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Optimum design of laterally-supported castellated beams using CBO algorithm

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Abstract. In this study, two common types of laterally supported castellated beams are considered as design problems: beams with hexagonal openings and beams with circular openings. The main goal of manufacturing these beams is to increase the moment of inertia and section modulus, which results in greater strength and rigidity. These types of open-web beams have found widespread use, primarily in buildings, because of great savings in materials and construction costs. Hence, the minimum cost is taken as the design objective function and the Colliding Bodies Optimization (CBO) method is utilized for obtaining the solution of the design problem. The design methods used in this study are consistent with BS5950 Part 1 and Part 3, and Euro Code 3. A number of design examples are considered to optimize by CBO algorithm. Comparison of the optimal solution of the CBO algorithm with those of the Enhanced Charged System Search (ECSS) method demonstrate the capability of CBO in solving the present type of design problem. It is also observed that optimization results obtained by the CBO algorithm for three design examples have less cost in comparison to the results of the ECSS algorithm. From the results obtained in this paper, it can be concluded that the use of beam with hexagonal opening requires smaller amount of steel material and it is superior to the cellular beam from the cost point of view.

Keywords: steel castellated beams; colliding bodies optimization; optimal design

1. Introduction

The production of structural beams with higher strength and lower cost has been studied by engineers since 1940. Due to the limitations on maximum allowable deflections, and the high strength properties of steel, it cannot always be utilized to the best advantage. As a result, several new methods have been developed for increasing the stiffness of beams without necessity to increase the weight of the required steel. Hence, castellated and cellular beams have been utilized extensively in recent years (Konstantinos and D’Mello 2012).

In design of steel structures, beams with web-opening are widely used to pass the under floor services ducts such as water pipes and air ducts. Castellated beams are varieties of girders that are manufactured by using an unaltered wide flange steel beam and cutting a certain pattern through its web, often in half-circle or half-hexagon patterns. The split halves are then offset and welded

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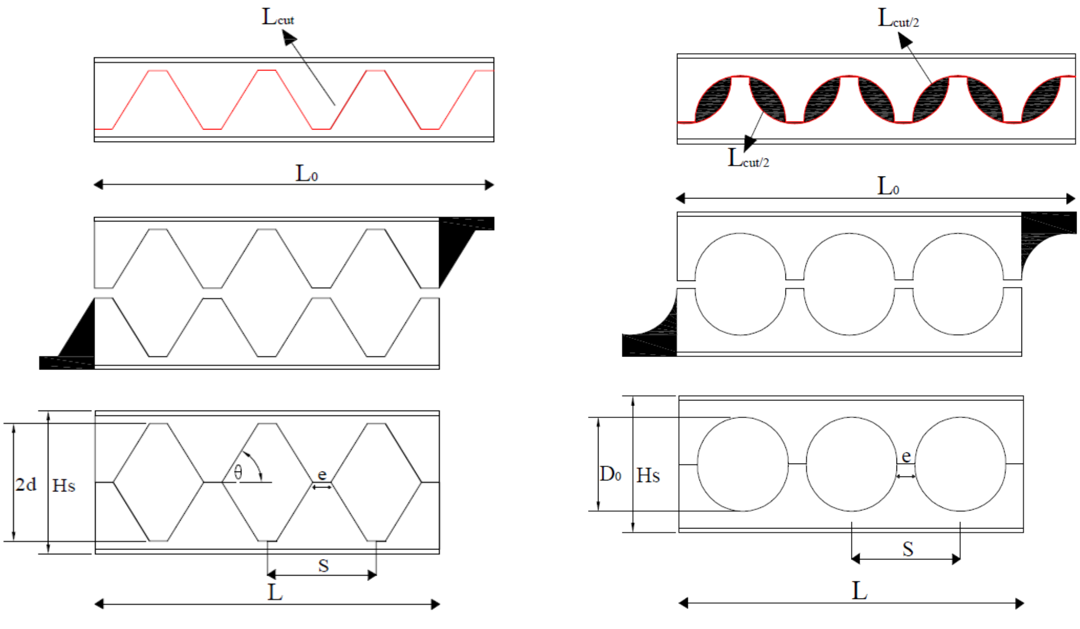
back together to form a deeper beam with full circular or hexagonal shaped web openings. Web-openings have been used for many years in structural steel beams in a great variety of applications because of the necessity and economic advantages. The main advantage of the steel castellated beam is that a designer can increase the depth of a beam to raise its strength without additional steel. The resulting castellated beam is approximately 50% deeper and much stronger than the original unaltered beam (Soltani *et al*. 2012, Zaarour and Redwood 1996, Redwood and Demirdjian 1998, Sweedan 2011). In recent years, a great deal of progress has been made in the design of steel beams with web-openings and a cellular beam is one of them. A cellular beam is the modern version of the traditional castellated beam, but with a far wider range of applications for floor beams. Cellular beams are steel sections with circular openings that are made by cutting a rolled beam web in a half circular pattern along its centerline and re–welding the two halves of hot rolled steel sections as shown in Fig. 1. This opening increases the overall beam depth, moment of inertia, and section modulus, without increasing the overall weight of the beam (Konstantinos and D'Mello 2011).

Recently, a new optimization method, so-called Colliding Bodies Optimization (CBO), is developed by Kaveh and Mahdavi (2014a, b) that utilizes simple formulation and it requires no parameter tuning. The main objective of this paper is to investigate the differences in cost associated with the castellated beams with hexagonal opening and cellular beams. Here, the CBO algorithm is utilized for optimization and cost of the beam is considered as the objective function.

In the first part of this paper, the design of castellated beam is introduced. In Section 2, optimum design of these beams is formulated based on The Steel Construction Institute Publication Number 100 and Euro Code 3. In Section 3, the CBO algorithm is briefly introduced,



(a) (b)

Fig. 1 (a) A castellated beam with hexagonal opening; (b) a castellated beam with circular opening or cellular beam and in Section 4, the cost of castellated beam as the design objective function is minimized. Design examples are provided in Section 5, and finally Section 6 concludes the paper.

