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Thermomechanical effects on the bending of antisymmetric cross-ply composite plates using a four variable sinusoidal theory

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Abstract. The thermomechanical bending response of anti-symmetric cross-ply composite plates is investigated by the use of the simple four variable sinusoidal plate theory. The theory accounts for sinusoidal distribution of transverse shear stress, and satisfies the free transverse shear stress conditions on the top and bottom surfaces of the plate without using shear correction factor. By dividing the transverse displacement into bending and shear parts, the number of unknowns and governing equations for the present theory is reduced, significantly facilitating engineering analysis. The validity of the present theory is demonstrated by comparison with solutions available in the literature. Numerical results are presented to demonstrate the behavior of the system. The influences of aspect ratio, side-to-thickness ratio, thermal expansion coefficients ratio and stacking sequence on the thermally induced response are studied. The present study is relevant to aerospace, chemical process and nuclear engineering structures which may be subjected to intense thermal loads.

Keywords: thermo-mechanical load; laminated plates; analytical modelling

1. Introduction

Laminated composite plates are widely used in the aerospace, automotive, marine, civil and other engineering applications because of advantageous features such as high ratio of stiffness and strength to weight and low maintenance cost. The physical properties, such as higher strength-to-weight ratio, stiffness-to-weight ratio and versatility, are achieved by combining different materials to meet specific requirement.

In the open literature, basically two different approaches were used in order to study laminated composite structures: equivalent single layer (ESL) theories and discrete layer theories. In the single layer theories laminated structures are assumed to be composed from one layer whereas in

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latter case each layer is considered in the analysis. The ESL theories can be divided into three main categories: classical plate theory (CPT), first order shear deformation theory (FSDT), and higher-order shear deformation theories (HSDTs). The CPT ignores shear deformation effects and provides reasonable results for thin laminates. However, it underestimates deflection and overestimates buckling load and frequency of moderately thick or thick laminates where shear deformation effects are more pronounced (Whitney and Leissa 1969, Grover *et al*. 2014, Nedri *et al*. 2014, Draiche *et al*. 2014, Mahi *et al*. 2015). The FSDT proposed by Reissner (1945) and Mindlin (1951) accounts for shear deformation effects by the way of linear variation of in-plane displacements through the thickness. Since the FSDT violates the equilibrium conditions on the top and bottom surfaces of the plate, a shear correction factor is required to compensate for the difference between actual stress state and assumed constant stress state (Castellazzi *et al*. 2013, Cui *et al*. 2011, Bouremana *et al*. 2013, Sadoune *et al*. 2014, Yaghoobi and Yaghoobi 2013, Berrabah *et al*. 2013, Tounsi *et al*. 2013a, b). The HSDTs account for shear deformation effects by higher-order variations of in-plane displacements or both in-plane and transverse displacements through the thickness, and satisfy the equilibrium conditions on the top and bottom surfaces of the plate without requiring any shear correction factors. For example, Ambartsumian (1958), proposed a transverse shear stress function in order to explain plate deformation. A similar method was used later by Soldatos and Timarci (1993), for dynamic analysis of laminated shells. Later some new functions were proposed by Reddy (1984), Karama *et al*. (2003), Zenkour (2004), Ait Atmane *et al*. (2010), Benachour *et al*. (2011), EL Meiche *et al*. (2011), and Ait Amar Meziane *et al*. (2014). Thermal flexural investigation of symmetric laminated plates subjected to a single sinusoidal thermal load is illustrated by Ali *et al*. (1999) by employing displacement-based higher-order theory. Wu *et al*. (2007) presented a global-local higher order theory including transverse normal deformation to study the thermal behavior of a laminated plate subjected to a single sinusoidal thermal load. Recently, new inverse hyperbolic theories were developed by Sahoo and Singh (2013) and Grover *et al*. (2013). An equivalent single-layer shear deformation theory is presented by Ghugal and Kulkarni (2013a, b) for evaluation of displacements and stresses of cross-ply laminated plates subjected to uniformly distributed nonlinear thermo-mechanical load. Ghugal and Kulkarni (2011) discussed thermal stresses in cross-ply laminated plates subjected to a sinusoidal thermal load through the thickness of the plate using refined shear deformation theory. For the evaluation of displacements, critical buckling temperature and stresses in functionally graded structures subjected to thermal and mechanical loadings, a two-dimensional higher-order deformation theory is developed by Matsunaga (1999), Bachir Bouiadjra *et al*. (2013), Bouderba *et al*. (2013), Tounsi *et al*. (2013c), Houari *et al*. (2013), Kettaf *et al*. (2013), Saidi *et al*. (2013), Ould Larbi *et al*. (2013), Khalfi *et al*. (2014), Zidi *et al*. (2014), Hebali *et al*. (2014), Belabed *et al*. (2014), Bousahla *et al*. (2014), Swaminathan and Naveenkumar (2014), Said *et al*. (2014), Bourada *et al*. (2015), Hamidi *et al*. (2015), Ait Yahia *et al*. (2015), Larbi Chaht *et al*. (2015), Belkorissat *et al*. (2015) and Bouchafa *et al*. (2015).

Among the aforementioned HSDTs, the Reddy's theory is the most widely used due to its high efficiency and simplicity. Since the in-plane displacements of the Reddy's theory are expanded as cubic function of the thickness coordinate, the equations of motion are more complicated than those of FSDT. Hence, there is a scope to develop an accurate theory, which is simple to use. In addition, the use of composite structures in environments with large temperature changes requires knowledge of thermally induced deflections and stresses. Further, thermal stresses are also induced during the fabrication of composite materials. For example, the static thermoelastic response of symmetric and antisymmetric cross-ply laminated plates has been discussed by Zenkour (2004) using the sinusoidal shear deformation theory (SSDT).

