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Seismic assessment of steel structures through a cumulative damage

R. Perera†, S. Gómez‡ and E. Alarcón‡†

Department of Structural Mechanics, Technical University of Madrid (UPM), Spain

Abstract. In the present work a constitutive model is developed which permits the assessment of the structural performance through a criterion based on cumulative damage. For it, a damage index is defined and is evaluated through the application of the Miner's rule in low-cycle fatigue. However, the damage index is not considered as a posteriori variable since is incorporated explicitly as an internal variable in the constitutive equations which produces a direct coupling between the damage and the structural mechanical behaviour allowing the possibility of considering as a whole different coupled phenomena. For the elaboration of this damage model, the concepts of the mechanics of continuum medium are applied on lumped dissipative models in order to obtain a coupled simplified model. As a result an elastoplastic model coupled with damage and fatigue damage is obtained.

Key words: continuum damage mechanics; low cycle fatigue; steel structures; seismic design; simplified model

1. Introduction

The modern approach to the seismic design of structures accounts for dissipation of the seismic energy input through plastic deformations. A ductile response is characterized by the structure's ability to undergo large inelastic displacements without loss in the load carrying capacity. The evaluation of the structural performance requires the definition of parameters to characterize the structural damage. Traditionally, ductility has been employed as the principal criterion for design. However, ductility does not account for the duration of ground shaking which is very important in inelastic design since the combined effects of ductility and energy absorption may lead to failure even at modest ductility demands. To suitably account for the seismic performance of structures in the design procedure, it is necessary to assess accurately the damage accumulation which progressively reduces the mechanical properties of structural components subjected to plastic strains.

Then, as an alternative to ductility based design, energy may be used as the basis for design and therefore it is convenient to use cumulative damage models to predict the probability of failure in cyclically loaded materials or structural elements. The simplest way to evaluate the cumulative damage using an energy approach consists on summing the inelastic deformations. However, this approach does not take into account the fact that the damage due to a large number of small plastic deformations may be less than one due to a smaller number of large plastic deformations. To overcome this problem,

†Assistant Professor
‡Graduate Research Assistant
‡†Professor

another way of thinking about energy is to use the concept of low cycle fatigue. Since the deformation histories are composed of random cycles, the structural damage is governed both by the maximum plastic displacement and by the dissipated energy. Then the low cycle fatigue approach appears to be a very interesting approach. In fact, during an earthquake, steel members and their connections, which influence strongly the behaviour of a steel structure, may be subjected to local buckling and low-cycle fatigue producing a ductility reduction and a possible premature failure. Therefore, an accurate assessment of the cumulative damage in structural members using damage indexes appears to be a key point in the design procedure of steel structures subjected to strain cycles in the plastic range during earthquakes.

In the last years the fatigue study has been reoriented through its incorporation in the Continuum Damage Mechanics (CDM) (Lemaitre and Chaboche 1985, Lemaitre 1996). The same concepts used in CDM to model ductile failure can be extended to low cycle fatigue damage processes, where plasticity is the key mechanism for crack initiation. Damage Mechanics deals with damage as a continuum variable and, because of it, CDM models including plasticity and damage can predict ductile crack initiation. An extension of themselves including the number of cycles could be suitable to simulate the low cycle fatigue damage. According to it, Chaboche (1985) developed a formulation for damaged materials where the fatigue phenomenon was incorporated in the CDM. However, only harmonic loads were considered being the hypotheses of fatigue cumulative damage suitable.

It is the purpose of this paper to develop a simplified cumulative damage model which permits the assessment of component reliability under arbitrary cyclic loading of the type experienced in severe earthquakes. To fulfill this objective a model is formulated according to the concepts and theories of the CDM and taking into account a similar approach to that developed in the lumped plasticity models. The combination of these two approaches, CDM and lumped models, produces a local damage model where damage indexes are defined at the ends of each structural element and are associated to the stiffness deterioration of these sections. Damage evolution is quantified using an extrapolation of the Palmgren-Miner relationship which is employed to characterize the fatigue damage accumulation. As a consequence a cumulative damage model results in which the coupling between damage and mechanical behaviour is explicitly considered but in a simplified way. Then, by comparison with other assessment procedures this approach presents the advantage that the potential damage in a structure is not evaluated with a postprocessor which seems a paradox since damage assessment would be performed with a structural model that assumes no structural damage. Besides, the model, such as it is presented, is applicable not only for harmonic loading but also for arbitrary loads as those appearing in earthquakes.