2. Design of castellated beams

Beams must be sufficiently strong to sustain the bending moments and shear forces produced by the applied loads. The performance of a beam dependents on the physical dimensions as well as the cross-section geometry and shape. Due to the presence of holes in the web, the structural behavior of castellated steel beam is different from that of the solid web beams. At present, there is no general accepted design method due to the complexity of the behavior of castellated beams and their associated modes of failure (Soltani *et al*. 2012). The strength of a beam with various web opening is determined by considering the interaction of the flexure and shear at the openings. There are many failure modes to be considered in the design of a beam with web opening consisting of Vierendeel mechanism, flexural mechanism, rupture of welded joints, and web post buckling. Lateral torsional buckling may also occur in an unrestrained beam. A beam is considered to be unrestrained when its compression flange is free to displace laterally and rotate. In this paper, it is assumed that the compression flange of the castellated beam is restrained by the floor system (the same as Saka (2009)). Therefore, the overall buckling strength of the castellated beam is omitted from the design consideration. The above mentioned modes are closely associated with beam geometry, shape parameters, type of loading, and provision of lateral supports. In the design of castellated beams, these cases should be considered (EN 1993-1-1 2005, Ward 1990, Erdal *et al*. 2011, Saka 2009, Raftoyiannis and Ioannidis 2006, British Standards 2000, LRFD-AISC 1986).

2.1 Overall beam flexural capacity

This mode of failure can occur when a section is subjected to pure bending. The span subjected to pure bending moment, the tee-sections above and below the holes yields in a manner similar to that of a plain webbed beam. Therefore, the maximum moment under applied external loading M_U should not exceed the plastic moment capacity M_P of the castellated beam (Ward 1990, Erdal *et al*. 2011).

$$M_U \le M_P = A_{LT} P_Y H_U \tag{1}$$

where A_{LT} is the area of lower tee, P_Y is the design strength of steel, and H_U is distance between center of gravities of upper and lower tees.

2.2 Beam shear capacity

In the design of castellated beams, it is necessary to control two modes of shear failures. The first one is the vertical shear capacity. The sum of the shear capacity of the upper and lower tees forms the shear capacity of the section and is checked using the following equations (Soltani *et al*. 2012, Erdal *et al*. 2011)

$$\begin{aligned} P_{VY} &= 0.6 P_Y \left(0.9 A_{WUL}\right) \quad \text{For circular opening} \\ P_{VY} &= \frac{\sqrt{3}}{3} P_Y \left(A_{WUL}\right) \quad \text{For hexagonal opening} \end{aligned} \tag{2}$$

The second one is the horizontal shear capacity. It is developed in the web post due to the change in axial forces in the tee-section as shown in Fig. 2. Web post with too short mid-depth welded joints may fail prematurely when horizontal shear exceed the yield strength. The horizontal shear capacity is checked using the following equations (Soltani *et al*. 2012, Erdal *et al*. 2011).

$$\begin{aligned} P_{VH} &= 0.6P_Y\left(0.9A_{WP}\right) \quad \text{For circular opening} \\ P_{VH} &= \frac{\sqrt{3}}{3}P_Y\left(A_{WP}\right) \quad \text{For hexagonal opening} \end{aligned} \tag{3}$$

where A_{WUL} is the total area of the webs of the tees and A_{WP} is the minimum area of web post.

2.3 Flexural and buckling strength of web post

It is assumed that the compression flange of the castellated beam are restrained by the floor system. Thus the overall buckling of the castellated beam is omitted from the design consideration. The web post flexural and buckling of capacity in castellated beam is given by (Soltani *et al*. 2012, Erdal *et al*. 2011)

$$\frac{M_{MAX}}{M_E} = [C_1.\alpha - C_2.\alpha^2 - C_3] \tag{4}$$

where M_{MAX} is the maximum allowable web post moment, and M_E is the web post capacity at critical section A-A shown in Fig. 2. C_1, C_2 and C_3 are constants obtained by following expressions

$$C_1 = 5.097 + 0.1464(\beta) - 0.00174(\beta)^2 \tag{5}$$

$$C_2 = 1.441 + 0.0625(\beta) - 0.000683(\beta)^2 \tag{6}$$

$$C_3 = 3.645 + 0.0853(\beta) - 0.00108(\beta)^2 \tag{7}$$

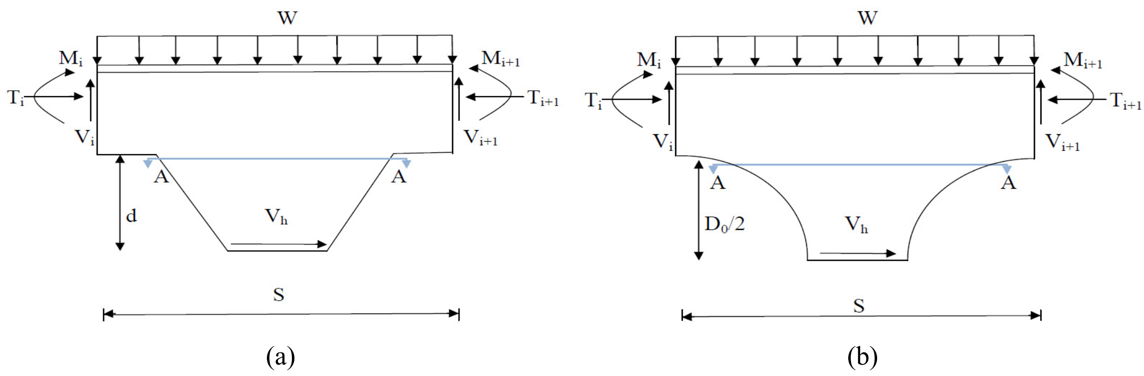


Fig. 2 Horizontal shear in the web post of castellated beams: (a) hexagonal opening; (b) circular opening

where $\alpha = \dfrac{S}{2d}$ for hexagonal openings, and $\alpha = \dfrac{S}{D_0}$ for circular openings, also $\beta = \dfrac{2d}{t_w}$ for hexagonal openings, and $\beta = \dfrac{D_0}{t_w}$ for circular openings, S is the spacing between the centers of holes, d is the cutting depth of hexagonal opening, D_0 is the holes diameter and t_w is the web thickness.

2.4 Vierendeel bending of upper and lower tees

Vierendeel mechanism is always critical in steel beams with web openings, where global shear force is transferred across the opening length, and the Vierendeel moment is resisted by the local moment resistances of the tee-sections above and below the web openings. A complete description of these mechanisms can be found in (Soltani *et al*. (2012) or Erdal *et al*. (2011)).

Vierendeel bending results in the formation of four plastic hinges above and below the web opening. The overall Vierendeel bending resistance depends on the local bending resistance of the web-flange sections. This mode of failure is associated with high shear forces acting on the beam. The Vierendeel bending stresses in the circular opening are obtained by using the Olander's approach, Fig. 3. The interaction between Vierendeel bending moment and axial force for the critical section in the tee should be checked as follows (Erdal *et al*. 2011)

$$\frac{P_0}{P_U} + \frac{M}{M_P} \leq 1.0 \tag{8}$$

where P_0 and M are the force and the bending moment on the section, respectively. P_U is equal to the area of critical section × P_Y, M_P is calculated as the plastic modulus of critical section × P_Y in plastic section or elastic section modulus of critical section × P_Y for other sections.