In the present work, a simple higher order shear deformation theory is developed for the thermo-mechanical bending analysis of laminated composite plates. The displacement field is chosen based on a *sinusoidal variation* of in-plane and transverse displacements through the thickness. The partition of the transverse displacement into the bending, and shear components leads to a reduction in the number of unknowns, and consequently, makes the new theory much more amenable to implementation. Closed-form solutions of simply supported antisymmetric cross-ply laminates are obtained and the results are compared with the existing solutions. The analysis is relevant to aerospace and nuclear engineering structures experiencing significant heat effects.

2. Problem formulation

Consider a rectangular laminated plate of length *a*, width *b* and uniform thickness *h* (see Fig. 1). The plate is composed of *n* orthotropic layers oriented at angles θ_1; θ_2; . . .; θ_n. The material of each layer is assumed to posses one plane of elastic symmetry parallel to the *x*-*y* plane. Perfect bonding between the orthotropic layers and temperature-independent mechanical and thermal properties are assumed. Let the plate be subjected to a transverse load $q(x, y)$ and temperature field $T(x, y, z)$.

2.1 Basic assumptions

The assumptions of the present theory are as follows

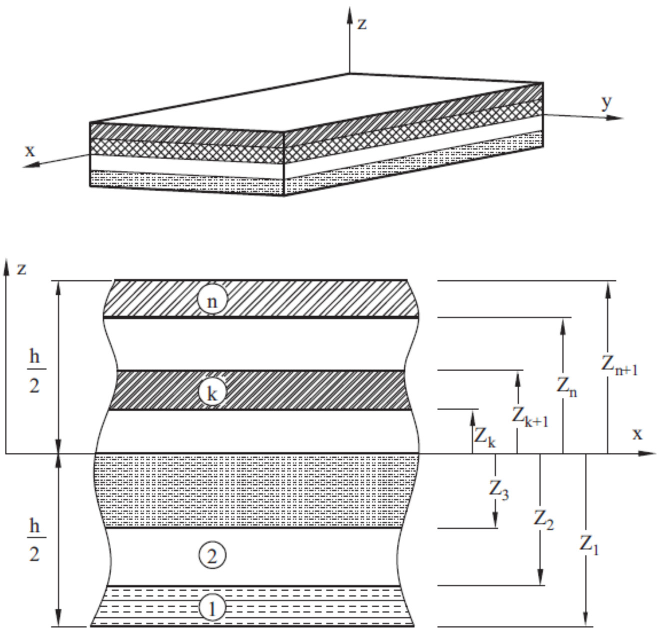

Fig. 1 Coordinate system and layer numbering used for a typical laminated plate

(i) The displacements are small in comparison with the plate thickness and, therefore, strains involved are infinitesimal.
(ii) The transverse displacement w includes two components of bending w_b, and shear w_s. These components are functions of coordinates x, y only

$$w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{1}$$

(iii) The transverse normal stress σ_z is negligible in comparison with in-plane stresses σ_x and σ_y.
(iv) The displacements u in x-direction and v in y-direction consist of extension, bending, and shear components.

$$u=u_0+u_b+u_s, \quad v=v_0+v_b+v_s \tag{2}$$

The bending components u_b and v_b are assumed to be similar to the displacements given by the classical plate theory. Therefore, the expression for u_b and v_b can be given as

$$u_b=-z\frac{\partial w_b}{\partial x}, \quad v_b=-z\frac{\partial w_b}{\partial y} \tag{3}$$

The shear components u_s and v_s give rise, in conjunction with w_s, to the trigonometric variations of shear strains γ_{xz}, γ_{yz} and hence to shear stresses τ_{xz}, τ_{yz} through the thickness of the plate in such a way that shear stresses τ_{xz}, τ_{yz} are zero at the top and bottom faces of the plate. Consequently, the expression for u_s and v_s can be given as

$$u_s=-f(z)\frac{\partial w_s}{\partial x}, \quad v_s=-f(z)\frac{\partial w_s}{\partial y} \tag{4}$$

where

$$f(z)=\left(z-\frac{h}{\pi}\sin\frac{\pi z}{h}\right) \tag{5}$$

2.2 Kinematics

Based on the assumptions made in the preceding section, the displacement field can be obtained using Eqs. (1)-(5) as

$$u(x,y,z)=u_0(x,y)-z\frac{\partial w_b}{\partial x}-f(z)\frac{\partial w_s}{\partial x} \tag{6a}$$

$$v(x,y,z)=v_0(x,y)-z\frac{\partial w_b}{\partial y}-f(z)\frac{\partial w_s}{\partial y} \tag{6b}$$

$$w(x,y,z)=w_b(x,y)+w_s(x,y) \tag{6c}$$

The kinematic relations can be obtained as follows

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} + z \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} + f(z) \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz} \\ \gamma_{xz} \end{Bmatrix} = g(z) \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} \tag{7}$$

where

$$\begin{Bmatrix} \varepsilon_x^0 \\ \varepsilon_y^0 \\ \gamma_{xy}^0 \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial u_0}{\partial x} \\ \dfrac{\partial v_0}{\partial x} \\ \dfrac{\partial u_0}{\partial y} + \dfrac{\partial v_0}{\partial x} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^b \\ k_y^b \\ k_{xy}^b \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_b}{\partial x^2} \\ -\dfrac{\partial^2 w_b}{\partial y^2} \\ -2\dfrac{\partial^2 w_b}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} k_x^s \\ k_y^s \\ k_{xy}^s \end{Bmatrix} = \begin{Bmatrix} -\dfrac{\partial^2 w_s}{\partial x^2} \\ -\dfrac{\partial^2 w_s}{\partial y^2} \\ -2\dfrac{\partial^2 w_s}{\partial x \partial y} \end{Bmatrix}, \quad \begin{Bmatrix} \gamma_{yz}^s \\ \gamma_{xz}^s \end{Bmatrix} = \begin{Bmatrix} \dfrac{\partial w_s}{\partial y} \\ \dfrac{\partial w_s}{\partial x} \end{Bmatrix} \tag{8}$$

