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A new first shear deformation beam theory based on neutral surface position for functionally graded beams

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Abstract. In this paper, a new first-order shear deformation beam theory based on neutral surface position is developed for bending and free vibration analysis of functionally graded beams. The proposed theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. The neutral surface position for a functionally graded beam which its material properties vary in the thickness direction is determined. Based on the present new first-order shear deformation beam theory and the neutral surface concept together with Hamilton's principle, the motion equations are derived. To examine accuracy of the present formulation, several comparison studies are investigated. Furthermore, the effects of different parameters of the beam on the bending and free vibration responses of functionally graded beam are discussed.

Keywords: functionally graded beam; first shear deformation theory; neutral surface position

1. Introduction

Functionally graded materials (FGM) structures are those in which the volume fractions of two or more materials are varied continuously as a function of position along certain dimension(s) of the structure to achieve a required function. Typically, FGMs are made from a mixture of ceramic and metal. The FGMs are widely used in mechanical, aerospace, nuclear, and civil engineering. Consequently, studies devoted to understand the static and dynamic behaviors of FGM beams, plates have being paid more and more attentions in recent years.

Due to increasing of FGM applications in engineering structures, many beam theories have been developed to predict the response of functionally graded (FG) beams. The classical beam theory (CBT) known as Euler–Bernoulli beam theory is the simplest one and is applicable to slender FG beams only. For moderately deep FG beams, the CBT underestimates deflection and overestimates natural frequency due to ignoring the transverse shear deformation effect (Yang and

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Chen 2008, Şimşek and Kocatürk 2009, Alshorbagy *et al*. 2011, Eltaher *et al*. 2012, Kaci *et al*. 2012). The first-order shear deformation beam theory (FBT) known as Timoshenko beam theory has been proposed to overcome the limitations of the CBT by accounting for the transverse shear deformation effect. In FBT (Li 2008, Sina *et al*. 2009, Şimşek and Yurtçu 2012, Sanjay Anandrao *et al*. 2012), the distribution of the transverse shear stress with respect to the thickness coordinate is assumed constant. Thus, a shear correction factor is required to compensate for the error because of this assumption in FBT (Menaa *et al*. 2012). To avoid the use of a shear correction factor and have a better prediction of response of FG beams, higher-order shear deformation theories have been proposed, notable among them are the parabolic shear deformation beam theory (PSDBT) of Reddy (Reddy 1984, Yesilce and Catal 2009, 2011, Yesilce 2010), the trigonometric shear deformation beam theory (TSDBT) of Touratier (1991), the hyperbolic shear deformation beam theory (HSDBT) of Soldatos (1992), the exponential shear deformation beam theory (ESDBT) of Karama *et al*. (2003). Based on the assumption of a higher-order variation of axial displacement through the depth of the beam various higher- order shear deformation theories are also developed (Kadoli *et al*. 2008, Şimşek 2009, 2010, Sallai *et al*. 2009, Li *et al*. 2010). However, studies in the literature show that FBT gives satisfactory results and it is very effective to investigate behavior of beams.

In this paper, a new first-order shear deformation beam theory (NFBT) for the bending and free vibration analysis of functionally graded beams is developed. This theory is based on assumption that the in-plane and transverse displacements consist of bending and shear components, in which the bending components do not contribute toward shear forces and, likewise, the shear components do not contribute toward bending moments. Since, the material properties of FG beam vary through the thickness direction, the neutral plane of such plate may not coincide with its geometric middle plane (Yahoobi and Feraidoon 2010, Asghari 2010, Eltaher *et al*. 2013). In addition, Zhang and Zhou (2008) and Larbi *et al*. (2013) show that the stretching – bending coupling in the constitutive equations of an FG beam does not exist when the coordinate system is located at the physical neutral surface of the plate. Therefore, the governing equations for the FG plate can be simplified. Based on the present theory and the exact position of neutral surface together with Hamilton's principle, the motion equations of the functionally graded beams are obtained. Analytical solutions for bending and free vibration are obtained for a simply supported FG beam. Numerical examples are presented to show the validity and accuracy of the present NFBT.