The plastic moment capacity of the tee-sections in castellated beams with hexagonal opening are calculated independently. The total of the plastic moment is equal to the sum of the Vierendeel resistances of the above and below tee-sections. The interaction between Vierendeel moment and shear forces should be checked by the following expression

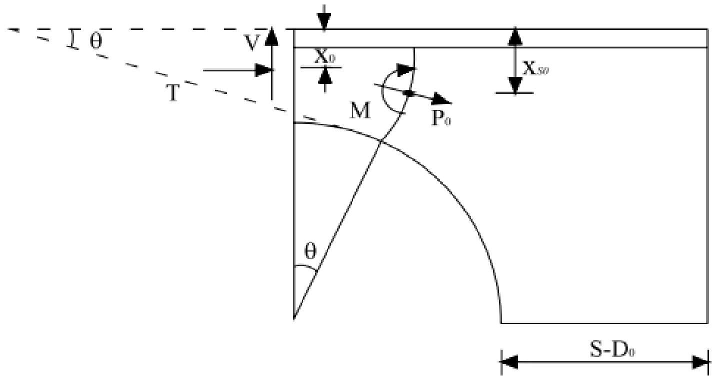


Fig. 3 Olander's curved beam approach

$$V_{OMAX}.e - 4M_{TP} \leq 0 \tag{9}$$

where V_{OMAX} and M_{TP} are the maximum shear force and the moment capacity of tee-section, respectively.

2.5 Deflection of castellated beam

Serviceability checks are of high importance in the design, especially in beams with web opening where the deflection due to shear forces is significant. The deflection of a castellated beam under applied load combinations should not exceed span/360. In castellated beams with circular opening, the deflection at each point is calculated by following expression

$$Y_{TOT} = Y_{MT} + Y_{WP} + Y_{AT} + Y_{ST} + Y_{SWP} \tag{10}$$

where Y_{MT}, Y_{WP}, Y_{AT}, Y_{ST} and Y_{SWP} are deflection due to bending moment in tee, deflection due to bending moment in web post of beam, deflection due to axial force in tee, deflection due to shear in tee and deflection due to shear in web post, respectively. These equations are provided in (Erdal *et al*. 2011).

For a castellated beam with hexagonal opening and length L subjected to transverse loading, the total deflection is composed by two terms: the first term corresponds to pure moment action f_b, and the second one corresponds to shear action f_s. Thus, the total deflection can be calculated by the following expression

$$f = f_b + f_s = c_1 L^3 + c_2 L \tag{11}$$

c_1 and c_2 are determined by means of a curve fitting technique (Raftoyiannis and Ioannidis 2006).

3. Optimum design problem of castellated beams

The main goal of producing and using castellated beams is to suppress the cost of the material by applying more efficient cross sectional shapes made from standard profiles in combination with aesthetic and architectural design considerations. Also, the web holes can be utilized for cross passing utility systems in building floors. There are many factors that require special considerations when estimating the cost of castellated steel beams, such as man-hours of fabrication, weight, price of web cutting and welding process. In this study, it is assumed that the costs associated with man-hours of fabrication for hexagonal and circular opening are identical. Thus, the objective function includes three parts: The beam weight, price of the cutting and price of the welding. The objective function can be expressed as

$$F_{\text{cost}} = \rho A_{\text{initail}}(L_0).P_1 + L_{cut}.P_2 + L_{weld}.P_3 \tag{12}$$

where P_1, P_2 and P_3 are the price of the weight of the beam per unit weight, length of cutting and welding for per unit length, L_0 is the initial length of the beam before castellation process, ρ is the density of steel, A_{initail} is the area of the selected universal beam section, L_{cut} and L_{weld} are the cutting length and welding length, respectively. The length of cutting is different for hexagonal and circular web-openings. The dimension of the cutting length is described by the following equations.

For circular opening

$$L_{cut} = \pi\, D_0.NH + 2e(NH+1) + \frac{\pi D_0}{2} + e \tag{13}$$

For hexagonal opening

$$L_{cut} = 2NH\left(e + \frac{d}{\sin(\theta)}\right) + 2e + \frac{d}{\sin(\theta)} \tag{14}$$

where NH is the total number of holes, e is the length of horizontal cutting of web, D_0 is the diameter of holes, d is the cutting depth, and θ is the cutting angle.

Also, the welding length for both of circular and hexagonal openings is determined by Eq. (15).

$$L_{weld} = e\,(NH+1) \tag{15}$$

As an example, in Fig. 1(a), the number of holes is equal to 3. Therefore, the total length of cutting can be expressed by the following equation

$$L_{cut} = 8e + 7\left(\frac{d}{\sin(\theta)}\right) \tag{16}$$

Similarly, for cellular beams, the same equations can be obtained. L_{cut} for both circular and hexagonal openings are shown in Fig. 1.

3.1 Design of castellated beam with circular opening

Design process of a cellular beam consists of three phases: The selection of a rolled beam, the selection of a suitable circular hole diameter, and the spacing between the center of holes or total number of holes in the beam as shown in Fig. 1, (Erdal *et al*. 2011, Saka 2009). Hence, the sequence number of the rolled beam section in the standard steel sections tables, the circular holes diameter and the total number of holes are taken as design variables in the optimum design problem. The optimum design problem formulated by considering the constraints explained in the previous sections can be expressed as the following:

Find an integer design vector $\{X\} = \{x_1, x_2, x_3\}^T$, where x_1 is the sequence number of the rolled steel profile in the standard steel section list, x_2 is the sequence number for the hole diameter which contains various diameter values, and x_3 is the total number of holes for the cellular beam (Erdal *et al*. 2011). Hence the design problem can be expressed as:

Minimize Eq. (12)

