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On the characteristics and seismic study of Hat Knee Bracing system, in steel structures

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Abstract. In this study, a new structural bracing system named 'Hat Knee Bracing' (HKB) is presented. In this structural system, a special form of diagonal braces, which is connected to the knee elements instead of beam-column joints, is investigated. The diagonal elements provide lateral stiffness during moderate earthquakes. However the knee elements, which is a fuse-like component, is designed to have one plastic joint in the knee elements for dissipation of the energy caused by strong earthquake. First, a suitable shape for brace and knee elements is proposed through elastic studying of the system and several practical parameters are established. Afterward, by developing applicable and highly accurate models in Drain-2DX, the inelastic behavior of the system is carefully considered. In addition, with inelastic study of the new bracing system and comparison with the prevalent Knee Bracing Frame system (KBF model) in nonlinear static and dynamic analysis, the seismic behavior of the new bracing system is reasonably evaluated.

Keywords: disposal knee brace; moment link; chevron knee brace; hat knee brace; optimal shape; energy dissipation.

1. Introduction

The seismic design of steel structures should satisfy two main criteria. These structures should have adequate "strength" as well as "stiffness" to control story drift in order to prevent damages to the structural elements during moderate excitations. The concentrically braced frames usually possess high stiffness. However, their poor ductility is owed to the buckling of the compression braces (Popov 1980). On the contrary, the steel moment resisting frames are supposed to exhibit acceptable ductility and energy dissipation capacity through flexural yielding in beams while their stiffness is limited (Elnashai and Sarno 2009, MoslehiTabar and Zahrai 2006). Nevertheless, the 1994 Northridge and 1995 Kobe earthquakes revealed serious damages to the conventional steel moment resisting frames (Engelhard and Husain 1993, Miller 1998, Mahin 1998). Therefore, it is fair to say that a combination of these two systems can produce a balance between requirements, concerning stiffness and energy dissipation capacity; (Clement and Martin 2004, Kigginsa and Uang 2006, Choi and Kim 2006, Fahnestock *et al*. 2007).

In the mid-1970s, Popov and Roeder (1978) did start working on a new idea of Eccentrically Braced

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Frames system (*EBFs*). This model was able to fulfill both seismic design criteria, which are "stiffness" and strength", economically; and were extensively employed as a common lateral load resisting system in the early 1980s. However, despite high seismic energy dissipation, this (*EBF*) system did have substantial disadvantages, and did experience several major drawbacks. For example, the energy dissipation capacity was provided through the shear links, which were the integral parts of the beam elements. This was leading to serious damage to floor beam after a severe earthquake; and causing repairs, which were very expensive and difficult. In addition, since the floor beams were mostly of heavy steel profiles, the braces should have also been proportionally strong elements, in order to be able to activate the shear link.

In 1986, Aristizabal-Ochoa (1986) did propose the Knee Braced Frame (*KBF*) as an alternative system. In this system, the knee element was functioning as a 'ductile fuse' to prevent collapse of the structure under extreme seismic excitations, by dissipating energy through flexural yielding. A diagonal brace connected at one end to the knee element, was providing most of the elastic lateral stiffness. In this system, however, the brace was not designed for compression and thus was allowed to buckle. Consequently, the hysteretic response of this structure was very similar to that of *CBF* with pinching in the hysteretic loops, which was not a desirable feature for energy dissipation. Subsequently, the system has been re-examined and modified by Balendra and his group of researchers (Balendra *et al.* 1990, Balendra *et al.* 1994, Balendra *et al.* 2001). The revised system was called the Ductile Knee Braced frame (*DKB*). In this system, the non-buckling diagonal brace did provide most of the lateral stiffness. Moreover, the flexural and/or shear yielding of the knee element was providing the ductility under a severe earthquake (Kim and Seo 2003, Blakeborough *et al*. 2002, Gerasimidis 2006). By this means, the damage was concentrated in a secondary member, which was easily possible to be repaired, at minimum cost.

The objective of this investigation is to present a new knee bracing system, which is called Hat Knee Bracing (*HKB*). Through the consideration of three independent parameters that cover any possible shape of the *HKB* frames and using elastic and inelastic analysis, the optimal shape of the knee and diagonal elements is ascertained. Furthermore, by means of the proposed optimal shape, presented in this paper, the nonlinear response of the new system is carefully demonstrated. Besides, the performances of this new system with respect to the similar KBF system in a nonlinear analytical model are compared.

2. Description

As shown in Fig. 1, the *HKB* system consists of a triangular knee and two diagonal elements. Two knee elements are rigidly joined to each other, and are connected to the beam by a hinge connection.

