1.05	12%
Fisher *et al.* (1978)	0.75/0.67*	**	0.89	12%
Load and (1986)	0.60	0.75	1.09	12%
Kulak *et al.* (1987)	0.62	0.70	1.07	14%
Load and (1993)	0.50	0.80	1.28	11%

*A325 / A490 Bolts
** Was not stated. Assumed to be 0.70 for evaluation purposes.

root area in the threaded portion of the bolt. Calculations based on these models are summarized in Table 2 along with statistical results from a comparison of test to predicted strength. There were a total of 71 single bolt lap-plate tests that failed by bolt shear.

Considering the ratio of test strength-to-predicted strength, all the models except Fisher *et al.* (1978) were conservative. The AISC Specification (*Load and* 1993) is purposefully conservative when applied to single bolts because the new model assumes long joint behavior that is generally less efficient than a single bolt joint. Overall, the model suggested by Fisher and Struik (1974) compared the best but, predictions using the AISC Specification (*Load and* 1986) are also satisfactory. The coefficient of variation (COV) for all the models are all slightly higher than the expected COV of 10% (Fisher *et al.* 1978).

Based on the above comparison, it appears that the average shearing strength of the bolts in the single bolt lap connections is slightly higher than would be expected based on equations developed from double shear bolt tests. One possible reason for this is an inclined shearing angle. A visual inspection of the bolts that sheared in the tests reported by Karsu (1995) showed that the shearing angle was inclined, similar to the tests reported by Munse, *et al.* (1954). A second possible reason for the increased load may be frictional forces between the plates resulting from tension in the bolt. The bolt tension could be a result of the original pre-tensioning or the result of prying forces developed by the deforming plates or some combination of these two.

Without additional testing and analysis, trying to include either of these possible effects to increase the bolt load capacity does not seem justifiable at this time. In general, it is believed that the model given in AISC Specification (*Load and* 1986) is sufficiently accurate.

5.2. Characterization of load-deformation behavior

Wallaert and Fisher (1965) conducted 174 elemental bolt shear tests. Single bolts were tested in double shear. Fisher (1965) developed the following expression to represent the load-deformation behavior of the bolt shear tests conducted by Wallaert and Fisher (1965):

$$R = R_{ult}\,[1 - e^{-\mu\Delta}]^{\lambda} \tag{4}$$

The equation parameters R_{ult}, μ, and λ were determined for a number of the bolt shear tests and these values were reported in Fisher (1965). The author recognized that R_{ult} corresponded well with the bolt shearing strength. The author also recognized that the parameter μ was primarily influenced by the type of connected material and that λ was basically unaffected by the type of connected material. It is believed that Eq. (4) can sufficiently approximate isolated bolt load-deformation behavior. The specific values of λ and μ are developed in the following section.

5.3. Equation parameters

Wallaert and Fisher (1965) reported tests that had bolts that were only tightened to the snug condition. The load-deformation behavior of these tests should be primarily comprised of the bolt and plate behavior (i.e. little if any influence by friction). The load vs. deformation for one of the tests with a snug-tight bolt was plotted using the curve parameters reported by Fisher (1965). Next, the plate load-deformation behavior was determined using the component behavior model previously discussed. The estimated plate deformations were subtracted from the test deformations (assumed given by the curve parameters) for each load. The remaining load-deformation behavior was then assumed to be that associated with the bolt alone. Based on a non-linear regression analysis of this load-deformation behavior, it was determined that a value of μ of approximately 34 and a value of λ of 1.0 seemed appropriate.

When λ has a value of 1.0 it can be shown that μ is a scaling factor for the initial stiffness of the load-deformation response (i.e., the initial stiffness is μ times the bolt strength R_{ult}). EC3 Annex J (1994) gives an estimate for initial bolt stiffness for a snug tight bolt in single shear. The exact expression given in EC3 Annex J (1994) can be rearranged in terms of bolt shear strength. When this is done the scaling factor for the initial stiffness is given as 52.2.

Based on the analysis of the tests reported by Fisher (1965) and the initial stiffness given by EC3 Annex J (1994), it appears that a value of μ somewhere between 34 and 52.2 and a value of λ of 1.0 is justifiable. To determine the most appropriate value of μ, the single bolt lap-plate connection tests with snug tight bolts were evaluated.

