Steel and Composite Structures, *Vol. 18*, *No. 6* (*2015*) *1391-1404*
DOI: http://dx.doi.org/10.12989/scs.2015.18.6.1391

Static analysis of a radially retractable hybrid grid shell in the closed position

Jianguo Cai [*1], Chao Jiang [1a], Xiaowei Deng [2b], Jian Feng [1c] and Yixiang Xu [3d]

[1] *Key Laboratory of C & PC Structures of Ministry of Education*, *National Prestress Engineering Research Center*, *Southeast University*, *Nanjing 210096*, *China*
[2] *Department of Structural Engineering*, *University of California*, *San Diego*, *USA*
[3] *Department of Civil Engineering*, *Strathclyde University*, *Glasgow G12 8QQ*, *United Kingdom*

(*Received May 29*, *2014*, *Revised November 16*, *2014*, *Accepted November 26*, *2014*)

Abstract. A radially retractable roof structure based on the concept of the hybrid grid shell is proposed in this paper. The single-layer steel trusses of the radially foldable bar structure are diagonally stiffened by cables, which leads to a single-layer lattice shell with triangular mesh. Then comparison between the static behavior between the retractable hybrid grid shell and the corresponding foldable bar shell with quadrangular mesh is discussed. Moreover, the effects of different structural parameters, such as the rise-to-span ratio, the bar cross section area and the pre-stress of the cables, on the structural behaviors are investigated. The results show that prestressed cables can strengthen the foldable bar shell with quadrangular mesh. Higher structural stiffness is anticipated by introducing cables into the hybrid system. When the rise-span ratio is equal to 0.2, where the joint displacement reaches the minimal value, the structure shape of the hyrbid grid shell approaches the reasonable arch axis. The increase of the section of steel bars contributes a lot to the integrity stiffness of the structure. Increasing cable sections would enhance the structure stiffness, but it contributes little to axial forces in structural members. And the level of cable prestress has slight influence on the joint displacements and member forces.

Keywords: hybrid structure; grid shell; retractable roof; foldable bar structure; static behavior

1. Introduction

Retractable roof structures, which enable the users of recreational facilities to enjoy open air as long as allowed by the weather, are playing an important role in the development of multi-functional sports and culture facilities to hold events such as Olympics, Tennis Professional tours and World Cup (Ishii 2000). With one or more degrees of freedom, deployable/foldable structures can expand to form a large opening space, which make them ideal for use as retractable roofs (Mao and Luo 2008, Van Mele *et al*. 2010). In the past two decades, many concepts of

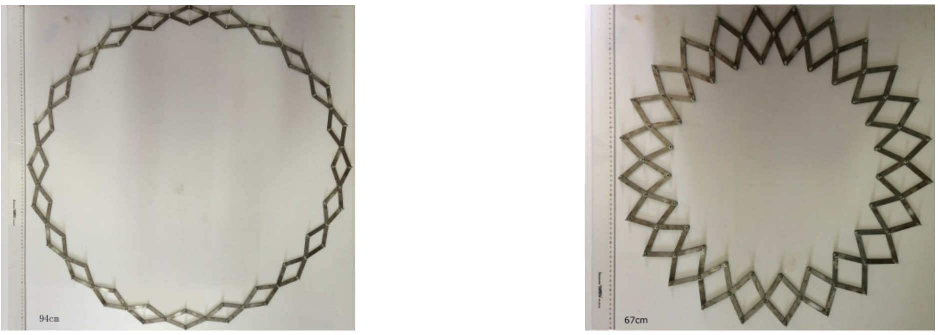
*Corresponding author, Associate Professor, E-mail: caijg_ren@hotmail.com; j.cai@seu.edu.cn
[a] Graduate Student, E-mail: jiangc_2008@126.com
[b] Ph.D., E-mail: x8deng@eng.ucsd.edu
[c] Professor, E-mail: fengjian@seu.edu.cn
[d] Lecturer, E-mail: yixiang.xu@strath.ac.uk

http://www.techno-press.org/?journal=scs&subpage=8 ISSN: 1229-9367 (Print), 1598-6233 (Online)

deployable structures have been proposed for retractable roof structures (De Temmerman *et al*. 2012).

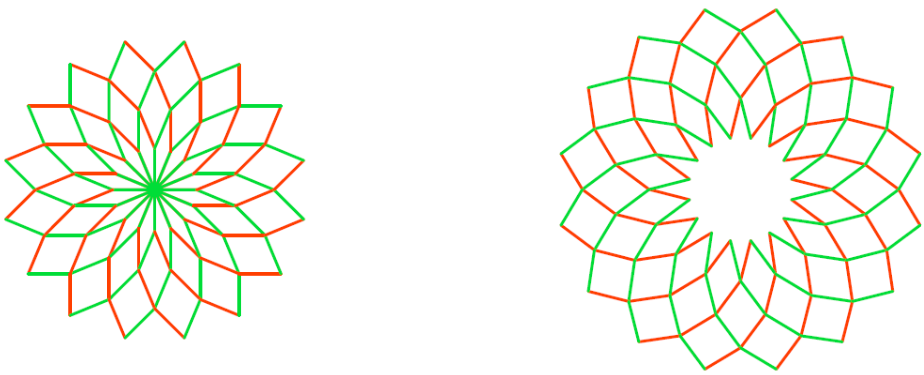
Among these concepts, there is a new type of retractable structures which use angulated scissor-like elements as basic structural elements (Hoberman 1990, 1991, 1992). The unit, which consists of a pair of identical angulated rods connected by a revolute joint, subtends a constant angle as their rods rotate while maintain the end pivots on parallel lines. Therefore, it can create a closed-loop mechanism, which is called Hoberman's Linkage as shown in Fig. 1. Then this concept was extended by You and Pellegrino (1997) who gave birth to a more general family of such structures called the foldable bar structures. One type of these foldable bar structures, as shown in Fig. 2, is made from multi-angulated rods similar to the angulated scissor-like elements introduced in Hoberman's linkage.

