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Application of a new extended layerwise approach to thermal buckling load optimization of laminated composite plates

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Abstract. This paper deals with the applicability of a new extended layerwise optimization method for thermal buckling load optimization of laminated composite plates. The design objective is the maximization of the critical thermal buckling of the laminated plates. The fibre orientations in the layers are considered as design variables. The first order shear deformation theory (FSDT) is used for the finite element solution of the laminates. Finally, the numerical analysis is carried out to show the applicability of extended layerwise optimization algorithm of laminated plates for different parameters such as plate aspect ratios and boundary conditions.

Keywords: laminated plates; thermal buckling load; finite element solution; extended layerwise optimization method; optimization

1. Introduction

The laminated composite plate is one of the important structural elements, which is widely used in a variety of high performance engineering systems including aircraft, submarine, automotive, naval and space structures. When the plate is subjected to temperature change, thermally induced compressive stresses are developed in the constraint plate due to thermo-elastic properties and consequently buckling takes place. Thin plate structure becomes unstable at relatively lower temperature and buckles in the elastic region. Hence, the thermal buckling represents an important parameter for consideration and plays the significant role in the design of the structures.

A considerable amount of literature exists on thermal buckling of laminated composite plates. For example, Shiau *et al.* (2010) studied thermal buckling behavior of composite laminated plates by making the use of finite element method. Lal *et al.* (2009) examined the effect of random system properties on thermal buckling load of laminated composite plates under uniform temperature rise. Vosoughi *et al.* (2011) investigated thermal postbuckling behavior of laminated composite skew plates. Akhras and Li (2010) extended the finite layer method to the thermal buckling analysis of piezoelectric antisymmetric angle-ply laminates. Rasid *et al.* (2011) improved thermal buckling and thermal post-buckling behaviours of laminated composite plates by embedding shape memory alloy wires within laminated composite plates. Ghomshei and

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Mahmoudi (2010) implemented differential quadrature method for analyzing the thermal buckling behavior of the symmetric cross-ply laminated rectangular thin plates subjected to uniform and/or non-uniform temperature fields.

The optimization of thermal buckling load for laminated composite plates has been the subject of significant research activities in recent years. Topal and Uzman (2008) investigated thermal buckling load optimization of laminated composite plates subjected to uniformly distributed temperature load. The objective function was to maximize the critical temperature capacity of laminated plates and the fibre orientation was considered as design variable. Topal and Uzman (2010) researched thermal buckling load optimization of symmetrically laminated angle-ply thin plates with centrally located different cutouts subjected to a uniform temperature load rise. Topal (2012) studied critical thermal buckling load optimization of symmetrically laminated four layered angle-ply plates with one or two different intermediate line supports. Spallino and Thierauf (2000) presented thermal buckling optimization of laminated composite plates subject to a temperature rise using evolution strategies. Singha *et al.* (2000) maximized buckling temperatures of graphite/epoxy laminated composite plates for a given total thickness considering fibre-directions and relative thicknesses of layers as design variables. Genetic algorithm was employed to optimize as many as ten variables for the five layered plates. Autio (2001) optimized behaviour of a laminated plate with given boundary temperatures and displacement constraints and the optimization problem was expressed in terms of lamination parameters. Mozafari *et al.* (2010) maximized thermal buckling loads of laminated composite plates for a given total thickness. Fibre directions and relative thickness of layers were considered as design variables. The imperialist competitive algorithm was employed to optimize as many as seven variables for the different layered plates. Chen *et al.* (2003) investigated design optimization for structural thermal buckling. The analysis of heat conduction, structural stress and buckling were considered at the same time in the design optimization procedure. The optimization model was constructed and solved by the sequential linear programming or sequential quadratic programming algorithm. Malekzadeh *et al.* (2012) applied the differential quadrature method in conjunction with the genetic algorithms to obtain the optimum buckling temperature of the laminated composite skew plates. Fares *et al.* (2004) presented a multiobjective optimization problem to determine the optimal layer thickness and optimal closed loop control function for a symmetric cross-ply laminate subjected to thermomechanical loadings. The optimization procedure aimed to maximize the critical combination of the applied edges load and temperature levels and to minimize the laminate dynamic response subject to constraints on the thickness and control energy. Lee *et al.* (1999) presented the design of a thick laminated composite plate subjected to a thermal buckling load under a uniform temperature distribution. In design procedures of composite laminated plates for a maximum thermal buckling load, golden section method was used as an optimization routine. Fares *et al.* (2005) presented design and control optimization to minimize the thermal postbuckling dynamic response and to maximize the buckling temperature level of composite laminated plates subjected to thermal distribution varying linearly through the thickness and arbitrarily with respect to the in-plane coordinates.

