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Strength and buckling of a sandwich beam with thin binding layers between faces and a metal foam core

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Abstract. The strength and buckling problem of a five layer sandwich beam under axial compression or bending is presented. Two faces of the beam are thin aluminium sheets and the core is made of aluminium foam. Between the faces and the core there are two thin binding glue layers. In the paper a mathematical model of the field of displacements, which includes a share effect and a bending moment, is presented. The system of partial differential equations of equilibrium for the five layer sandwich beam is derived on the basis of the principle of stationary total potential energy. The equations are analytically solved and the critical load is obtained. For comparison reasons a finite element model of the beam is formulated. For the case of bended beam the static analysis has been performed to obtain the stress distribution across the height of the beam. For the axially compressed beam the buckling analysis was carried out to determine the buckling load and buckling shape. Moreover, experimental investigations are carried out for two beams. The comparison of the results obtained in the analytical and numerical (FEM) analysis is shown in graphs and figures. The main aim of the paper is to present an analytical model of the five layer beam and to compare the results of the theoretical, numerical and experimental analyses.

Keywords: sandwich structure; buckling; metal foam; mathematical model

1. Introduction

Sandwich structures with a metal foam core are a subject of contemporary studies. These structures are characterized by impact and heat resistance, acoustic and vibration reduction and easy assembly. Because of the above excellent properties these structures are widely used in aerospace, automotive, rail and shipbuilding industry. Plantema (1966) and Allen (1969) described the bases of the theory of sandwich structures. Noor *et al*. (1996) and Vinson (2001) presented strength and stability problems of sandwich structures. Grigolyuk and Chulkov (1973) provided the first hypothesis of cross section deformations of sandwich structures. Wang *et al*. (2000) discussed the higher order hypotheses including shearing of beams and plates. Carrera (2000) formulated the zig-zag hypotheses for multilayered plates. Jasion *et al*. (2012) studied analytically,

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numerically and experimentally the global and local buckling-wrinkling of the face sheets of sandwich beams. Jasion and Magnucki (2013) investigated the global buckling phenomenon of an axially loaded sandwich beam-column with a metal foam core. Jasion and Magnucki (2012) analysed the local buckling problem of sandwich beams under pure bending. Kasprzak and Ostwald (2006) presented a generalization of the hypotheses of deformations. Chakrabarti *et al.* (2011) developed a new FE model based on higher order zig-zag theory for the static analysis of laminated sandwich beams with a soft core. Magnucka-Blandzi (2008 and 2009) presented a theoretical study on dynamic stability of a metal foam circular plates. Magnucka-Blandzi (2011) compared the results of vibration problem of a sandwich beams for the three different Timoshenko hypotheses of deformation. Vlasov and Leont'ev (1960) discussed in detail the theory of elastic foundation and stability problems of beams, plates and shells on elastic foundations. Magnucki *et al.* (2006) carried out analytical investigations of bending and buckling of a rectangular plate made of a porous material. Chen and Yu (2000) presented the numerical simulation and analysis of the elastic-plastic beam-on-foundation. Kesler and Gibson (2002), Steeves and Fleck (2004) and Qin and Wang (2009) presented analytical models of collapse mechanisms of sandwich beams under transverse force. Rakow and Wass (2005) presented mechanical properties of an aluminium foam under shear. Magnucka-Blandzi and Magnucki (2007) and Magnucki *et al.* (2011) described strength and buckling problems of sandwich beams with a metal foam core. Magnucki *et al.* (2013a and 2013b) presented the strength analysis of a simply supported five layer sandwich beams with metal foam core and a beams with corrugated main core. Zenkert (1991) presented strength of sandwich beams with debondings in the interface between the face and the core. Burlayenko and Sadowski (2009, 2010) studied influence of skin/core debonding on free vibration behaviour of foam cored sandwich plates. Jakobsen *et al*. (2008) and Zhang and Wang (2011) presented delamination of interface layered structures on an elastic foundation.

This paper is devoted to the buckling and strength analysis of a five layer sandwich beam. The goal is to elaborate a mathematical model of this beam and to check the influence of the binding glue layers on the strength of this structure.