However, if this foldable bar structures is used as a retractable roof structure, Teall (1996) found that the deformation of the structure, even for a three layer foldable bar structure with a span of 2 m, was quite large under self-weight. This leads to the problem that the stiffness of the structure is too small to resist the external loads. Mao and Luo (2008) pointed out that the previous model was mounted on a set of pinned columns at the positions of outmost hinges which led to a large deformation. They proposed two support systems to work together with the foldable bar



(a) (b)

Fig. 1 Hoberman's Linkage



(a) (b)

Fig. 2 Foldable bar structures

structure, which may bear the external loads more effectively. They drew a conclusion that that the stiffness of the retractable roof structure can meet the requirement of practical application by employing a proper support system. But these support systems are complex.

To advance the practical application of the foldable bar structure in long-span retractable roofs, this paper proposed a hybrid foldable bar structure. El-Sheikh (2001a, b) pointed out that a single-layer lattice shell with quadrangular mesh lack integration between its segments, which is much less stiff in the plane compared to the single-layer lattice shell with triangular mesh. Moreover, the stability of shells becomes most dependent on the joint rigidity. Cables, which are light, can also be constructed to a structural system with a rational layout of members so as to make the best use of individual material properties (Hosozawa *et al*. 1999). To achieve high performance of the retractable roof structure, the single-layer steel trusses are diagonally stiffened by cables. Note the cables under compression will not become slack when the vault is pre-stressed before applying external loads. The static and dynamic behavior of the barrel vaults and domes based on this concept have been studied by many researchers (Schlaich and Schober 1997, Bulenda and Knippers 2001, Glymph *et al*. 2004, Cai *et al*. 2012, 2013, Feng *et al*. 2012, 2013, Shon *et al*. 2013). However, in these studies, most of the researchers assumed that the structures were not deployable and the steel trusses were rigidly jointed.

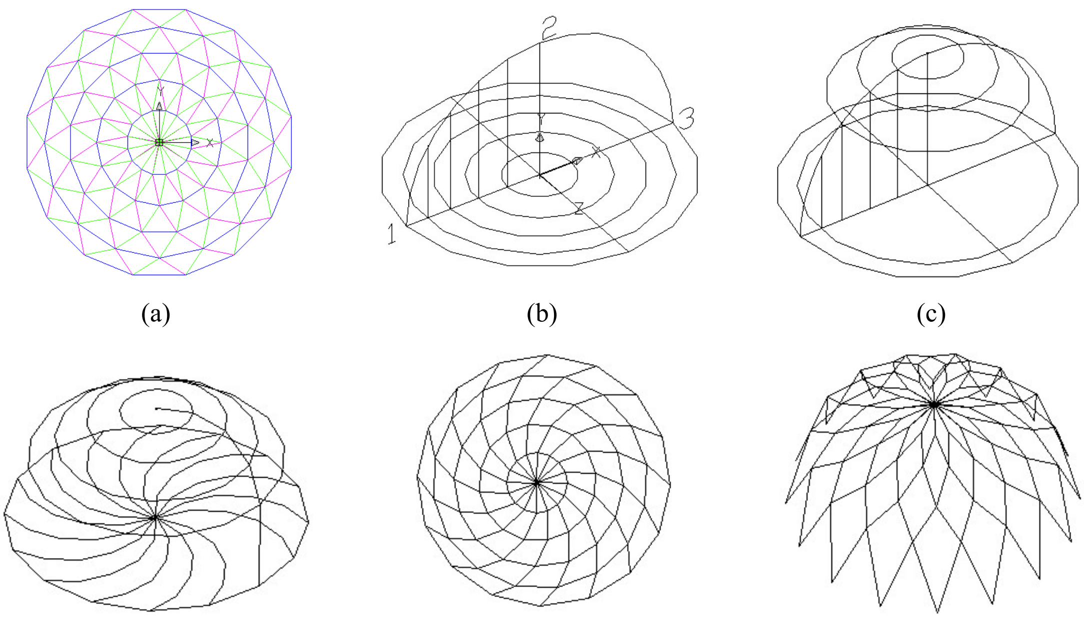
This paper addresses several key issues of the radially retractable hybrid grid shell. Firstly, the structural system was introduced in Section 2. Then a comparison of the static behavior between the retractable hybrid grid shell and the corresponding foldable bar shell with quadrangular mesh is discussed in Section 3. Section 4 gives a parametric study of the static behavior. Areas of steel bars and cables, pre-stresses of cables, and rise-span ratios are considered. Section 5 draws some conclusions.

2. Structural system

Normally, the dome structure has a better structural stiffness than the planar one. Therefore, the two-dimensional foldable bar structures are extend to curved structures, which is obtained by projecting any two-dimensional structure onto a surface with the required shape as shown in Fig. 3. Thus, each multi-angulated rod can be curved out of plane. Of course, all the connectors between the multi-angulated rods should be perpendicular to the plane of projection.

In this paper, the single layer latticed shell structure of the hybrid grid shell given in Cai *et al*. (2012) is replaced by the radially retractable roof structure shown in Fig. 3, which leads to the radially retractable hybrid grid shell given in Fig. 4. The typical joint of the system is shown in Fig. 5. In the deployment progress, the structure unfolds gradually with the driving of support joints of foldable bar structures (as shown in Fig. 6). And the radial cables are in slack while the length of hoop cables keeps increasing in this period. Similarly, in the folding progress, hoop cables can be considered as active cables and radial cables are passive cables. Then when the structure is totally closed, prestress will be imposed on the hybrid grid shell if active cables are remain stretched. Therefore, it should be that only the single layer grid shell resists external loads in the moving process of the structure. Only when the structure is in the fully closed state, the foldable bar structure and the cable net will work together as the cable supported grid shell structure.