and

$$g(z) = 1 - \frac{df(z)}{dz} = \cos\left(\frac{\pi z}{h}\right) \tag{9}$$

2.3 Constitutive and governing equations

The stress–strain relationships, accounting for transverse shear deformation and thermal effects, in the plate coordinates for the *k*th layer can be expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \varepsilon_x - \alpha_x T \\ \varepsilon_y - \alpha_y T \\ \gamma_{xy} - \alpha_{xy} T \end{Bmatrix}^{k)} \quad \text{and} \quad \begin{Bmatrix} \tau_{yz} \\ \tau_{zx} \end{Bmatrix}^{(k)} = \begin{bmatrix} \overline{Q}_{44} & \overline{Q}_{45} \\ \overline{Q}_{45} & \overline{Q}_{55} \end{bmatrix}^{(k)} \begin{Bmatrix} \gamma_{yz} \\ \gamma_{zx} \end{Bmatrix}^{(k)} \tag{9}$$

where $T = T(x, y, z)$ is the temperature distribution; and (α_x; α_y; α_{xy}) are the thermal expansion coefficients in the plate coordinates, and are related to the coefficients (α_L; α_T; 0) in the material principal directions. $\overline{Q}_{ij}$ are the transformed elastic coefficients are the transformed material constants given as (Bogdanovich and Pastore 1996)

$$\begin{aligned}
\overline{Q}_{11} &= Q_{11}\cos^2\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\sin^4\theta \\
\overline{Q}_{12} &= (Q_{11} + Q_{22} - 4Q_{66})\sin^2\theta\cos^2\theta + Q_{12}(\sin^4\theta + \cos^4\theta) \\
\overline{Q}_{22} &= Q_{11}\sin^2\theta + 2(Q_{12} + 2Q_{66})\sin^2\theta\cos^2\theta + Q_{22}\cos^4\theta \\
\overline{Q}_{16} &= (Q_{11} - Q_{12} - 2Q_{66})\sin\theta\cos^3\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin^3\theta\cos\theta \\
\overline{Q}_{26} &= (Q_{11} - Q_{12} - 2Q_{66})\sin^3\theta\cos\theta + (Q_{12} - Q_{22} + 2Q_{66})\sin\theta\cos^3\theta \\
\overline{Q}_{66} &= (Q_{11} + Q_{22} - 2Q_{12} - 2Q_{66})\sin^2\theta\cos^2\theta + Q_{66}(\sin^4\theta + \cos^4\theta) \\
\overline{Q}_{44} &= Q_{44}\cos^2\theta + Q_{55}\sin^2\theta \\
\overline{Q}_{45} &= (Q_{55} - Q_{44})\cos\theta\sin\theta \\
\overline{Q}_{55} &= Q_{55}\cos^2\theta + Q_{44}\sin^2\theta
\end{aligned} \tag{10}$$

where Q_{ij} are the (plane stress-reduced) material stiffness of the lamina

$$Q_{11}=\frac{E_{xx}}{1-\nu_{xy}\nu_{yx}},\quad Q_{12}=\frac{\nu_{xy}E_y}{1-\nu_{xy}\nu_{yx}},\quad Q_{22}=\frac{E_y}{1-\nu_{xy}\nu_{yx}},\quad Q_{66}=G_{xy},\quad Q_{44}=G_{yz},\quad Q_{55}=G_{xz} \tag{11}$$

in which E_x and E_y are Young's moduli in the x and y material principal directions, respectively; ν_{xy} and ν_{yx} are Poisson's ratios; and G_{xy}; G_{yz} and G_{xz} are shear moduli in the x–y, y–z and x–z surfaces, respectively.

The stress and moment resultants of a laminated composite plate made up of n layers of orthotropic laminae can be obtained by integrating (9) over the thickness, and are written as

$$\begin{Bmatrix} N \\ M^b \\ M^s \end{Bmatrix}=\begin{bmatrix} A & B & B^s \\ B & D & D^s \\ B^s & D^s & H^s \end{bmatrix}\begin{Bmatrix} \varepsilon \\ k^b \\ k^s \end{Bmatrix}-\begin{Bmatrix} N^T \\ M^{bT} \\ M^{sT} \end{Bmatrix},\quad S=A^s\gamma, \tag{12}$$

in which

$$N=\{N_x,N_y,N_{xy}\}^t,\quad M^b=\{M_x^b,M_y^b,M_{xy}^b\}^t,\quad M^s=\{M_x^s,M_y^s,M_{xy}^s\}^t \tag{13a}$$

$$\varepsilon=\{\varepsilon_x^0,\varepsilon_y^0,\gamma_{xy}^0\}^t,\quad k^b=\{k_x^b,k_y^b,k_{xy}^b\}^t,\quad k^s=\{k_x^s,k_y^s,k_{xy}^s\}^t \tag{13b}$$

$$A=\begin{bmatrix} A_{11} & A_{12} & A_{16} \\ A_{12} & A_{22} & A_{26} \\ A_{16} & A_{26} & A_{66} \end{bmatrix},\quad B=\begin{bmatrix} B_{11} & B_{12} & B_{16} \\ B_{12} & B_{22} & B_{26} \\ B_{16} & B_{26} & B_{66} \end{bmatrix},\quad D=\begin{bmatrix} D_{11} & D_{12} & D_{16} \\ D_{12} & D_{22} & D_{26} \\ D_{16} & D_{26} & D_{66} \end{bmatrix} \tag{13c}$$

$$B^s=\begin{bmatrix} B_{11}^s & B_{12}^s & B_{16}^s \\ B_{12}^s & B_{22}^s & B_{26}^s \\ B_{16}^s & B_{26}^s & B_{66}^s \end{bmatrix},\quad D^s=\begin{bmatrix} D_{11}^s & D_{12}^s & D_{16}^s \\ D_{12}^s & D_{22}^s & D_{26}^s \\ D_{16}^s & D_{26}^s & D_{66}^s \end{bmatrix},\quad H^s=\begin{bmatrix} H_{11}^s & H_{12}^s & H_{16}^s \\ H_{12}^s & H_{22}^s & H_{26}^s \\ H_{16}^s & H_{26}^s & H_{66}^s \end{bmatrix} \tag{13d}$$

