Steel and Composite Structures, Vol.14, No. 3 (2013) 243-265
DOI: http://dx.doi.org/10.12989/scs.2013.14.3.243

Distortional and local buckling of steel-concrete composite box-beam

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(Received September 14, 2009, Revised January 12, 2012, Accepted December 13, 2012)

Abstract. Distortional and local buckling are important factors that influences the bearing capacity of steel-concrete composite box-beam. Through theoretical analysis of distortional buckling forms, a stability analysis calculation model of composite box beam considering rotation of steel beam top flange is presented. The critical bending moment calculation formula of distortional buckling is established. In addition, mechanical behaviors of a steel beam web in the negative moment zone subjected separately to bending stress, shear stress and combined stress are investigated. Elastic buckling factors of steel web under different stress conditions are calculated. On the basis of local buckling analysis results, a limiting value for height-to thickness ratio of a steel web in the elastic stage is proposed. Numerical examples are presented to verify the proposed models.

Keywords: steel-concrete composite box-beam; distortional buckling; local buckling; negative moment

1. Introduction

Beam stability is a focus to which engineers pay much attention, because it affects the safety of a structure and the ability of a member to attain its full capacity. In comparison with a steel structure, a steel-concrete composite beam can achieve increased global stability and local stability because of the constraint of concrete slab. However, it is still no necessary to consider the buckling problem of a steel-concrete composite beam, particularly with respect to the negative moment region. Narayanan (1988) systematically studied the possible local buckling of a steel beam web when the continuous composite beam is subjected to dynamic loading. He pointed out that local buckling of a steel beam web can lead to great deformation of the flange. Fukumoto and Kubo (1997) adopted a linear elastic finite element method to calculate global lateral buckling moment of the compressive bottom flange. Because the model which they adopted ignored the constraint action of the web steel plate to compressive flange lateral buckling, it is conservative. Fan *et al.* (2004) discussed the elastic buckling analysis of a steel beam web under combined stress loads of flexure, axial pressure and shear. The elastic local buckling coefficient for complicated stress conditions was calculated.

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As previously mentioned, most research has concentrated on local buckling of web in the positive moment zone. With respect to the negative moment zone, the web not only can sustain local buckling but also global buckling. Few researchers have considered this aspect of the problem. Also, current codes and standards only provide stability calculation and corresponding construction measures for composite beams in positive moment zone. In order to improve the current codes, it is essential to study the stability problems of the composite beam in the negative moment zone.

In this paper, on the basis of the stability theory and experiment results of other researchers, a calculation model for composite box-beam stability is presented. Distortional buckling is studied using energy method. Considering not only the lateral bending and rotation of bottom flange but also rotation of top flange, the calculation formula of critical bearing moment of a composite box-beam is obtained. For the sake of deducing the formula for corresponding local buckling critical stress and limit value of height-to-thickness ratio, a local buckling model under different loads is established. Finally, numerical examples are presented to demonstrate the application of the proposed models.

2. Model and Assumptions

2.1 Distortional buckling of steel-concrete composite box-beam

Steel-concrete composite box-beams are made of welded steel plates and concrete slab with stud connection. Because of the large bending and torsional stiffness of the concrete slab, composite box-beam in the positive bending zone needs not be checked for global stability. However, for large-span composite box-beams with high-narrow steel section, if the steel beam in negative bending zone has no lateral support or the space between lateral support is too large, the compressive flange and web of the steel box-beam may twist and deviate from the loading plane. Thus, load supporting capability may be lost. This phenomenon is called global distortional buckling. For the sake of developing a distortional buckling model some assumptions are made:

(1) Materials are isotropic and elastic without considering initial defects and residual stress.

(2) The composite box-beam has uniform cross section. The lateral flexural deformation of box-beam is small, and the torsional deformation is not taken into account.

(3) Because of the great in-plane flexural stiffness of composite beam, the influences of in-plane flexural deformation on lateral bending can be ignored.

(4) The top flange of the steel beam cannot generate lateral deformation, but it can twist.

(5) Ignoring the bending effect of concrete in tension, only the bending effect of the reinforcing steel bar in the concrete slab is considered.

On the basis of above assumptions, when composite beam suffers elastic buckling, the lateral deformation of bottom flange is assigned u_B, and the torsional angles of two ends of bottom flange are assigned as ϕ_i and ϕ_j .Since it is assumed that the lateral deformation of steel beam top flange is fully constrained by concrete slab, the torsional angles of both top flanges ϕ_n and ϕ_m are assigned. System of coordinates is shown in Fig.1.

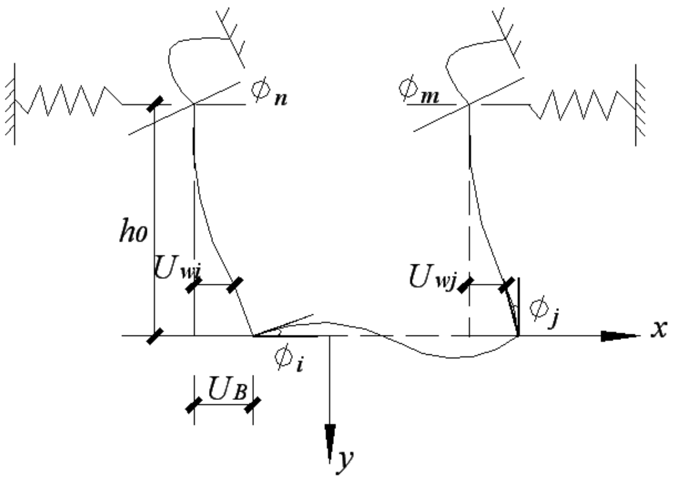


Fig.1 Distortional buckling model of composite box-beam

When there are small transversal deformation u_B and torsional deformation ϕ_i of steel beam bottom flange and torsional deformation ϕ_n of steel beam top flange, the steel webs will yield out-plane bending deformation u_{wi} and u_{wj}. The lateral bending deformation of left steel beam web u_{wi} can be expressed as cubic polynomial function about u_B、ϕ_i and ϕ_n as following

$$u_{wi} = f_1(y)u_B(z) + f_2(y)\phi_i(z) + f_3(y)\phi_n(z) \tag{1}$$

Because the lateral bending deformation of right steel beam web u_{wj} has the same expression about u_B、ϕ_j and ϕ_m, it is displayed as follows

$$u_{wj} = f_1(y)u_B(z) + f_2(y)\phi_j(z) + f_3(y)\phi_m(z) \tag{2}$$

$$\varsigma = f_4(x)\phi_i(z) + f_5(x)\phi_j(z) \tag{3}$$

Deformation compatibility of left web

$$\left.\begin{aligned} &\left.\frac{\partial u_{wi}(y,z)}{\partial y}\right|_{y=0} = \phi_i \\ &u_{wi}(y,z)\big|_{y=0} = u_B \\ &\left.\frac{\partial u_{wi}(y,z)}{\partial y}\right|_{y=-h_0} = \phi_n \\ &u_w(y,z)\big|_{y=-h_0} = 0 \end{aligned}\right\} \tag{4}$$

Deformation compatibility of right web

$$\left.\begin{aligned} &\left.\frac{\partial u_{\mathrm{wj}}(y,z)}{\partial y}\right|_{y=0} = \phi_{\mathrm{j}} \\ &\left.u_{\mathrm{wj}}(y,z)\right|_{y=0} = u_{\mathrm{B}} \\ &\left.\frac{\partial u_{\mathrm{wj}}(y,z)}{\partial y}\right|_{y=-h_0} = \phi_{\mathrm{n}} \\ &\left.u_{\mathrm{w}}(y,z)\right|_{y=-h_0} = 0 \end{aligned}\right\} \tag{5}$$

Deformation compatibility of bottom flange

$$\left.\begin{aligned} &\left.\frac{\partial \varsigma(x,z)}{\partial x}\right|_{x=-\frac{b}{2}} = \phi_{\mathrm{i}} \\ &\left.\varsigma(x,z)\right|_{y=0} = 0 \\ &\left.\frac{\partial \varsigma(x,z)}{\partial x}\right|_{x=\frac{b}{2}} = \phi_{\mathrm{j}} \\ &\left.\varsigma(x,z)\right|_{y=-h_0} = 0 \end{aligned}\right\} \tag{6}$$

Substituting Eqs. (1)-(3) into deformation compatibility conditions Eqs. (4)-(6), the deformation curves of webs and bottom flange are obtained.