Besides as the role of active and passive elements, the cables stiffen the retractable dome in two different ways. First, for the pin-connected four bar linkage formed by angulated scissor-like elements, the cables transform the un-stiffened kinematic system into a statically determinate or



(a) (b) (c)

(d) (e) (f)

Fig. 3 Foldable dome extended from the two-dimensional structure

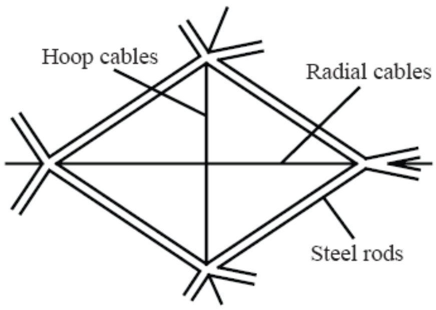


Fig. 4 Radially retractable hybrid grid shell

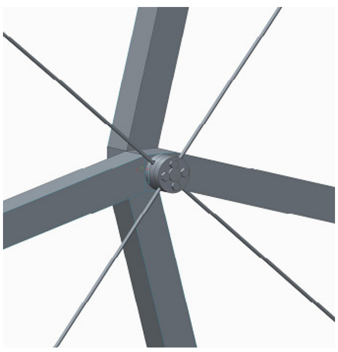
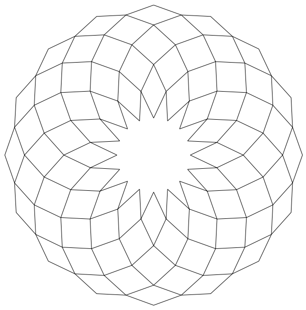
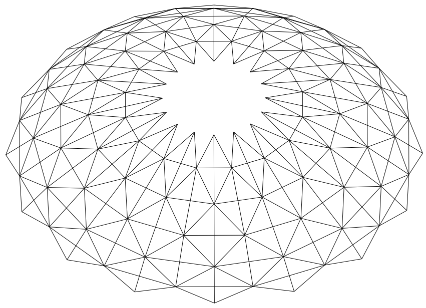


Fig. 5 The typical joint of the system



(a) Foldable bar structures



(b) Retractable hybrid grid shell

Fig. 6 Deployment state of the system

indeterminate structure. For a single-layer lattice structure, the cables transform the system into a triangular meshed structure. The other way of stiffening the hybrid system by cables is that applying pretensions of cables transforms unfavorable compressive stresses into favorable tension stresses.

3. Structural behavior

3.1 Finite element model

An example of the hybrid grid shell shown in Fig. 7 will be studied in this paper. In this model, every multi-angulated rod has six segments, whose projected length in the horizontal plane is 4 m. And the inclined angle between adjacent segments is 157.5°. The span of the grid shell is 38 m with the rise/span ratio of 0.2. Other geometric parameters are given in Fig. 8.

The symmetrical load case g + s (dead load + snow load) has been taken into account in all the following computations. The dead load g consists of a self-weight of 0.5 kN/m^2 including all the beams and cables. The snow load is applied to the top surface of the structure in the vertical direction with a magnitude of 0.5 kN/m^2.

In this paper, the finite element software ANSYS is used to analyze the structure and the geometric nonlinearity is fully considered. The angulated rods are simulated by BEAM188. Joints

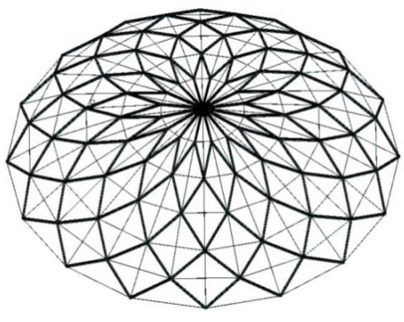
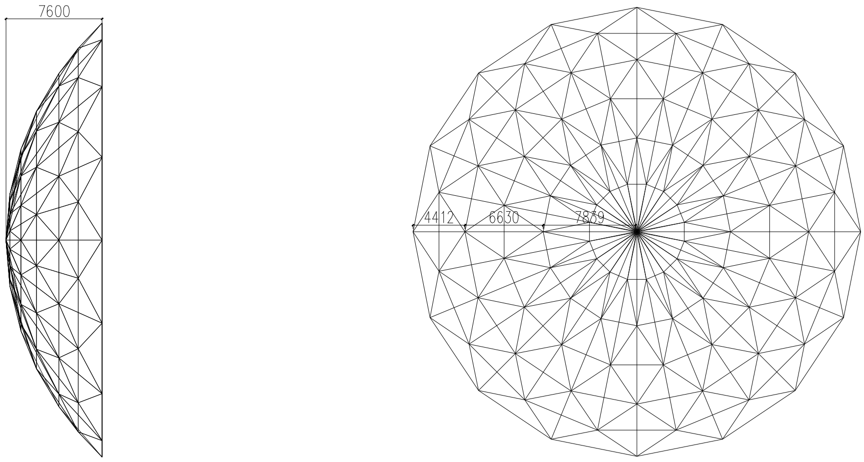


Fig. 7 3D hybrid grid shell model



(a) (b)

Fig. 8 Geometric parameters of the hybrid grid shell (mm)

of these multi-angulated rods are pinned with the release of rotational degree of freedom and the boundary of the hybrid grid shell is fixed circularly. Members of multi-angulated rods adopt the box beam with Q345B steel, with dimensions of cross section 200 mm × 150 mm × 10 mm (height × width × wall thickness). The yield stress of Q345B steel is 345 MPa. Additionally, the cross section area of the cable, which is simulated by tension-only bar element LINK10, is 78.5 mm^2 and the cable with initial prestress of 100 MPa won't yield. The Young's modulus of steel bars and cables are 210 MPa and 180 MPa respectively.