A simply supported sandwich beam consists of five layers: two thin faces (aluminium sheets) of a thin t_f, one core (aluminium foam) of a thin t_c and two thin binding layers (glue) of a thin t_b. The beam of the length L and the width b carries a compressive axial force F_0 or concentrated force F_1 as shown in Fig. 1. The force F_1 is located in the middle of the beam.

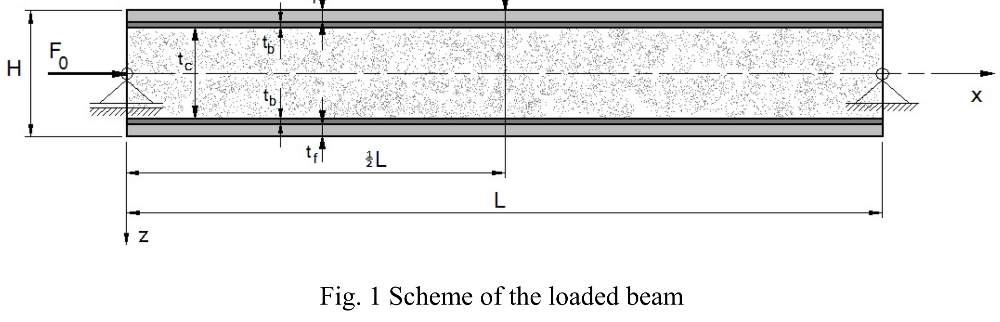


Fig. 1 Scheme of the loaded beam

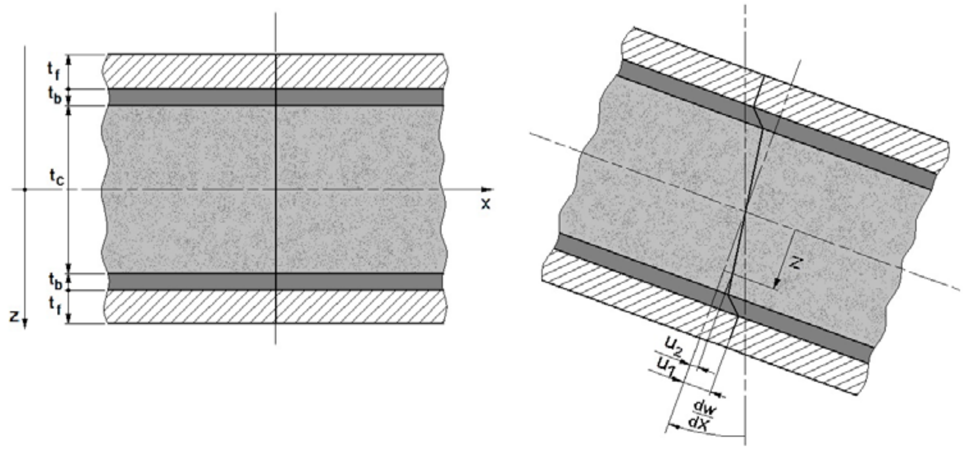


Fig. 2 Scheme of displacements – the hypothesis for the beam

2. Analytical analysis

The deformation of the flat cross section of the five layers beam is shown in Fig. 2. The field of displacements is formulated as follows

1. the upper face $-(1/2+x_1+x_2) \le \zeta \le -(1/2+x_1)$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx}+\psi_1(x)\right], \tag{1}$$

2. the upper binding layer $-(1/2+x_1) \le \zeta \le -1/2$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx}+\psi_2(x)-\frac{1}{x_1}\left(\zeta+\frac{1}{2}\right)(\psi_1(x)-\psi_2(x))\right], \tag{2}$$

3. the core $-1/2 \le \zeta \le 1/2$

$$u(x,\zeta) = -t_c\zeta\left[\frac{dw}{dx}-2\psi_2(x)\right], \tag{3}$$

4. the lower binding layer $1/2 \le \zeta \le 1/2+x_1$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx}-\psi_2(x)-\frac{1}{x_1}\left(\zeta-\frac{1}{2}\right)(\psi_1(x)-\psi_2(x))\right], \tag{4}$$

5. the lower face $1/2+x_1 \le \zeta \le 1/2+x_1+x_2$

$$u(x,\zeta) = -t_c\left[\zeta\frac{dw}{dx} - \psi_1(x)\right], \tag{5}$$

where

$$x_1 = t_b/t_c \ , \ \ x_2 = t_f/t_c \ , \ \ \zeta = z/t_c \ , \ \ \psi_1(x) = u_1(x)/t_c \ , \ \ \psi_2(x) = u_2(x)/t_c.$$