The final value of μ was determined by calibrating the predicted load-deformation behavior for single bolt lap-plate connections against the test data reported by Sarkar and Wallace (1992). The load-deformation response for each of these tests was simulated using the plate-bolt-plate springs in series. The plate spring behaviors were approximated using the plate behavior model discussed previously and the bolt spring behavior was approximated using Eq. (4) with R_{ult} equal to the test strength and λ equal to 1.0. The best value of μ was then determined through numerical analysis. Based on this analysis, a final value of μ equal to 50 seemed most appropriate and agrees well with the value derived from EC3 Annex J (1994).

5.4. Failure deformation

The last step in characterizing the bolt component behavior was to determine the deformation in the bolt at failure. First, the failure deformations of the test specimens reported by Wallaert and Fisher (1965) with A514 steel plates were considered. It was assumed that the majority of the deformation at failure in these specimens was deformation in the bolt and not in the plate bearing (because of the extremely high plate strength). Based on the results of these tests, the bolt deformation at failure was found to be approximately 1/8-in. Measurements were made on sheared bolts from the tests conducted by Karsu (1995). These measurements confirmed that for A325 bolts a bolt deformation of about 1/8-in. at failure is a reasonable value.

6. Plate friction load-deformation behavior

Frank and Yura (1981) conducted 77 elemental slip tests using steel plates with blasted surfaces and single bolts in double shear. A special test setup that insured the only resistance to load was the frictional resistance between the plates was used. A typical load-deformation response has been reproduced from Frank and Yura (1981) and is presented in Fig. 5.

There are two important observations based on the test behavior presented in Fig. 5. First, the test specimen exhibited a linear behavior up to very near the slipping load. Second, after the slipping load was reached the load resistance degraded with increased slip. Based on these observations it appears that there are three characteristic stages of behavior associated with the load-slip response: initial stiffness, slip load, and post slip behavior. The only literature identified deals with the slip load and is discussed in the following section.

6.1. Existing methods for predicting slip load

Fisher *et al*. (1978) performed a statistical study of the slip resistance associated with the use of high strength bolts. The results showed that the average slip resistance of a high strength bolt with a single shear plane in mild steels with clean mill scale surfaces and where the bolts had been tightened by turn-of-nut method is given by

$$R_n = \alpha A_{bt} F_{ub} \tag{5}$$

Where α was 0.33 and 0.29 for A325 and A490 bolts respectively. The COV was determined as 24% for both A325 and A490 bolts. A_{bt} is the tension area of a bolt usually taken as 75% of the gross area of the bolt "A_b".

6.2. Quantification of characteristic behavior based on test results

The frictional behavior for each of the single bolt lap-plate tests with fully tensioned bolts reported by

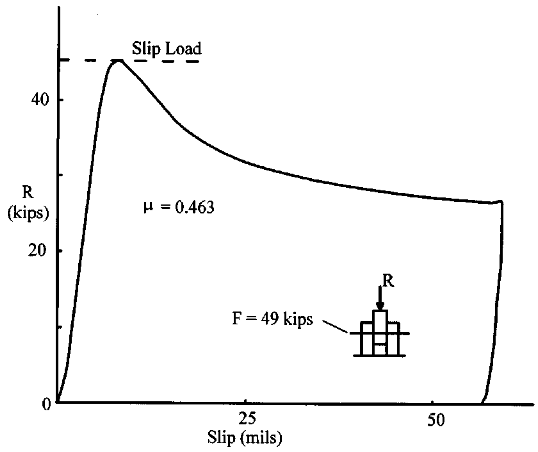

Fig. 5 Frictional load-slip behavior (Frank and Yura 1981)