3.2 Static behavior analysis

The static behavior analysis of structures, which offers the fundamental theoretical base for structure design as well as essential references for other analysis such as dynamic behavior analysis, stability analysis etc., is an important part of structural analysis. There are some literatures aimed at the theoretical and experimental research on static behavior of hybrid grid shells. However, studies focused on the radially retractable hybrid grid shell are lack. Therefore, in order to fit the continuous development of the engineering practice, static behavior of the radially retractable cable supported grid shell would be investigated in this paper to draw relevant general conclusions.

According to the whole construction process of the hybrid grid shell, force states of the structure can be divided into three parts: zero state, initial state, load state. Zero state is the state bearing self-weight before installation, initial state denotes the state after the finish of installation and prestress action and load state begins once the structure bears other external loads except self-weight.

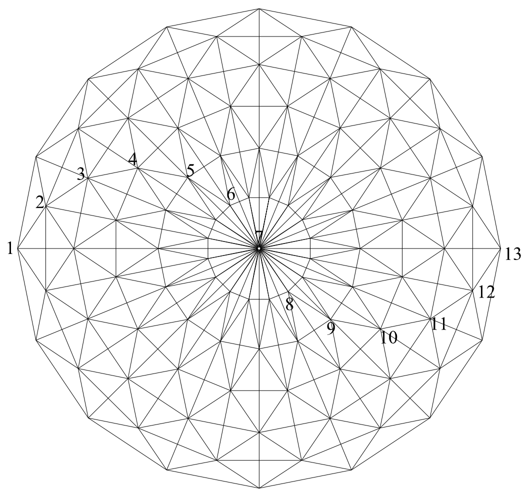
In order to better reflect the influence of external loads on each parameter of the structure, resistance of the structure to external loads should be taken into consideration. Then the joint displacements and steel bar forces picked up from the static behavior analysis of the structure are the values under the load state minus those under the initial state, leading to a better understanding on the structural ability resisting external loads. Thus the study of static behavior on the structure can be divided into three steps:

(1) Calculate the member displacement and force under the initial state;
(2) Calculate the member displacement and force under the load state;
(3) Subtract results of the first two steps to obtain the structure response only under external loads (without considering self-weight and prestress).

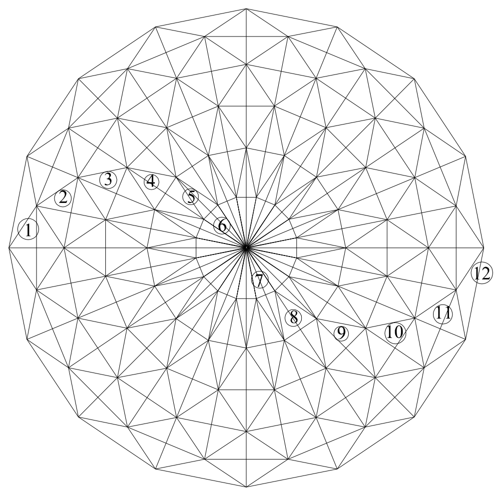
The typical nodes and elements of the hybrid grid shell structure, which are shown in Fig. 9, are chosen to describe the static behavior of the structure. Anti-symmetrical load case is an important factor that affects the static behavior of hybrid barrel vaults. One type of anti-symmetrical load is the half-span load which can be resulted during construction or from snow. The anti-symmetrical load case g + s / 2 (g – dead load; and s – snow load, uniformly distributed over half of the span) is considered in this study. The half-span snow load is applied in the left half-span.

In order to better understand the effect of the prestress in the retractable hybrid grid shell on the static behavior, the joint displacements and bar axial forces of the hybrid grid shell and general grid shell without cable are compared under the symmetric loading case g + s (Load 1) and the anti-symmetric loading case g + s / 2 (Load 2), respectively.

Fig. 10 shows the vertical joint displacement comparison between the foldable bar structures and retractable hybrid grid shell. It can be found that the distribution of joint displacements between the two structures is almost identical, and the only difference is the absolute values. Due to the existence of prestressed cables under both symmetric and anti-symmetric load cases, joint displacements of the structure decrease significantly. The displacements of any joint in the foldable bar structure envelope the corresponding value of the retractable hybrid grid shell. Fig. 11 are axial stresses comparison diagrams of steel bars for the foldable bar structure and retractable hybrid grid shell. It can be obtained that the axial stresses in the steel bars of the retractable hybrid grid shell are slightly smaller than those of the foldable bar structure for the most part of the typical elements. For the anti-symmetric load case, the steel bar stresses have a remarkable change in the central part of the foldable bar structure. However, the retractable hybrid grid shell has a more smooth behavior.