Deformation curve of left web

$$\begin{aligned} u_{\mathrm{wi}} = &\left[1-3(\frac{y}{h_0})^2-2(\frac{y}{h_0})^3\right]u_{\mathrm{B}}(z)+h_0\left[\frac{y}{h_0}+2(\frac{y}{h_0})^2+(\frac{y}{h_0})^3\right]\phi_{\mathrm{i}}(z) \\ &+h_0\left[(\frac{y}{h_0})^2+(\frac{y}{h_0})^3\right]\phi_{\mathrm{n}}(z) \end{aligned} \tag{7}$$

Deformation curve of right web

$$\begin{aligned} u_{\mathrm{wj}} = &\left[1-3(\frac{y}{h_0})^2-2(\frac{y}{h_0})^3\right]u_{\mathrm{B}}(z)+h_0\left[\frac{y}{h_0}+2(\frac{y}{h_0})^2+(\frac{y}{h_0})^3\right]\phi_{\mathrm{j}}(z) \\ &+h_0\left[(\frac{y}{h_0})^2+(\frac{y}{h_0})^3\right]\phi_{\mathrm{m}}(z) \end{aligned} \tag{8}$$

Deformation curve of bottom flange

$$\varsigma=\left(\frac{b}{8}-\frac{1}{4}x-\frac{1}{2b}x^2+\frac{1}{b^2}x^3\right)\phi_i(z)+\left(-\frac{b}{8}-\frac{1}{4}x+\frac{1}{2b}x^2+\frac{1}{b^2}x^3\right)\phi_j(z) \tag{9}$$

According to the buckling model, the total strain energy of a composite box-beam taking into account the rotation of top flange includes: lateral bending strain energy U_1, torsinal strain energy of steel beam top flanges U_2, U_3, bending strain energy of bottom flange U_4, strain energy of concrete spring restraint of top flanges U_5, $U_6$and lateral bending strain energy U_i,U_j.

Lateral bending strain energy of the bottom flange

$$U_1=\frac{1}{2}\int_0^l EI_{y\mathrm{b}}u_{\mathrm{B},zz}{}^2 d_z \tag{10}$$

Torsional strain energy of the left top flange

$$U_2=\frac{1}{2}\int_0^l GJ_{\mathrm{t}}\phi_{\mathrm{n},z}{}^2 d_z \tag{11}$$

Torsional strain energy of the right top flange

$$U_3=\frac{1}{2}\int_0^l GJ_{\mathrm{t}}\phi_{\mathrm{m},z}{}^2 d_z \tag{12}$$

Vertical deformation energy of the bottom flange is obtained according to small deflection theory of thin plate as follows

$$\begin{aligned}U_4=\frac{Et_{\mathrm{f}}^3}{24(1-\mu^2)}\int_0^l\Bigg[&\frac{4}{b}\phi_{\mathrm{i}}^2+\frac{4}{b}\phi_i\phi_{\mathrm{j}}+\frac{4}{b}\phi_{\mathrm{j}}^2-\frac{4\mu b}{15}\phi_i\phi_{i,zz}+\frac{\mu b}{15}\phi_i\phi_{j,zz}\\&+\frac{\mu b}{15}\phi_j\phi_{i,zz}-\frac{4\mu b}{15}\phi_j\phi_{j,zz}-\frac{4\mu b}{15}\phi_{\mathrm{j}}\phi_{j,zz}+\frac{b^3}{105}\phi_{i,zz}^2-\frac{b^3}{70}\phi_{i,zz}\phi_{j,zz}\\&+\frac{b^3}{105}\phi_{j,zz}^2+\frac{4(1-\mu)b}{15}\phi_{i,z}^2-\frac{2(1-\mu)b}{15}\phi_{i,z}\phi_{j,z}+\frac{4(1-\mu)b}{15}\phi_{i,z}^2\Bigg]d_z\end{aligned} \tag{13}$$

Strain energy of concrete spring restraint to left steel top flange

$$U_5=\frac{1}{2}\int_0^l k\phi_{\mathrm{n}}{}^2 d_z \tag{14}$$

Strain energy of concrete spring restraint to right steel top flange

$$U_6=\frac{1}{2}\int_0^l k\phi_{\mathrm{m}}{}^2 d_z \tag{15}$$

The term k is the spring constant of rotation spring ,Hu(1996)

$$k = \frac{k_1 k_2}{k_1 + k_2} \tag{16}$$

Where, $k_1 = \dfrac{\alpha E_{\mathrm{cm}} I_{\mathrm{c2}}}{a}$ is the rotation spring constant of concrete slab; $k_2 = \dfrac{1}{4}\dfrac{E_{\mathrm{a}}}{1-\mu_{\mathrm{a}}^2}\dfrac{t_{\mathrm{w}}^3}{h_{\mathrm{s}}}$ is the rotation spring constant of steel beam web; α is span influence coefficient; a is span of concrete slab; $E_{\mathrm{cm}}I_{\mathrm{c2}}$ is mid-span average bending rigidity per unit of width of concrete slab considering the transverse reinforcement effect in concrete slab and ignoring the concrete effect of tensile zone; h_s is distance between the top flange gravity axis and the bottom flange gravity axis of steel beam; E_a is elasticity modulus of steel beam.

Lateral bending strain energy is found form Eq. (17)

$$U_{\mathrm{w}} = \frac{Et_{\mathrm{w}}^3}{24(1-\mu^2)}\int_0^l \int_{-h_0}^0 \left[u_{\mathrm{w},yy}^2 + 2\mu u_{\mathrm{w},yy} u_{\mathrm{w},zz} + u_{\mathrm{w},zz}^2 + 2(1-\mu)u_{\mathrm{w},yz}^2\right] d_y d_z \tag{17}$$

Substituting Eq. (7) and Eq. (8) separately into Eq. (17), lateral bending strain energy of left web (U_i) and right web (U_j) are obtained. Total strain energy U of the composite box beam considering rotation of top flange is the summation of equations form Eq. (10) to Eq. (17).

$$U = U_1 + U_2 + U_3 + U_4 + U_5 + U_6 + U_i + U_j$$

The total external work is equal to

$$\begin{aligned} W = {} & \frac{1}{2}\int_0^l \int_{A_{\mathrm{b}}} \sigma_{\mathrm{b}z}\left(u_{\mathrm{B},z}^2 + \varsigma_{\mathrm{B},z}^2\right) d_A d_z + \frac{1}{2}\int_0^l \int_{A_{\mathrm{t}}} \sigma_{\mathrm{t}z}\left(x^2\phi_{n,z}^2 + x^2\phi_{m,z}^2\right) d_A d_z \\ & + \frac{1}{2}\int_0^l \int_{A_{\mathrm{w}}} \sigma_{\mathrm{w}z} u_{\mathrm{w},z}^2 d_A d_z \end{aligned} \tag{18}$$