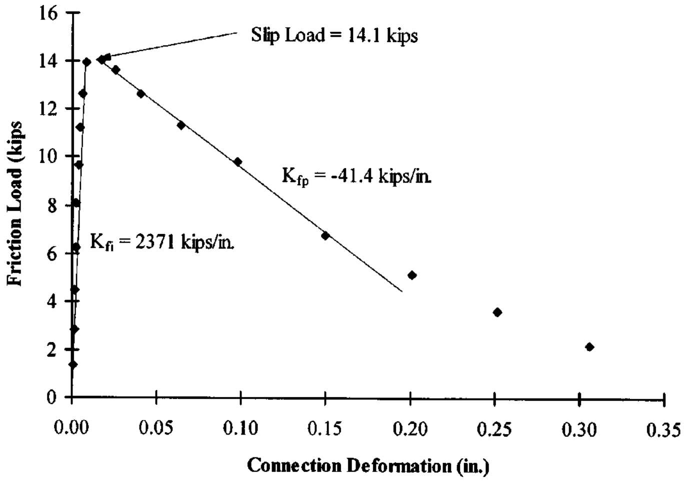

Fig. 6 Experimental friction load-deformation behavior test 4 (Gillett 1978)

Karsu (1995) and Gillett (1978) was determined. This was done by approximating the plate-bolt-plate behavior with the plate and bolt models developed previously. This approximate plate-bolt-plate behavior was subtracted from the test behavior. It is assumed that the remaining load-deformation behavior is the frictional behavior. An example of this behavior is shown in Fig. 6.

The basic shape of the load-deformation response in Fig. 6 is similar to that reported by Frank and Yura (1981). The only significant difference between the above isolated behavior and that reported by Frank and Yura (1981) is that the post slip load resistance continues to degrade until there is little or no frictional load transfer.

Based on the above results, a bi-linear model of the frictional behavior has been developed. This model is shown graphically in Fig. 7. Values of the initial stiffness (K_{fi}), the post stiffness (K_{fp}), and the slip load (R_f) determined from the test data were used to develop equations to predict these quantities. First, based on a combination of the AISC Specification (*Load and* 1993) requirements for bolt tightening and the recommended coefficients for A325 and A490 bolts given by Fisher *et al.* (1978), the following expression for the slip load was derived.

$$R_f = \alpha\,(0.7\,F_{ub})\,(0.75\,A_b)\,\mu \tag{6}$$

where

α = 1.0 for A325 bolts and 0.88 for A490 bolts

μ = Friction coefficient (0.33 for clean mill scale surfaces)

A comparison of slip loads based on the test data with predicted slip loads based on Eq. (6) gives an average value of 1.09 with a COV of 22%. This value of the COV is large; however, it is consistent with the value reported by Fisher *et al.* (1978).

Second, the deformation when slip started to occur was determined to have an average value 0.0076-in. with a COV of 47%. The initial frictional stiffness (K_{fi}) is determined by dividing R_f by 0.0076-in.

Third, the post slip stiffness (K_{fp}) was related to the combined thickness of t_1 and t_2. This relationship is best represented by determining the deformation at which the frictional resistance could be assumed

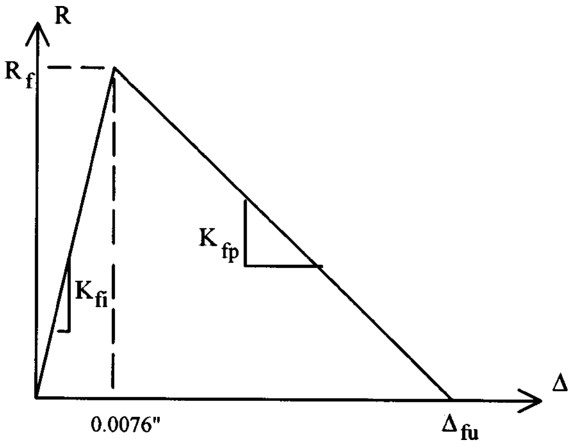

Fig. 7 Bi-linear representation of friction load-deformation behavior

to be zero (Δ_{fu}).

$$\Delta_{fu}\left|\begin{array}{ll} (t_1 + t_2) < 0.5'' & \Delta_{fu} = 0.4'' \\ 0.5'' \le (t_1 + t_2) \le 0.5'' & \Delta_{fu} = 0.4'' - (t_1 + t_2 - 0.5)0.3 \\ 1.5''(t_1 + t_2) & \Delta_{fu} = 0.1'' \end{array}\right. \quad (7)$$

The post slip stiffness is then approximated by dividing R_f by Δ_{fu}.