On the other hand, layerwise optimization (LO) approach was introduced by Narita who applied this method to the optimization of the laminates. Narita started with a predetermined number of layers in symmetric formation and systematically found the optimal fibre orientations from the outer to the inner layers. His study was restricted by predetermined number of layers. Topal (2012) studied frequency optimization of laminated composite plates using a new extended layerwise optimization method. On the other hand, this paper deals with a new extended layerwise optimization method for thermal buckling load optimization of laminated plates. Furthermore, this algorithm has no limitations on the number of layers. The design objective is the maximization of the critical thermal buckling load. The first order shear deformation theory is used for finite element solution of laminates. The design variable is the fiber orientations. Finally, the numerical analysis is carried out to show the applicability of extended layerwise optimization algorithm of laminated plates for different parameters such as plate aspect ratios and boundary conditions.

2. Basic equations

Consider a laminated composite plate of uniform thickness h, having a rectangular plan axb as shown in Fig. 1. The individual layers are assumed to be homogeneous and orthotropic.

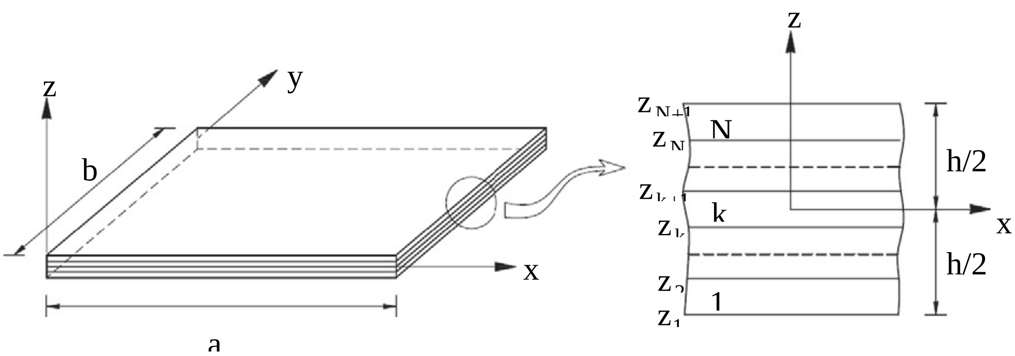

Fig. 1. Geometry and coordinate system of a rectangular laminated composite plate

The displacement field for the first order shear deformation theory can be expressed as

$$
\begin{aligned}
u(x,y,z) &= u_o(x,y) + z\psi_x(x,y) \\
v(x,y,z) &= v_o(x,y) + z\psi_y(x,y) \qquad (1) \\
w(x,y,z) &= w_o(x,y)
\end{aligned}
$$

where u, v and w are the displacements of a general point in the x, y and z directions respectively. The parameters u_o, v_o are the inplane displacements and w_o is the transverse displacement of a point on the laminate middle plane. The functions ψ_x and ψ_y are the rotations of the normal to the laminate middle plane about x- and y-axes, respectively. The displacement vector at the mid-plane can be defined as

$$
\bar{d} = \left\{u_o, v_o, w_o, \psi_x, \psi_y\right\}^T \qquad (2)
$$

Substituting Eq. (1) into the general linear strain-displacement relations, the following relations are obtained.

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} = \begin{vmatrix} \dfrac{\partial u_o}{\partial x} \\ \dfrac{\partial v_o}{\partial y} \\ \dfrac{\partial u_o}{\partial y} + \dfrac{\partial v_o}{\partial x} \end{vmatrix} - z \begin{vmatrix} \dfrac{\partial \psi_x}{\partial x} \\ \dfrac{\partial \psi_y}{\partial y} \\ \dfrac{\partial \psi_x}{\partial y} + \dfrac{\partial \psi_y}{\partial x} \end{vmatrix}, \quad \begin{vmatrix} \gamma_{yz} \\ \gamma_{xz} \end{vmatrix} = \begin{vmatrix} \dfrac{\partial w}{\partial y} - \psi_y \\ \dfrac{\partial w}{\partial x} - \psi_x \end{vmatrix} \tag{3}$$

The stress-strain relations accounting for thermal effects for the *kth* lamina in the element co-ordinates (x,y,z) are written as

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{11} & \bar{Q}_{12} & \bar{Q}_{16} \\ \bar{Q}_{12} & \bar{Q}_{22} & \bar{Q}_{26} \\ \bar{Q}_{16} & \bar{Q}_{26} & \bar{Q}_{66} \end{pmatrix}_{(k)} \begin{pmatrix} \varepsilon_x - \alpha_x \Delta T \\ \varepsilon_y - \alpha_y \Delta T \\ \varepsilon_{xy} - \alpha_{xy} \Delta T \end{pmatrix} \tag{4}$$

$$\begin{pmatrix} \tau_{yz} \\ \tau_{xz} \end{pmatrix}_{(k)} = \begin{pmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{pmatrix}_{(k)} \begin{pmatrix} \gamma_{yz} \\ \gamma_{xz} \end{pmatrix} \tag{5}$$

where $\overline{Q}_{ij}$ is the transformed reduced stiffnesses, α_x , α_y , α_{xy} are the coefficients of thermal expansion and ΔT is the uniform constant temperature difference.