Strains of the layers of the beam are defined by the geometric relationship in the following form

1. the upper face

$$\varepsilon_x = -t_c\left[\zeta\frac{d^2w}{dx^2} + \frac{d\psi_1(x)}{dx}\right], \ \ \gamma_{xz} = 0, \tag{6}$$

2. the upper binding layer

$$\varepsilon_x = -t_c\left[\zeta\frac{d^2w}{dx^2} + \frac{d\psi_2(x)}{dx} - \frac{1}{x_1}\left(\zeta + \frac{1}{2}\right)\left(\frac{d\psi_1(x)}{dx} - \frac{d\psi_2(x)}{dx}\right)\right], \ \ \gamma_{xz} = \frac{1}{x_1}\left[\psi_1(x) - \psi_2(x)\right], \tag{7}$$

3. the core

$$\varepsilon_x = -t_c\zeta\left[\frac{d^2w}{dx^2} + 2\frac{d\psi_2(x)}{dx}\right], \ \ \gamma_{xz} = 2\psi_2(x), \tag{8}$$

4. the lower binding layer

$$\varepsilon_x = -t_c\left[\zeta\frac{d^2w}{dx^2} + \frac{\psi_2(x)}{dx} - \frac{1}{x_1}\left(\zeta - \frac{1}{2}\right)\left(\frac{d\psi_1(x)}{dx} - \frac{d\psi_2(x)}{dx}\right)\right], \ \ \gamma_{xz} = \frac{1}{x_1}\left[\psi_1(x) - \psi_2(x)\right], \tag{9}$$

5. the lower face

$$\varepsilon_x = -t_c\left[\zeta\frac{d^2w}{dx^2} - \frac{d\psi_1(x)}{dx}\right], \ \ \gamma_{xz} = 0, \tag{10}$$

The physical relationships, according to Hooke's law, for individual layers are

$$\sigma_x = E\varepsilon_x \ , \quad \tau_{xz} = G\gamma_{xz} \tag{11}$$

The bending moment of any cross section of the beam

$$\begin{aligned} M_b(x) = \int_A \sigma_x z dA = -bt_c^3\Bigg\{&\left(2E_f c_{2f} + 2E_b c_{2b} + \frac{1}{12}E_c\right)\frac{d^2w}{dx^2} \\ &-\left[E_f c_{1f} + E_b\frac{x_1}{6}(3+4x_1)\right]\frac{d\psi_1}{dx} - \left[\frac{1}{6}E_c + \frac{1}{6}E_b x_1(3+2x_1)\right]\frac{d\psi_2}{dx}\Bigg\} \end{aligned} \tag{12}$$

where

$$c_{1b} = x_1(1+x_1), \quad c_{2b} = \frac{1}{12}x_1\left(3+6x_1+4x_1^2\right), \quad c_{1f} = x_2(1+2x_1+x_2),$$

$$c_{2f} = \frac{1}{12}x_2\left(12x_1(1+x_1+x_2)+3+6x_2+4x_2^2\right).$$

The transverse force of any cross section of the beam

$$Q(x) = \int_A \tau_{xz}dA = 2bt_c\left[G_b\psi_1(x) + (G_c - G_b)\psi_2(x)\right] \tag{13}$$

where

$$G_c = \frac{E_c}{2(1+\nu_c)}\ , \quad G_b = \frac{E_b}{2(1+\nu_b)}.$$

2.1 Equations of equilibrium

The potential energy of the elastic strain of the beam is

$$U_\varepsilon = \frac{1}{2}\int_V (\varepsilon_x\sigma_x + \gamma_{xz}\tau_{xz})dV = \frac{1}{2}bt_c\int_0^L \left(f_{Ef} + f_{Eb} + f_{Ec}\right)dx \tag{14}$$

where

$$f_{Ef} = 2E_f t_c^2\left[c_{2f}\left(\frac{d^2w}{dx^2}\right)^2 - c_{1f}\frac{d^2w}{dx^2}\frac{d\psi_1}{dx} + x_2\left(\frac{d\psi_1}{dx}\right)^2\right]$$