The frame shown in Fig. 1, has a height signified by H and a width labeled as B. The height and width of the triangular knee are also described by h and b as indicated in the Fig. 1, as well. The elastic lateral stiffness of the frame, K, can further be expressed as a function of frame geometry and the member properties, as follows

$$K = f(I_c, A_c, I_b, A_b, I_k, A_k, A_{br}, b, h, H, B, E) \tag{1}$$

Where, E is the Young's modulus of elasticity, A_{br}, A_c, A_b and A_k are the cross-sectional area of brace, column, beam and knee, respectively. In addition, I_c, I_b and I_k are the second moment of area of column, beam and knee, respectively. Through the dimensional analysis, similar to the procedure presented by Blendra *et al*. (1991), after dropping the insignificant parameters, Eq. (1) can be re-written in a more compact form as follows

$$K/(EI_c/H^3) = f(I_k/I_c,(A_{br}/l_{br})/(I_c/H^3),b/B,h/H,B/H) \tag{2}$$

Where, l_{br} is the length of the brace. Based on Eq. (2), for the *KBF* system, the influence of each parameter in the right side of Eq. (2) on the stiffness of the frame can easily be demonstrated, when the other parameters are assumed constant. Several studies have been carried out by Balendra *et al*. (1991) to determine the effect of the right side parameters of Eq. (2) for the *KBF* system in the elastic region. By using two parameters from right side of Eq. (2) and establishing new parameter, which will be described in Eqs. (3) to (5), three parameters entirely cover any possible shape of the *HKB* system. Referring to Fig. 1, the aforementioned parameters are described as follows

$$\alpha = BE/BO \tag{3}$$

$$\beta = b/B \text{ or } BC/AD \tag{4}$$

$$\gamma = h/H \tag{5}$$

Sectional properties of the structural elements are required to study the *HKB* system. By defining an

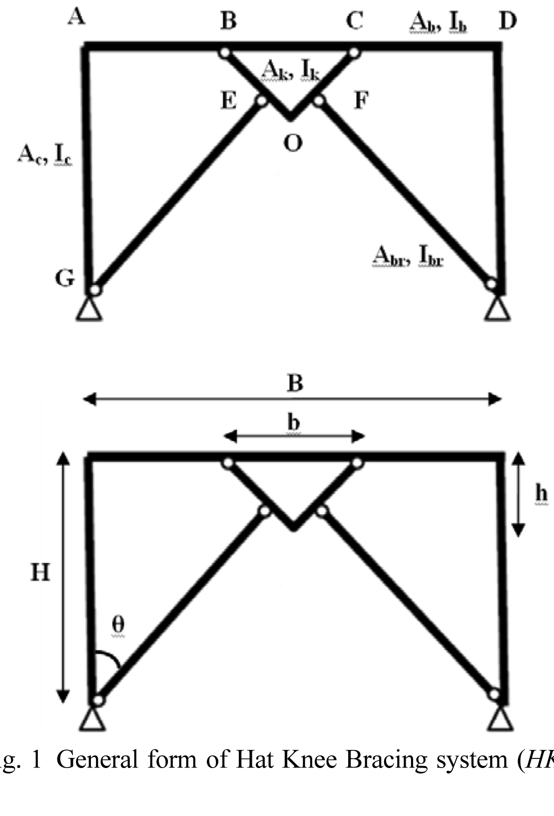

Fig. 1 General form of Hat Knee Bracing system (*HKB*)

Table 1 Definition of the example structure

Number of bays	3	Number of stories	1
B (cm)	6	α	0.5
H (cm)	3	DL (ton/m)	4
b (cm)	2	LL (ton/m)	1
h (cm)	1	Seismic coefficient	0.14

Table 2 Example structure's design results

A_c (cm^2) = 39.10 cm^2	I_c (cm^4) = 3892 cm^4
A_b (cm^2) = 84.50 cm^2	I_b (cm^4) = 23130 cm^4
A_k (cm^2) = 99.75 cm^2	I_k (cm^4) = 3427 cm^4
A_{br} (cm^2) = 43.44 cm^2	

example structure as illustrated in Table(1), and employing the new bracing system with reasonable dimensions, the elements of structure are designed based on *AISC-LRFD*(American Institute of Steel Construction) and *IBC*2010(International Building Code 2010) codes. The earthquake load applied for design of the frame is obtained with respect to the IBC code. The frame design is accomplished based on *AISC* standard. The knee elements are designed weaker compared to diagonal elements. The results are illustrated in the Table 2.

3. Elastic analysis

By employing sectional property of the example of structural elements, it is fairly attempted to determine the best shape of the *HKB* positioning according to the system stiffness. By change of α, β and γ parameters, the angle of brace members is changed and it results in alternation of the frames stiffeners. In order to omit these influences, a modification on frames lateral drift and stiffness is implemented, through the application of the following equations. In this study, several required modifications on the frames lateral stiffness are practically implemented, which lead to the drift modification as follows

$$K_m = K_i / Sin(\theta) \quad (6)$$

$$\Delta_m = \Delta_i * Sin(\theta) \quad (7)$$

In the illustrated structure, the diagonal elements are designed at the starting value of $\theta = 45^o$. However, when the three principal parameters change during this parametric study, the value of θ is also altered according to the Eqs. (3) to (5). Besides, the amount of K_m and Δ_m are further modified, when θ varies accordingly.