The sum of strain energy and external work is called total potential energy of distortional buckling of composite box beam

$$\Pi = U + W \tag{19}$$

In accordance with principle of minimum potential energy (Xia and Pan 1987), the critical moment of a composite box beam under equal end moment considering top flange rotation buckling can be obtained by imposing the stationarity of the action functional

$$\delta\Pi = 0 \tag{20}$$

Five balance equations are derived from Eq. (20) as follows:

$$\left[EI_{\mathrm{yb}}+\frac{13Eh_0t_{\mathrm{w}}^3}{210(1-\mu^2)}\right]u_{\mathrm{B}}^{IV}-\frac{11Eh_0^2t_{\mathrm{w}}^3}{2520(1-\mu^2)}\left(\phi_{\mathrm{i}}^{IV}+\phi_{\mathrm{j}}^{IV}\right)-\frac{11Eh_0^2t_{\mathrm{w}}^3}{5040(1-\mu^2)}\left(\phi_{\mathrm{n}}^{IV}+\phi_{\mathrm{m}}^{IV}\right)$$
$$-\left[\frac{2Et_{\mathrm{w}}^3}{5(1-\mu^2)h_0}-\frac{(35y_{\mathrm{c}}A_{\mathrm{b}}+26y_{\mathrm{c}}h_0t_{\mathrm{w}}-6h_0^2t_{\mathrm{w}})M_x}{35I}\right]u_{\mathrm{B}}''+\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}\right.$$
$$\left.-\frac{(22y_{\mathrm{c}}-7h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]\left(\phi_{\mathrm{i}}''+\phi_{\mathrm{j}}''\right)+\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}-\frac{(13y_{\mathrm{c}}-6h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]\left(\phi_{\mathrm{n}}''+\phi_{\mathrm{m}}''\right)$$
$$+\frac{2Et_{\mathrm{w}}^3}{(1-\mu^2)h_0^3}u_{\mathrm{B}}-\frac{Et_{\mathrm{w}}^3}{2(1-\mu^2)h_0^2}\left(\phi_{\mathrm{i}}+\phi_j+\phi_n+\phi_m\right)=0 \tag{21a}$$

$$\frac{11Eh_0^2t_{\mathrm{w}}^3}{2520(1-\mu^2)}u_{\mathrm{B}}^{IV}-\frac{Eh_0^3t_{\mathrm{w}}^3+Et_{\mathrm{f}}^3b^3}{1260(1-\mu^2)}\phi_{\mathrm{i}}^{IV}+\frac{Eb^3t_{\mathrm{f}}^3}{1680(1-\mu^2)}\phi_{\mathrm{j}}^{IV}$$
$$+\frac{Eh_0^3t_{\mathrm{w}}^3}{1680(1-\mu^2)}\phi_{\mathrm{n}}^{IV}-\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}-\frac{(22y_{\mathrm{c}}-7h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]u_{\mathrm{B}}''$$
$$+\left[\left[-\frac{(8y_{\mathrm{c}}-3h_0)h_0^3t_{\mathrm{w}}M_x}{840I}-\frac{b^2A_by_{\mathrm{c}}M_x}{105I}+\frac{Eh_0t_{\mathrm{w}}^3+Et_{\mathrm{f}}^3b}{45(1+\mu)}\right]\right.\phi_{\mathrm{i}}''$$
$$-\left[\frac{Et_{\mathrm{f}}^3b}{180(1-\mu^2)}-\frac{b^2A_by_{\mathrm{c}}M_x}{140I}\right]\phi_{\mathrm{j}}''-\left[\frac{Eh_0t_{\mathrm{w}}^3}{180(1+\mu)}-\frac{(2y_{\mathrm{c}}-h_0)h_0^3t_{\mathrm{w}}M_x}{280I}\right]\phi_{\mathrm{n}}''$$
$$+\frac{Et_{\mathrm{w}}^3}{2(1-\mu^2)h_0^2}u_B-\frac{Et_{\mathrm{w}}^3b+Et_{\mathrm{f}}^3h_0}{3(1-\mu^2)h_0b}\phi_{\mathrm{i}}-\frac{Et_{\mathrm{f}}^3}{6(1-\mu^2)b}\phi_{\mathrm{j}}-\frac{Et_{\mathrm{w}}^3}{6(1-\mu^2)h_0}\phi_{\mathrm{n}}=0 \tag{21b}$$

$$\frac{11Eh_0^2t_{\mathrm{w}}^3}{2520(1-\mu^2)}u_{\mathrm{B}}^{IV}+\frac{Eb^3t_{\mathrm{f}}^3}{1680(1-\mu^2)}\phi_{\mathrm{i}}^{IV}-\frac{Eh_0^3t_{\mathrm{w}}^3+Et_{\mathrm{f}}^3b^3}{1260(1-\mu^2)}\phi_{\mathrm{j}}^{IV}$$
$$+\frac{Eh_0^3t_{\mathrm{w}}^3}{1680(1-\mu^2)}\phi_{\mathrm{m}}^{IV}-\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}-\frac{(22y_{\mathrm{c}}-7h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]u_{\mathrm{B}}''$$
$$-\left[\frac{Et_{\mathrm{f}}^3b}{180(1-\mu^2)}-\frac{b^2A_by_{\mathrm{c}}M_x}{140I}\right]\phi_{\mathrm{i}}''$$
$$+\left[\left[-\frac{(8y_{\mathrm{c}}-3h_0)h_0^3t_{\mathrm{w}}M_x}{840I}-\frac{b^2A_by_{\mathrm{c}}M_x}{105I}+\frac{Eh_0t_{\mathrm{w}}^3+Et_{\mathrm{f}}^3b}{45(1+\mu)}\right]\right.\phi_{\mathrm{j}}''$$
$$-\left[\frac{Eh_0t_{\mathrm{w}}^3}{180(1-\mu^2)}-\frac{(2y_{\mathrm{c}}-h_0)h_0^3t_{\mathrm{w}}M_x}{280I}\right]\phi_{\mathrm{m}}''$$
$$+\frac{Et_{\mathrm{w}}^3}{2(1-\mu^2)h_0^2}u_B-\frac{Et_{\mathrm{f}}^3}{6(1-\mu^2)b}\phi_{\mathrm{i}}-\frac{Et_{\mathrm{w}}^3b+Et_{\mathrm{f}}^3h_0}{3(1-\mu^2)h_0b}\phi_{\mathrm{j}}-\frac{Et_{\mathrm{w}}^3}{6(1-\mu^2)h_0}\phi_{\mathrm{m}}=0 \tag{21c}$$

$$\begin{aligned}&\frac{13Eh_0^2t_{\mathrm{w}}^3}{5040(1-\mu^2)}u_{\mathrm{B}}^{IV}+\frac{Eh_0^3t_{\mathrm{w}}^3}{1680(1-\mu^2)}\phi_{\mathrm{i}}^{IV}-\frac{Eh_0^3t_{\mathrm{w}}^3}{1260(1-\mu^2)}\phi_{\mathrm{n}}^{IV}\\&-\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}-\frac{(13y_{\mathrm{c}}-6h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]u_{\mathrm{B}}''-\left[\frac{Et_{\mathrm{f}}^3b}{180(1-\mu^2)}-\frac{(2y_{\mathrm{c}}-h_0)h_0^3t_{\mathrm{w}}M_x}{280I}\right]\phi_{\mathrm{i}}''\\&+\left[\frac{Eh_0t_{\mathrm{w}}^3}{45(1+\mu)}+GJ_{\mathrm{t}}-\frac{(y_{\mathrm{c}}-h_0)b_t^3t_fM_x}{12I}-\frac{(8y_{\mathrm{c}}-5h_0)h_0^3t_{\mathrm{w}}M_x}{840I}\right]\phi_{\mathrm{n}}''\\&+\frac{Et_{\mathrm{w}}^3}{2(1-\mu^2)h_0^2}u_B-\frac{Et_{\mathrm{f}}^3}{6(1-\mu^2)h_0}\phi_{\mathrm{i}}-\left[k+\frac{Et_{\mathrm{w}}^3}{3(1-\mu^2)h_0}\right]\phi_{\mathrm{n}}=0\end{aligned}\tag{21d}$$