7. Parametric model of lap-plate load-deformation behavior

The basic connection components required to implement the component model of a single bolt lap-plate connection were developed in the previous sections. The component model is ideally applicable to a broad spectrum of connection parameters and is not, in general, restricted to the range of parameters for which there are complete connection tests. The modeling is limited by the individual component parameters, particularly if empirical relationships are used.

Parametric equations are typically easy to use but are limited to the range of parameters tested. However, given the large number of tests collected in this report and the wide range and number of parameters included in the tests, the development of parametric equations seems like a reasonable way of providing a second method by which the load-deformation behavior can be approximated. Because the majority of the test data collected was for tests with fully tensioned bolts, parameter equations are only developed for connections with fully tensioned bolts. Snug-tight bolts are generally recommended for use in shear connections because of the added cost of fully tightening the bolts, given that the additional bolt tension does not enhance the ultimate strength of the bolt. However, the authors feel that the benefits of using PR connection component stiffness to minimize beam deflections due to concrete placement offsets the additional cost of fully tightening the bolts.

The simplest method of representing the non-linear load-deformation behavior of the single bolt lap-plate connections is with a continuous non-linear parametric equation. The Richard Equation was chosen for this application (Richard and Elsalti 1991). A graphical representation of the Richard

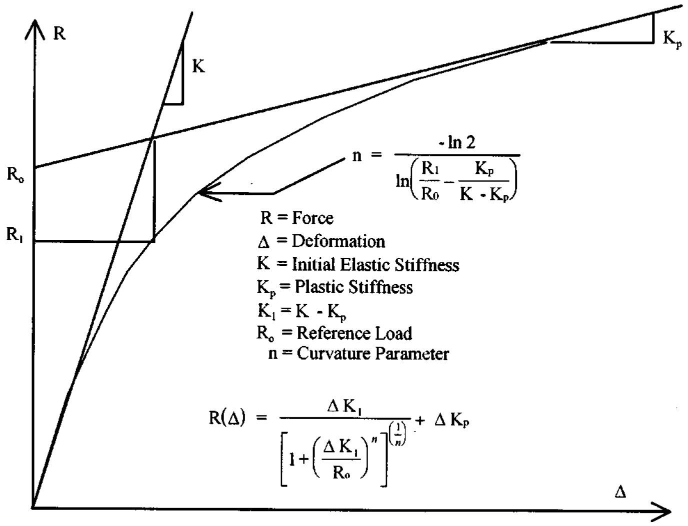

Fig. 8 The Richard equation (Rrchard and Elsalti 1991)

Equation along with definitions of the equation parameters is presented in Fig. 8. To determine relationships between the connection parameters and the equation parameters, a detailed graphical and numerical study of the test data was conducted. Based on this study the following relationships were determined (note: all units are in kips and inches):

$$R_n = R_{np} \leq R_{nb} \tag{8}$$

$$K = 5751\ t_1 d_b + 1213 \tag{9}$$

$$K_p = 9\ \{R_{np} / R_{nb}\}^{2.9} \tag{10}$$

$$R_{transition} = 0.14\ F_{ub}\ d_b^2 + 12 t_1 / d_b \leq R_n \tag{11}$$

$$R_o = R_n - 0.25\ K_p \geq R_{transition} \tag{12}$$

$$R_1 = R_{transition}\ (t_2 / t_1)^{0.1} \leq 0.98\ R_o \tag{13}$$

$$n = \frac{-\ln(2)}{\ln\left(\dfrac{R_1}{R_o} - \dfrac{K_p}{K - K_p}\right)} \leq 3 \tag{14}$$

Where:

R_{np} = The lowest plate strength of the two plates
R_{nb} = Bolt strength
t_1 = Thickness of the thinner plate
t_2 = Thickness of the thinner plate

In the above equations, upper and lower bounds have been placed on some of the load constants to avoid having predicted loads above the nominal strength of the connection (i.e., the increased strength over the plate strength resulting from friction, which was seen for thin plate combinations, is ignored). In addition, only positive plastic slopes are assumed.