Two different types of framing with various B/H ratios are defined to evaluate the probable different behaviors of this bracing systems. Therefore, Frame A for $B/H < 1$ and Frame B for $B/H > 1$ are selected, respectively, as displayed in Fig. 2.

During this parametric study, the values of the Eqs. (3) to (5) are considered in the following range:

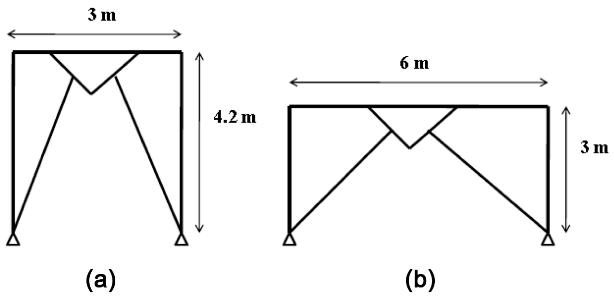

Fig. 2 General Form of tall and wide *HKB* frames

$0 < \alpha$ and $\beta < 1$, and $0 < \gamma < 0.50$. The elastic analysis results under unit lateral load(1 ton) at the beam level for the Frames types (A) and (B) with the various values of the parameters α, β and γ are demonstrated in Figs. 3 and 4. The extensive study of the graphs presented in these two figures reveals the following important points:

- The maximum stiffness is reached when approximately $0.8 < \alpha < 1$.
- When the amount of α is selected within the aforementioned range, the stiffness does not vary significantly for the various value of β.
- The behavior of the both frames is nearly the same for the smaller values of γ.

As illustrated in Fig. 3, for different values of γ, the maximum stiffness is obtained, by considering α