$$\begin{aligned}&\frac{13Eh_0^2t_{\mathrm{w}}^3}{5040(1-\mu^2)}u_{\mathrm{B}}^{IV}+\frac{Eh_0^3t_{\mathrm{w}}^3}{1680(1-\mu^2)}\phi_{\mathrm{j}}^{IV}-\frac{Eh_0^3t_{\mathrm{w}}^3}{1260(1-\mu^2)}\phi_{\mathrm{m}}^{IV}\\&-\left[\frac{(1+5\mu)Et_{\mathrm{w}}^3}{60(1-\mu^2)}-\frac{(13y_{\mathrm{c}}-6h_0)h_0^2t_{\mathrm{w}}M_x}{420I}\right]u_{\mathrm{B}}''-\left[\frac{Et_{\mathrm{f}}^3b}{180(1-\mu^2)}-\frac{(2y_{\mathrm{c}}-h_0)h_0^3t_{\mathrm{w}}M_x}{280I}\right]\phi_{\mathrm{j}}''\\&+\left[\frac{Eh_0t_{\mathrm{w}}^3}{45(1+\mu)}+GJ_{\mathrm{t}}-\frac{(y_{\mathrm{c}}-h_0)b_t^3t_fM_x}{12I}-\frac{(8y_{\mathrm{c}}-5h_0)h_0^3t_{\mathrm{w}}M_x}{840I}\right]\phi_{\mathrm{m}}''\\&+\frac{Et_{\mathrm{w}}^3}{2(1-\mu^2)h_0^2}u_B-\frac{Et_{\mathrm{f}}^3}{6(1-\mu^2)h_0}\phi_{\mathrm{j}}-\left[k+\frac{Et_{\mathrm{w}}^3}{3(1-\mu^2)h_0}\right]\phi_{\mathrm{m}}=0\end{aligned}\tag{21e}$$

Assuming the lateral displacement and rotation angle of steel beam bottom flange as well as rotation angle of steel beam top flange are half-sine wave curves, the deformations of distortional buckling of composite box beam can be expressed as (Xia and Pan 1987)

$$\begin{Bmatrix}u_{\mathrm{B}}\\\phi_{\mathrm{i}}\\\phi_{\mathrm{j}}\\\phi_n\\\phi_m\end{Bmatrix}=\begin{Bmatrix}C_1\\C_2\\C_3\\C_4\\C_5\end{Bmatrix}\sin\frac{n\pi z}{l}\tag{22}$$

Wherein, C_1, C_2, C_3, C_4, C_5 are the amplitudes of the deformations separately. n is the number of half-sine waves of deformations within the l length buckling range. Its value is determined by trial method on condition that the minimum M_x is obtained. For ease of calculation, equivalent coefficient η_1 of steel beam bottom flange to y axis moment of inertia and geometric properties coefficients of cross section β_1, β_2, β_3, β_4, β_5, β_6, β_7 as well as R_1, R_2, R_3, R_4, R_5, R_6, R_7 are introduced as following:

$$\eta_1 = 1 + \frac{13h_0 t_{\mathrm{w}}^3}{210(1-\mu^2)I_{y\mathrm{b}}} \tag{23}$$

$$\beta_1 = \frac{35y_{\mathrm{c}}A_{\mathrm{b}} + 26y_{\mathrm{c}}h_0 t_{\mathrm{w}} - 6h_0^2 t_{\mathrm{w}}}{35I} \tag{24a}$$

$$\beta_2 = \frac{(22y_c - 7h_0)h_0^2 t_w}{420I} \tag{24b}$$

$$\beta_3 = \frac{8y_c h_0^3 t_w - 3h_0^4 t_w + 8b^2 A_b y_c}{840I} \tag{24c}$$

$$\beta_4 = \frac{b^2 A_b y_c}{140I} \tag{24d}$$

$$\beta_5 = \frac{(13y_c - 6h_0)h_0^2 t_w M_x}{420I} \tag{24e}$$

$$\beta_6 = \frac{(2y_c - h_0)h_0^3 t_w M_x}{280I} \tag{24f}$$

$$\beta_7 = \frac{(y_c - h_0)b_t^3 t_f}{12I} + \frac{(8y_c - 5h_0)h_0^3 t_w}{840I} \tag{24g}$$

$$R_1 = E\eta_1 I_{yb}\left(\frac{n\pi}{l}\right)^2 + \frac{2Et_w^3}{5(1-\mu^2)h_0} + \frac{2Et_w^3}{(1-\mu^2)h_0^3}\left(\frac{l}{n\pi}\right)^2 \tag{25a}$$

$$R_2 = \frac{11Eh_0^2 t_w^3}{2520(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{(1+5\mu)Et_w^3}{60(1-\mu^2)} + \frac{Et_w^3}{2(1-\mu^2)h_0^2}\left(\frac{l}{n\pi}\right)^2 \tag{25b}$$

$$R_3 = \frac{E(h_0^3 t_w^3 + t_f^3 b^3)}{1260(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{E(h_0 t_w^3 + bt_f^3)}{45(1-\mu^2)} + \frac{E(t_w^3 b + h_0 t_f^3)}{3(1-\mu^2)h_0 b}\left(\frac{l}{n\pi}\right)^2 \tag{25c}$$

$$R_4 = \frac{Eb^3 t_f^3}{1680(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{Et_f^3 b}{180(1-\mu^2)} - \frac{Et_f^3}{6(1-\mu^2)b}\left(\frac{l}{n\pi}\right)^2 \tag{25d}$$

$$R_5 = \frac{13Eh_0^2 t_w^3}{5040(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{(1+5\mu)Et_w^3}{60(1-\mu^2)} + \frac{Et_w^3}{2(1-\mu^2)h_0^2}\left(\frac{l}{n\pi}\right)^2 \tag{25e}$$

$$R_6 = \frac{Eh_0^3 t_w^3}{1680(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{Eh_0 t_w^3}{180(1-\mu^2)} + \frac{Et_w^3}{6(1-\mu^2)h_0}\left(\frac{l}{n\pi}\right)^2 \tag{25f}$$

$$R_7 = \frac{Eh_0^3 t_w^3}{1260(1-\mu^2)}\left(\frac{n\pi}{l}\right)^2 + \frac{Eh_0 t_w^3}{45(1-\mu^2)} + GJ_t + \left[k + \frac{Et_w^3}{3(1-\mu^2)h_0}\right]\left(\frac{l}{n\pi}\right)^2 \tag{25g}$$

Simplified the Eq. (21), the matrix equations are obtained as

$$\mathbf{A}(\mathrm{M_x})*\mathbf{C}=0 \tag{26}$$

C is the vector $[C_1, C_2, C_3, C_4, C_5]$ and **A**(M_x) is the matrix about M_x. This is a system of homogeneous linear equations. In order to get a nontrivial solution, the determinant of coefficient $|\mathbf{A}(M_x)|$ must equal to zero. Therefore, five solutions of M_x are obtained and the minimum one is the critical moment.

$$M_{x1} = \frac{-A_5 + \sqrt{A_5^2 - 4A_6A_4}}{2A_6} \tag{27a}$$

$$M_{x2} = \frac{-A_5 - \sqrt{A_5^2 - 4A_6A_4}}{2A_6} \tag{27b}$$

$$M_{x3} = Z_1 - \frac{1}{3}\frac{A_2}{A_3} \tag{27c}$$

$$M_{x4} = Z_2 - \frac{1}{3}\frac{A_2}{A_3} \tag{27d}$$

$$M_{x5} = Z_3 - \frac{1}{3}\frac{A_2}{A_3} \tag{27e}$$

Wherein:

$$A_0 = -R_1R_4R_7 + R_1R_3R_7 + 2R_5^2R_4 - R_1R_6^2 - 2R_2^2R_7 - 2R_5^2R_3 - 4R_2R_5R_6 \tag{28a}$$

$$\begin{aligned} A_1 =& 4R_2R_5\beta_6 - 2R_5^2\beta_4 + R_1R_4\beta_7 + 2R_5^2\beta_3 - \beta_1R_3R_7 + 4R_2\beta_2R_7 + 2R_2^2\beta_7 - 4R_5\beta_5R_4 + \beta_1R_4R_7 \\ & - R_1\beta_3R_7 + 4R_2\beta_5R_6 + 4R_5\beta_5R_3 + \beta_1R_6^2 - R_1R_3\beta_7 + 2R_1R_6\beta_6 + 4\beta_2R_5R_6 + R_1\beta_4R_7 \end{aligned} \tag{28b}$$