$$f_{Eb} = 2E_b t_c^2\left[c_{2b}\left(\frac{d^2w}{dx^2} - \frac{1}{x_1}\frac{d\psi_1}{dx} + \frac{1}{x_1}\frac{d\psi_2}{dx}\right)^2 + \frac{c_{1b}}{2x_1}\left(\frac{d^2w}{dx^2} - \frac{1}{x_1}\frac{d\psi_1}{dx} + \frac{1}{x_1}\frac{d\psi_2}{dx}\right)\right.$$
$$\left.\times\left(\frac{d\psi_1}{dx} - (1+2x_1)\frac{d\psi_2}{dx}\right) + \frac{1}{4x_1}\left(\frac{d\psi_1}{dx} - (1+2x_1)\frac{d\psi_2}{dx}\right)^2\right] + \frac{2}{x_1}G_b\left[\psi_1(x) - \psi_2(x)\right]^2,$$

$$f_{Ec} = \frac{1}{12}E_c t_c^2\left[\frac{d^2w}{dx^2} - 2\frac{d\psi_2}{dx}\right]^2 + 4G_c\psi_2^2(x)$$

The work of the external load is

$$W = \int_0^L qwdx + \frac{1}{2}F_0\int_0^L\left(\frac{dw}{dx}\right)^2 dx \tag{15}$$

The system of three partial differential equations obtained from the principle of stationary total potential energy $\delta(U_\varepsilon - W) = 0$, after integrating over the thickness of the beam and integrating by parts over the length of the beam, takes the following form

$$
\delta w) \qquad
\begin{aligned}
& bt_c^3\left\{\left(2E_f c_{2f}+2E_b c_{2b}+\frac{1}{12}E_c\right)\frac{d^4 w}{dx^4}\right. \\
& -\left[E_f c_{1f}+\frac{1}{6}E_b x_1(3+4x_1)\right]\frac{d^3\psi_1}{dx^3} \\
& \left.+-\frac{1}{6}\left[E_c+E_b x_1(3+2x_1)\right]\frac{d^3\psi_2}{dx^3}\right\}=q-F_0\frac{d^2 w}{dx^2}
\end{aligned}
\qquad (16)
$$

$$
\delta\psi_1) \qquad
\begin{aligned}
& \left[E_f c_{1f}+\frac{1}{6}E_b x_1(3+4x_1)\right]\frac{d^3 w}{dx^3} \\
& -2\left(E_f x_2+\frac{1}{3}E_b x_1\right)\frac{d^3\psi_1}{dx^3}-\frac{1}{3}E_b x_1\frac{d^2\psi_2}{dx^2} \\
& +\frac{2G_b}{x_1 t_c^2}\left[\psi_1(x)-\psi_2(x)\right]=0
\end{aligned}
\qquad (17)
$$

$$
\delta\psi_2) \qquad
\begin{aligned}
& \frac{1}{6}\left[E_c+E_b x_1(3+2x_1)\right]\frac{d^3 w}{dx^3} \\
& -\frac{1}{3}E_b x_1\frac{d^2\psi_1}{dx^2}-\frac{1}{3}\left(E_c+2E_b x_1\right)\frac{d^2\psi_2}{dx^2} \\
& +\frac{2G_b}{x_1 t_c^2}\psi_1(x)+\frac{1}{t_c^2}\left(4G_b+\frac{2}{x_1}G_b\right)\psi_2(x)=0
\end{aligned}
\qquad (18)
$$

The first Eq. (16) of the system is equivalent to the bending moment (12). Therefore, for further analysis purpose the system of three Eqs. (12), (17) and (18) is applied.

2.2 The strength of the bended beam

The simply supported sandwich beam is loaded by force F_1 ($F_0 = 0$, see Fig. 1). The three unknown functions are assumed in the following forms

$$
\begin{gathered}
w(\xi)=-4w_1(\xi-1)-4w_2\left(\xi^2-\frac{3}{4}\right)\xi\,, \\
\psi_1(x)=\psi_{10}+\psi_{11}\xi+\psi_{12}\xi^2\ , \quad \psi_2(x)=\psi_{20}+\psi_{21}\xi+\psi_{22}\xi^2
\end{gathered}
\qquad (19)
$$

where $\xi = x / L$ is the dimensionless coordinate.