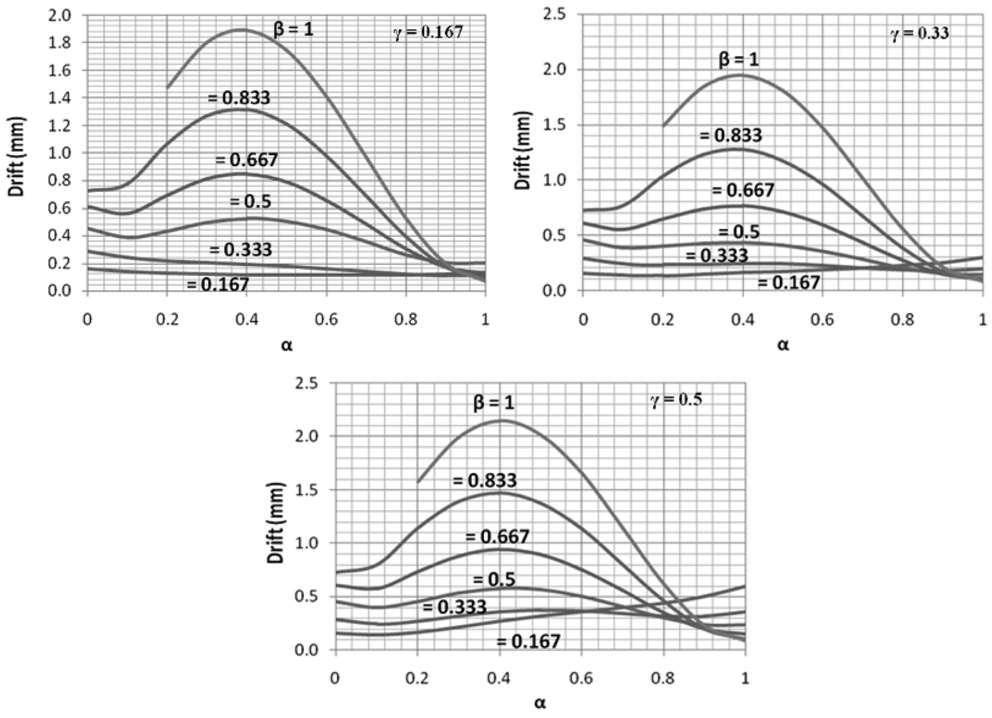

Fig. 3 Modified drifts of wide frames with various value of γ

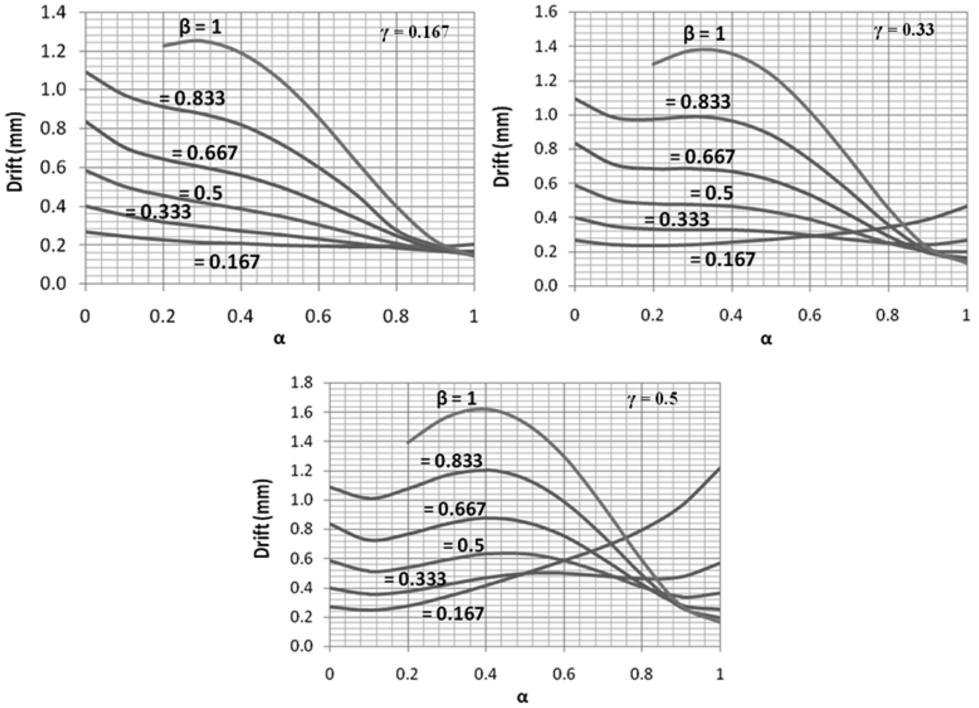

Fig. 4 Modified drift of tall frames with various value of γ

at the interval of [0.8-1]. In addition, the results in Fig. 3 reveal that when α parameter is chosen in this interval, frames with various value of β have approximately the same stiffness. The same graphs for the tall frames (type B) are also demonstrated in Fig. 4. As presented in this figure, the behavior of frames with smaller values of γ is as identical as the short (Type A frames). According to Figs. 3 and 4, the stiffness in frames with smaller values of β has more independency to α parameter with minimal variation. Finally, by using the interval of [0.8-1] for α and [0.1-0.3] for γ (optimum region), one can assume that stiffness is nearly irrelevant to β.

Furthermore, in this investigation, an extensive parametric study on finding the optimum region and/or characteristics of the *HKB* according to the different ratio of ρ = (Width)/(Height) is successfully performed.

Elastic analysis reveals that for all the frames defined in Table 3, the value of α which leads to the highest independency of stiffness compared to β is reasonably related to γ. This value of α changes from [0.9 to 0.75], when γ varied from [0.1 to 0.3]. Therefore, due to the aforementioned results, it is strongly suggested that the value of α and γ should be taken equal to 0.85 and 0.20, respectively.

Table 3 Dimensions of frames for more study on optimum region

ρ	H (m)	B (m)	ρ	H (m)	B (m)
0.6	270	450	1.4	490	350
0.8	320	400	1.6	480	300
1.0	400	400	1.8	540	300
1.2	420	350	2.0	600	300

4. Inelastic analysis

Consequently, nonlinear analysis of the new knee bracing system is performed, in order to investigate nonlinear behavior of the *HKB* system. One of the Balendra's frames examined through the numerical and experimental model is being considered as a "Base frame"; in order to have a criterion to evaluate the *HKB* system (Balendra *et al*. 1994). To perceive the influence of β parameter on nonlinear behavior of the *HKB* system, two frames with various values of β parameter (Frame I with $\beta < 0.5$ and Frame II with $\beta > 0.5$) are defined, as shown in Fig. 6. The general properties of the models are presented in Table 4; moreover, additional details of elements dimensions are illustrated in Fig 6. In addition, the material properties are shown in Table 5.The section of all elements in Frames I and II is as the same as section of elements in the Base frame, except knee element of Frame I which is defined weaker in order to achieve lateral stiffness similar to the Base frame. Lateral elastic stiffness of Frames I and II is considered equal to the Base frame as well. The axial deformation of the beam is further presumed to be insignificant. The buckling of the braces is prevented in all models.

Since the knee elements are designed to yield at moment mode, the moment-axial force interaction

Table 4 General properties of the *KBF* and *CKB* models

Type of the element	Moment yield condition
General properties of the frame	Section
Beam	WF 100*100*17.2 kg/m
Column	WF 125*125*23.8 kg/m
Brace	[100*50*9.36 kg/m]
Panel width (B)	320 cm
Panel height (H)	280 cm

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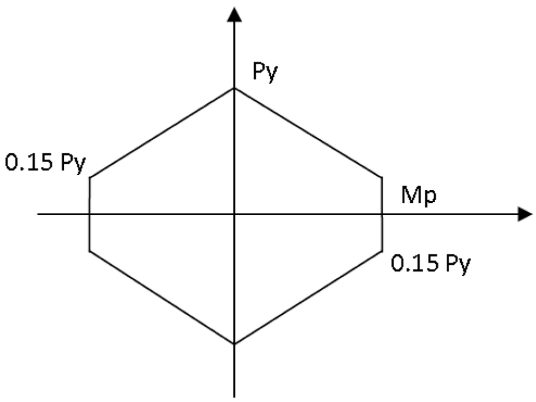