Subjected to

$$g_1 = 1.08 \times D_0 - S \le 0 \tag{17}$$

$$g_2 = S - 1.60 \times D_0 \le 0 \tag{18}$$

$$g_3 = 1.25 \times D_0 - H_S \le 0 \tag{19}$$

$$g_4 = H_S - 1.75 \times D_0 \leq 0 \tag{20}$$

$$g_5 = M_U - M_P \leq 0 \tag{21}$$

$$g_6 = V_{MAXSUP} - P_V \leq 0 \tag{22}$$

$$g_7 = V_{OMAX} - P_{VY} \leq 0 \tag{23}$$

$$g_8 = V_{HMAX} - P_{VH} \leq 0 \tag{24}$$

$$g_9 = M_{A-AMAX} - M_{WMAX} \leq 0 \tag{25}$$

$$g_{10} = V_{TEE} - 0.50 \times P_{VY} \leq 0 \tag{26}$$

$$g_{11} = \frac{P_0}{P_U} + \frac{M}{M_P} - 1.0 \leq 0 \tag{27}$$

$$g_{12} = Y_{MAX} - L/360 \leq 0 \tag{28}$$

where t_W is the web thickness, H_S and L are the overall depth and the span of the cellular beam, and S is the distance between centers of holes. M_U is the maximum moment under the applied loading, M_P is the plastic moment capacity of the cellular beam, V_{MAXSAP} is the maximum shear at support, V_{OMAX} is the maximum shear at the opening, V_{HMAX} is the maximum horizontal shear, $M_{A\text{–}AMAX}$ is the maximum moment at A-A section shown in Fig. 2. M_{WMAX} is the maximum allowable web post moment, V_{TEE} represents the vertical shear on the tee at $\theta = 0$ of web opening, P_0 and M are the internal forces on the web section as shown in Fig. 3, and Y_{MAX} denotes the maximum deflection of the cellular beam (Erdal *et al*. 2011, LRFD-AISC 1986).

3.2 Design of castellated beam with hexagonal opening

In the design of castellated beams with hexagonal openings, the design vector includes four design variables: The selection of a rolled beam, the selection of a cutting depth, the spacing between the center of holes or total number of holes in the beam, and the cutting angle as shown in Fig. 2. Hence the optimum design problem formulated by the following expression:

Find an integer design vector $\{X\} = \{x_1, x_2, x_3, x_4\}^T$ where x_1 is the sequence number of the rolled steel profile in the standard steel section list, x_2 is the sequence number for the cutting depth which contains various values, x_3 is the total number of holes for the castellated beam and x_4 is the cutting angle. Thus, the design problem turns out to be as follows:

Minimize Eq. (12)

Subjected to

$$g_1 = d - \frac{3}{8}.(H_S - 2t_f) \leq 0 \tag{29}$$

$$g_2 = (H_S - 2t_f) - 10 \times (d_T - t_f) \leq 0 \tag{30}$$

$$g_3 = \frac{2}{3}.d.\cot\phi - e \le 0 \tag{31}$$

$$g_4 = e - 2d.\cot\phi \le 0 \tag{32}$$

$$g_5 = 2d.\cot\phi + e - 2d \le 0 \tag{33}$$

$$g_6 = 45^\circ - \phi \le 0 \tag{34}$$

$$g_7 = \phi - 64^\circ \le 0 \tag{35}$$

$$g_8 = M_U - M_P \le 0 \tag{36}$$

$$g_9 = V_{MAXSUP} - P_V \le 0 \tag{37}$$

$$g_{10} = V_{OMAX} - P_{VY} \le 0 \tag{38}$$

$$g_{11} = V_{HMAX} - P_{VH} \le 0 \tag{39}$$

$$g_{12} = M_{A-AMAX} - M_{WMAX} \le 0 \tag{40}$$

$$g_{13} = V_{TEE} - 0.50 \times P_{VY} \le 0 \tag{41}$$

$$g_{14} = V_{OMAX}.e - 4M_{TP} \le 0 \tag{42}$$

$$g_{15} = Y_{MAX} - L/360 \le 0 \tag{43}$$

where t_f is the flange thickness, d_T is the depth of the tee-section, M_P is the plastic moment capacity of the castellated beam, M_{A-AMAX} is the maximum moment at A-A section shown in Fig. 2, M_{WMAX} is the maximum allowable web post moment, V_{TEE} represents the vertical shear on the tee, M_{TP} is the moment capacity of tee-section, and Y_{MAX} denotes the maximum deflection of the castellated beam with hexagonal opening (Soltani *et al*. 2012).

4. The colliding bodies optimization method

Nature has always been a major source of inspiration to engineers and natural philosophers and many meta-heuristic approaches are inspired by solutions that nature herself seems to have chosen for hard problems. The collision is a natural occurrence that happens between objects, bodies, cars, etc. The Colliding bodies optimization algorithm is one of the recently developed meta-heuristic search methods (Kaveh and Mahdavai 2014a, b). It is a population-based search approach, where each agent (CB) is considered as a colliding body with mass *m*. The idea of the CBO algorithm is based on observation of a collision between two objects in one-dimension; in which one object collide with other object and they moves toward minimum energy level.

Fig. 4 The collision between two bodies; (a) before the collision; (b) after the collision

4.1 Collision laws

In physics, collisions between bodies are governed by: (i) laws of momentum; and (ii) laws of energy. When a collision occurs in an isolated system, Fig. 4, the total momentum and energy of the system of object is conserved.

The conservation of the total momentum requires the total momentum before the collision to be the same as the total momentum after the collision, and can be expressed as

$$m_1 v_1 + m_2 v_2 = m_1 v_1' + m_2 v_2' \tag{44}$$

Likewise, the conservation of the total kinetic energy is expressed by

$$\frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2 = \frac{1}{2} m_1 v_1'^2 + \frac{1}{2} m_2 v_2'^2 + Q \tag{45}$$

where v_1 is the initial velocity of the first object before impact, v_2 is the initial velocity of the second object before impact, v'_1 is the final velocity of the first object after impact, v'_2 is the final velocity of the second object after impact, m_1 is the mass of the first object, m_2 is the mass of the second object, and Q is the loss of kinetic energy due to impact.

The velocity after a one-dimensional collision can be obtained as

$$v_1' = \frac{(m_1 - \varepsilon m_2) v_1 + (m_2 + \varepsilon m_2) v_2}{m_1 + m_2} \tag{46}$$

$$v_2' = \frac{(m_2 - \varepsilon m_1) v_2 + (m_1 + \varepsilon m_1) v_1}{m_1 + m_2} \tag{47}$$

where ε is the coefficient of restitution (COR) of two colliding bodies, defined as the ratio of relative velocity of separation to relative velocity of approach

$$\varepsilon = \frac{|v_2' - v_1'|}{|v_2 - v_1|} = \frac{v'}{v} \tag{48}$$

According to the coefficient of restitution, two special cases of collision can be considered as

- A perfectly elastic collision is defined as the one in which there is no loss of kinetic energy in the collision ($Q = 0$ & $\varepsilon = 1$). In reality, any macroscopic collision between objects will convert some kinetic energy to internal energy and other forms of energy. In this case, after collision the velocity of separation is high.