Substituting the functions (19) into three Eqs. (12), (17) and (18) one obtains the unknown functions

$$
\psi_{11}=\psi_{21}=0\,,\ \psi_{12}=\psi_{22}=0\,,\quad w_1=0\,, \qquad (20)
$$

$$
w_2=\frac{F_1 L^3}{48D}\ , \quad D=bt_c^3\left(2E_f c_{2f}+2E_b c_{2b}+\frac{1}{12}E_c\right)\,, \qquad (21)
$$

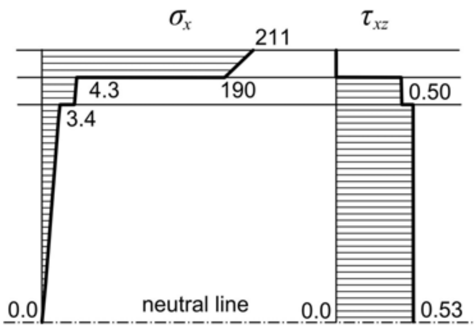


Fig. 3 Distribution of normal σ_x and shear τ_{xz} stresses across the sandwich beam section

$$\psi_2(x) = \psi_{20} = \frac{6t_c^2}{G_c L^3}\left[E_f c_{1f} + E_b x_1 (1 + x_1) + \frac{1}{6}E_c\right] w_2 \ , \tag{22}$$

$$\psi_1(x) = \psi_{10} = \frac{12 x_1 t_c^2}{G_b L^3}\left[E_f c_{1f} + \frac{1}{6}E_b x_1 (3 + 4x_1)\right] w_2 + \psi_{20}. \tag{23}$$

After transformations, stresses of each layer of the beam are

1. the upper face

$$\sigma_x = \frac{1}{4}F_1 L \frac{E_f t_c}{D}\zeta \ , \quad \tau_{xz} = 0 \ , \tag{24}$$

2. the upper binding layer

$$\sigma_x = \frac{1}{4}F_1 L \frac{E_b t_c}{D}\zeta \ , \quad \tau_{xz} = \frac{1}{2}F_1 \frac{t_c^2\left[E_f c_{1f} + \frac{1}{6}E_b x_1 (3 + 4x_1)\right]}{2D} \ , \tag{25}$$

3. the core

$$\sigma_x = \frac{1}{4}F_1 L \frac{E_c t_c}{D}\zeta \ , \quad \tau_{xz} = \frac{1}{2}F_1 \frac{t_c^2\left[E_f c_{1f} + E_b x_1 (1 + x_1) + \frac{1}{6}E_c\right]}{2D} \ , \tag{26}$$

4. the lower binding layer

$$\sigma_x = \frac{1}{4}F_1 L \frac{E_b t_c}{D}\zeta \ , \quad \tau_{xz} = \frac{1}{2}F_1 \frac{t_c^2\left[E_f c_{1f} + \frac{1}{6}E_b x_1 (3 + 4x_1)\right]}{2D} \ , \tag{27}$$

5. the lower face

$$\sigma_x = \frac{1}{4}F_1 L \frac{E_f t_c}{D}\zeta \ , \quad \tau_{xz} = 0 \, . \tag{28}$$

The example of numerical calculations for the sandwich beam is shown below (based on Eqs. (24-28)). The parameters of the beam are: thickness of the faces $t_f = 1$ mm, thickness of the core $t_c = 17.8$ mm, thickness of the binding layers $t_b = 0.1$ mm, Young modulus of the faces $E_f = 65600$ MPa, Young modulus of the core $E_c = 1200$ MPa, Young modulus of the binding layers $E_b = 1500$ MPa, the length $L = 800$ mm, and the width $b = 50$ mm, Poison ratios $\nu_c = \nu_b = 0.3$, $F_1 =$ kN. In Fig. 3 the stress distribution across the section of the beam is shown.

2.3 Buckling of the axially compressed beam

The simply supported sandwich beam is compressed by axial force F_0 ($F_1 = 0$, see Fig.1). The system of equilibrium Eqs. (12), (17), and (18) is approximately solved.

The next three unknown functions are assumed in the following forms

$$w(x) = w_a \sin\left(\frac{\pi x}{L}\right), \quad \psi_1(x) = \psi_{a1} \cos\left(\frac{\pi x}{L}\right) \text{ and } \psi_2(x) = \psi_{a2} \cos\left(\frac{\pi x}{L}\right), \tag{29}$$

where w_a, ψ_{a1}, ψ_{a2} are parameters of the functions.