$$\begin{aligned} A_2 =& -4R_2\beta_2\beta_7 + \beta_1\beta_3R_7 - 2\beta_5^2R_3 - \beta_1R_4\beta_7 - 2\beta_2^2R_7 - \beta_1\beta_4R_7 - R_1\beta_6^2 - 2\beta_1R_6\beta_6 - 4\beta_2\beta_5R_6 \\ & - R_1\beta_4\beta_7 + R_1\beta_3\beta_7 - 4R_5\beta_5\beta_3 - 4\beta_2R_5\beta_6 + \beta_1R_3\beta_7 - 4R_2\beta_5\beta_6 + 2\beta_5^2R_4 + 4R_5\beta_5\beta_4 \end{aligned} \tag{28c}$$

$$A_3 = 2\beta_5^2\beta_3 + 2\beta_2^2\beta_7 + \beta_1\beta_6^2 - \beta_1\beta_3\beta_7 - 2\beta_5^2\beta_4 + 4\beta_2\beta_5\beta_6 + \beta_1\beta_4\beta_7 \tag{28d}$$

$$A_4 = R_7R_4 + R_7R_3 - R_6^2 \tag{28e}$$

$$A_5 = -(R_7\beta_3 + R_7\beta_4 + R_4\beta_7 + R_3\beta_7 - 2R_6\beta_6) \tag{28f}$$

$$A_6 = (-\beta_6^2 + \beta_7\beta_3 + \beta_7\beta_4) \tag{28g}$$

$$Z_1 = 2\sqrt{|B_1|}\cos\theta \tag{29a}$$

$$Z_2 = \sqrt{|B_1|}\cos(\frac{2\pi}{3} + \theta) \tag{29b}$$

$$Z_3 = \sqrt{|B_1|}\cos(\frac{2\pi}{3} - \theta) \tag{29c}$$

$$\cos 3\theta = -\frac{B_0}{|B_1|^{3/2}} \tag{30}$$

$$B_1 = \frac{1}{3}\frac{A_1}{A_3} - \frac{1}{9}\left(\frac{A_2}{A_3}\right)^2 \tag{31a}$$

$$B_0 = \frac{1}{27}\left(\frac{A_2}{A_3}\right)^3 - \frac{A_1 + A_2}{6A_3^2} + \frac{A_0}{2A_3} \tag{31b}$$

2.2 Local buckling of steel-concrete composite box-beam

2.2.1 Buckling model

When a continuous composite box-beam is subjected to adverse loads, negative moment will develop at the intermediate support. In this case, the steel beam web near the intermediate support is in compression state. As soon as the critical pressure is reached, it leads to local buckling of the web, which reduces the ultimate capacity of the composite box-beam. Local buckling of a continuous composite box-beam happens mainly in the local zone of the steel beam web near the intermediate support. Therefore, for the purpose of preventing local buckling, the height-thickness ratio of steel beam web is controlled in design. To derive local buckling equations, the buckling model is shown in Fig.2.

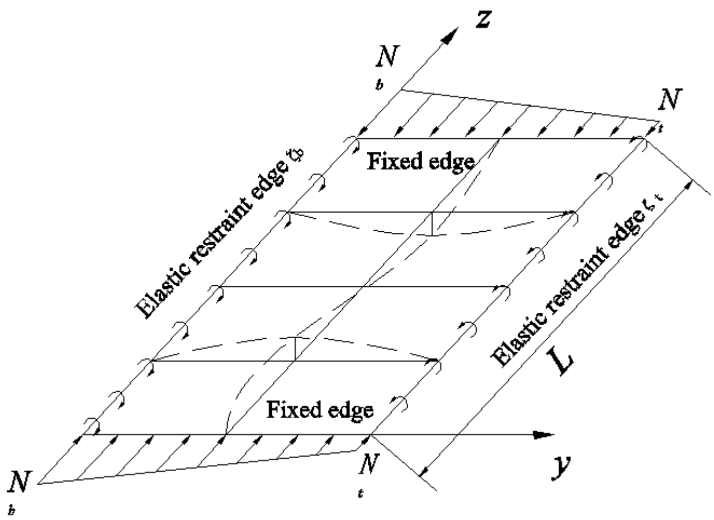


Fig.2 Local buckling model of steel beam web

Assuming the web displacement function as follows

$$\omega(y,z)=\left\{\frac{y}{h_0}+\gamma_1\left(\frac{y}{h_0}\right)^2+\gamma_2\left(\frac{y}{h_0}\right)^3+\gamma_3\left(\frac{y}{h_0}\right)^4\right\}\sum_{m=1}^{\infty}C_m\sin\frac{m\pi z}{l} \tag{32}$$

The first part of the displacement function is a quartic polynomial satisfying non-loading side displacement boundary conditions, and the second part is a trigonometric harmonic function satisfying loading side displacement boundary conditions.

Boundary conditions of loading sides:

$$\left.\begin{aligned}\omega(0,y)&=0\\ \omega(l,y)&=0\end{aligned}\right\} \tag{33}$$

Boundary conditions of non-loading sides

$$\left.\begin{aligned}&\omega(z,0)=0\\ &\omega(z,h_0)=0\\ &M(z,0)=-D\left(\left.\frac{\partial^2\omega}{\partial y^2}\right|_{y=0}\right)=\zeta_b\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=0}\right)\\ &M(z,h_0)=-D\left(\left.\frac{\partial^2\omega}{\partial y^2}\right|_{y=h_0}\right)=\zeta_t\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=h_0}\right)\end{aligned}\right\} \tag{34}$$

Wherein, $D=\dfrac{Et_{\mathrm{w}}^3}{12(1-\mu^2)}$ is the bending rigidity of steel beam web; ζ_b , ζ_t is restrained rotation rigidity of non-loading side.

Substituting Eq. (30) into Eq. (32), the buckling displacement function is obtained

$$\begin{aligned}\omega(y,z)=&\left\{\frac{y}{h_0}+\frac{\chi_b}{2}\left(\frac{y}{h_0}\right)^2+\frac{-(12+5\chi_b+\chi_b\chi_t+3\chi_t)}{(6+\chi_t)}\left(\frac{y}{h_0}\right)^3\right.\\ &\left.+\frac{(6+2\chi_b+\frac{\chi_b\chi_t}{2}+2\chi_t)}{(6+\chi_t)}\left(\frac{y}{h_0}\right)^4\right\}\sum_{m=1}^{\infty}C_m\sin\frac{m\pi z}{l}\end{aligned} \tag{35}$$

$\chi_b=\zeta_b/D$, $\chi_t=\zeta_t/D$ are dimensionless parameters.

2.2.2 Local buckling of web under non-uniform compression

The in-plane load is considered as a linear distribution along the loading side as shown in Fig.2, its expression is

$$N_x = N_b(1-\lambda y/h_0) \tag{36}$$

N_x is the pressure per unit length on loading side; N_b , N_t is the maximum and minimum pressure separately per unit length on loading side; $\lambda = \frac{(N_b - N_t)}{N_b}$ is the pressure change rate per unit length along plate width.