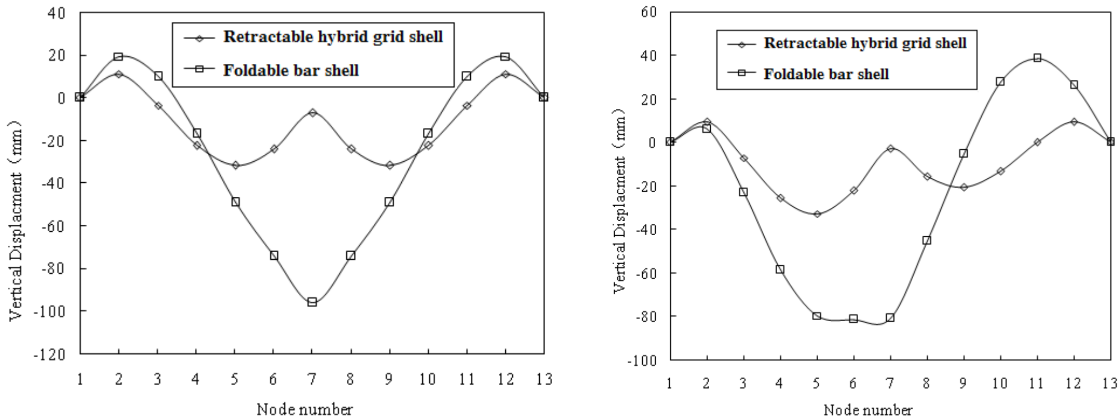


(a) Joint numbers



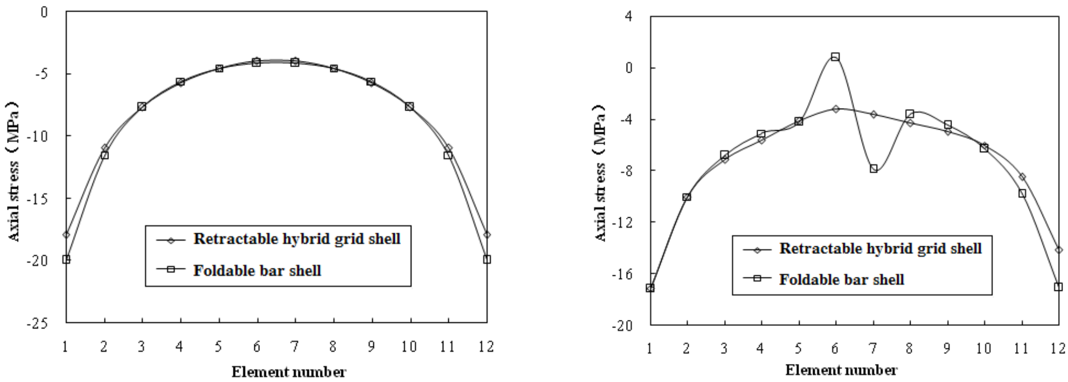
(b) Element numbers

Fig. 9 The design model of the hybrid barrel vault with typical node and element numbers



(a) Load 1 (b) Load 2

Fig. 10 Nodal displacements of the foldable bar structure and retractable hybrid grid shell



(a) Load 1 (b) Load 2

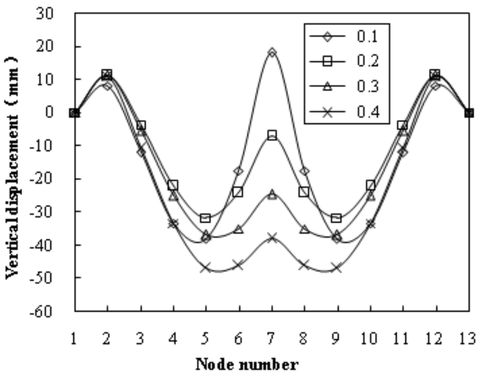
Fig. 11 Steel bar axial stresses of the foldable bar structures and retractable hybrid grid shell

4. Parameter analyses

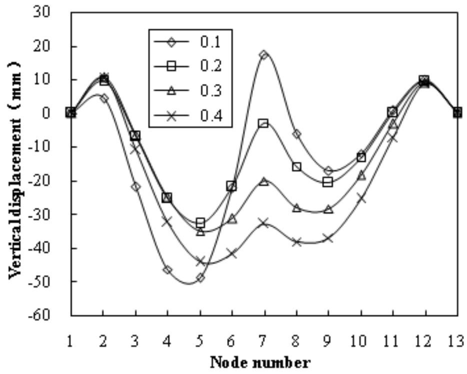
4.1 Influence of the rise-span ratio

The rise-span ratio deeply influences the rationality of arch structures. When the arch shape approaches the reasonable arch axis, the arch only bears axial force with the maximum structure stiffness. Then when the arch shape largely deviates from the axis, the arch effect of the structure can be ignored. As well, the grid shell has the similar phenomenon.

By changing the rise of the primary model in 4 cases, four hybrid grid shell models can be constructed. The joint vertical displacements of structures with the rise-span ratio of 0.10, 0.20, 0.30, and 0.40 are given in Fig. 12. It can be seen from this figure that the joint displacements around the supports of the structure are almost approximate while the difference is large in the mid-span. The vertical displacement of the structure under both load case reaches the minimum

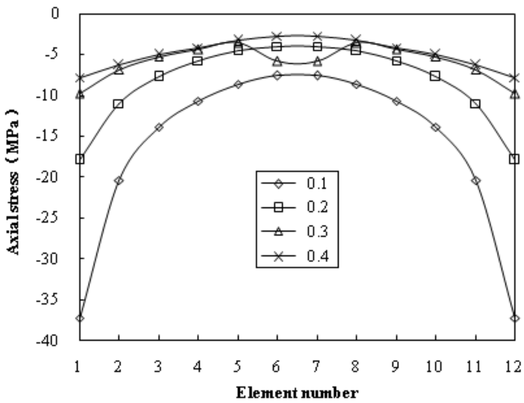


(a) Load 1

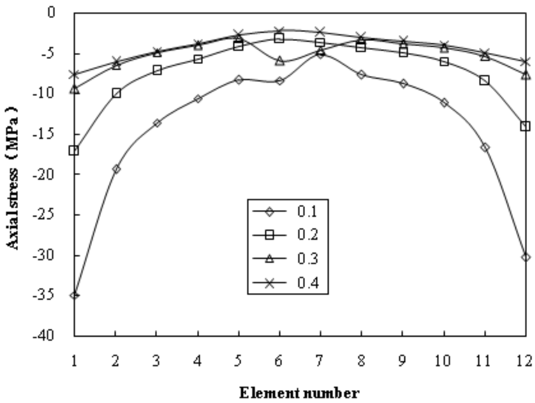


(b) Load 2

Fig. 12 Influence of rise-span ratios on the nodal displacement



(a) Load 1



(b) Load 2

Fig. 13 Influence of rise-span ratios on the axial stress of steel bars

when the rise-span ratio is equal to 0.2, which means the shape of the grid shell approaches the reasonable arch axis. Then for the Load 1, the vertical displacement of each joint in the structure increases when the rise-span ratio is less or greater than 0.2. For the anti-symmetric load case, the trend of the joint displacement on the left half-span, which is considered the snow load, is the same as that for Load 1. On the other half-span, the joint displacement increase with the increase of the rise-span ratio.