8. Evaluation of load-deformation models

In the previous sections of this paper a component model and a parametric model of a single bolt lap-plate connection were developed. In this section previously existing models for predicting the load-deformation behavior are presented. This is followed by a numerical evaluation of the accuracy with which each model is able to predict the experimental load-deformation results.

8.1. Existing models

A model for the load-deformation behavior of high strength bolts is given in the AISC Manual Vol. II (*Manual of* 1994). This model is used for determining the strength of eccentric loaded bolted connections and is given by:

$$R = R_{ult}(1 - e^{-\mu\Delta})^{\lambda} \tag{15}$$

where:
μ = 10
λ = 0.55
R_{ult} = Bolt strength
e = Base of natural logarithm

Eq. (15) was developed by Fisher (1965) and will be referred to as the Fisher Equation from here on. The values of the coefficients were determined by Crawford and Kulak (1971) based on six identical elemental bolt tests.

A second model was developed in Karsu (1995). This model uses the Richard Equation (Richard and Elsalti 1991) with four different sets of equation parameter coefficients. The coefficient values depended on the plate thickness of the thinner plate in the connection (t_1) and/or whether bolt or plate failure occurred. The coefficient values are summarized in Table 3. These coefficients are based on data that was normalized by the test strength; consequently, it is necessary to multiply the resulting value from the Richard Equation by the plate or bolt strength to obtain the estimated load.

8.2. Benchmarks for evaluation of models

In the following section, each of the methods for approximating the load-deformation behavior of single bolt lap-plate connections are evaluated against the test data. This evaluation is made by using each method to calculate the connection load at each experimental load-deformation point. The ratio of

Table 3 Normalized richard equation coefficients (Karsu 1995)

Failure & Plate Thickness	K	K_p	R_o	n
Plate Failure				
t_1 = 0.125-in.	25.42	-0.2260	1.234	1.56
t_1 = 0.25-in.	20.34	-0.0286	1.070	1.11
t_1 = 0.375-in.	20.14	0.0368	1.020	1.11
Bolt Failure	26.30	0.0610	1.130	0.66

the test load over the predicted load was then determined. The COV as well as the L_2 Norm were then determined for each group of data. The L_2 Norm is the square root of the sum of the squares of the test load minus the predicted load for a given deformation point.

When evaluating large sets of test data in this manner described above, it is difficult to interpret how well one method works when one considers the natural scatter that is inherent in experimental test data. To provide some basis of comparison, two different benchmark evaluations were conducted to determine variations and norms by which the results of the other methods could be compared.

It is assumed that the best any method could come to approximating the load-deformation behavior is if that method were able to predict the coefficients, for either the Richard Equation or Fisher Equation, that would minimize the L_2 Norm for each group of identical tests. The first benchmark is based on this assumption. Non-linear regression was used to determine the best (minimize the L_2 Norm) equation coefficients for the Richard and Fisher Equations for each group of identical tests. Next, using these coefficients, the load at each test deformation was calculated. These loads were then compared to the test loads to determine ratio of test over predicted and values of the COV and the L_2 Norm. This first benchmark will be referred to as Benchmark Level 1.

The second benchmark makes some adjustment to include the inaccuracies in predicting connection strength. The assumption is that if the basic shape of the load-deformation curve is correct but the calculated connection strength is wrong, then the variation and norm values will be larger than if the connection strength had been accurately estimated. To determine what part of the variation and norm values are attributable to inaccurate strength predictions, a second set of benchmark values were calculated. These were determined by multiplying the original estimated loads (at each test deformation from the Benchmark Level 1 study) by the ratio of connection predicted/test strength. These new values were then evaluated to determine revised COV and L_2 Norm values. The second benchmark is referred to as Benchmark Level 2.

8.3. Evaluation of models

The component, parametric, and existing models for predicting the load-deformation behavior of a single bolt lap-plate connection were evaluated against the experimental load-deformation data as described above. The average value of test load over predicted load, COV, and L_2 Norm for each method are presented in Table 4. Because of the way the benchmarks were determined, the models are grouped under Richard Equation methods or Fisher Equation methods with the exception of the component model, which does not use a continuous non-linear analytical curve.