The stress resultants $\{N\}$, stress couples $\{M\}$ and transverse shear stress resultants $\{Q\}$ are

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} dz, \quad \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \sigma_x \\ \sigma_y \\ \sigma_{xy} \end{Bmatrix} z dz, \quad \begin{Bmatrix} Q_x \\ Q_y \end{Bmatrix} = \int_{-h/2}^{h/2} \begin{Bmatrix} \tau_{xz} \\ \tau_{yz} \end{Bmatrix} dz \tag{7}$$

In Eq. (7), K is the shear correction factor. In this study, the shear correction factor is taken 5/6.

3. Finite element formulation

In this study, nine noded Lagrangian rectangular plate elements having five degrees of freedom are used for the finite element solution of the laminated plates. The interpolation function of the displacement field is defined as

$$\begin{pmatrix} u \\ v \\ w \\ \psi_x \\ \psi_y \end{pmatrix} = \sum_{i=1}^{n} N_i d_i \tag{8}$$

where d_i, N_i and n are the nodal variables, the interpolation function and total number of nodals per element, respectively. The stiffness matrix of the plate is obtained by using the minimum potential energy principle. Bending stiffness $[K_b]$, shear stiffness $[K_s]$ and geometric stiffness $[K_g]$ can be calculated as below

$$
\begin{aligned}
[K_b] &= \int_A [B_b]^T [D_b][B_b] dA \\
[K_s] &= \int_A [B_s]^T [D_s][B_s] dA \\
[K_g] &= \int_A [B_g]^T [D_g][B_g] dA
\end{aligned}
\tag{9}
$$

where

$$
[D_b] = \begin{bmatrix} A_{ij} & 0 \\ 0 & D_{ij} \end{bmatrix}, \ [D_s] = \begin{bmatrix} k_1^2 A_{44} & A_{45} \\ A_{45} & k_2^2 A_{55} \end{bmatrix}, \ [D_g] = \begin{bmatrix} \bar{N}_1 & \bar{N}_{12} \\ \bar{N}_{12} & \bar{N}_2 \end{bmatrix}
\tag{10}
$$

A_{ij} and D_{ij} can be calculated as follows

$$
\begin{aligned}
(A_{ij}, D_{ij}) &= \int_{-h/2}^{h/2} \bar{Q}_{ij} (1, z^2) dz \qquad (i, j=1, 2, 6) \\
A_{ij} &= \int_{-h/2}^{h/2} \bar{Q}_{ij} dz \qquad (i, j=4, 5)
\end{aligned}
\tag{11}
$$

In Eq. (10), k_1^2 and k_2^2 are the shear correction factors and, in this study the shear correction factor is assumed 5/6.

The discrete eigenvalue equation of the static buckling problem of laminates can be derived as

$$
([K_b + K_s] - \lambda [K_g])\{u\} = 0 \tag{12}
$$

Calculating the critical buckling temperature of buckling due to thermal load is two stage processes. For a specified rise ΔT in temperature the thermal loads are computed and a linear static analysis is carried out to determine the thermal stress resultants. These stress resultants are then used to compute the geometric stiffness matrice, which subsequently used in Eq. (12), to determine the least eigenvalue, λ, and the associated mode shape. The critical buckling temperature, T_{cr}, is calculated as follows

$$
T_{cr} = \lambda \Delta T \tag{12}
$$

In this study, subspace iteration technique is applied to obtain the numerical solutions of the problem.