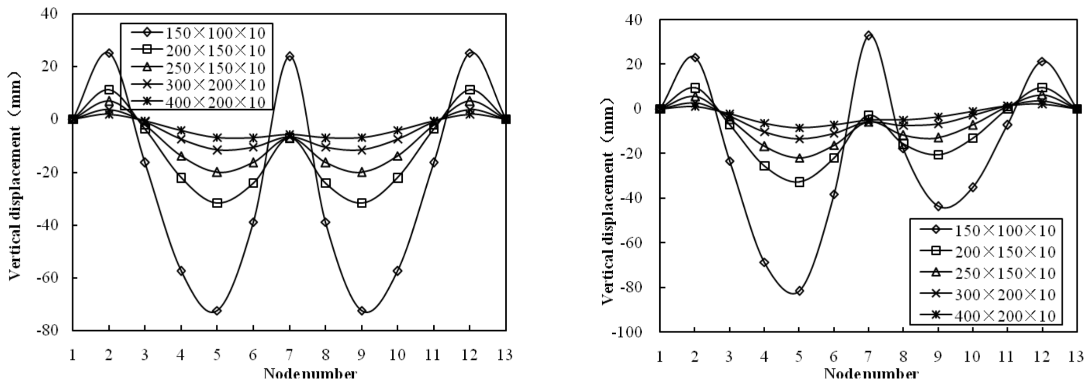
Fig. 13 shows the axial stress of steel bars for the structure with different rise-span ratios. It is obvious that the rise-span ratio has a great influence on the axial forces of members, which decrease with the increase of rise-span ratios under both load cases.

4.2 Influence of the steel bar section

Various steel bar sections (150 mm × 100 mm × 10 mm, 200 mm × 150 mm × 10 mm, 250 mm × 150 mm × 10 mm, 300 mm × 200 mm × 10 mm, 400 mm × 200 mm × 10 mm) are chosen to studied the effect of bar section on static behavior. The joint vertical displacements of the structure are presented in Fig. 14. It can be obtained the vertical stiffness of the structure has a great improvement with the increase of the steel bar section. The reason is that most of the structural stiffness of the retractable hybrid grid shell is supplied by steel bars. Therefore, the stiffness improvement of steel bars contributes a lot to the rise of the whole stiffness of the structure. However, on the other hand, as the global structural stiffness increases more, its influence on the vertical displacement of the structure becomes weaker. Fig. 15 shows the bar axial stress of the structures with different steel bar sections. It can be observed the axial stress of the structure members has a corresponding decrease with the increase of steel bar sections.

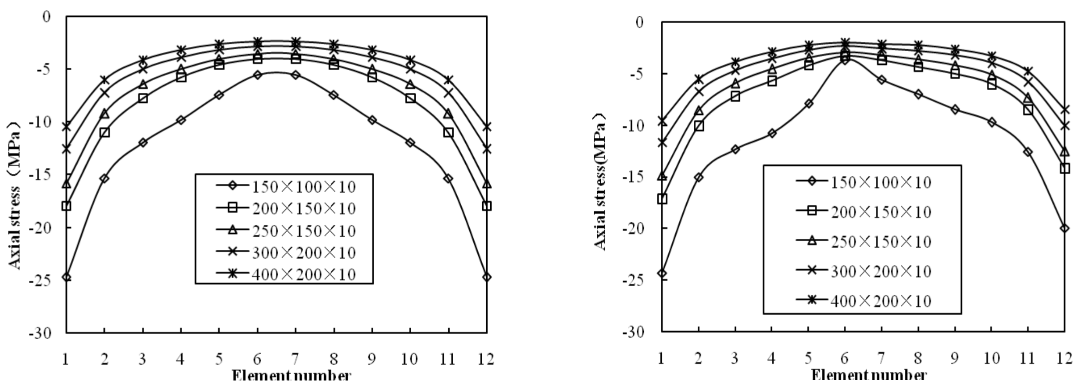
4.3 Influence of the cable area

Different cable sections with diameters of 8 mm, 10 mm, 12 mm, 15 mm, and 20 mm are adopted to investigate the cable area on the static behavior of retractable hybrid grid shell. The