2. Theoretical formulations

A beam made of functionally graded materials with a uniform cross-section of area A, height h, and length L is considered here. The Cartesian coordinate system, (x, y, z), with the origin at the left end of the beam is used in this analysis. The xoy plane is taken to be the undeformed mid-plane of the beam, the x axis coincides with the centroidal axis of the beam, and the z axis is perpendicular to the $x - y$ plane. Due to asymmetry of material properties of FG beams with respect to middle plane, the stretching and bending equations are coupled. But, if the origin of the coordinate system is suitably selected in the thickness direction of the FG beam so as to be the neutral surface, the properties of the FG beam being symmetric with respect to it. To specify the position of neutral surface of FG beams, two different planes are considered for the measurement of z, namely, z_{ms} and z_{ns} measured from the middle surface and the neutral surface of the beam, respectively, as depicted in Fig. 1.

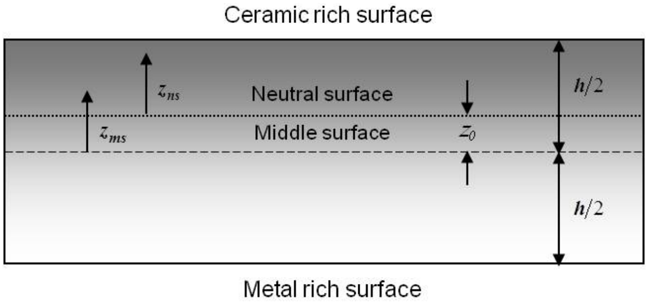

Fig. 1 The position of middle surface and neutral surface for a functionally graded beam

The volume-fraction of ceramic V_C is expressed based on z_{ms} and z_{ns} coordinates as

$$V_C = \left(\frac{z_{ms}}{h} + \frac{1}{2}\right)^k = \left(\frac{z_{ns} + z_0}{h} + \frac{1}{2}\right)^k \tag{1}$$

where k is the power law index which takes the value greater or equal to zero and z_0 is the distance of neutral surface from the mid-surface. Material non-homogeneous properties of a functionally graded material beam may be obtained by means of the Voigt rule of mixture (Suresh and Mortensen 1998). Thus, using Eq. (1), the material non-homogeneous properties of FG beam P, as a function of thickness coordinate, become

$$P(z) = P_M + P_{CM}\left(\frac{z_{ns} + z_0}{h} + \frac{1}{2}\right)^k, \quad P_{CM} = P_C - P_M \tag{2}$$

where P_M and P_C are the corresponding properties of the metal and ceramic, respectively. In the present work, we assume that the elasticity modules E and the mass density ρ are described by Eq. (2), while Poisson's ratio v, is considered to be constant across the thickness (Sallai *et al*. 2009). Based on the physical neutral surface concept put forward by Larbi *et al*. (2013), the physical neutral surface of an FG beam is given by

$$z_0 = \frac{\int_{-h/2}^{h/2} E(z_{ms}) z_{ms} dz_{ms}}{\int_{-h/2}^{h/2} E(z_{ms}) dz_{ms}} \tag{3}$$

It is clear that the parameter z_0 is zero for homogeneous isotropic beams, as expected.

2.1 The conventional first-order shear deformation beam theory (FBT)

The displacements of a material point located at (x, z) in the beam according to the conventional first-order shear deformation theory is given by (Şimşek 2010)

$$u(x,z,t) = u_0(x,t) - z\frac{\partial w_0}{\partial x} + z\,\theta_x \tag{4a}$$

$$w(x,z,t) = w_0(x,t) \tag{4b}$$

where, u, w are displacements in the x, z directions, u_0 and w_0 are midplane displacements, θ_x is the transverse shear strain of any point on the neutral axis.