Fig. 5 Yield interaction surface for the beam-column elements

Table 5 Material properties of three models

E (kg/cm^2)	1.90 E6	*Strain hardening ratio*	0.01
σ_y (kg/cm^2)	3.50 E3	*Poisson ratio*	0.30

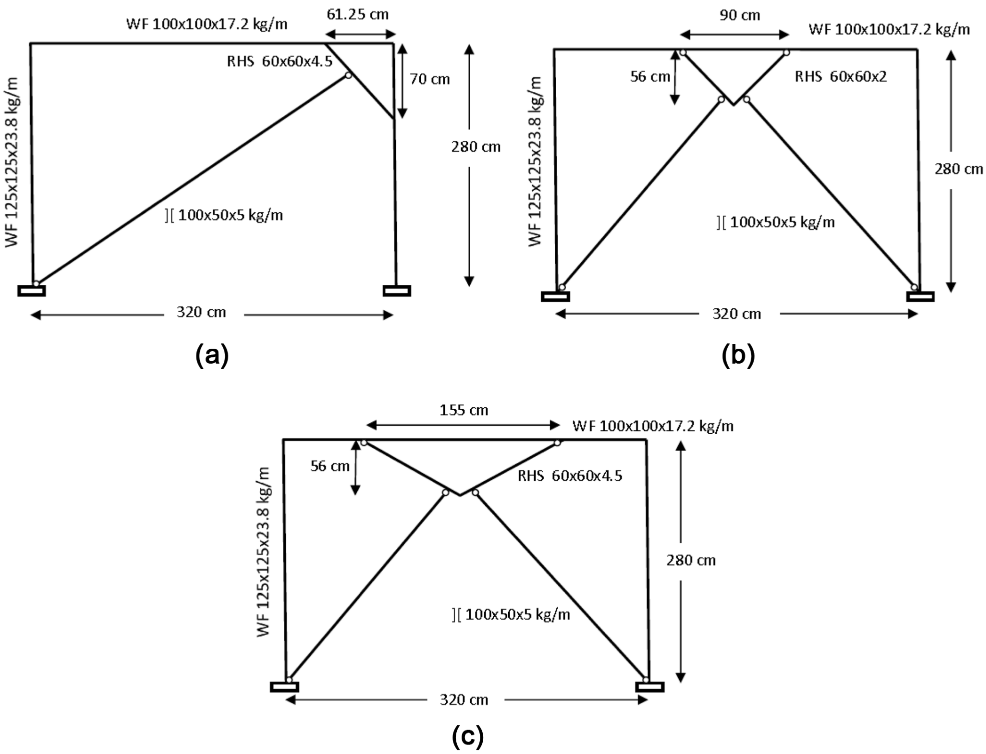

Fig. 6 Base frame, Frame I and II dimensions and section properties (a) Base frame (b) Frame I (c) Frame II

riteria depicted in Fig. 5, is utilized for these elements and other beam-column elements such as beams and columns (Balendra *et al*. 1994, Lotfollahi 2003).

4.1 Inelastic static analysis

Expressions for the yield displacement and maximum ductility of a single-story *HKB* frame is presented in Fig. 1 and can be expressed as a function of the variables of the frame as follows

$$\Delta_y = f\{I_c, A_c, I_b, A_b, I_k, A_k, A, b, h, H, B, \alpha, E, M_{ky}\} \tag{8}$$

$$\mu_m = g\{I_c, A_c, I_b, A_b, I_k, A_k, A, b, h, H, B, E, M_{ky}, \alpha, M_{by}, M_{cy}, P_{cr}\} \tag{9}$$

Where, M_{by}, M_{cy} and M_{ky} are the yield moments of the beam, column and knee, respectively; and P_{cr} is the buckling load of the brace. Other parameters are the same as Eq. (1) and (3). By Normalizing and dropping the parameters containing P_{cr}, as it is assumed no buckling occurs in the brace, Eqs. (8, 9) can be re-written as follows

$$\begin{aligned}\Delta_y/H = (f(I_k/I_c, I_b/I_c, (A_{br}/l_{br})/(I_c/H^3), b/B, h/H, B/H, \alpha, M_{ky}/(EI_c/H^3) \\ , M_{by}/(EI_c/H^3), M_{cy}/(EI_c/H^3))\end{aligned} \tag{10}$$

Table 6 Summary of yielding points of the frames in pushover analysis

Element type	Base frame	Frame I	Frame II
Knee	0.47 cm-5.08 ton 0.52 cm-5.08 ton 0.64 cm-5.39 ton	5.52 cm-10.98 ton	5.64 cm-11.06 ton
Beam	0.46 cm-5.68 ton	6.30 cm-11.31 ton	6.43 cm-11.40 ton
Column	0.68 cm-7.31 ton	6.37 cm-13.29 ton	6.43 cm-13.32 ton