- An inelastic collision is the one in which part of the kinetic is changed to some other form of energy in the collision. Momentum is conserved in inelastic collisions (as it is for elastic collision), but one cannot track the kinetic energy through the collision since some of it is converted to other forms of energy. In this case, coefficient of restitution does not equal to one ($Q \neq 0$ & $\varepsilon \leq 1$). Here, after collision the velocity of separation is low.

For most of the real objects, ε is between 0 and 1.

4.2 The CBO algorithm

The Colliding Bodies Optimization is one of the recently developed meta-heuristic algorithms (Kaveh and Mahdavai 2014a, b). In this method, each solution candidate X_i is considered as a colliding body (CB). The massed objects are composed of two main equal groups; i.e., stationary and moving objects, where the moving objects move to follow stationary objects and a collision occurs between pairs of objects. This is done for two purposes: (i) to improve the positions of moving objects; (ii) to push stationary objects towards better positions. After the collision, the new positions of the colliding bodies are updated based on the new velocity by using the collision laws as discussed in Section 4.1.

The pseudo-code for the CBO algorithm can be summarized as follows:

Step 1: Initialization. The initial positions of CBs are determined randomly in the search space

$$x_i^0 = x_{\min} + rand.(x_{\max} - x_{\min}) \qquad i = 1, 2, ..., n \tag{49}$$

where x_i^0 determines the initial value vector of the ith CB. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable values vectors of variables, respectively; *rand* is a random number in the interval [0,1]; and n is the number of CBs.

Step 2: Determination of the body mass for each CB. The magnitude of the body mass for each CB is defined as

$$m_k = \frac{\frac{1}{fit(k)}}{\sum_{i=1}^{n} \frac{1}{fit(i)}}, \qquad k = 1, 2, ..., n \tag{50}$$

where *fit* (i) represents the objective function value of the ith agent and n is the population size. Obviously a CB with good values exerts a larger mass than the bad ones. Also, for maximizing the objective function, the term $\frac{1}{fit(i)}$ is replaced by *fit* (i).

Step 3: Arrangement of the CBs. The arrangement of the CBs objective function values is performed in ascending order (Fig. 5(a)). The sorted CBs are equally divided into two groups:

- The lower half of CBs (stationary CBs); These CBs are good agents which are stationary and the velocity of these bodies before collision is zero. Thus

$$v_i = 0 \qquad i = 1, 2, ..., \frac{n}{2} \tag{51}$$

- The upper half of CBs (moving CBs): These CBs move toward the lower half. Then, according to Fig. 5(b), the better and worse CBs, i.e., agents with upper fitness value of each group will collide together. The change of the body position represents the velocity of these bodies before collision as

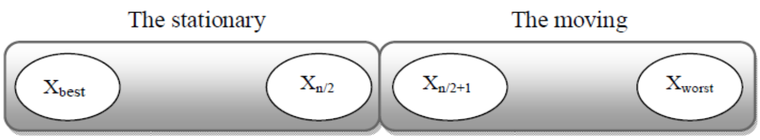
$$v_i = x_i - x_{i-\frac{n}{2}} \qquad i = \frac{n}{2}+1, ..., n \tag{52}$$

where v_i and x_i are the velocity and position vector of the ith CB in this group, respectively; $x_{i-\frac{n}{2}}$ is the ith CB pair position of x_i in the previous group.

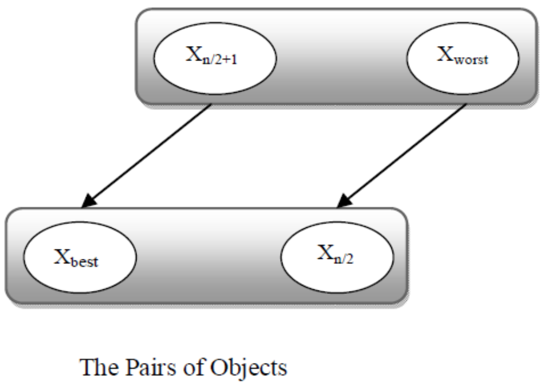
Step 4: Calculation of the new position of the CBs. After the collision, the velocity of bodies in each group is evaluated using Eq. (46), Eq. (47) and the velocities before collision. The velocity of each moving CB after the collision is

$$v'_i = \frac{(m_i - \varepsilon m_{i-\frac{n}{2}})v_i}{m_i + m_{i-\frac{n}{2}}} \qquad i = \frac{n}{2}+1, ..., n \tag{53}$$

where v_i and v'_i are the velocity of the ith moving CB before and after the collision, respectively;



(a)



(b)

Fig. 5 (a) The sorted CBs in an increasing order; (b) the pairs of objects for the collision

m_i is the mass of the ith CB; $m_{i-\frac{n}{2}}$ is mass of the ith CB pair. Also, the velocity of each stationary CB after the collision is

$$v'_i = \frac{(m_{i+\frac{n}{2}} + \varepsilon m_{i+\frac{n}{2}})v_{i+\frac{n}{2}}}{m_i + m_{i+\frac{n}{2}}} \qquad i = 1,\ldots,\frac{n}{2} \tag{54}$$

where $v_{i+\frac{n}{2}}$ and v_i are the velocity of the ith moving CB pair before and the ith stationary CB after the collision, respectively; m_i is mass of the ith CB; $m_{i+\frac{n}{2}}$ is mass of the ith moving CB pair. As mentioned previously, ε is the coefficient of restitution (COR) and for most of the real objects, its value is between 0 and 1. It defined as the ratio of the separation velocity of two agents after collision to the approach velocity of two agents before collision. In the CBO algorithm, this index is used to control of the exploration and exploitation rate. For this goal, the COR is decreases linearly from unit to zero. Thus, ε is defined as

$$\varepsilon = 1 - \frac{iter}{iter_{\max}} \tag{55}$$

where *iter* is the actual iteration number and $iter_{\max}$ is the maximum number of iterations, with COR being equal to unit and zero representing the global and local search, respectively.

New positions of CBs are obtained using the generated velocities after the collision in position of stationary CBs.