In general, the component method does the best job of predicting the load-deformation behavior and has values of the COV and L_2 Norm that are in the same range as the Benchmark Level 2 values (using the Richard Equation methods). The parametric method also provides good estimates with less complexity than the component method. The Fisher Equation methods generally had higher values of the COV and L_2 Norm because the equation lacks the ability to model the descending branch of the load-deformation behavior that was prevalent in thin plate combinations.

9. Evaluation of deformation at failure

It is important to be able to estimate the connection deformation at failure. This is a primary measure of the overall ductility of the connection. An evaluation of how well the component method was able to

Table 4 Evaluation of load-deformation models (ratio of test over predicted)

Method	Fully Tensioned Bolts			Sung Tight Bolts		
	Average	COV	L_2 Norm(kips)	Average	COV	L_2 Norm(kips)
Component Method	1.02	20%	140	0.92	25%	43
Richard Equation Methods						
Parametric	1.09	21%	167	-	-	-
Karsu Unified Curves	1.20	30%	231	-	-	-
Benchmark Level 1	0.99	11%	52	-	-	-
Benchmark Level 2	1.04	16%	160	-	-	-
Fisher Equation Methods						
AISC Vol II	1.67	41%	339	0.88	31%	63
Benchmark Level 1	0.96	23%	85	1.00	14%	13
Benchmark Level 2	0.99	31%	189	1.13	17%	40

predict the deformation at connection failure is presented in the following sections.

9.1. Data for evaluation

The deformation at failure was determined for each set of test data reported by Karsu (1995), Gillett (1978), and Sarkar and Wallace (1992). The deformation at failure was defined as the test deformation just prior to a significant loss in load carrying capacity resulting from a plate or bolt failure.

When a bolt failure occurs the deformation at failure is easily defined. However, when plate failure occurs the deformation at failure is less easily defined because of the long plastic plateaus. In addition, many of the tests were stopped before any reduction in load carrying capacity was observed. These tests do not provide useful data for evaluating the deformation at failure. Consequently, it is convenient to separate the tests into bolt failures and plate failures. When considering bolt failures, all of the failure deformations were used in the evaluation and development of models. When considering plate failures, only the tests with bearing/tearout or splitting failures were considered. Of these test results, only the tests reported by Karsu (1995) were used. This is because all but two of the tests reported by Sarkar and Wallace (1992) failed by bolt shear and the tests reported by Gillett (1978) were typically stopped at a deformation limit of around 0.3-in. In many of the tests reported by Gillett (1978) it is believed that additional deformations could have been sustained without a significant loss of load capacity.

9.2. Component model prediction of deformation at failure

The component model can be used to determine the deformation at failure by combining the deformations of the plate-bolt-plate spring series. This is done by pre-determining which of the elements will control the strength of the series. The failure deformation of the controlling element is then determined based on the behavior models developed previously. The deformation in the remaining elements can then be determined by back-substituting the failure load into the behavior models. The deformation of all three elements is then combined to provide an estimate of the connection failure deformation.

9.3. EValuation of component model failure deformation predictions

Failure deformations for tests failing by plate bearing/tearout or splitting and reported by Karsu (1995) were calculated using the component model. The average test over predicted ratio for the component model was 1.23 with a COV of 23%. Failure deformations for tests failing by bolt shear and reported by Karsu (1995), Gillett (1978), and by Sarkar and Wallace (1992) were calculated using the component model. The average test over predicted ratio for the component model was 1.06 with a COV of 47%.

10. Conclusions

10.1. Summary

The objective of the research presented in this paper was to develop two models for approximating the load-deformation behavior of a single bolt lap-plate connection: a component model and a parametric model.

Data from four independent testing programs that studied single bolt lap-plate connections was collected. This data was then used to develop and or evaluate behavior models for plate bearing, bolt shearing and plate friction. These behavior models were then combined in a component model to predict the load-deformation behavior of the single bolt lap-plate connection. The experimental test data was also used to develop a less general, but simpler parametric model of the load-deformation behavior of the single bolt lap-plate connection.