2.2 Basic assumptions of the new first-order shear deformation beam theory (NFBT)

The assumptions of the present theory (NFBT) are as follows

(i) The origin of the Cartesian coordinate system is taken at the neutral surface of the FG beam.
(ii) The displacements are small in comparison with the beam depth and, therefore, strains involved are infinitesimal.
(iii) The transverse normal stress σ_z is negligible in comparison with axial stress σ_x.
(iv) A line, which is normal to the mid-surface of plate before deformation, remains straight (i.e., may or may not be normal to the mid-surface of the plate) after deformation. Therefore, this theory assumes constant transverse shear stress and it needs a shear correction factor in order to satisfy the beam boundary conditions on the lower and upper surface.
(v) The transverse displacement w includes two components of bending w_b and shear w_s. These components are functions of coordinate x, and time t only.

2.2.1 Kinematics and constitutive equations

Based on the assumptions made in the preceding section, the displacement field can be obtained as follows

$$u(x, z_{ns}, t) = u_0(x,t) - z_{ns}\frac{\partial w_b}{\partial x} \tag{5a}$$

$$w(x, z_{ns}, t) = w_b(x,t) + w_s(x,t) \tag{5b}$$

where, u, w are displacements in the x, z directions, u_0, w_b and w_s are mid-plane displacements.

The strains associated with the displacements in Eq. (5) are

$$\varepsilon_x = \varepsilon_x^0 + z_{ns}\, k_x^b \quad \text{and} \quad \gamma_{xz} = \gamma_{xz}^s \tag{6a}$$

where

$$\varepsilon_x^0 = \frac{\partial u_0}{\partial x}, \quad k_x^b = -\frac{\partial^2 w_b}{\partial x^2}, \quad \gamma_{xz}^s = \frac{\partial w_s}{\partial x} \tag{6b}$$

By assuming that the material of FG beam obeys Hooke's law, the stresses in the beam become

$$\sigma_x = Q_{11}(z_{ns})\,\varepsilon_x \quad \text{and} \quad \tau_{xz} = Q_{55}(z_{ns})\,\gamma_{xz} \tag{7a}$$

where

$$Q_{11}(z_{ns}) = E(z_{ns}) \quad \text{and} \quad Q_{55}(z_{ns}) = \frac{E(z_{ns})}{2(1+\nu)} \tag{7b}$$

2.2.2 Equations of motion

Hamilton's principle is used herein to derive the equations of motion. The principle can be stated in analytical form as (Reddy 2002)

$$\delta\int_{t_1}^{t_2}(U+V-K)dt=0 \tag{8}$$

where t is the time; t_1 and t_2 are the initial and end time, respectively; δU is the virtual variation of the strain energy; δV is the virtual variation of the potential energy; and δK is the virtual variation of the kinetic energy. The variation of the strain energy of the beam can be stated as

$$\begin{aligned}\delta U&=\int_0^L\int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0}(\sigma_x\delta\varepsilon_x+\tau_{xz}\delta\gamma_{xz})dz_{ns}dx\\&=\int_0^L\left(N\frac{d\delta u_0}{dx}-M_b\frac{d^2\delta w_b}{dx^2}+Q\frac{d\delta w_s}{dx}\right)dx\end{aligned} \tag{9}$$

where N, M_b, and Q are the stress resultants defined as

$$(N,M_b)=\int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0}(1,z_{ns})\,\sigma_x dz_{ns}\quad\text{and}\quad Q=\int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0}\tau_{xz}dz_{ns} \tag{10}$$

The variation of the potential energy by the applied transverse load q can be written as

$$\delta V=-\int_0^L q(\delta w_b+\delta w_s)dx \tag{11}$$

The variation of the kinetic energy can be expressed as

$$\begin{aligned}\delta K&=\int_0^L\int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0}\rho(z_{ns})[\dot{u}\delta\dot{u}+\dot{w}\delta\dot{w}]dz_{ns}dx\\&=\int_0^L\left\{I_0[\dot{u}_0\delta\dot{u}_0+(\dot{w}_b+\dot{w}_s)(\delta\dot{w}_b+\delta\dot{w}_s)]-I_1\left(\dot{u}_0\frac{d\delta\dot{w}_b}{dx}+\frac{d\dot{w}_b}{dx}\delta\dot{u}_0\right)+I_2\left(\frac{d\dot{w}_b}{dx}\frac{d\delta\dot{w}_b}{dx}\right)\right\}dx\end{aligned} \tag{12}$$