When the web yields buckling deformation, the bending elastic strain energy in plate is

$$\begin{aligned} U_\mathrm{e} &= \frac{Et_\mathrm{w}^3}{24(1-\mu^2)}\int_0^l\int_{-h_0}^0 \left[u_{\mathrm{w},yy}^2 + 2\mu u_{\mathrm{w},yy}u_{\mathrm{w},zz} + u_{\mathrm{w},zz}^2 + 2(1-\mu)u_{\mathrm{w},yz}^2\right]d_y d_z \\ &= \frac{D}{2}\int_0^l\int_{-h_0}^0 \left[u_{\mathrm{w},yy}^2 + 2\mu u_{\mathrm{w},yy}u_{\mathrm{w},zz} + u_{\mathrm{w},zz}^2 + 2(1-\mu)u_{\mathrm{w},yz}^2\right]d_y d_z \end{aligned} \tag{37}$$

Because of the constraint of rotation along non-loading sides, the strain energy which store in equivalent spring is

$$U_\Gamma = \frac{1}{2}\int\left[\zeta_b\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=0}\right)^2 + \zeta_t\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=h_0}\right)^2\right]d_\Gamma \tag{38}$$

The work of in-plane load V is

$$V = \frac{1}{2}N_b\int\int\left(1-\lambda\frac{y}{h_0}\right)\left(\frac{\partial\omega}{\partial z}\right)^2 d_y d_z \tag{39}$$

The total potential energy is the summation of strain energy and external work

$$\Pi = U_e + U_\Gamma - V \tag{40}$$

According to principle of minimum potential energy, substituting Eqs. (34)-(37) into Eq. (38), we obtain

$$\begin{aligned} \delta\Pi &= \delta U_e + \delta U_\Gamma - \delta V \\ &= D\left[\frac{\pi^4 h_0 A_1}{5040 l^3\left(6+\chi_t\right)^2}\sum C_m m^4\delta C_m + \frac{lA_2}{10\left(6+\chi_t\right)^2 h_0^3}\sum C_m\delta C_m \right.\\ &\left. + \frac{2\pi^2 A_3}{420\left(6+\chi_t\right)^2 h_0 L}\sum C_m m^2\delta C_m\right] + \frac{Dl}{2h_0^3}\left[\frac{A_4+A_5}{\left(6+\chi_t\right)^2}\right]\sum C_m\delta C_m \\ &+ \frac{\pi^2 h_0 N_b A_6}{10080 h_0\left(6+\chi_t\right)^2}\sum C_m m^2\delta C_m = 0 \end{aligned} \tag{41}$$

The parameters in Eq. (41) are as follows:

$$A_1 = \beta_1 + \beta_2\chi_b + \beta_3\chi_b^2 \tag{42a}$$

$$A_2 = \beta_4 + \beta_5\chi_b + \beta_6\chi_b^2 \tag{42b}$$

$$A_3 = \beta_7 + \beta_8\chi_b + \beta_9\chi_b^2 \tag{42c}$$

$$A_4 = (6+\chi_b)^2\chi_t \tag{42d}$$

$$A_5 = (6+\chi_t)^2\chi_b \tag{42e}$$

$$A_6 = \beta_{10} + \beta_{11}\chi_b + \beta_9\chi_b^2 \tag{42f}$$

$$\beta_1 = 4(1116 + 285\chi_t + 19\chi_t^2) \tag{43a}$$

$$\beta_2 = 1140 + 272\chi_t + 17\chi_t^2 \tag{43b}$$

$$\beta_3 = 76 + 17\chi_t + \chi_t^2 \tag{43c}$$

$$\beta_4 = 36(24 + 6\chi_t + \chi_t^2) \tag{43d}$$

$$\beta_5 = 4(54 + 9\chi_t + 2\chi_t^2) \tag{43e}$$

$$\beta_6 = 36 + 8\chi_t + \chi_t^2 \tag{43f}$$

$$\beta_7 = 72(51 + 13\chi_t + \chi_t^2) \tag{43g}$$

$$\beta_8 = 3(312 + 70\chi_t + 5\chi_t^2) \tag{43h}$$

$$\beta_9 = 72 + 15\chi_t + \chi_t^2 \tag{43i}$$

$$\beta_{10} = 4464(2-\lambda) + 24(95-44\lambda)\chi_t + 2(76-33\lambda)\chi_t^2 \tag{43j}$$

$$\beta_{11} = 24(95-51\lambda) + 272(2-\lambda)\chi_t + 2(17-8\lambda)\chi_t^2 \tag{43k}$$

$$\beta_{12} = (15-86\lambda) + (34-18\lambda)\chi_t + 2(2-\lambda)\chi_t^2 \tag{43l}$$

Because of the arbitrariness of δC_i, linear system of equations about displacement function coefficients is obtained

$$\left[k_{ij}\right]\left\{C_j\right\} = \left\{0\right\} \tag{44}$$

In order to get nontrivial solution of above equations, determinant of coefficients matrix must equal to zero

$$\left[k_{ij}\right] = 0 \tag{45}$$

Therefore, critical load of plate buckling is calculated and the computing formula of plate yield strength is

$$N_b = \frac{10080\pi^2 D}{A_6 h_0^2}\left[\frac{h_0^2 m^2 A_1}{5040 l^2} + \frac{l^2 A_2}{10 h_0^2 m^2 \pi^4} + \frac{2A_3}{420\pi^2} + \frac{l^2 A_4}{2h_0^2 m^2 \pi^4} + \frac{l^2 A_5}{2h_0^2 m^2 \pi^4}\right] \tag{46}$$

Assuming the length-width ratio is

$$\rho = \frac{l}{h_0} \tag{47}$$

Then the coefficient k is obtained from

$$k = \frac{10080\pi^2 D}{A_6 h_0^2}\left[\frac{m^2 A_1}{5040\rho^2} + \frac{\rho^2 A_2}{10m^2\pi^4} + \frac{2A_3}{420\pi^2} + \frac{\rho^2 A_4}{2m^2\pi^4} + \frac{\rho^2 A_5}{2m^2\pi^4}\right] \tag{48}$$

In order to get the minimum value of the coefficient, the following equation must be true

$$\frac{\partial k}{\partial \rho} = 0 \tag{49}$$

Simplifying Eq. (49), the critical yield strength coefficient is obtained

$$k_t = \frac{24}{A_6\pi^2}\left\{2A_3 + \sqrt{(56 + \frac{7}{32})A_1 A_7}\right\} \tag{50a}$$

$$A_7 = A_2 + 5\left(A_4 + A_5\right) \tag{50b}$$

In accordance with formula of critical stress as follows

$$\sigma_{\mathrm{cr}} = k_{\mathrm{t}}\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_{\mathrm{w}}}{h_0}\right)^2 \geq f_y \tag{51}$$

Finally, limit value of height-thickness ratio in the negative moment zone web under non-uniform compression is obtained

$$\frac{h_0}{t_{\mathrm{w}}} \leq \sqrt{\frac{k_{\mathrm{t}}\pi^2 E}{12\left(1-\nu^2\right)f_y}} \tag{52}$$

2.2.3 Local buckling of web pure shearing

Local buckling of steel beam web subjected to pure shearing is an important classical problem. The force diagram of web is shown in Fig.3. The web is long and narrow and length l is much longer than width h_0. The shear force of web middle plane is N_{xy}.

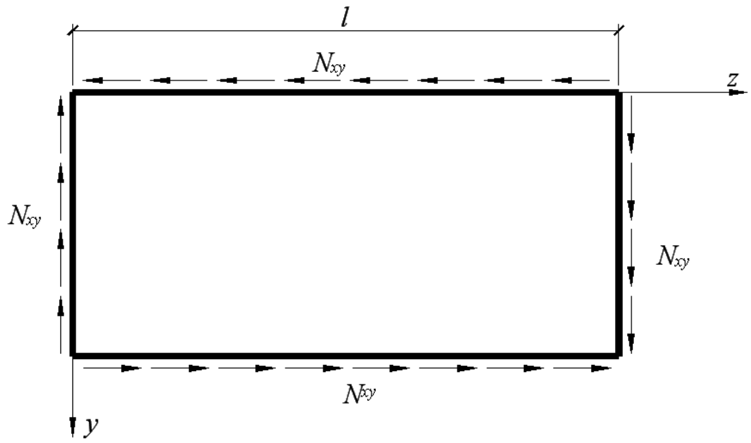


Fig.3 Pure shearing buckling model of steel beam web

The work of web middle plane shear force *Nxy* is

$$
\begin{aligned}
V &= \frac{1}{2}\iint 2N_{yz}\frac{\partial\omega}{\partial y}\frac{\partial\omega}{\partial z}d_y d_z \\
&= \frac{1}{2}\iint 2\tau_{yz}\frac{\partial\omega}{\partial y}\frac{\partial\omega}{\partial z}t_w d_y d_z
\end{aligned}
\tag{53}
$$