$$S=\{S_{xz}^s,S_{yz}^s\}^t,\quad \gamma=\{\gamma_{xz},\gamma_{yz}\}^t,\quad A^s=\begin{bmatrix} A_{44}^s & A_{45}^s \\ A_{45}^s & A_{55}^s \end{bmatrix} \tag{13e}$$

and stiffness components and inertias are given as

$$\left(A_{ij},B_{ij},D_{ij},B_{ij}^s,D_{ij}^s,H_{ij}^s\right)=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\overline{Q}_{ij}^{(k)}\left(1,z,z^2,f(z),z\,f(z),f^2(z)\right)dz,\quad (i,j)=(1,2,6) \tag{14a}$$

$$A_{ij}^{s} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \overline{Q}_{ij}^{(k)} [g(z)]^2 dz, \quad (i,j) = (4,5) \tag{14b}$$

Note that, z_k represents the distance from the mid-plane to the lower surface of the kth layer. The stress and moment resultants, $N_x^T, N_y^T; \ldots$ etc., due to thermal loading are defined by

$$\begin{Bmatrix} N_1^T, M_1^{bT}, M_1^{sT} \\ N_2^T, M_2^{bT}, M_2^{sT} \\ N_6^T, M_6^{bT}, M_6^{sT} \end{Bmatrix} = \sum_{k=1}^{n} \int_{z_k}^{z_{k+1}} \begin{bmatrix} \overline{Q}_{11} & \overline{Q}_{12} & \overline{Q}_{16} \\ \overline{Q}_{12} & \overline{Q}_{22} & \overline{Q}_{26} \\ \overline{Q}_{16} & \overline{Q}_{26} & \overline{Q}_{66} \end{bmatrix}^{(k)} \begin{Bmatrix} \alpha_x \\ \alpha_y \\ \alpha_{xy} \end{Bmatrix} (1, z, f(z)) T dz \tag{15}$$

Consistent with the present unified plate theory, the temperature variation through the thickness is assumed to be

$$T(x,y,z) = T_1(x,y) + \frac{z}{h} T_2(x,y) + \frac{1}{\pi} \sin\left(\frac{\pi z}{h}\right) T_3(x,y), \tag{16}$$

The governing equations of equilibrium can be derived by using the principle of virtual displacements. The equilibrium equations associated with the present four variable sinusoidal plate theory are

$$\begin{aligned}
&\delta u_0 : \ \frac{\partial N_x}{\partial x} + \frac{\partial N_{xy}}{\partial y} = 0 \\
&\delta v_0 : \ \frac{\partial N_{xy}}{\partial x} + \frac{\partial N_y}{\partial y} = 0 \\
&\delta w_b : \ \frac{\partial^2 M_x^b}{\partial x^2} + 2\frac{\partial^2 M_{xy}^b}{\partial x \partial y} + \frac{\partial^2 M_y^b}{\partial y^2} + q = 0 \\
&\delta w_s : \ \frac{\partial^2 M_x^s}{\partial x^2} + 2\frac{\partial^2 M_{xy}^s}{\partial x \partial y} + \frac{\partial^2 M_y^s}{\partial y^2} + \frac{\partial S_{xz}^s}{\partial x} + \frac{\partial S_{yz}^s}{\partial y} + q = 0
\end{aligned} \tag{17}$$

Eq. (17) can be expressed in terms of displacements (u_0, v_0, w_b and w_s) by substituting for the stress resultants from Eq. (12). For homogeneous laminates, the equations of motion (17) take the form

$$\begin{aligned}
&A_{11}\frac{\partial^2 u_0}{\partial x^2} + 2A_{16}\frac{\partial^2 u_0}{\partial x \partial y} + A_{66}\frac{\partial^2 u_0}{\partial y^2} + A_{16}\frac{\partial^2 v_0}{\partial x^2} + (A_{12} + A_{66})\frac{\partial^2 v_0}{\partial x \partial y} + A_{26}\frac{\partial^2 v_0}{\partial y^2} \\
&- B_{11}\frac{\partial^3 w_b}{\partial x^3} - 3B_{16}\frac{\partial^3 w_b}{\partial x^2 \partial y} - (B_{12} + 2B_{66})\frac{\partial^3 w_b}{\partial x \partial y^2} - B_{26}\frac{\partial^3 w_b}{\partial y^3} \\
&- B_{11}^s\frac{\partial^3 w_s}{\partial x^3} - 3B_{16}^s\frac{\partial^3 w_s}{\partial x^2 \partial y} - (B_{12}^s + 2B_{66}^s)\frac{\partial^3 w_s}{\partial x \partial y^2} - B_{26}^s\frac{\partial^3 w_s}{\partial y^3} = p_1
\end{aligned} \tag{18a}$$

$$\begin{aligned}
&A_{16}\frac{\partial^2 u_0}{\partial x^2}+\left(A_{12}+A_{66}\right)\frac{\partial^2 u_0}{\partial x\partial y}+A_{26}\frac{\partial^2 u_0}{\partial x^2}+A_{66}\frac{\partial^2 v_0}{\partial x^2}+2A_{26}\frac{\partial^2 v_0}{\partial x\partial y}+A_{22}\frac{\partial^2 v_0}{\partial y^2}\\
&-B_{16}\frac{\partial^3 w_b}{\partial x^3}-\left(B_{12}+2B_{66}\right)\frac{\partial^3 w_b}{\partial x^2\partial y}-3B_{26}\frac{\partial^3 w_b}{\partial x\partial y^2}-B_{22}\frac{\partial^3 w_b}{\partial y^3}\\
&-B_{16}^s\frac{\partial^3 w_s}{\partial x^3}-B_{22}^s\frac{\partial^3 w_s}{\partial y^3}-\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 w_s}{\partial x^2\partial y}-3B_{26}^s\frac{\partial^3 w_s}{\partial x\partial y^2}=p_2
\end{aligned}\tag{18b}$$