4. Extended layerwise optimization algorithm

In this paper, the objective of extended layerwise optimization algorithm is finding the optimum stacking sequence $[\theta_1/\theta_2/\theta_3/..../\theta_N]_{s,opt}$ for the maximum fundamental frequency of laminated plates which can be determined sequentially in the order from the outermost to the innermost layer. The current algorithm basically is illustrated in Fig. 2. The aim of this algorithm is the introduction of new layers in the stack that serve to improve the frequency criterion under consideration. However, there is a major difference in the procedure adopted with that of Narita [11-12], in that herein no predetermined number of layers is assumed a priori. Here, the new layers are introduced on the mid-surface of the laminate whose optimal orientation are determined with no limitations as to their number. Another reason for the outward ordering of the successive layers is to place the most effective ones furthest away from the mid-surface. The steps of this algorithm can be expressed as below:

I-1. Assuming constant total laminate thickness equal to h, θ_1 is found so that it would possess the best critical thermal buckling load. The search for optimal angle is done exhaustively in the 0° to $+90^\circ$ domain in increments of $\Delta\theta$ selected.

I-2 Addition of the new layer into the stack which would cause the previously determined layer's thickness to reduce to half and be placed on the top of the stack.

I-N: In this step, by the introduction of the Nth layer based on the same criterion of choice, the thickness of the N-1 layers previously determined would decrease to h/2N. Finally, the new layer must show non-negative improvement of the critical thermal buckling load criterion. The process stops when this improvement becomes less than a predefined value. At the end of stage, a laminate of 2N-layers with the best posture for the critical thermal buckling load criterion is available.

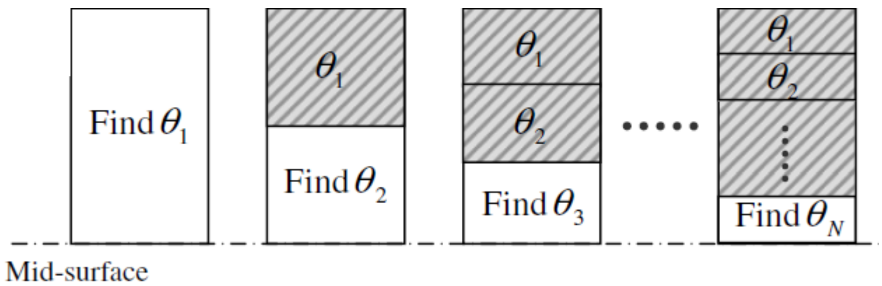

Fig. 2. Stepwise of extended layerwise optimization algorithm

The optimal design problem can be stated mathematically as follows

$$\begin{aligned} &\text{Find:} \quad [\theta_1/\theta_2/\theta_3/..../\theta_N]_s \\ &\text{Maximize } (T_{cr})_{max} = \max_{\theta} T_{cr}(\theta) \\ &\text{Subject to } 0^o \le \theta_k \le 90^o \end{aligned} \tag{13}$$

The critical thermal buckling load for a given fibre orientation is determined from the finite element solution of the eigenvalue problems given by Eq. (12). The optimization procedure involves the stages of evaluating the critical thermal buckling load and improving the fiber orientation θ to maximise T_{cr}. Thus, the computational solution consists of successive stages of analysis and optimization until a convergence is obtained and the optimal angle θ_{opt} is determined within a specified accuracy.

5. Numerical results and discussion

In order to show the applicability of this algorithm, the optimization results of the laminated plates are given for T300/5208 graphite/epoxy material. The material properties are given as below

$E_1 = 181 GPa$, $E_2 = 10.3 GPa$, $G_{12} = 7.17 GPa$, ν_{12} =0.28, $\alpha_1 = 0.02 x 10^{-6}\ {}^\circ C^{-1}$, $\alpha_2 = 22.5 x 10^{-6}\ {}^\circ C^{-1}$

In this study, the increments of $\Delta\theta$ is equal to 5°. The nondimensional thermal buckling load parameter is defined as

$$\overline{T}_{cr} = \alpha_o T_{cr} x 10^3 \tag{14}$$

where $\alpha_o = 10^{-6} /^o C$.

In this study, firstly a convergence study is performed to determine the appropriate finite element mesh to be used in the thermal buckling load analysis of the laminated plate model. Four meshes are developed, with increasing numbers of elements in the x and y directions. In the numerical analysis, simply supported 4-layered cross ply-ply $\left(0^\circ/90^\circ/0^\circ/90^\circ\right)$ laminated plates are investigated (b/h=10). The material properties are given as below

As it can be seen from Table 1, there is only a 0.18% difference between the loads calculated for mesh 15x15 and mesh 20x20. This indicates that mesh 20x20 is capable of performing the analysis within a reasonable degree of accuracy.