Table 7 Knee yielding range

Element type	Base frame	Frame I	Frame II
Knee yielding range	5.05 cm-5.90 ton	5.84 cm-6.27 ton	5.69 cm-5.98 ton

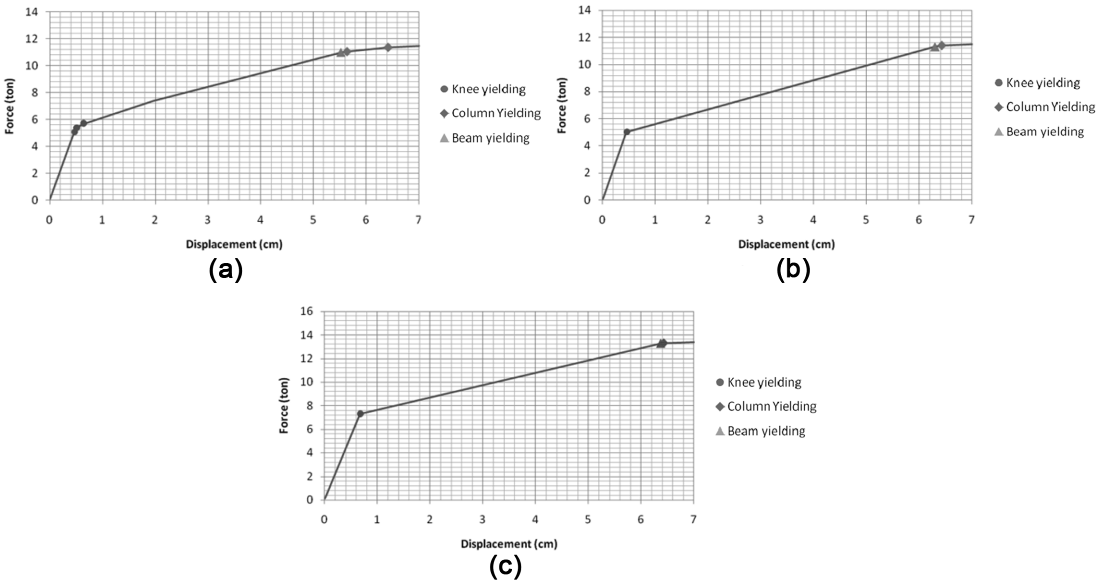

Fig. 7 Load–displacement graph for (a) Base frame, (b) Frame I (c) Frame II

$$\mu_m = g(I_k/I_c, I_b/I_c, (A_{br}/l_{br})/(I_c/H^3), b/B, h/H, B/H, \alpha, M_{ky}/(EI_c/H^3), M_{by}/(EI_c/H^3), M_{cy}/(EI_c/H^3)) \tag{11}$$

By considering suitable displacement incremental sizes (0.1 mm), which create convergence during analysis (Fillippou *et al*. 1992), pushover test is performed on all the three models. It can be observed that the imposed displacement is increased to 7 cm, which is equal to 2.5 percent of the frames height. Load-displacement graphs for the Base frame, Frames I and II are illustrated in Fig. 7. Furthermore, summary of the static nonlinear analysis for all the models is depicted in Table 6. In fact, the result can be compared, as shown in Fig .8. As it can be observed in Table 6 and Fig. 8, the elastic capacity of the Frame II is higher than the Base frame and Frame I. In addition, the elastic and inelastic stiffness of all frames are very similar to one another. In the Frames I and II, the displacement and the corresponding

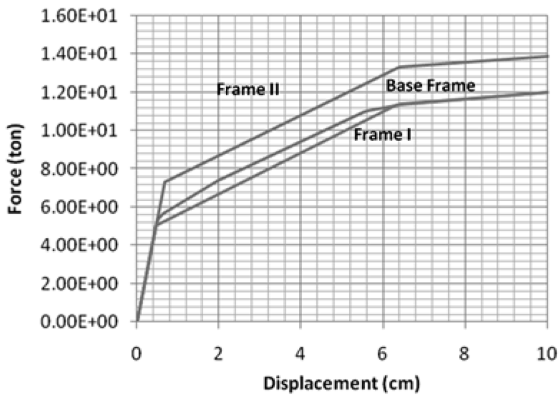

Fig. 8 Compression between model's push over analysis results

load of beam yielding point on pushover graph has more distance to knee yielding point, in comparison with the Base frame. Knee yielding ranges of all frames are compared in Table 7.

4.2 Inelastic dynamic analysis

The presented knee element model in this study can further be examined through the dynamic analysis. This dynamic analysis is performed with different values of β parameter under cyclic loading and in the attendance of the properties provided in Tables 4, 5 and Fig. 6. In this study, viscous damping of 2% of the critical damping is prescribed for all models. A sinusoidal base excitation is considered for the inelastic test. The excitation frequency is 20 rad/sec with initial amplitude of 2.25 m/sec^2. This amplitude is increased every five cycles by 0.79 m/s increment (Lotfollahi and Mofid 2006). The analysis is performed under the displacement control method with suitable time incremental size of 0.01 second and is completed in ten seconds (Balendra *et al*. 1994). The summary of the results analysis for the Base frame and the Frames I and II are shown in Tables 8. It is noticeable that in all frames, the beam and columns remain elastic within the 10 seconds duration.