The bending moment for this load case is written in the form

$$M(x) = F_0 \cdot w(x). \tag{30}$$

Substitution of functions (29) and (30) into Eqs. (12), (17) and (18) leads to the set of three homogeneous equations in relations to parameters w_a, ψ_{a1}, ψ_{a2}. The nontrivial solution of this set exists on condition that the determinant of the set is equal to zero. Thus

$$\det\begin{vmatrix} \alpha_{11} - f_0 & -\alpha_{12} & -\alpha_{13} \\ \alpha_{21} & -\alpha_{22} & -\alpha_{23} \\ \alpha_{31} & -\alpha_{32} & -\alpha_{33} \end{vmatrix} = 0, \tag{31}$$

where

$$f_0 = \frac{F_0}{bt_c^3}, \quad \alpha_{11} = \left(2E_f c_{2f} + 2E_b c_{2b} + \frac{1}{12}E_c\right)\left(\frac{\pi}{L}\right)^2, \quad \alpha_{12} = \left[E_f c_{1f} + \frac{1}{6}E_b x_1 (3 + 4x_1)\right]\frac{\pi}{L},$$

$$\alpha_{13} = \frac{1}{6}\left[E_c + E_b x_1 (3 + 2x_1)\right]\frac{\pi}{L}, \quad \alpha_{21} = \left[E_f c_{1f} + \frac{1}{6}E_b x_1 (3 + 4x_1)\right]\left(\frac{\pi}{L}\right)^3,$$

$$\alpha_{22} = 2\left(E_f x_2 + \frac{1}{3}E_b x_1\right)\left(\frac{\pi}{L}\right)^2 + \frac{2G_b}{x_1 t_c^2}, \quad \alpha_{23} = \frac{1}{3}E_b x_1 \left(\frac{\pi}{L}\right)^2 - \frac{2G_b}{x_1 t_c^2},$$

$$\alpha_{31} = \left[E_f c_{1f} + E_b x_1 (1 + x_1) + \frac{1}{6}E_c\right]\left(\frac{\pi}{L}\right)^3, \quad \alpha_{32} = \left(2E_f x_2 + E_b x_1\right)\left(\frac{\pi}{L}\right)^2,$$

$$\alpha_{33} = \left[\left(\frac{1}{3}E_c + E_b x_1\right)\left(\frac{\pi}{L}\right)^2 + \frac{4G_c}{t_c^2}\right].$$

The solution of the Eq. (31) gives the critical compressive axial force in the following form

$$F_{0,CR} = bt_c^3 f_0 , \tag{32}$$

where

$$f_0 = \frac{(\alpha_{31}\alpha_{22} - \alpha_{21}\alpha_{32})\alpha_{13} + (\alpha_{11}\alpha_{32} - \alpha_{31}\alpha_{12})\alpha_{23} + (\alpha_{21}\alpha_{12} - \alpha_{11}\alpha_{22})\alpha_{33}}{\alpha_{32}\alpha_{23} - \alpha_{22}\alpha_{33}} .$$

The values of the critical loads obtained for the family of sandwich beams are shown in the following section, in which the comparison with the FEM analysis results is presented.

3. FEM analysis

The finite element model of the five layer sandwich beam has been elaborated. It consisted of the core modelled with the use of 3D brick elements, two binding layers for which the same elements have been used and two faces modelled with the use of 2D shell elements. The faces were offset from the glue layers about half of the thickness. Between particular layers the tie conditions have been imposed. Because of the symmetry of the problem only the quarter of the beam has been modelled.

As to the bended beam the static analysis has been performed as a result of which the normal and shear stress distribution has been obtained. The buckling analysis has been carried out for axially compressed beams. The results of both analyses are shown in Fig. 4.