The new positions of each moving CB is

$$x_i^{new} = x_{i-\frac{n}{2}} + rand \circ v'_i \qquad i = \frac{n}{2} + 1,\ldots,n \tag{56}$$

where x_i^{new} and v'_i are the new position and the velocity after the collision of the ith moving CB, respectively; $x_{i-\frac{n}{2}}$ is the old position of the ith stationary CB pair. Also, the new positions of stationary CBs are obtained by

$$x_i^{new} = x_i + rand \circ v'_i \qquad i = 1,\ldots,\frac{n}{2} \tag{57}$$

where x_i^{new}, x_i and v'_i are the new position, old position and the velocity after the collision of the ith stationary CB, respectively. *rand* is a random vector uniformly distributed in the range (-1, 1) and the sign "∘" denotes an element-by-element multiplication.

Step 5: *Termination* criterion control. Steps 2-4 are repeated until a termination criterion is satisfied. It should be noted that, the status of a body (stationary or moving body) and its numbering are changed in two subsequent iterations.

The flowchart of the CBO algorithm is shown in Fig. 6.

5. Design examples

In this section, in order to compare fabrication cost of the castellated beams with circular and hexagonal holes, three beams are selected from literature. Here, it is assumed that the compression flanges of the castellated beams are restrained by the floor system. Therefore, the overall buckling is prevented.

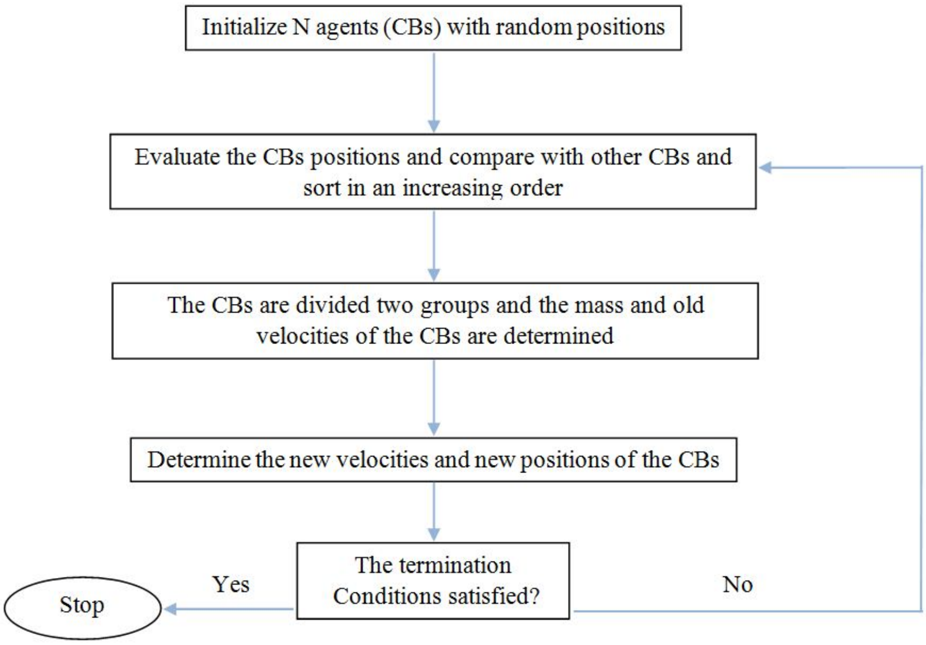


Fig. 6 The flowchart of the CBO

The CBO algorithm is used for optimizing these examples to show the efficiency of the new optimization algorithm. Among the steel section list of British Standards, 64 Universal Beam (UB) sections starting from 254×102×28 UB to 914×419×388 UB are selected to constitute the discrete set for steel sections from which the design algorithm selects the sectional designations for the castellated beams. In the design pool of diameters, 421 values are arranged which varies between 180 and 600 mm with an increment of 1 mm. Also, for cutting depth of hexagonal opening, 351 values are considered which varies between 50 and 400 mm with an increment of 1 mm and cutting angle changes from 45 to 64. Another discrete set is arranged for the number of holes (Erdal *et al*. 2011, Saka 2009). Likewise, in all the design problems, the coefficients P_1, P_2 and P_3 in the objective function are considered 0.85, 0.30 and 1.00, respectively.

5.1 Castellated beam with 4-m span

As the first design example, a simply supported beam with 4m span is selected as shown in Fig. 7. The beam is subjected to 5 kN/m dead load including its own weight. A concentrated live load of 50 KN also acts at mid-span of the beam, and the allowable displacement of the beam is limited to 12 mm. The modulus of elasticity is taken as 205 kN/mm^2, and Grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. The number of CBs is taken as 50 and the maximum number of iterations is considered as 200.

Table 1 represents the design variables and the cost of the castellated beam with 4m span obtained by two meta-heuristic methods. The optimum result for castellated beam with hexagonal hole is equal in two ways, but it is apparent from Table 1 that the CBO algorithm gives better results than ECSS (Kaveh and Talatahari 2010, Kaveh and Talatahari 2011a, b, Kaveh and

Table 1 Optimum designs of the castellated beams with 4m span

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO algorithm	UB 305×102×25	125	14	57	89.78	Hexagonal
ECSS algorithm (Kaveh and Shokohi 2014)	UB 305×102×25	125	14	57	89.78	
CBO algorithm	UB 305×102×25	244	14	_	91.14	Circular
ECSS algorithm (Kaveh and Shokohi 2014)	UB 305×102×25	248	14	_	96.32	

Table 2 Optimum designs of the castellated beams with 8m span

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO algorithm	UB 610×229×101	243	14	59	718.93	Hexagonal
ECSS algorithm (Kaveh and Shokohi 2014)	UB 610×229×101	246	14	59	719.47	
CBO algorithm	UB 610×229×101	487	14	_	721.55	Circular
ECSS algorithm	UB 610×229×101	487	14	_	721.55	

Shokohi 2014) for cellular beams. Also, it is observed that the castellated beam with hexagonal opening have less cost in comparing with the cellular beam. In this problem, the dimension of the span is short, hence shear capacity is very important in optimum design of this beam and it is the most effective factor in the design of this example.

Fig. 8 shows the convergence of CBO algorithm for design of castellated beams with different openings.