It is reasonable to claim that energy dissipation of a system can convincingly be one of the greatest criteria for the structural seismic performance. In this study, the energy dissipation of frames is directly computed through multiplying plastic rotation at the corresponding moment. Formulation of this method is expressed in Eq. (12)

$$E_H = \Sigma_0^T((\Delta\theta_p)_t \cdot (M_p)_t) \tag{12}$$

Where, E_H is the inelastic strain energy or hysteretic energy; $(\Delta\theta_p)_t$ is plastic rotation of yielding point of the element in the time interval at time t; and $(M_p)_t$ is moment of element in yielding point at time t.

Table 8 Summary of the nonlinear dynamic analysis results

Parameter	Base frame	Frame I	Frame II
Δ_y	0.47 cm	0.46 cm	0.68 cm
Δ_{max}	4.05 cm	4.08 cm	4.16 cm
μ	8.61 cm	8.86	6.11

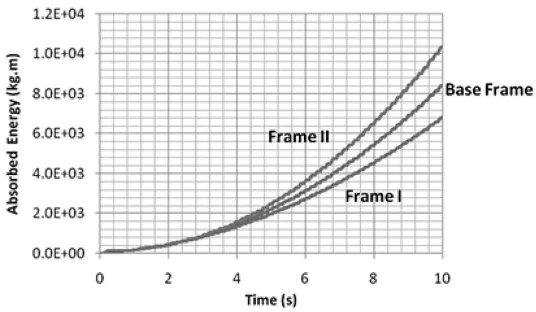

Fig. 9 Cumulative input energy and hysteretic energy dissipation

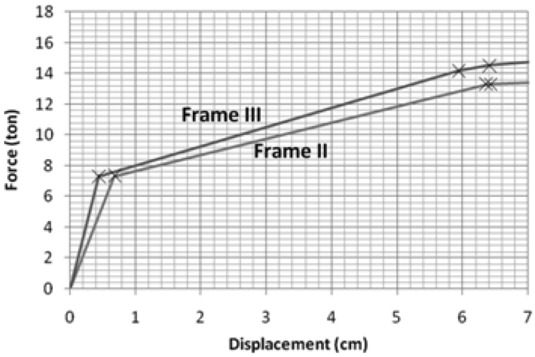

Fig. 10 Comparison between pushover analysis result of Frames II and III

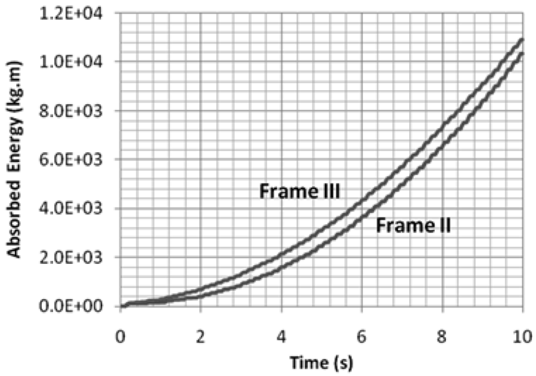

Fig. 11 Comparison between energy dissipation of Frames II and III

Hysteretic absorbed energy for all models is computed according to Eq. (12), and the summary of results for the Base frame and the Frames I and II are displayed in Fig. 9. It can considerably be observed that a higher energy is absorbed through Frame II, in comparison with the Base frame and Frame I.

A significant difference between energy dissipation of Frames I and II is not expected. However, this unrepentant outcome can be occurred as result of using a weaker knee element in Frame I. To substantiate the aforementioned point, a new frame similar to Frame I is defined. The new frame, with

Table 9 Summary of yield points in pushover analysis of the Frames III

Frame type	Knee Yielding	Beam Yielding	Column Yielding
Frame III	0.44 cm-7.31 ton	5.95 cm-14.18 ton	6.42 cm-14.54 ton

Table 10 Maximum plastic rotation of the hinges

Frame type	Maximum plastic rotation
Base frame	0.12 rad
Frame I	0.22 rad
Frame II	0.16 rad
Frame III	0.21 rad

replacement of knee element section with knee section of the Base frame, is re-named as Frame III. Subsequently, by using nonlinear static and dynamic analysis of the new Frame III, a comparison between Frames II and III is furnished. The pushover and energy dissipation graphs of the Frame III are depicted in Figs. 10 and 11, respectively.

As illustrated in Fig. 10, due to the increment in knee section properties, elastic and inelastic stiffness of the Frame III is increased. In addition, yielding range of Frame III is nearly 20 percent more than Frame II and yielding of column in both frames occurred at the same drift. According to Fig. 11, the energy dissipation in Frame III is more than Frame II.

The maximum plastic rotation for each frame that occurs in knee element is shown in Table 10. Moreover, it is noteworthy that the maximum plastic rotation in Base frame is less than other frames. These values for Frames I, II and III are about 85, 35 and 75 percent more than Base frame, respectively.