The total potential energy is

$$
\begin{aligned}
\Pi &= U_e + U_\Gamma - V \\
&== \frac{D}{2}\int_0^l\int_0^{h_0}\left[u_{\mathrm{w},yy}^2 + 2\mu u_{\mathrm{w},yy}u_{\mathrm{w},zz} + u_{\mathrm{w},zz}^2 + 2(1-\mu)u_{\mathrm{w},yz}^2\right]d_y d_z \\
&+ \frac{1}{2}\int\left[\zeta_b\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=0}\right)^2 + \zeta_t\left(\left.\frac{\partial\omega}{\partial y}\right|_{y=h_0}\right)^2\right]d_\Gamma \\
&- \frac{A_8}{60\left(6+\chi_t\right)^2}\tau_{yz}t_w\sum_{m=1}\sum_{n=1}C_mC_n\frac{mn}{m^2-n^2}
\end{aligned}
\tag{54}
$$

Wherein

$$
A_8 = \left(h_0-1\right)\left(8\chi_t + \chi_b\chi_t + 72 + \chi_b\right)\left(6+\chi_t\right) \tag{55}
$$

According to principle of minimum potential energy $\delta\Pi = 0$, critical shear stress is calculated

$$
\tau_{cr} = \frac{90\sqrt{P_1P_2}}{A_8}\frac{D\pi^2}{h_0^2t_w} \tag{56}
$$

Then the yield coefficient of web under pure shearing is

$$k_s = \frac{90\sqrt{P_1 P_2}}{A_8} \tag{57}$$

Shear stress is

$$\tau_{\mathrm{cr}} = \frac{k_2 \pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_{\mathrm{w}}}{h_0}\right)^2 \tag{58}$$

Finally, the height-thickness ratio of web under pure shearing is

$$\frac{h_0}{t_{\mathrm{w}}} \le \sqrt{\frac{k_{\mathrm{s}} \pi^2 E}{12\left(1-\nu^2\right) f_y}} \tag{59}$$

2.2.4 Local buckling of web under combination of non-uniform compression and pure shearing

For a simply supported composite beam, usually the middle span section bears large moment and small shear force, and support section bears large shear force and small moment. But when using a multi-span continuous beam, the section near middle support is subjected to both large moment and large shear force. Since it is a complicated strain condition, considering the influences of both moment and shear force on web buckling is necessary. Fig.4 shows the strain condition of the web before buckling: eccentric compression stress is linear along direction of web height; shearing strength is uniformly distributed.

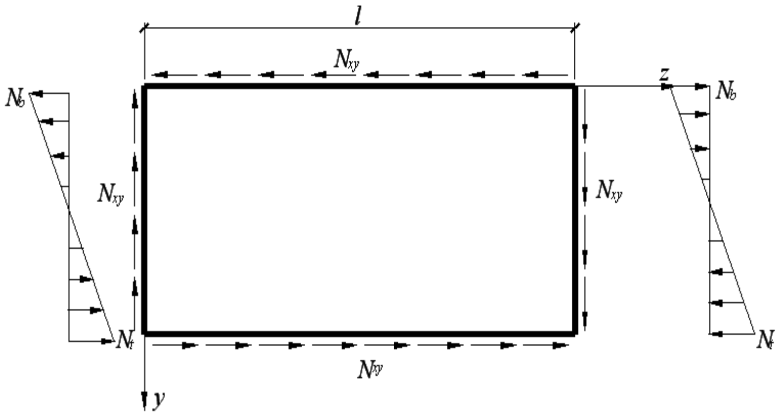


Fig.4 Buckling model of steel beam web under combined stresses

When axial compressive force and moment as well as shear force are presented, approximate calculation formula Eq. (58) for elastic buckling is presented by CHWALAE(Chen 1996).

$$\left(\frac{\tau}{\tau_{\mathrm{cr}}}\right)^2+\left(1-\frac{\lambda}{2}\right)\frac{\sigma}{\sigma_{\mathrm{cr}}}+\frac{\lambda}{2}\left(\frac{\sigma}{\sigma_{\mathrm{cr}}}\right)^2\leq 1 \tag{60}$$

The term σ is the maximum compressive stress on the web edge; λ is the stress gradient of web; σ_{cr}、 τ_{cr} are the buckling stresses separately when the web is under non-uniform compression and pure shear force.

When the web reaches critical state under the action of combined stresses, the elastic buckling stress σ_{cs} can be expressed as maximum compressive stress σ of web edge from Eq. (58). Then the general expression of buckling stress is

$$\sigma_{\mathrm{cs}}=\frac{k\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_{\mathrm{w}}}{h_0}\right)^2 \tag{61}$$

Wherein, k is the buckling coefficient of web under combined forces

$$k=\frac{k_t k_s\left[\sqrt{\left(1+\frac{\lambda}{2}\right)^2 k_s^2+4k_t^2\gamma^2}-\left(1-\frac{\lambda}{2}\right)k_s\right]}{2k_t^2\gamma^2+\lambda k_s^2} \tag{62}$$

Form the above equations, the limit value of web height-to-thickness ratio under combined forces is obtained

$$\frac{h_0}{t_{\mathrm{w}}}\leq\sqrt{\frac{k\pi^2 E}{12\left(1-\nu^2\right)f_y}} \tag{63}$$

3. Numerical examples and discussions

3.1 Example1

Consider a multi-span continuous composite box-beam. The sectional dimensions are shown in Fig.5. Assuming the negative moment span of composite box-beam suffered pure bending, the global stability is calculated and the residual stress is ignored. The grade of concrete is C40.The span of negative moment is l=4000mm.

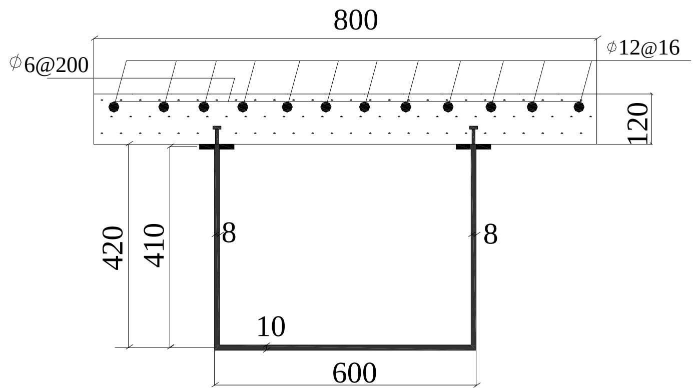


Fig.5 Section dimension of composite box-beam

a. Sectional geometric properties are obtained according to Papp et. al (2001) : h_0=410mm，t_{bf}=10mm，b_{bf}=600mm，t_w=8mm，A_b=6000mm^2，A_t=2400mm^2，A_w=6400mm^2，A_s=2412.7mm^2，y_s=507.3mm，y_c= 204.498mm，I= 658894284.8mm^4，I_{yb}= 180000000mm^4;

b.Materialproperties:E=2.06×10^{5}N/mm^2,μ=0.3;G=0.79×10^{5}N/mm^2,k=7.00408868×10^{5}N•mm

c. Parameters：η_1= 1.000079335，η_2= 1.000081035，β_1= 0.002268535，β_2= 0.007915931，β_3= 6.789101245，β_4= 4.788494998. β_5= 0.000964487，β_6= -0.003000559，β_7= -0.861563164，β_8= 0.002269559，β_9= 0.004944781，β_{10}= 6.787325523

(1) Numerical method proposed above

According to the condition of minimum M_x, when composite box-beam is buckling, using trial method the number of deformation half sine waves is 9.

Coefficients:R_1=1853024775,R_2=15978382.91,R_3=10733035577,R_4=950299856.9,R_5=14240020.82,R_6=-441387636.3,R_7=25109409187.Therefore, the critical moment of composite box-beam distorsional bending and rotation buckling is M_{cr}=1.009916537×10^{9}N•mm.