where dot-superscript convention indicates the differentiation with respect to the time variable t; $\rho(z_{ns})$ is the mass density; and (I_0, I_1, I_2) are the mass inertias defined as

$$(I_0,I_1,I_2)=\int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0}(1,z_{ns},z_{ns}^2)\rho(z_{ns})dz_{ns} \tag{13}$$

Substituting the expressions for δU, δV, and δK from Eqs. (9), (11), and (12) into Eq. (8) and integrating by parts versus both space and time variables, and collecting the coefficients of δu_0, δw_b, and δw_s, the following equations of motion of the functionally graded beam are obtained

$$\delta u_0 : \ \frac{dN}{dx} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} \tag{14a}$$

$$\delta w_b : \ \frac{d^2 M_b}{dx^2} + q = I_0(\ddot{w}_b + \ddot{w}_s) + I_1 \frac{d\ddot{u}_0}{dx} - I_2 \frac{d^2 \ddot{w}_b}{dx^2} \tag{14b}$$

$$\delta w_s : \ \frac{dQ}{dx} + q = I_0(\ddot{w}_b + \ddot{w}_s) \tag{14c}$$

Eq. (14) can be expressed in terms of displacements (u_0, w_b, w_s) by using Eqs. (5), (6), (7) and (10) as follows

$$A_{11} \frac{\partial^2 u_0}{\partial x^2} = I_0 \ddot{u}_0 - I_1 \frac{d\ddot{w}_b}{dx} \tag{15a}$$

$$-D_{11} \frac{\partial^4 w_b}{\partial x^4} + q = I_0(\ddot{w}_b + \ddot{w}_s) + I_1 \frac{d\ddot{u}_0}{dx} - I_2 \frac{d^2 \ddot{w}_b}{dx^2} \tag{15b}$$

$$A_{55}^s \frac{\partial^2 w_s}{\partial x^2} + q = I_0(\ddot{w}_b + \ddot{w}_s) \tag{15c}$$

where A_{11}, D_{11}, and A_{55}^s, are the beam stiffness, defined by

$$\left(A_{11}, D_{11}\right) = \int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0} Q_{11}\left(1, z_{ns}^2\right) dz_{ns} \tag{16a}$$

and

$$A_{55}^s = \int_{-\frac{h}{2}-z_0}^{\frac{h}{2}-z_0} Q_{55}(z_{ns})^2 \, dz_{ns}, \tag{16b}$$

3. Analytical solution

The equations of motion admit the Navier solutions for simply supported beams. The variables u_0, w_b, w_s can be written by assuming the following variations

$$\begin{Bmatrix} u_0 \\ w_b \\ w_s \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} U_m \cos(\lambda x)\, e^{i\omega t} \\ W_{bm} \sin(\lambda x)\, e^{i\omega t} \\ W_{sm} \sin(\lambda x)\, e^{i\omega t} \end{Bmatrix} \tag{17}$$

where U_m, W_{bm}, and W_{sm} are arbitrary parameters to be determined, ω is the eigenfrequency associated with mth eigenmode, and $\lambda = m\pi / L$. The transverse load q is also expanded in Fourier series as

$$q(x) = \sum_{m=1}^{\infty} Q_m \sin(\lambda x) \tag{18}$$

where Q_m is the load amplitude calculated from

$$Q_m = \frac{2}{L}\int_0^L q(x)\sin(\lambda x)dx \tag{19}$$

The coefficients Q_m are given below for some typical loads. For the case of a sinusoidally distributed load, we have