5. Conclusions

1. The investigation of the results reveals the optimum shape can successfully be achieved when:
   * The amount of γ varies between 0.1 and 0.3;
   * The amount of α varies between 0.8 and 1.0;
   * In addition, in order to have higher stiffness in frames (with smaller values of β) and higher energy absorption in frames (with larger value of β), it is highly recommend that value of β parameter is selected about 0.5;
2. The proposed *HKB* model that is presented in this study is an applicable system with a larger stiffness and a higher rate in dissipating of the input energy.

References

American Institute of Steel Construction (2010), “Seismic provisions for structural steel buildings”, Chicago (IL, USA), AISC.

American Institute of Steel Construction (2010), “Specification for structural steel buildings”, Chicago (IL, USA), AISC.

Aristizabal-ochoa, J. D. (1986), "Disposable knee bracing: improvement in seismic design of steel frames", *J. Struct. Eng.*, ASCE, **112**(7), 1544-1552.

Balendra, T., Lee S. L. and Lim E. L. (1994), "Ductile knee braced frames with shear yielding knee for seismic resistant structures", *Eng. Struct.*, **16**(4), 263-269.

Balendra T., Liaw, C. Y. and Sam, MT. (1990), "Diagonal brace with ductile anchor for a seismic steel frame", *Earthq. Eng. Struct. D.*, **19**(6), 847-858.

Balendra, T., Liaw, C. Y., Sam, M. T. and Seng, L. L. (1991), "Preliminary studies into the behavior of knee braced frames subject to seismic loading", *Eng. Struct.*, **13**, 67-74.

Balendra, T., Xiao, Y. and Yu, C.Y. (2001), "An economical structural system for wind and earthquake loads", *Eng. Struct.*, **23**, 491-501.

Blakeborough, A., Bourahla, N., Clement, D. and William, M. S. (2002), "Seismic behavior of knee braced frames", *Proceedings of the Institution of Civil Engineers, Structures and Buildings*, **152**(2), 147-155.

Choi, H. and Kim, J. (2006), "Energy-based seismic design of buckling-restrained braced frames using hysteretic energy spectrum", *Eng. Struct.*, **28**(2), 304-311.

Clement, D. E. and Martin S. Williams (2004), "Seismic design and analysis of a knee braced frame building", *J. Earthq. Eng.*, **8**(4), 523-543.

Elnashai, A. S. and Sarno, L. D. (2009), "Bracing systems for seismic retrofitting of steel frames", *J. Constr. Steel. Res.*, **65**(2), 452-465.

Engelhard, M. D. and Husain, A. S. (1993), "Cyclic-loading performance of welded flange bolted web connections", *J. Struct. Eng.*, AISC, **119**(12), 3537-3550.

Fahnestock, L., Ricles, J.M. and Sause, R. (2007), "Experimental Evaluation of a Large-Scale Buckling-Restrained Braced Frame", *J. Struct. Eng.*, AISC, **133**(9), 1205-1214.

Filippou, F. C., Powel, G. H. and Prakash, V. (1992), "DRAIN-2DX: base program user guide", *Structural engineering mechanics and materials*, Report No. UCB/SEMM-92/29, Berkeley, University of California.

Gerasimidis, S. (2006), "Application of knee-bracing system on high-rise buildings", Thesis for Master's Degree, *Massachusetts Institute of Technology*, Department of Civil and Environmental Engineering.

Kim, J. and Seo, Y. (2003), "Seismic design of steel structures with buckling-restrained knee braces", *J. Constr. Steel. Res.*, **59**, 1477-1497.

Kigginsa, S. and Uang, C. M. (2006), "Reducing residual drift of buckling-restrained braced frames as a dual system", *Eng. Struct.*, **28**(11), 1525-1532.

Lotfollahi, M. (2003), "Thesis for Master's Degree", Department of Civil Engineering, Sharif University of Technology, Tehran, Iran.

Lotfollahi, M. and Mofid, M. (2006), "On the characteristics of new ductile knee bracing systems", *J. Constr. Steel. Res.*, **62**, 71-81.

Mahin, S. A. (1998), "Lessons from damage to steel buildings during the Northridge earthquake", *Eng. Struct.*, **20**(4-6), 261-270.

Miller, D. K. (1998), "Lessons learned from the Northridge earthquake", *Engineering Structures,* **20**(4-6), 249-260.

MoslehiTabar, A and Zahrai, S. M. (2006), "Cyclic behavior of steel braced frames having shear panel system", *Asian journal of civil engineering (building and housing*, **7**(1), 13-26.

Popov, E. P. (1980), "Seismic behavior of structural sub-assemblage", *Journal of Structural Division*, AISC, Vol. **106**(7), 1451-1474.

Popov E. P. and Roeder, C. W. (1978), "Eccentrically braced steel frames for earthquakes", *J. Struct. Division*, ASCE, **104**(3), 391-412.

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