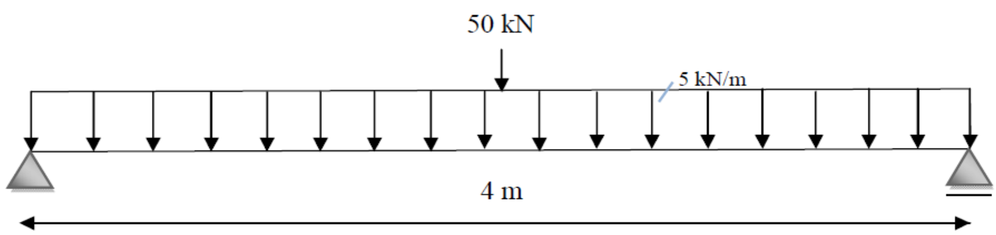
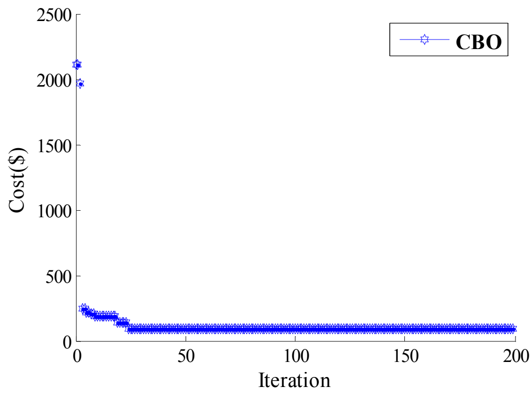
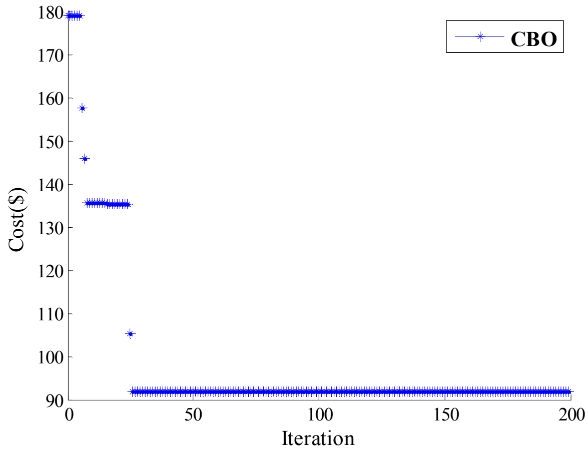


Fig. 7 Simply supported beam with 4m span



(a)



(b)

Fig. 8 Variation of minimum cost versus the number of iterations for 4m span castellated beam. (a) Castellated beam with hexagonal opening; (b) castellated beam with circular opening

5.2 Castellated beam with 8 m span

In the second problem the CBO algorithm is used to design a simply supported castellated beam with a span of 8 m. The beam carries a uniform dead load of 0.40 kN/m, which includes its own weight. The beam is also subjected to two concentrated loads consisting of a dead load of 70 kN and a live load of 70 kN, as shown in Fig. 9. The allowable displacement of the beam is limited to 23 mm. The modulus of elasticity is taken as 205 kN/mm^2 and Grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. The number of CBs is taken as 50. The maximum number of iterations is considered 200.

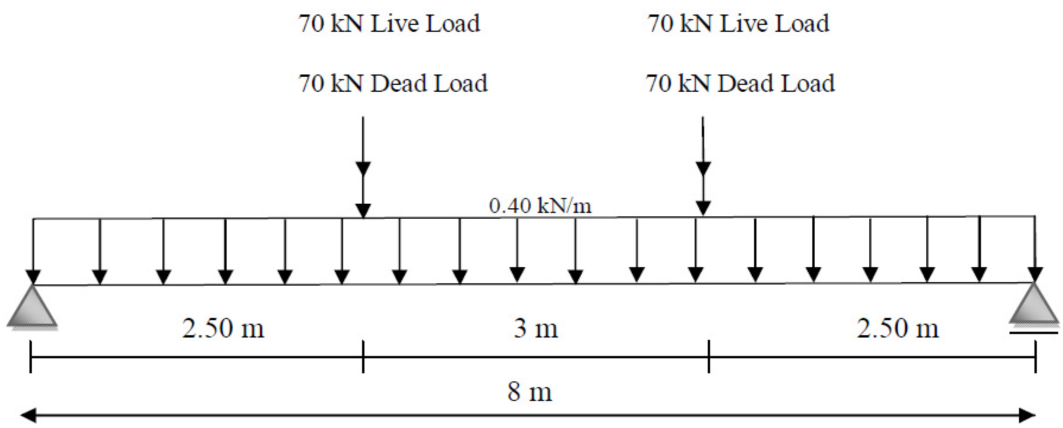
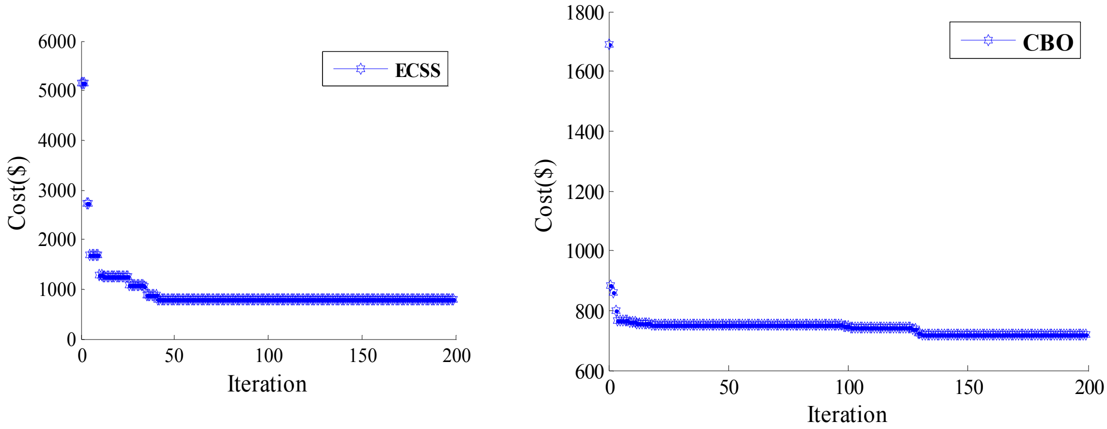


Fig. 9 A simply supported beam with 8m span



(a) (b)

Fig. 10 Variation of minimum cost versus the number of iterations for a hexagonal beam with 8 m span: (a) ECSS algorithm; (b) CBO algorithm

The beam with 8 m span is separately designed by colliding bodies optimization method and ECSS algorithm. The maximum number of iteration is considered the same for both methods. In design of the beam with hexagonal hole, the corresponding cost obtained by the ECSS is equal to 719.47$ while this value is equal to 718.93$ for the CBO algorithm. As a result the performance of the CBO method is better than the ECSS algorithm in this design example. The cellular beam designed by both algorithms has the minimum cost of 721.55$. These results show that the beam with hexagonal opening has less cost than cellular beam and it is a more appropriate option in this case. The maximum value of the strength ratio is equal to 0.99 for both hexagonal and circular beams, and it is show that these constraints are dominant in the design.