$$\begin{aligned}
&B_{11}\frac{\partial^3 u_0}{\partial x^3}+3B_{16}\frac{\partial^3 u_0}{\partial x^2\partial y}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{26}\frac{\partial^3 u_0}{\partial y^3}\\
&+B_{16}\frac{\partial^3 v_0}{\partial x^3}+\left(B_{12}+2B_{66}\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+3B_{26}\frac{\partial^3 v_0}{\partial x\partial y^2}+B_{22}\frac{\partial^3 v_0}{\partial y^3}\\
&-D_{11}\frac{\partial^4 w_b}{\partial x^4}-4D_{16}\frac{\partial^4 w_b}{\partial x^3\partial y}-2\left(D_{12}+2D_{66}\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-4D_{26}\frac{\partial^4 w_b}{\partial x\partial y^3}-D_{22}\frac{\partial^4 w_b}{\partial y^4}\\
&-D_{11}^s\frac{\partial^4 w_s}{\partial x^4}-4D_{16}^s\frac{\partial^4 w_s}{\partial x^3\partial y}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-4D_{26}^s\frac{\partial^4 w_s}{\partial x\partial y^3}-D_{22}^s\frac{\partial^4 w_s}{\partial y^4}=p_3
\end{aligned}\tag{18c}$$

$$\begin{aligned}
&B_{11}^s\frac{\partial^3 u_0}{\partial x^3}+3B_{16}^s\frac{\partial^3 u_0}{\partial x^2\partial y}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 u_0}{\partial x\partial y^2}+B_{26}^s\frac{\partial^3 u_0}{\partial y^3}\\
&+B_{16}^s\frac{\partial^3 v_0}{\partial y^3}+\left(B_{12}^s+2B_{66}^s\right)\frac{\partial^3 v_0}{\partial x^2\partial y}+3B_{26}^s\frac{\partial^3 v_0}{\partial x\partial y^2}+B_{22}^s\frac{\partial^3 v_0}{\partial y^3}\\
&-D_{11}^s\frac{\partial^4 w_b}{\partial x^4}-4D_{16}^s\frac{\partial^4 w_b}{\partial x^3\partial y}-2\left(D_{12}^s+2D_{66}^s\right)\frac{\partial^4 w_b}{\partial x^2\partial y^2}-4D_{26}^s\frac{\partial^4 w_b}{\partial x\partial y^3}-D_{22}^s\frac{\partial^4 w_b}{\partial y^4}\\
&-H_{11}^s\frac{\partial^4 w_s}{\partial x^4}-4H_{16}^s\frac{\partial^4 w_b}{\partial x^3\partial y}-2\left(H_{12}^s+2H_{66}^s\right)\frac{\partial^4 w_s}{\partial x^2\partial y^2}-4H_{26}^s\frac{\partial^4 w_b}{\partial x\partial y^3}-H_{22}^s\frac{\partial^4 w_s}{\partial y^4}\\
&+A_{55}^s\frac{\partial^2 w_s}{\partial x^2}+A_{44}^s\frac{\partial^2 w_s}{\partial y^2}+2A_{45}^s\frac{\partial^2 w_s}{\partial x\partial y}=p_4
\end{aligned}\tag{18d}$$

where $\{p\} = \{p_1, p_2, p_3, p_4\}^t$ is a generalized force vector given by

$$\begin{aligned}
&p_1=\frac{\partial N_x^T}{\partial x}+\frac{\partial N_{xy}^T}{\partial y}, && p_2=\frac{\partial N_{xy}^T}{\partial x}+\frac{\partial N_y^T}{\partial y},\\
&p_3=q-\frac{\partial^2 M_x^{bT}}{\partial x^2}-2\frac{\partial^2 M_{xy}^{bT}}{\partial x\partial y}-\frac{\partial^2 M_y^{bT}}{\partial y^2}, && p_4=q-\frac{\partial^2 M_x^{sT}}{\partial x^2}-\frac{\partial^2 M_y^{sT}}{\partial y^2}
\end{aligned}\tag{19}$$

3. Analytical solutions for antisymmetric cross-ply laminates

The Navier approach is employed to obtain the closed-form solutions of the partial differential equations in Eq. (18) for simply supported rectangular plates. For antisymmetric cross-ply laminates, the following plate stiffnesses are identically zero

$$\begin{aligned} &A_{16} = A_{26} = D_{16} = D_{26} = D_{16}^s = D_{26}^s = H_{16}^s = H_{26}^s = 0 \\ &B_{12} = B_{16} = B_{26} = B_{66} = B_{12}^s = B_{16}^s = B_{26}^s = B_{66}^s = 0 \\ &B_{22} = -B_{11};\ B_{22}^s = -B_{11}^s \end{aligned} \tag{20}$$

The following boundary conditions for antisymmetric cross-ply laminates can be written as

$$v_0 = w_b = w_s = \frac{\partial w_b}{\partial y} = \frac{\partial w_s}{\partial y} = N_x = M_x^b = M_x^s = 0 \quad \text{at} \quad x = 0,\ a \tag{21a}$$

$$u_0 = w_b = w_s = \frac{\partial w_b}{\partial x} = \frac{\partial w_s}{\partial x} = N_y = M_y^b = M_y^s = 0 \quad \text{at} \quad y = 0,\ b \tag{21b}$$

To solve this problem, Navier assumed that the transverse mechanical and temperature loads, q, T_1, T_2, and T_3 in the form of a double trigonometric series as

$$\begin{Bmatrix} q \\ T_1 \\ T_2 \\ T_3 \end{Bmatrix} = \begin{Bmatrix} q_0 \\ t_1 \\ t_2 \\ t_3 \end{Bmatrix} \sin(\lambda\, x)\sin(\mu\, y) \tag{22}$$

where $\lambda = m\pi / a$, $\mu = n\pi / b$, and m and n are mode numbers. q_0, t_1, t_2 and t_3 are constants.