$$m = 1 \quad \text{and} \quad Q_1 = q_0 \tag{20a}$$

and for the case of uniform distributed load, we have

$$Q_m = \frac{4q_0}{m\pi}, \quad (m = 1,3,5....) \tag{20b}$$

Substituting the expansions of u_0, w_b, w_s, and q from Eqs. (17) and (18) into Eq. (15), the analytical solutions can be obtained from the following equations

$$\left(\begin{bmatrix} S_{11} & 0 & 0 \\ 0 & S_{22} & 0 \\ 0 & 0 & S_{33} \end{bmatrix} - \omega^2 \begin{bmatrix} m_{11} & m_{12} & 0 \\ m_{12} & m_{22} & m_{23} \\ 0 & m_{23} & m_{33} \end{bmatrix} \right) \begin{Bmatrix} U_m \\ W_{bm} \\ W_{sm} \end{Bmatrix} = \begin{Bmatrix} 0 \\ Q_m \\ Q_m \end{Bmatrix} \tag{21}$$

where

$$S_{11} = A_{11}\lambda^2, \quad S_{22} = D_{11}\lambda^4, \quad S_{33} = A_{55}^s\lambda^2 \tag{22a}$$

$$m_{11} = m_{23} = m_{33} = I_0, \quad m_{12} = -I_1\lambda, \quad m_{22} = I_0 + I_2\lambda^2 \tag{22b}$$

4. Results and discussion

In this study, bending and free vibration analysis of simply supported FG beams by the present method is suggested for investigation.

The FG beam is taken to be made of aluminum and alumina with the following material properties

Ceramic (P_C: Alumina, Al_2O_3): E_c = 380 GPa; ν = 0.3; ρ_c = 3960 kg/m^3.

Metal (P_M: Aluminium, Al): E_m = 70 GPa; ν = 0.3; ρ_m = 2707 kg/m^3.

We take the shear correction factor K = 5/6 in both FBT and NFBT. For convenience, the following dimensionless forms are used

4.1 Bending analysis

The non-dimensional displacements and stresses obtained using the present new first-order shear deformation beam theory (NFBT) for FG beams with different values of power law index k and length-to-depth ratio L/h are compared with CBT, FBT and the analytical solutions given by Li *et al*. (2010) in Table 1. As can be seen the results of NFBT are in good agreement with those of FBT and Li *et al*. (2010).

In Figs. 2 and 3 we present the evolution of the axial displacement u and the axial stresses σ_x across the depth of the FG beam under uniform load. A comparison with FBT is also shown in these figures using different values of the power law index k. It is seen that both NFBT and FBT give identical results. It can be seen from Fig. 2 that the increase of the power law index k leads to an increase of the axial displacement u and especially at the top and bottom of the beam. In Fig. 3,

Table 1 Nondimensional deflections and stresses of FG beams under uniform load q_0