(2) British steel structure institute method (Lawson and Rackham 1989)

$$M_{cr} = \frac{\pi E I_z D}{2L^2} + \frac{GJ}{D} + \frac{E t_w^3 L^2}{4\pi^2 D^2} \tag{64}$$

Wherein, D is the height of steel beam section; EI_z is the bending rigidity of pressurized flange about z axis.Therefore, the critical moment of composite box-beam distorsional bending and rotation buckling is M_{cr}= 1.808878983×10^{9}N•mm.

(3) Equivalent I-steel bottom flange distorsional bending buckling method (Li 2006)

Assuming the left and right web of box-beam concentrate into I-steel beam in the middle, the critical moment of distorsional buckling of equivalent I-steel beam is M_{cr}= 7.3842×10^{8}N•mm.

(4) Finite element method using the ANSYS program.

In this computational model, the concrete slab is represented by solid65 element. The steel box beam is made of shell43 finite elements. Conducting eigenvalue buckling analysis in the ANSYS program, the critical moment of composite box-beam distortional buckling is obtained as M_{cr}= 1.0000×10^9N•mm. And the buckling mode is the same as the assumed model in Fig.1.

Results obtained by above methods are summarized in Table 1

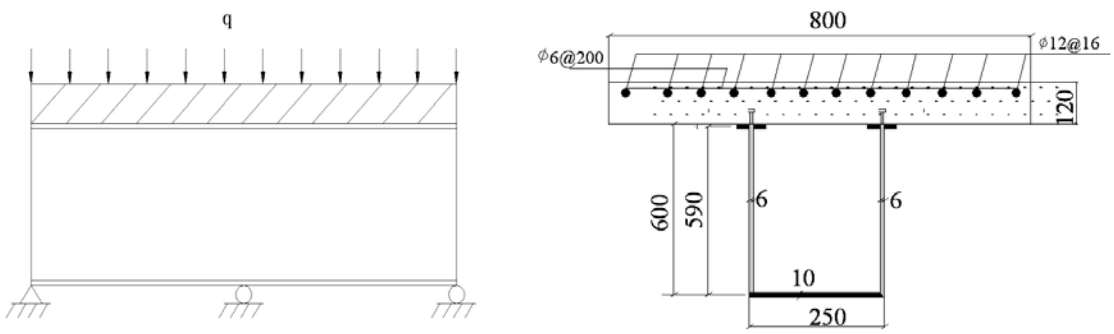
Table 1 Comparison of calculation results (Unit: N·mm)

Calculation method	Method proposed in this paper	British Steel Structure Institute method	Equivalent I-steel beam method	ANSYS program
Critical moment	1.0099×10^9	1.8089×10^9	7.3842×10^8	1.0000×10^9

The method of the British Steel Structure Institute assumes the concrete slab as perfectly rigid. On the basis of the energy equations, the critical buckling moment M_{cr} is calculated considering Saint-Venant rotation of the whole cross-section. However, the method in this paper uses lateral deformation and rotation of the bottom flange as well as rotation of the top flange to define web deformation. It also assumes non-deformation of concrete slab. It can be seen form the results the British Steel Structure Institute design method result is greater than the proposed method. This means British Steel Structure Institute method will lead to the possibility that composite box-beam will yield advanced distortional buckling. The result computed by method in this paper is much greater than by literature (Li 2006) method. It can be concluded that the proposed model of composite box-beam has provides greater bearing capacity compared with the method of literature (Li 2006). Known from the Table1, the results using proposed method in this paper agree well with that using ANSYS program. Therefore, the result is reasonable and the proposed method is verified.

3.2 Example 2

In this example, a two equal span continuous composite box-beam is investigated to evaluate the local stability of a steel beam web in negative moment region. The length of span is l = 12m. Construction diagram and sectional dimension are shown in Fig.6.Concrete slab is C30 cast-in-place concrete and steel beam is Q235 steel. The connector is the stud whose diameter is 16mm.The composite box-beam is subjected to a uniform distributed load q＝15KN/m.



(a) (b)

Fig.6 Construction diagram a) and Section dimension b) of composite box-beam

From the shear force and moment distribution diagram of continuous composite box-beam, the least favorable section is near the middle support.

(1) Numerical method proposed in above section

For convenience, assuming $\chi_b = \chi, \chi_t = \chi$. In accordance with literature (Li 2006), the flange-to-web constraint coefficient χ under non-uniform compression is 1.51; the flange-to-web constraint coefficient χ under pure shearing action is 1.51. Using Eq. (60), the buckling coefficient is obtained as k=21.6524. According to Eq. (61), the limit value of web height-to-thickness ratio in negative bending region is

$$\frac{h_0}{t_{\mathrm{w}}} = 97 \le \sqrt{\frac{k\pi^2 E}{12\left(1-\nu^2\right) f_y}} = 130.96 \tag{65}$$

It is not necessary to provide a stiffening rib.

(2) Code for design of steel structures method (GB50017-2003 2003)

For the purpose of verifying the local stability of steel beam web, method in code for design of steel structure is adopted. Height-to-thickness ratio of web is

$$80 < \frac{h_0}{t_{\mathrm{w}}} = 97 < 170 \tag{66}$$

It is necessary to provide a stiffening rib.

(3) Method of literature (Fan *et al.* 2004)

In literature (Fan *et al.* 2004), elastic buckling coefficient of web when there is no stiffening rib is

$$k_e = \frac{4.812\lambda^2 - 6.233\lambda^2\gamma + 5.012}{\lambda^{0.133}} \tag{67}$$

Therefore, elastic height-to-thickness ratio is

$$\frac{h_0}{t_w} \le \sqrt{\frac{k_e \pi^2 E}{12(1-\upsilon^2)\sigma_1}} = 431.5\sqrt{\frac{k_e}{\sigma_1}} = 146 \tag{68}$$

Critical buckling coefficients k_t, k_s are unrelated to the number of buckling half-sine waves, but depend on constraint coefficients χ_b, χ_t. Compared with code for design of steel structures, the proposed method is more reasonable when calculating web height-to-thickness ratio. The proposed method is to the benefit of reducing steel quantity and more close to practical situation. Although the results of proposed method and literature (Fan *et al.* 2004) method are close to each other, there are still some differences between these two methods. In literature (Fan *et al.* 2004), calculation formula of elastic buckling coefficient k_e is very complicated and it demands the elastic buckling stress of web should be greater than maximum compressive stress. However, the proposed method requires elastic buckling stress of web should greater than yield strength.

Compared with literature method, it is simpler and safer. Ignoring the actual stress state of composite box-beam web and providing stiffening rib only according to construction requirements, could produce difficulties in design and construction execution and become uneconomical. It is considered to be more reasonable to determine the critical height-to-thickness ratio of web according to the actual stress state of the composite beam. The elastic critical height-to-thickness ratio of continuous composite box-beam web is mainly influenced by bending stress and secondly by shear stress.

4. Conclusions

Through the theoretical analysis of steel-concrete composite box-beam, the main conclusions that can be drawn from this investigation are:

(1) According to global buckling forms of steel-concrete composite box beams, distortional buckling model considering rotation of steel beam top flange is established. Critical moment calculation formula is deduced using energy method. Numerical example demonstrates that the proposed model has better reliability and is more economical compared with existing methods.

(2) The local buckling model of composite box-beam web under non-uniform compression、pure shear stress and combined stresses is proposed. Critical buckling stress formula under different stress conditions is presented. Elastic buckling factor k of steel beam web under complicated stress conditions is obtained. Height-to-thickness ratio of composite box-beam web in elastic stage which has no stiffening rib is also calculated. Numerical results show that calculation method of existing codes and specifications can relax the restriction of height-to-thickness ratio. The proposed model of local buckling provided significant reference value to reasonable optimized design of composite box-beam in negative bending region.

Acknowledgements

This research is supported by the Nature Science Fund of China (50778177), the Outstanding Youth Fund of Hunan Province of China (07JJ1009) & the New Century Elitist of Ministry of Education of China.

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