Following the Navier solution procedure, we assume the following solution form for u_0, v_0, w_b and w_s that satisfies the boundary conditions

$$\begin{Bmatrix} u_0 \\ v_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty}\sum_{n=1}^{\infty} \begin{Bmatrix} U_{mn} \cos(\lambda\, x)\sin(\mu\, y) \\ V_{mn} \sin(\lambda\, x)\cos(\mu\, y) \\ W_{bmn} \sin(\lambda\, x)\sin(\mu\, y) \\ W_{smn} \sin(\lambda\, x)\sin(\mu\, y) \end{Bmatrix}, \tag{23}$$

where U_{mn}, V_{mn}, W_{bmn}, and W_{smn} are arbitrary parameters to be determined subjected to the condition that the solution in Eq. (23) satisfies governing Eqs. (18). One obtains the following operator equation

$$[C]\{\Delta\} = \{P\}, \tag{24}$$

where $\{\Delta\} = \{U, V, W_b, W_s\}^t$ and $[C]$ is the symmetric matrix given by

$$[C]=\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{12} & a_{22} & a_{23} & a_{24} \\ a_{13} & a_{23} & a_{33} & a_{34} \\ a_{14} & a_{24} & a_{34} & a_{44} \end{bmatrix}, \tag{25}$$

in which

$$\begin{gathered} a_{11}=-(A_{11}\lambda^2+A_{66}\mu^2) \\ a_{12}=-\lambda\,\mu\left(A_{12}+A_{66}\right) \\ a_{13}=-\lambda^3 B_{11} \\ a_{14}=\lambda^3 C_{11}+2\lambda\mu^2 C_{33}+\lambda\mu^2 C_{12} \\ a_{22}=-\lambda^2 A_{33}-\mu^2 A_{22} \\ a_{23}=\mu^3 B_{22}+2\mu\lambda^2 B_{33}+\mu\lambda^2 B_{12} \\ a_{24}=\mu^3 C_{22}+2\mu\lambda^2 C_{33}+\mu\lambda^2 C_{12} \\ a_{33}=-(D_{11}\lambda^4+2(D_{12}+2D_{66})\lambda^2\mu^2+D_{22}\mu^4) \\ a_{34}=-(D_{11}^s\lambda^4+2(D_{12}^s+2D_{66}^s)\lambda^2\mu^2+D_{22}^s\mu^4) \\ a_{44}=-(H_{11}^s\lambda^4+2(H_{12}^s+2H_{66}^s)\lambda^2\mu^2+H_{22}^s\mu^4+\lambda^2 A_{22}^s+\mu^2 A_{11}^s) \end{gathered} \tag{26}$$

The components of the generalized force vector $\{P\} = \{P_1, P_2, P_3, P_4\}^t$ are given by

$$\begin{aligned} P_1 &= -\lambda\left(A_1^T t_1 mn+\frac{B_1^T}{h}t_2 mn+\frac{B_1^{Ts}}{h}t_3 mn\right) \\ P_2 &= -\mu\left(A_2^T t_1 mn+\frac{B_2^T}{h}t_2 mn+\frac{B_2^{Ts}}{h}t_3 mn\right) \\ P_3 &= \left(\lambda^2 B_1^T+\mu^2 B_2^T\right)t_1 mn+\frac{1}{h}\left(\lambda^2 D_1^T+\mu^2 D_2^T\right)t_2 mn+\frac{1}{h}\left(\lambda^2 D_1^{Ts}+\mu^2 D_2^{Ts}\right)t_3 mn-q_0 \\ P_4 &= -\left(\lambda^2 B_1^{Ts}+\mu^2 B_2^{Ts}\right)t_1 mn-\frac{1}{h}\left(\lambda^2 D_1^{Ts}+\mu^2 D_2^{Ts}\right)t_2 mn-\frac{1}{h}\left(\lambda^2 H_1^{Ts}+\mu^2 H_2^{Ts}\right)t_3 mn-q_0 \end{aligned} \tag{27}$$

$$\left(A_i^T, B_i^T, D_i^T, B_i^{Ts}, D_i^{Ts}, H_i^{Ts}\right)=\sum_{k=1}^{n}\int_{z_k}^{z_{k+1}}\left(\alpha_x\overline{Q}_{1i}^{(k)}+\alpha_y\overline{Q}_{2i}^{(k)}\right)\left(1, z, z^2, f(z), z\,f(z), f^2(z)\right)dz, \quad (i,j)=(1,2)$$

4. Numerical results and discussion

In this section, various numerical examples are described and discussed to verify the accuracy of the present theory. The thermo-mechanical bending behaviors of the simply-supported anti-symmetric cross-ply composite plates are considered. Computations were carried out for the fundamental mode (i.e., $m = n = 1$). All of the lamina are assumed to be of the same thickness and made of the same orthotropic material. In all problems, the lamina properties are assumed to be

$$E_x = 25\times10^6 \text{ psi},\quad E_x = 10^6 \text{ psi},\quad G_{xy} = G_{xz} = 0.5\times10^6 \text{ psi},\quad G_{yz} = 0.2\times10^6 \text{ psi},\quad \nu_{xy} = 0.25.$$

Note that, values of α_x ($\equiv \alpha_1$) and α_y ($\equiv \alpha_2$) are given during the discussion of material results. We will assume in all of the analyzed cases (unless otherwise stated) that $a / h = 10$, $a / b = 1$, $t_1 = 1$, and $\alpha_2 / \alpha_1 = 3$.

The thermomechanical bending analysis of the present example problem has been studied by several authors, and their results were used here for further comparison purposes. Zenkour (2004) presented a generalized HSDT and presented the closed-form solution of a plate subjected to thermal and mechanical loads, thus providing benchmark results. Reddy and Hsu (1980) developed for thermal bending of laminated plates, a finite-element model that possesses competitive accuracy.