k	Method	$L/h=5$				$L/h=20$			
		$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$	$\bar{w}$	$\bar{u}$	$\bar{\sigma}_x$	$\bar{\tau}_{xz}$
0	Li *et al*. (2010)	3.1657	0.9402	3.8020	0.7500	2.8962	0.2306	15.0130	0.7500
	NFBT	**3.1657**	**0.9209**	**3.7500**	**0.5990**	**2.8963**	**0.2303**	**15.0000**	**0.5993**
	FBT	3.1657	0.9209	3.7500	0.5990	2.8963	0.2303	15.0000	0.5993
	CBT	2.8783	0.9211	3.7500	–	2.8783	0.2303	15.0000	–
0.5	Li *et al*. (2010)	4.8292	1.6603	4.9925	0.7676	4.4645	0.4087	19.7005	0.7676
	NFBT	**4.8347**	**1.6331**	**4.9206**	**0.6270**	**4.4648**	**0.4083**	**19.6825**	**0.6266**
	FBT	4.8347	1.6331	4.9206	0.6270	4.4648	0.4083	19.6825	0.6266
	CBT	4.4401	1.6331	4.9206	–	4.4401	0.4083	19.6825	–
1	Li *et al*. (2010)	6.2599	2.3045	5.8837	0.7500	5.8049	0.5686	23.2054	0.7500
	NFBT	**6.2600**	**2.2722**	**5.7959**	**0.5988**	**5.8050**	**0.5681**	**23.1834**	**0.5995**
	FBT	6.2600	2.2722	5.7959	0.5988	5.8050	0.5681	23.1834	0.5995
	CBT	5.7746	2.2722	5.7959	–	5.7746	0.5680	23.1834	–
2	Li *et al*. (2010)	8.0602	3.1134	6.8812	0.6787	7.4415	0.7691	27.0989	0.6787
	NFBT	**8.0307**	**3.0741**	**6.7678**	**0.5101**	**7.4400**	**0.7686**	**27.0704**	**0.5102**
	FBT	8.0307	3.0741	6.7678	0.5101	7.4400	0.7686	27.0704	0.5102
	CBT	7.4003	3.0740	6.7676	–	7.4003	0.7685	27.0704	–
5	Li *et al*. (2010)	9.7802	3.7089	8.1030	0.5790	8.8151	0.9133	31.8112	0.5790
	NFBT	**9.6484**	**3.6496**	**7.9427**	**0.3926**	**8.8068**	**0.9120**	**31.7710**	**0.3927**
	FBT	9.6484	3.6496	7.9427	0.3926	8.8068	0.9120	31.7710	0.3927
	CBT	8.7508	3.6496	7.9428	–	8.7508	0.9124	31.7711	–
10	Li *et al*. (2010)	10.8979	3.8860	9.7063	0.6436	9.6879	0.9536	38.1372	0.6436
	NFBT	**10.7194**	**3.8098**	**9.5231**	**0.4288**	**9.6770**	**0.9524**	**38.0915**	**0.4292**
	FBT	10.7194	3.8098	9.5231	0.4288	9.6770	0.9524	38.0915	0.4292
	CBT	9.6072	3.8097	9.5228	–	9.6072	0.9524	38.0913	–

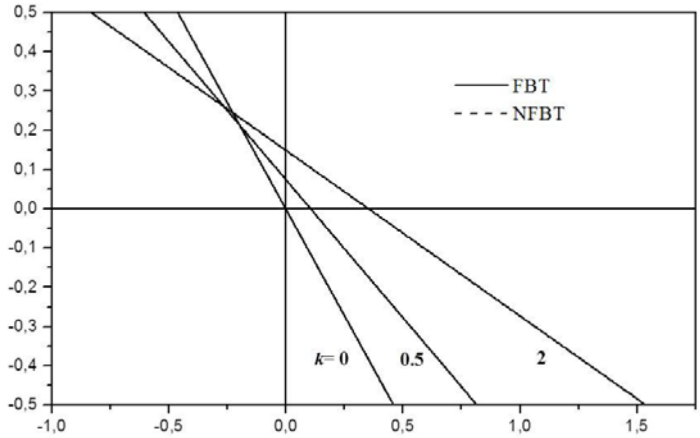

Fig. 2 The variation of the axial displacement $\overline{u}$ through-the-thickness of a FG beam ($L = 10h$)

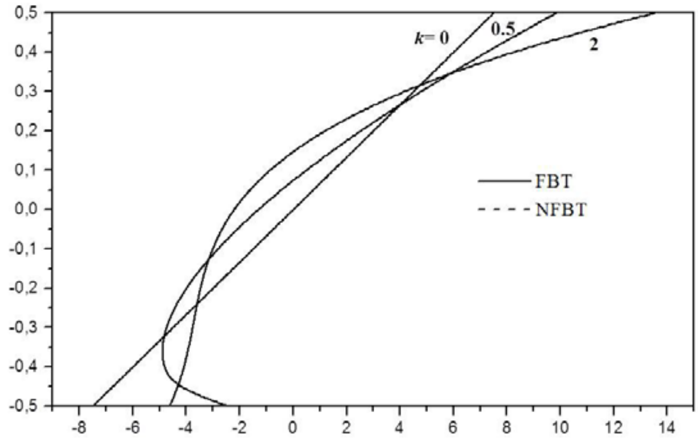

Fig. 3 The variation of the axial stress $\overline{\sigma}_x$ through-the-thickness of a FG beam ($L = 10h$).