The basic aspects of the model are discussed in detail in the next section for damage indexes based only in ductility design. The expansion of the model for the assessment of cumulative damage of steel structures is shown in the third section where the application for seismic loading is included also. Numerical results are shown in the forth section and, finally, proper conclusions are reflected in the last part of the paper.

2. Elastoplastic damage model

2.1. Constitutive equations

As above mentioned, in the development of an elastoplastic damage model the concepts of the CDM were adopted. For it, a damage internal variable is introduced to characterize the degradation of material properties related to the initiation, growth and coalescence of microcracks. The influence of damage on the elastic behaviour is taken into account through the strain equivalence principle (Lemaitre 1996). According to this principle, if E denotes the undamaged stiffness, the unloading stiffness of the damaged material is defined by $E(1\text{-}d)$ such that

$$\varepsilon - \varepsilon^p = \frac{\sigma}{E(1-d)} \tag{1}$$

where d is an isotropic damage variable.

From Eq. (1), it can be observed that damage in CDM is related to the degradation of materials resulting in a stiffness reduction and its value is bounded by 0 and 1. Ideally, a value of damage equal to one should constitute failure while a value of zero corresponds to an undamaged material.

If it is assumed that all microcracks close upon unloading, no permanent deformation remains after the complete unloading. However, the truly reversible elastic strain (ε^e) is obtained by

$$\varepsilon^e = \frac{\sigma}{E} \tag{2}$$

The rest of the strain is actually the inelastic strain ε^d due to the microcrack opening during the loading process and bring in the effect of the degradation of elastic properties (Ortiz 1985, Ju 1989):

$$\varepsilon^d = \frac{\sigma}{E(1-d)} - \frac{\sigma}{E} = \frac{\sigma d}{E(1-d)} \tag{3}$$

This result is consistent with the principal phenomena observed in the response of concrete under uniaxial loading.

Eq. (3) can be specified for the particular case of a truss member. Denoting by N, δ^d and d_a the axial force, the damage elongation and the axial damage, respectively, it follows that

$$\delta^d = \frac{NL}{EA}\frac{d_a}{(1-d_a)} \tag{4}$$

This result constitutes the basis of the model proposed by Cipollina *et al.* (1995). According to this model, using a similar approach to that employed in lumped plasticity models and in order to include the damage effects it is assumed, that not only the plastic deformations but also the damage deformations are concentrated at the hinges, i.e., all the dissipative phenomena occur at the hinges.

This assumption allows the simulation of the degradation of frames using a simplified model. For this model, a frame member is idealized by an elastic element considering the dissipative effects lumped at its ends (Fig. 1). More details about this mechanical model can be found in Cohn and Franchi (1979).

Eq. (4) can be generalized in order to take into account the flexural damage effects in a frame member. Thus, being the stress distribution for each element described by a three component vector, $q=[M_i, M_j, N]^T$, collecting the bending moments at the two ends and the axial force (Fig. 2), which is