Table 1 presents results of non-dimensionalized center deflections $\overline{w} = 10wh/(\alpha_1\overline{T}_2a^2)$ of two-layer cross-ply (0°/90°) plates due to thermal loading. After a detailed comparison analysis, it can be concluded that the present four variable sinusoidal plate theory gives very closed results to the values obtained by the conventional sinusoidal plate theory (SSDT) obtained by Zenkour (2004).

In Table 2, the following non-dimensionalized deflection $\widetilde{w}$ of two-layer cross-ply (0°/90°) plates subjected to combined loading is used (see Reddy and Hsu (1980))

$$\widetilde{w} = w\left[\frac{q_0a^4}{h^3\zeta} + \frac{\alpha_1\overline{T}_2a^2}{10h}\right]^{-1} \quad \text{with} \quad \zeta = \frac{1}{12}\pi^4\left[4G_{xy} + \frac{E_x + (1+\nu_{xy})E_y}{1-\nu_{xy}\nu_{yx}}\right]^{-1}$$

Table 1 Nondimensional center deflections $\overline{w} = 10wh/(\alpha_1\overline{T}_2a^2)$ of cross-ply square plates (0°/90°) subjected to thermal loading ($\overline{T}_3 = 0$)

a / h	Exact [(a)]	Present	HSDT [(b)]	SSDT [(b)]
100	1.6765	1.6766	1.6766	1.6766
50	1.6765	1.6767	1.6767	1.6767
25	1.6765	1.6771	1.6770	1.6771
20	1.6765	1.6774	1.6773	1.6774
12.5	1.6765	1.6789	1.6786	1.6789
10	1.6765	1.6802	1.6798	1.6802
6.25	1.6765	1.6858	1.6848	1.6858
5	1.6765	1.6910	1.6894	1.6910

[(a)] Reddy and Hsu (1980)
[(b)] Zenkour (2004)

Table 2 Nondimensional center deflections $\tilde{w}$ of cross-ply square plates (0°/90°) subjected to combined loading (q_0 = 100, $\overline{T}_2 = 100$, $\overline{T}_3 = 0$, $\alpha_1 = 10^{-6}$)

a / h	Exact [(a)]	Present	HSDT [(b)]	SSDT [(b)]
100	2.4451	2.4481	2.4481	2.4481
50	2.4597	2.4585	2.4586	2.4584
25	2.5083	2.4999	2.5006	2.4996
20	2.5443	2.5309	2.5321	2.5304
12.5	2.7001	2.6650	2.6679	2.6636
10	2.8438	2.7885	2.7927	2.7859
6.25	3.4666	3.3186	3.3273	3.3090
5	4.0415	3.8013	3.8120	3.7821

[(a)] Reddy and Hsu (1980)
[(b)] Zenkour (2004)

The obtained results are tabulated in Table 2 and are compared to those predicted using various theories (Zenkour 2004) and the solution of Reddy and Hsu (1980). It can be seen that the present theory with only four unknowns agree extremely well with those obtained in (Zenkour 2004).

The variation of non-dimensionalized vertical displacement w versus the ratio a/h for anti-symmetric two- and four-layer cross-ply square plates is shown in Figs. 2 and 3, respectively. An interesting result deduced from Figs. 2 and 3 is that the vertical displacement $\overline{w}$ is independent of the side to-thickness ratio for the case of the CPT. On the other hand, with the consideration of the shear deformation effect, all responses of the present theory, HSDT, SSDT, and FSDT become dependent on the side to-thickness ratio. It is known that the dependency of the responses on the

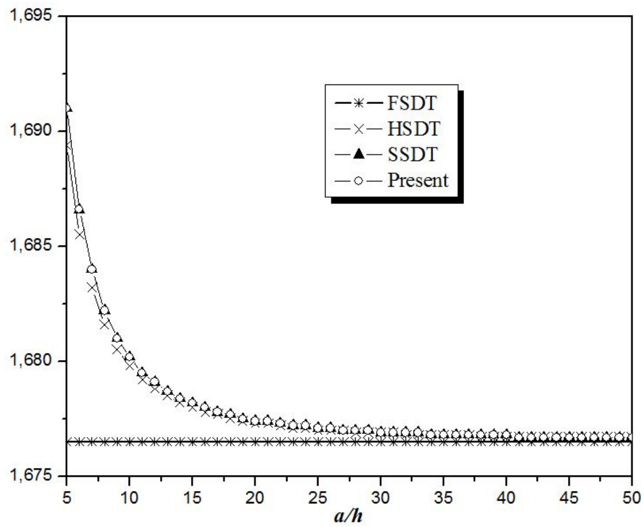

Fig. 2 Effect of thickness on the dimensionless deflection $\overline{w}$ of a two-layer, anti-symmetric cross-ply (0/90) square plate (t_3 = 0)

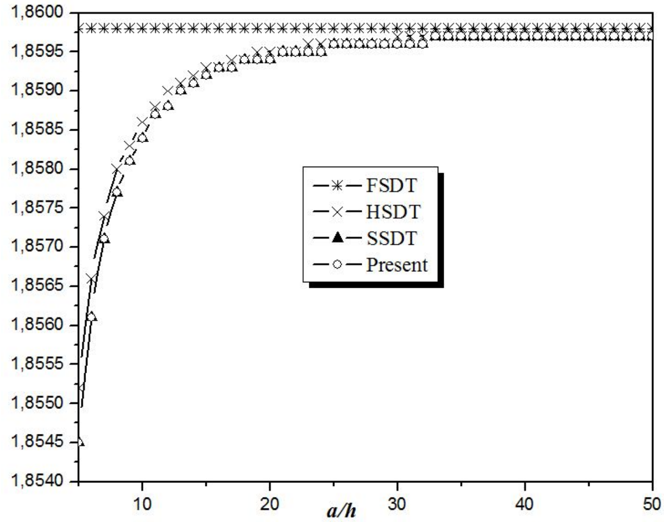

Fig. 3 Effect of thickness on the dimensionless deflection $\overline{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)_2$ square plate ($t_3 = 0$)

side to-thickness ratio for the present theory, HSDT, SSDT, and FSDT is uniquely due to the effect of shear deformation. The obtained results are compared with those generated by HSDT, SSDT, and FSDT as is shown in Figs. 2 and 3. In addition, it is seen that the vertical displacement $\overline{w}$ decreases with increasing the side to-thickness ratio for two-layer plates, whereas for four-layer plates ones the increase in vertical displacement due to the same theories is shown.