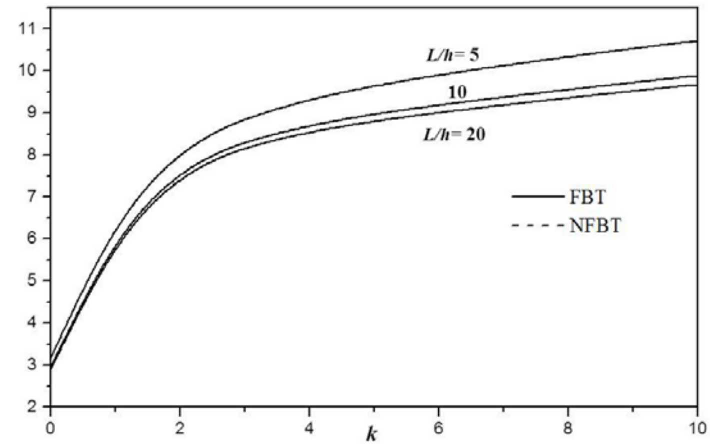

Fig. 4 Variation of the nondimensional deflection $\overline{w}$ of FG beam with power law index k and length-to-height ratio L/h

the axial stress σ_x is tensile at the top surface and compressive at the bottom surface. The homogeneous ceramic beam (k = 0) yields the maximum compressive stresses at the bottom surface and the minimum tensile stresses at the top surface of the beam. The effect of the power law index k on the dimensionless deflection w is presented in Fig. 4 for several different values of the length-to-depth ratio. When k is increased, the deflection w rises monotonically. The reason is that the larger k is, the more the volume fraction of aluminium is or the richer aluminium is, so that such an FG beam is more compliant and more flexible. As is shown in Figs. 2 and 3, both NFBT and FBT give identical results for the deflection w.

4.2 Free vibration

For the verification purpose, the nondimensional fundamental frequencies ω of FG beams obtained by the present theory (NFBT) are compared with those given by Şimşek (2010) and FBT for different values of power law index k and length-to-depth ratio L / h and the results are presented in Table 2. It can be observed that present results obtained using NFBT are in an excellent agreement to those predicted using the various theories used by Şimşek (2010). In addition, both the present NFBT and FBT (Şimşek 2010) give identical results for the nondimensional fundamental frequencies ω.

Figs. 5 and 6 show the variation of the fundamental frequency of simply supported FG beam with length-to-height ratio and the power-law exponent, respectively by using CBT and different shear deformation beam theories (NFBT, FBT, PSDBT and TSDBT). It can be seen that the agreement between the present results (NFBT) and those obtained using other shear deformation

Table 2 Variation of fundamental frequency $\bar{\omega}$ with the power-law index for FG beam

L / h	Theory	k					
		0	0.5	1	2	5	10
5	PSDBT[(a)]	5.1527	4.4111	3.9904	3.6264	3.4012	3.2816
	TSDBT[(a)]	5.1531	4.4114	3.9907	3.6263	3.3998	3.2811
	HSDBT[(a)]	5.1527	4.4111	3.9904	3.6265	3.4014	3.2817
	ESDBT[(a)]	5.1542	4.4122	3.9914	3.6267	3.3991	3.2813
	FBT[(a)]	5.1525	4.4083	3.9902	3.6344	3.4312	3.3134
	NFBT	**5.1525**	**4.4079**	**3.9902**	**3.6344**	**3.4312**	**3.3134**
	CBT[(a)]	5.3953	4.5936	4.1484	3.7793	3.5949	3.4921
20	PSDBT[(a)]	5.4603	4.6516	4.2050	3.8361	3.6485	3.5390
	TSDBT[(a)]	5.4604	4.6516	4.2051	3.8361	3.6484	3.5390
	HSDBT[(a)]	5.4603	4.6516	4.2050	3.8361	3.6485	3.5390
	ESDBT[(a)]	5.4604	4.6517	4.2052	3.8362	3.6483	3.5390
	FBT[(a)]	5.4603	4.6514	4.2051	3.8368	3.6509	3.5416
	NFBT	**5.4603**	**4.6509**	**4.2051**	**3.8368**	**3.6509**	**3.5416**
	CBT[(a)]	5.4777	4.6646	4.2163	3.8472	3.6628	3.5547