The two component and parametric models along with existing models for predicting the load-deformation behavior of the single bolt lap-plate connection were evaluated against the experimental test data for accuracy and precision.

10.2. Conclusions

An existing plate bearing behavior model was evaluated. This evaluation showed that using the bearing/tearout strength based on the AISC Specification (*Load and* 1993) provided an accurate and reasonably precise estimate of the experimental strength of connections that had plate failures.

Five existing methods for estimating the bolt shear strength of a high strength bolt in single shear were evaluated. Based on this evaluation, the bolt shear strength values based on the AISC Specification (*Load and* 1986) provide an accurate; however, slightly conservative estimate of the experimental bolt shear strength values. Strength estimates were improved by using an ultimate bolt shear strength to tensile strength ratio of 0.62 as recommended by Fisher and Struik (1974). This ratio is slightly higher than the 0.60 value used in the AISC Specification (*Load and* 1986)**.** An existing bolt shear load-deformation equation developed by Fisher (1965) was shown to provide a reasonable estimate for the shape of the load-deformation test data when equation parameters based on linear regression analysis and similar to those recommended by EC3 Annex J (1994) are used. Bolt shear failure was found to occur at an average bolt shear deformation of 1/8-inch.

An evaluation of the plate friction behavior showed that a bi-linear load-deformation model provided a reasonable estimate of this behavior. The slip load was found to be consistent with the load prediction based on the AISC Specification (*Load and* 1993) and slip occurred at an average deformation of 0.0076-inches. After slip occurred, the test data showed that the frictional resistance between the two plates tended to degrade substantially and could be approximated as degrading to zero friction resistance.

A comparison between the component model, the parametric model, and the experimental test data showed that these models had good correlation with the test data. Comparisons between the experimental connection deformation at failure and the deformation at failure predicted by the component model showed that the component model provided, on average, a conservative estimate of the failure deformation.

10.3. Recommendations

The bolt shear strength predicted using the AISC Specification (*Load and* 1986) was shown to be approximately 9% conservative. The AISC Specification (*Load and* 1986) bolt strength was based on bolt tests of bolts in double shear. One possible reason for the apparent increase in experimental bolt shear strength compared to the specification equation could be an inclined shear angle. The plates in a single bolt lap-plate connection tend to deform under load (because of the eccentric load transfer). This deformation tends to force the bolt into a combined tension and shear failure that results in an apparent higher shear capacity. The writers recommend that a research study that isolates this particular aspect be conducted to better quantify the shear strength of bolts in these types of connections.

The plate friction behavior was based on the assumption that the bearing behavior of the bolts and plates could be satisfactorily approximated with the component method. A much better understanding of this behavior could be obtained based on the load-displacement histories of actual friction tests such as those conducted by Frank and Yura (1981). It is recommended that the data from the Frank and Yura (1981) tests be obtained. This data was not included in the report by Frank and Yura (1981) nor in the thesis that the report was based on (Perry 1981). In addition, new tests considering thinner plates and possibly specially designed lap-plate connection tests that avoid initial bearing should be conducted. The data from the Frank and Yura (1981) tests and the new tests could be used to develop a better understanding of the friction behavior. Also, literature from the area of tribology should be consulted. A brief literature review in this area produced at least one paper (Simkins 1967) that may provide some insight into the pre- and post-slip frictional behaviors.

There were only 16 lap-plate connection tests with bolts in the snug tight condition. Additional tests should be conducted. These tests would provide a better basis for evaluation of models for predicting the load-deformation behavior. In addition, they could be used to gain a better understanding of the bolt component load-deformation behavior. Finally, when combined with the database of connection tests that had fully tightened bolts a much better understanding of the frictional component behavior could be obtained.

It has been shown that the shape of the load-deformation behavior and the deformation at failure are not constant values; however, this is the assumption made when using the current ultimate strength method for analysis of eccentrically loaded bolt groups. An analytical study of the effect of varying shape and failure deformation on the load capacity of eccentric bolt groups should be conducted to determine if using constant shape and failure deformation values provides sufficient accuracy and safe results. The parameter model based on the Richard Equation (Richard and Elsalti 1991) could be used for this study.

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