The effect of the ratio of thermal expansion coefficients (α_2 / α_1) on the bending response of

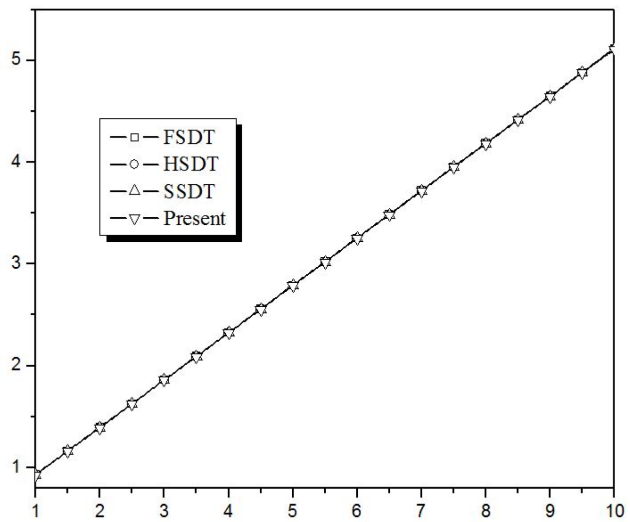

Fig. 4 Effect of the ratio of thermal expansion coefficients α_2 / α_1 on the dimensionless deflection $\overline{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)_2$ square plate ($t_3 = 0$)

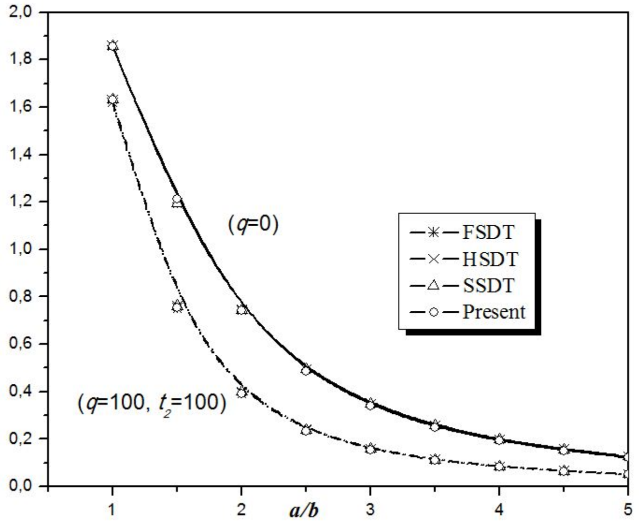

Fig. 5 Effect of aspect ratio on the dimensionless combined deflection $\overline{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)_2$ square plate ($t_3 = 0$)

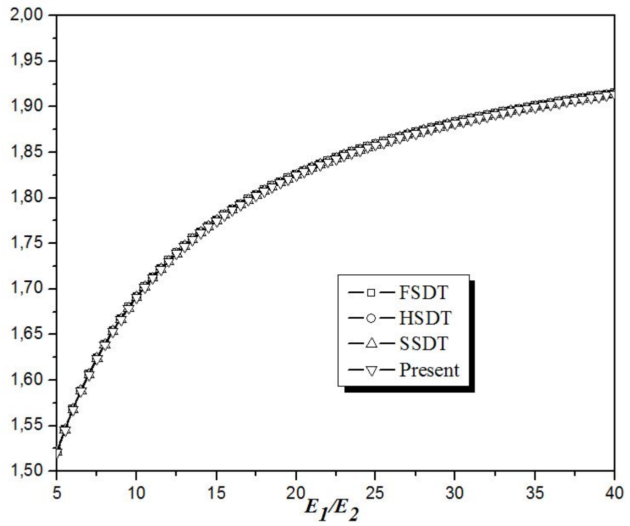

Fig. 6 The effect of material anisotropy E_1 / E_2 on the dimensionless deflection $\overline{w}$ of a four-layer, anti-symmetric cross-ply $(0/90)_2$ square plate ($t_3 = 0$)

anti-symmetric four-layer cross-ply square plate is demonstrated in Fig. 4. It can be seen that the vertical displacement is linearly proportional to the α_2 / α_1 ratio.

Fig. 5 demonstrates the effects of the aspect ratio (a / b) on the non- dimensionalized vertical displacement $\overline{w}$ of anti-symmetric four-layer cross-ply square plate subjected to linear temperature distribution and/or mechanical loading. It is found that the aspect ratio effect is more pronounced on the thermal bending deflection $\overline{w}$ (q = 0) of a plate under non-uniform temperature distribution.

The effect of the modulus ratio (E_1 / E_2) on the bending response of anti-symmetric four-layer cross-ply square plate is shown in Fig. 6. It can be deduced that the bending response of the composite plate depends strongly on the material anisotropy of the layer.

5. Conclusions

A simple four variable sinusoidal plate theory has been successfully developed for the thermo-mechanical of simply supported laminated plates. The theory accounts for the shear deformation effects without requiring a shear correction factor. By dividing the transverse displacement into bending, shear and stretching components, the number of unknowns and governing equations of the present theory is reduced to four and is therefore less than alternate theories. The accuracy and efficiency of the present theory has been demonstrated for thermo-mechanical bending behavior of antisymmetric cross-ply laminates.

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