[(a)] Taken from Şimşek (2010)

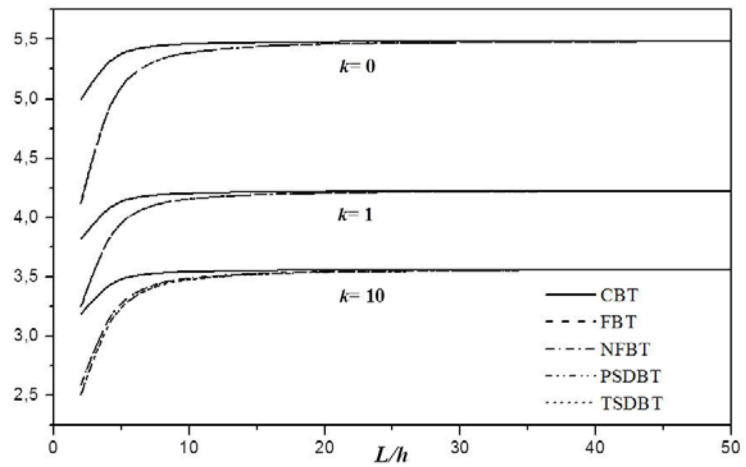

Fig. 5 Variation of the nondimensional fundamental frequency $\bar{\omega}$ of FG beam with length-to-height ratio L/h and power law index k

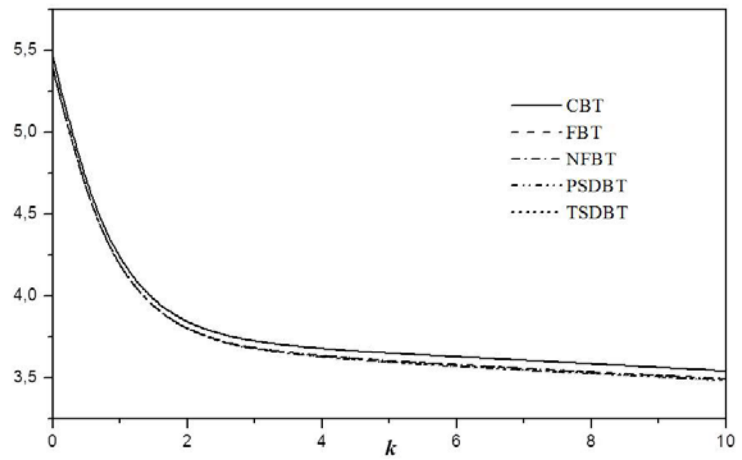

Fig. 6 Variation of the nondimensional fundamental frequency $\bar{\omega}$ of FG beam versus power law index k ($L = 10h$)

theories is satisfactory. From these figures, it can be observed also that there is a remarkable difference between the frequencies of CBT and those of shear deformable beam theories when the slenderness ratio of the FG beam is less than $L/h = 20$.

5. Conclusions

A new first-order shear deformation beam theory (NFBT) was proposed to analyse static and dynamic behaviour of functionally graded beams. Based on the present beam theory and the neutral surface concept, the equations of motion are derived from Hamilton's principle. The effectiveness of the theory is brought out by applying them for static as well as dynamic analysis. The results obtained using this new theory, are found to be in excellent agreement with previous studies. Unlike the conventional first shear deformation theory, the proposed first shear deformation theory contains only four unknowns. In conclusion, it can be said that the proposed theory NFBT is not only accurate but also efficient in predicting the static and dynamic behaviour of functionally graded beams.

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