Table 1 Mesh convergence study of the present study for simply supported 4-layered cross ply-ply $\left(0^\circ/90^\circ/0^\circ/90^\circ\right)$ laminated plates

Mesh	Critical temperature $T_{cr}(^\circ C)$
5x5	$5.246x10^4$
10x10	$4.983x10^4$
15x15	$4.949x10^4$
20x20	$4.940x10^4$

In this section, the convergence behavior and accuracy of the present study are investigated. A single thin (b/h=40) clamped laminated square plate ($\theta = 45^\circ$) is considered to compare the present study with the literature results. The material properties are given as below:

$E_1 = 76\text{GPa}$, $E_2 = 5.5\text{GPa}$, $G_{12} = G_{13} = 2.30\text{GPa}$, $G_{23} = 1.5\text{GPa}$, $\nu_{12} = 0.34$, $\alpha_1 = -4x10^{-6}\ {}^\circ C^{-1}$, $\alpha_2 = 79x10^{-6}\ {}^\circ C^{-1}$

Table 2 Convergence study of the present study for a clamped square laminated plate

Critical Temperature	Huang and Tauchert (1992)	Kabir *et al.* (2003)	Present study
$T_{cr}(^\circ C)$	129.91	131.55	130.04

As it can be seen from Table 2, the results obtained for critical buckling temperature are in very close aggrement with the literature results.

Table 3 shows the stepwise results of this algorithm for simply supported square laminated plates (b/h=25). The first column indicates the number of steps and the second column indicates the optimum fibre orientations in the layers. The third column and fourth column show the critical thermal buckling loads and the increases in critical thermal buckling loads between the steps, respectively. The stopping criterion for $\Delta\overline{T}_{cr}$ is taken as 0.005.

Table 3. Stepwise results for critical thermal buckling load for simply supported laminated plates (a/b=1, b/h=25)

Step	Stacking order	$\overline{T}_{cr}$	$\Delta\overline{T}_{cr}$
1	[0] or [90]	1.6009	-
2	[0/45] or [90/45]	1.8903	0.289
3	[0/45/45] or [90/45/45]	1.8956	0.005

In Table 4, effect of plate aspect ratio (a/b) on the optimum results using extended layerwise optimization approach is illustrated. As seen from Table 2, the optimum stacking sequences and the number of layers are the same for $a/b\rangle 1$. On the other hand, the critical thermal buckling load decreases with increase in the plate aspect ratio because of diminishing of the plate rigidity.

Table 4 Effect of plate aspect ratio (a/b) on the optimum results for simply supported laminated plates (b/h=25)

a/b	Optimum stacking sequence	$\overline{T}_{cr}$
1	[0/45/45] or [90/45/45]	1.8956
1.5	[90]	1.0884
2	[90]	0.8954
2.5	[90]	0.8034
3	[90]	0.7557

Extended layerwise optimization algorithm may be applied to laminated plates with any combinations of simple support (S), clamped support (C), and free edge (F). Different combinations of the boundary conditions are considered in this study. For example, a clamped-simple-clamped-simple (CSCS) is a specimen with clamped supported on x=0 and x=a, and simple supported on y=0 and y=b, respectively. In Table 5, effect of different boundary conditions on the optimum results are given using extended layerwise optimization approach (b/h=25, a/b=1). As seen from Table 5, the maximum and minimum critical thermal buckling loads are obtained for (SFCF) and (SCCF) boundary conditions, respectively. It is obvious from the results that, the optimum stacking sequences and the number of layers can be changeable for different boundary conditions.

Table 5 Effect of boundary conditions on the optimum results for square laminated plates (b/h=25, a/b=1)

Boundary conditions	Optimum stacking sequence	$\overline{T}_{cr}$
(SSSS)	[0/45/45] or [90/45/45]	1.8956
(CSCS)	[0/90/0/0/0/0/0/0/0/0/0]	7.8609
(CCCC)	[0/90/90/0/0]	3.2427
(CFCF)	[0]	3.5058
(CFFF)	[0]	4.7251
(CCFF)	[90/0/0/0]	26.8995
(SFCF)	[90]	1212.70 40
(SSSF)	[90]	1.8328
(SCSF)	[90/0/90/90/90/90/90]	23.1651
(SSCF)	[0/90]	15.7081
(SCCF)	[0]	1.5939
(SSSC)	[90]	2.2696
(SSCC)	[90/0]	10.0142

6. Conclusions

In this paper, the applicability of a new extended layerwise optimization method on critical thermal buckling load optimization of laminated composite plates is investigated. The design objective is the maximization of the critical thermal buckling load. The fibre orientations in the layers are considered as design variables. The aim of this algorithm is the introduction of new layers in the stack that serve to improve the critical thermal buckling load criterion under consideration. The limited set of results presented in this paper suggests that the extended layerwise optimization method is an effective technique for determining the optimum laminate lay-ups in laminated plates in spite of increase of the computational effort and time. On the other hand, the optimum stacking sequences and the number of layers can be changeable for different boundary conditions.

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