The dimensions of the beam as well as the material properties were the same as in the example considered in the Subsection 2.2. Additionally, for axially compressed beams, different values of the Young modulus for the core were considered, that is 50, 300, 600 and 1200 MPa. Good agreement can be seen between the stress distributions shown in Figs. 3 and 4(a). Maximum normal stresses are: for the faces $\sigma_{\max}^{(Face-Anal)} = 211$ MPa and $\sigma_{\max}^{(Face-FEM)} = 214$ MPa, for the binding layers $\sigma_{\max}^{(B-L-Anal)} = 4.3$ MPa and $\sigma_{\max}^{(B-L-FEM)} = 4.17$ MPa, for the core $\sigma_{\max}^{(Core-Anal)} = 3.4$ MPa and $\sigma_{\max}^{(Core-Anal)} = 3.05$ MPa. Maximum shear stresses are: for the binding layers $\tau_{\max}^{(B-L-Anal)}$

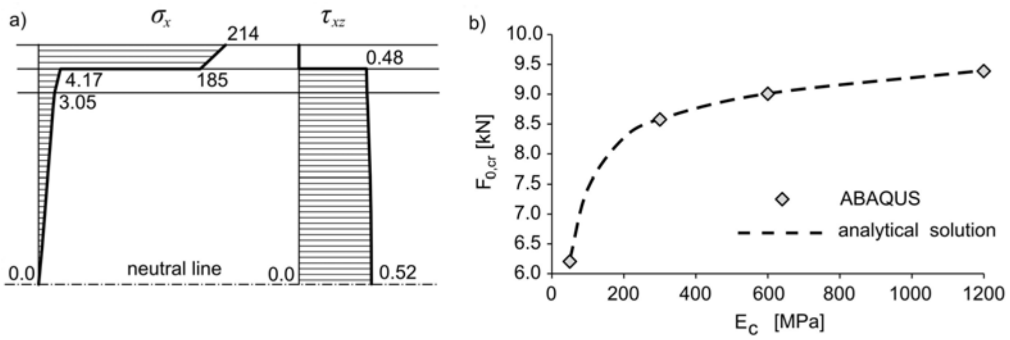


Fig. 4 (a) Distribution of normal σ_x and shear τ_{xz} stresses across the bended sandwich beam; (b) critical loads for the compressed sandwich beam

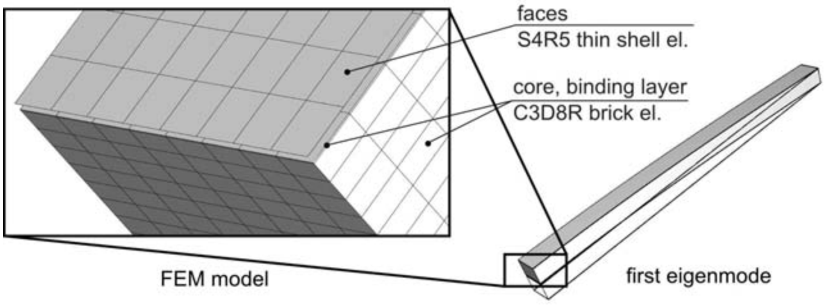
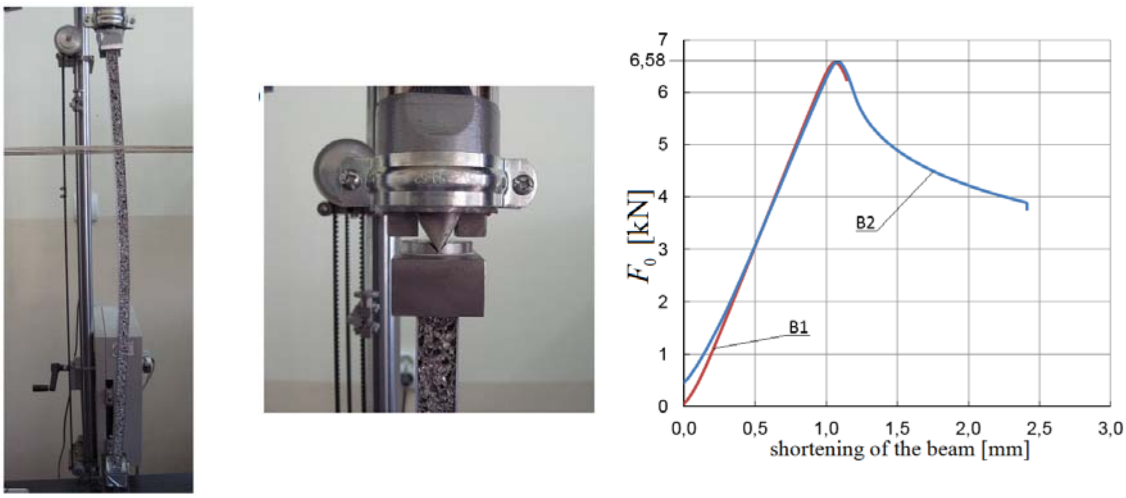