Fig. 10 shows the convergence of the ECSS and CBO algorithms for design of a hexagonal beam with 8-m span.

5.3 Castellated beam with 9 m span

The beam with 9 m span is considered as the third example of this study in order to compare the minimum cost of the castellated beams with hexagonal and circular openings. The beam caries a uniform load of 40 kN/m including its own weight and two concentrated loads of 50 kN as shown in Fig. 11. The allowable displacement of the beam is limited to 25 mm. The modulus of elasticity is taken as 205 kN/mm^2, and grade 50 is selected for the steel of the beam which has the design strength of 355 MPa. Similar to the two previous examples, the number of CBs is taken as 50 and the maximum number of iterations is considered 200.

Table 3 compares the results obtained by the CBO with those of the ECSS algorithm. In the optimum design of castellated beam with hexagonal hole, ECSS algorithm selects 684×254×125 UB profile, 13 holes, and 277 mm for the cutting depth and 56 for the cutting angle. The minimum cost of design is equal to 995.97$. Also, in the optimum design of cellular beam, the ECSS algorithm selects 684×254×125 UB profile, 14 holes and 539 mm for the holes diameter. It is observed from Table 3 that the optimal design has the minimum cost of 993.79$ for beam with

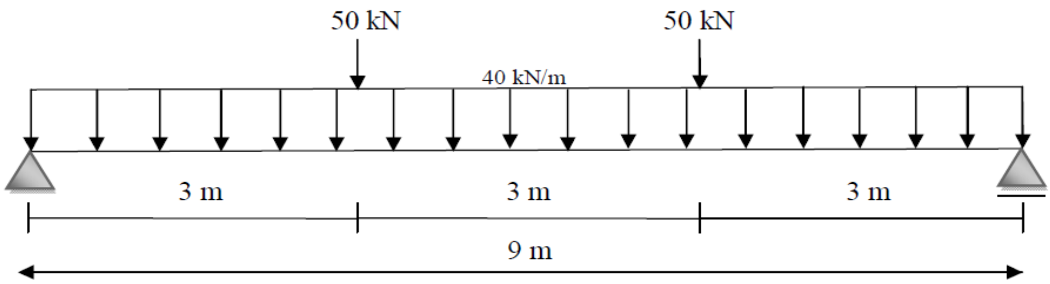


Fig. 11 Simply supported beam with 9m span

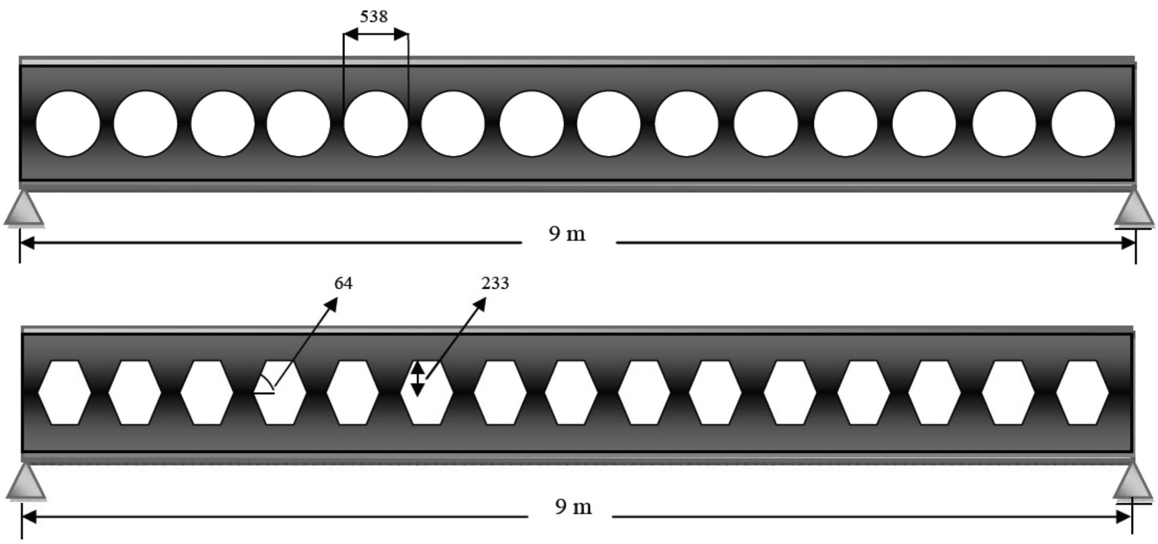


Fig. 12 Optimum profiles of the castellated beams with cellular and hexagonal openings obtaind by the CBO method

Table 3 Optimum designs of the castellated beams with 9 m span

Method	Optimum UB section	Hole diameter or cutting depth (mm)	Total number of holes	Cutting angle	Minimum cost ($)	Type of the hole
CBO algorithm	UB 684×254×125	233	15	64	993.79	Hexagonal
ECSS algorithm (Kaveh and Shokohi 2014)	UB 684×254×125	277	13	56	995.97	
CBO algorithm	UB 684×254×125	538	14	_	997.57	Circular
ECSS algorithm (Kaveh and Shokohi 2014)	UB 684×254×125	539	14	_	998.94	

hexagonal holes and it is obtained by the CBO algorithm. In cellular beam, the maximum value of deflection of the beam is smaller than its upper bound. This show that the strength criteria are dominant in the design of this beam and it is related to the Vierendeel mechanism. Similar to the cellular beam, in castellated beam with hexagonal opening, the strength constraints are dominant in the design process. The maximum ratio of these criteria is equal to 0.99 for the Vierendeel mechanism.

The optimum shapes of the hexagonal and circular openings are illustrated separately in Fig. 12.

6. Conclusions

In this paper, the CBO algorithm is utilized for optimum design of three castellated beams selected from literature. Beams with hexagonal and circular openings are considered as web-opening of castellated beams. The cost of the beam is considered as the objective function. A comparison of the optimal solution is performed between the CBO algorithm and ECSS method.

It is observed that the optimization results obtained from CBO algorithm for most of the design examples have less cost in comparison to the results of the ECSS algorithm. Also, from the results obtained in this paper, it can be concluded that the use of beam with hexagonal opening can lead to the use of less steel material and it is better than cellular beam from the cost point of view. It should be noted that performance of any meta-heurastic algorithm depends on the selection of appropriate values for its parameters. The recently developed algorithm, CBO, utilizes simple formulation and its application requires no parameter selection. This algorithm does not have internal parameter beside the COR. This feature of CBO is a definite strength of this algorithm. The results indicate the high capability of the CBO algorithm in finding the optimum solution.

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