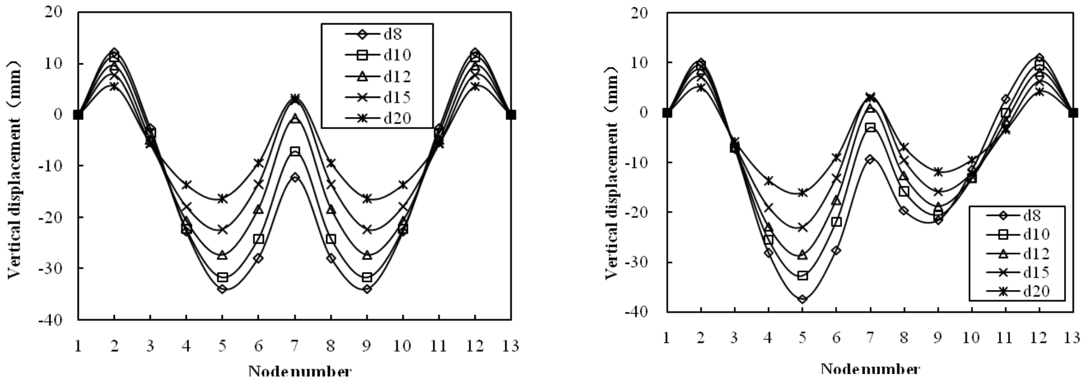
(a) Load 1 (b) Load 2

Fig. 14 Influence of steel bar cross section area on the nodal displacement



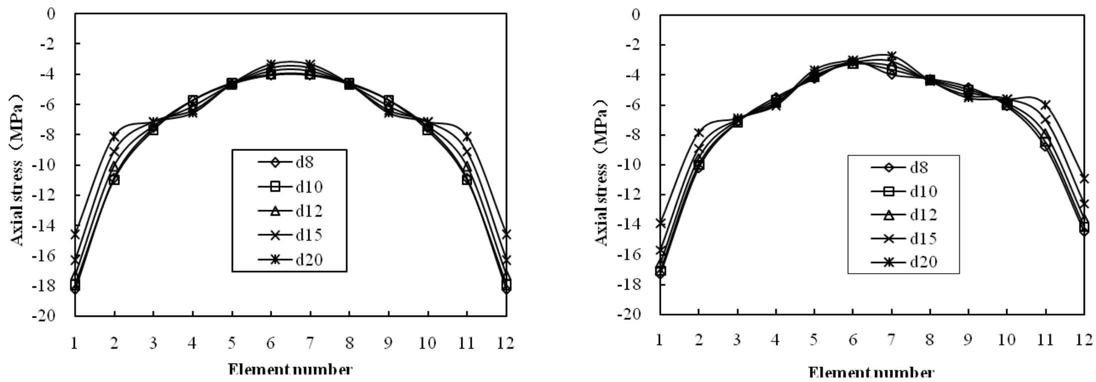
(a) Load 1 (b) Load 2

Fig. 15 Influence of steel bar cross section area on its axial stress



(a) Load 1 (b) Load 2

Fig. 16 Influence of the cable cross section area on the nodal displacement



(a) Load 1 (b) Load 2

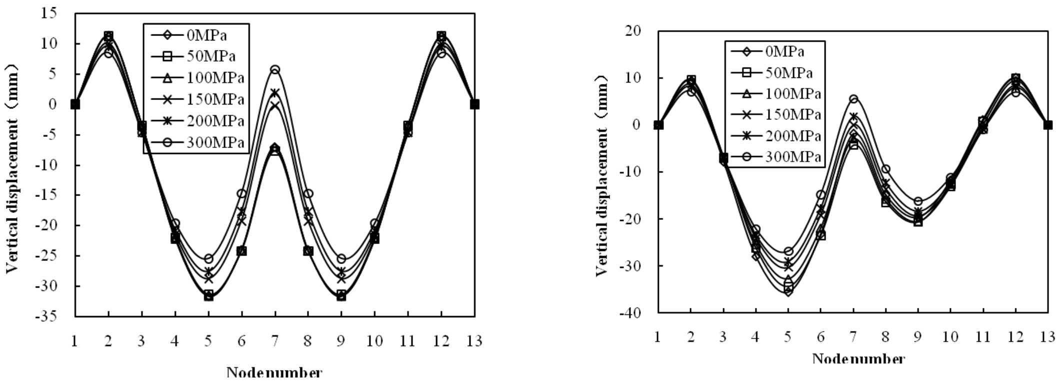
Fig. 17 Influence of the cable cross section area on the axial stress of steel bars

joint vertical displacements with different cable areas are shown in Fig. 16. It can be easily observed, under Load 1 and Load 2, the joint vertical displacements decrease with the increase of the cable area. With the rise of the cable section, the structural stiffness increases correspondingly, and the joint displacements of the structure decrease accordingly.

Fig. 17 describes the axial stress change of steel bars in the structure with different cable areas. It can be found that the cable area has a weak influence on the axial stress of steel bars under both load cases.

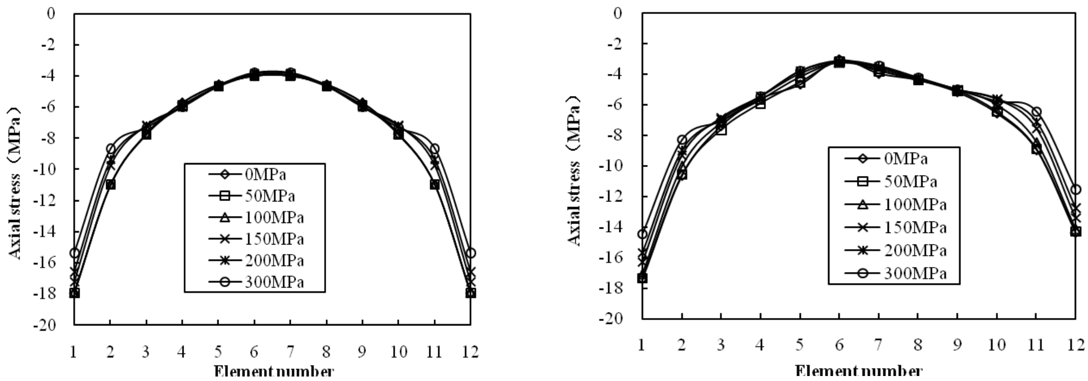
4.4 Influence of the initial prestress of cables

Different levels of initial prestresses are set to be 0 MPa, 50 MPa, 100 MPa, 150 MPa and 200 MPa, respectively. The variation of the vertical joint displacements of the structure with different cable initial pretress is shown in Fig. 18. It can be observed, under Load 1 and Load 2 cases, the cable initial prestress has a slight influence on the structure and the displacement curve shapes are



(a) Load 1 (b) Load 2

Fig. 18 Influence of the cable initial prestress on the nodal displacement



(a) Load 1 (b) Load 2

Fig. 19 Influence of the initial prestress on the axial stress of steel bars

similar. Fig. 19 shows axial stresses of steel bars in the structure with different cable initial prestress. Just like the result of displacement values, the cable initial prestress contributes little to axial stresses in steel bars. With the initial prestress ranged from 0 MPa to 200 MPa, axial stresses in members chosen under both load cases are nearly close. Therefore, it can be concluded that the cable initial prestress contributes little to the structure.