Fig. 5 Scheme of the FEM model of the sandwich beam

$= 0.5\,\text{MPa}$ and $\tau_{\max}^{(B-L-FEM)} = 0.48\,\text{MPa}$, for the core $\tau_{\max}^{(Core-Anal)} = 0.53\,\text{MPa}$ and $\tau_{\max}^{(Core-Anal)} = 0.52\,\text{MPa}$. Similarly, the difference between the buckling load obtained from Eq. (32) and that from FEM analysis is small – less than 1%. In Fig. 5 the buckling mode as well as the details of the FEM model are given.

4. Experimental investigations

In the experimental investigations a two sandwich beams with a metal foam core were axially compressed on the universal testing machine Zwick Z100/TL3S. The test stand is shown in Fig. 6(a). The dimensions of a cross-section of two beams are as follows: the width b = 50 mm, the



(a) (b) (c)

Fig. 6 (a) Test stand; (b) buckled beam; (c) support and results total thickness h = 20 mm, thickness of the faces t_f = 1 mm. The faces were made of aluminium and the core was aluminium foam. Particular layers of the beam was glued together – thickness of the binding layers t_b = 0.1 mm. To both ends of the beam the steel elements have been added to provide a proper support conditions, what is shown in Fig. 6(b). The distance between supported ends of the beams was 892 mm for the first beam (B1) and 860 mm for the second one (B2). The material constants for the aluminium alloy of the faces was E_f = 65600 MPa, and for the binding glue layers: E_b = 1500 MPa. The Young modulus for the aluminium foam core, based on the experimental tests described in details in the monograph by Magnucki and Szyc (2012), is E_c = 216 MPa.

Beam compression process was recorded – the axial displacement-shortening and the axial force F_0 have been measured. The obtained results are given in Fig. 6(c) in the form of curves showing the relation between the axial load and the shortening of the beam. Since in the initial stage of the test of the sample B2 some substantial slip occurred, the corresponding curve given in Fig. 6(c) has been moved to the left to coincide with the other curve.

The critical load for both beams were almost the same and had a value about $F_{0,CR}^{(Exp)}$ = 6.58 kN. Including Young's modulus E_c = 216 MPa and the length L = 892 mm of the beam, the critical force obtained analytically is $F_{0,CR}^{(Anal)}$ = 6.83 kN. A good agreement can be seen between the critical loads obtained from these methods – the difference is about 4%.

5. Conclusions

In the paper the strength and stability of a sandwich beam has been analysed. The beam consists of a light core made of aluminium foam and two thin aluminium faces. The faces are glued to the core with thin binding layers. The glue is treated as a separate layer. The beam is considered then as a five layer sandwich beam.

Two load cases have been taken into consideration. The first one is a beam simply supported and loaded with a concentrated force placed in the mid-length of the beam. For this case the strength of the beam has been analysed. From the analytical model the normal and share stress distribution can be obtained for individual layers of the sandwich beam (see Figs. 3 and 4(a)).

In the second load case the beam is loaded with an axial force. For this case the equations of equilibrium have been derived on the basis of the principle of stationary total potential energy. These equations enable to obtain the value of the critical load (see Fig. 4(b)).

Additionally the finite element model has been formulated. The core and the binding layers have been modelled with the use of 3D brick elements. For the faces 2D shell elements has been chosen. The strength and buckling analyses have been conducted for a family of sandwich beams. The results obtained from the FEM analysis have been compared with these given by the analytical model proposed in the paper. The parameters of the beam taken as an example are given in Subsection 2.2 and Section 3. A good agreement can be seen between the buckling loads obtained from both models – the difference is less than 1%. As to the strength analysis the maximum normal stresses in the upper face are 211 MPa, according to the analytical model, and 214 MPa according to the FEM model. The maximum shear stresses are: 0.53 MPa and 0.52 MPa, respectively. Furthermore, experimental tests have been carried out for axially compressed beams. The values of the critical loads obtained from each method correspond to each other very well. The discrepancies are smaller than 4%.

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