5. Conclusions

This paper introduces the conception of hybrid grid shells into the radially retractable roof system.

The mechanical characteristics of the hybrid structure were investigated. Comprehensive static analysis was carried out on the foldable bar structures and the retractable hybrid grid shell. Then a numerical parametric analysis of the hybrid structure was performed. Based on the results, the following conclusions are drawn:

- It can be found that effects of prestressed cables strengthen the strucutre. The hybrid system constructed by introducing cables has smaller structural deformations and lower member forces in comparison with the foldable bar structure.
- The displacement of each joint in the structure increases when the rise-span ratio is larger or less than 0.2. Axial forces in steel bars decrease with the increase of the rise-span ratio.
- As most of the structural stiffness in the hybrid grid shell supplied by steel bars, the increase of the section of steel bars contributes a lot to the integrity stiffness of the structure. And axial stresses in structure members decrease correspondingly with the increase of steel bar sections.
- Increasing cable sections contributes little to axial forces in members. And the level of initial prestresses in cables has no remarkable influence on the joint displacements and member forces.

Acknowledgments

The work presented in this article was supported by the National Natural Science Foundation of China (Grant No. 51308106, No. 51278116 and No. 51378104), the Natural Science Foundation of Jiangsu Province (Grant No. BK20130614), the Specialized Research Fund for the Doctoral Program of Higher Education (Grant No. 20130092120018) and a Project Funded by the Priority Academic Program Development of Jiangsu Higher Education Institutions. Authors also thank the anonymous reviewers for their valuable comments and thoughtful suggestions which improved the quality of the presented work.

References

Bulenda, T. and Knippers, J. (2001), "Stability of grid shells", *Compos. Struct.*, **79**(12), 1161-1174.

Cai, J.G., Xu, Y.X., Wang, F., Feng, J. and Zhang, J. (2012), "A computational study of the static and dynamic response of a hybrid barrel vault structure", *Adv. Steel Constr.*, **8**(4), 317-330.

Cai, J.G., Xu, Y.X., Feng, J. and Zhang, J. (2013), "Design and analysis of a glass roof structure", *Struct. Design Tall Spec. Build*, **22**(8), 677-686.

De Temmerman, N., Alegria Mira, L. and Vergauwen, A. (2012), "Feasibility of the Universal Scissor Component (USC): Building a full-scale deployable dome", *J. Int. Assoc. Shell Spatial Struct.*, **53**(4), 227-236.

El-Sheikh, A. (2001a), "Configurations of single-Layer barrel vaults", *Adv. Struct. Eng.*, **4**(2), 53-64.

El-Sheikh, A. (2001b), "Performance of single-layer barrel vaults with different configurations", *Int. J. Space Struct.*, **16**(2), 111-123.

Feng, R.Q., Ye, J.H. and Yao, B. (2012), "Evaluation of the buckling load of an elliptic paraboloid cable-braced grid shell using the continuum analogy", *J. Eng. Mech.*, **138**(12), 1468-1478.

Feng, R.Q., Yao, B. and Ye, J.H. (2013), "Stability of lamella cylinder cable-braced grid shells", *J. Constr. Steel Res.*, **88**, 220-230.

Glymph, J., Shelden, D., Ceccato, C., Mussel, J. and Schober, H. (2004), "A parametric strategy for free-form glass structures using quadrilateral planar facets", *Automation Constr.*, **13**(2), 187-202.

Hoberman, C. (1990), *Reversibly Expandable Doubly-Curved Truss Structures*, US Patent 4; 942, 700.

Hoberman, C. (1991), *Radial Expansion Retraction Truss Structure*, US Patent 5; 024, 031.

Hoberman, C. (1992), "Art and science of folding structures", *Sites*, **24**, 61-69.

Hosozawa, O., Shimamura, K. and Mizutani, T. (1999), "The role of cables in large span spatial structures: Introduction of recent space structures with cables in Japan", *Eng. Struct.*, **21**(8), 795-804.

Ishii, K. (2000), *Structural Design of Retractable Roof Structures*, WIT Press, Southampton, Boston, MA, USA.
Mao, D. and Luo, Y. (2008), "Analysis and design of a type of retractable roof structure", *Adv. Struct. Eng.*, **11**(4), 343-354.
Schlaich, J. and Schober, H. (1997), "Glass roof for the Hippo house at the Berlin zoo", *Struct. Eng. Int.*, **7**(4), 252-254.
Shon, S.D., Lee, S.J. and Lee, K.G. (2013), "Characteristics of bifurcation and buckling load of space truss in consideration of initial imperfection and load mode", *J. Zhejiang Univ.-Sci. A*, **14**(3), 206-218.
Teall, M.J. (1996), "Deployable roof structures", Masters Dissertation; University of Cambridge, UK.
Van Mele, T., De Temmerman, N., De Laet, L. and Mollaert, M. (2010), "Scissor-hinged retractable membrane structures", *Int. J. Struct. Eng.*, **1**(3/4), 374-396.
You, Z. and Pellegrino, S. (1997), "Foldable bar structures", *Int. J. Solids Struct.*, **34**(15), 1825-1847.

CC