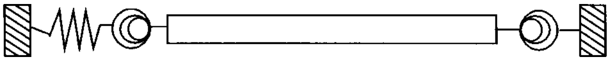

Fig. 1 Mechanical model

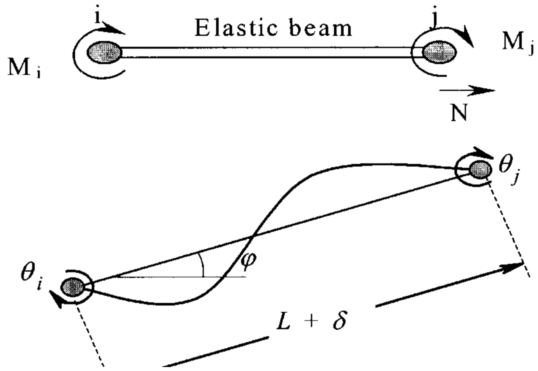

Fig. 2 Generalized stresses and strains for the model

associated to the corresponding kinematic variables $u=[\theta_i, \theta_j, \delta]^T$, the constitutive equations expressing the relations between flexural moments and the corresponding rotations due to damage, $u^d = [\theta_i^d, \theta_j^d, \delta^d]^T$, are obtained as:

$$\theta_i^d = \frac{d_i}{1-d_i}\frac{L}{4EI}M_i \tag{5}$$

$$\theta_j^d = \frac{d_j}{1-d_j}\frac{L}{4EI}M_j \tag{6}$$

being d_i and d_j the damage variables due to flexural effects at both ends of the member. Therefore, the damage vector for each member will be defined as $D^T=(d_i, d_j, d_a)$. More details about this formulation can be found in Florez-Lopez (1995) and Perera *et al.* (1998).

The extension of the constitutive model for cyclic and seismic loading is direct. For it, two sets of scalar damage variables are defined in order to consider positive and negative actions. The corresponding constitutive equations are given by

$$u - u^p = F(D^+)\langle q\rangle_+ + F(D^-)\langle q\rangle_- \tag{7}$$

where $<q>_+$ and $<q>_-$ are the positive and negative parts of q. According to this formulation a unilateral behavior under cyclic loading is assumed in the sense that the damage originated by positive actions has no influence on the behaviour in compression and viceversa which is an idealization.

2.2. Plastic dissipative potential

Since the proposed model is derived within the framework of thermodynamics of irreversible processes the evolution of the damage and plastic strain internal variables is formulated consistently with this framework.

In order to obtain the plastic evolution law it is necessary to define a plastic dissipative potential. For it, by analogy with the effective stress concept proposed by Rabotnov (1968), the three component stress vector q proposed in the last subsection can be redefined as an effective generalized stress vector using the following expression:

$$\tilde{q} = \left(\frac{M_i}{1-d_i}, \frac{M_j}{1-d_j}, \frac{N}{1-d_a}\right)^T \tag{8}$$

According to the strain equivalence principle, any constitutive equation for a damaged material may be derived in the same way as for a virgin material replacing the usual stress by the effective stress (Lemaitre 1996). Therefore the plastic dissipation potential for each plastic hinge of the member may be expressed using the same expression employed for undamaged materials replacing the moment by the corresponding effective moment. Then, when damage occurs, if it is not considered the effect of the axial plastic strains, the plastic function can be written at each end as:

$$f_i = \left|\frac{M_i}{1-d_i} - X_i\right| - M_y \tag{9}$$

where X_i is the kinematic hardening term and M_y is the yield moment.

To define the evolution of X, the following expression is proposed:

$$X = X_\infty(1 - e^{-\alpha\theta^p}) \tag{10}$$

being X_∞ and α parameters to be identified for each material and geometry; from the expression it can be observed that X increases in a nonlinear way with the plastic strain and tends to saturate to some value X_∞ with a velocity controlled by the value of α.

Being defined the plastic potential, the Principle of Maximum Plastic Dissipation implies the normality of the plastic flow rule in the generalized stress space:

$$du^p = d\lambda^p \frac{\partial f}{\partial q} \tag{11}$$

where $d\lambda^p$ is a plastic parameter which can be obtained enforcing the plastic consistency condition.

Similarly, a damage potential based on the energy release rate can be defined, such as it appears in Perera *et al*. (1998), to formulate the damage evolution in order to obtain a damage flow rule. However, a cumulative law is employed such as it is going to be presented in the next section.

3. Cumulative damage law

To completely define the model, the damage evolution law has to be specified. In cyclically loaded materials it is convenient to use cumulative damage models. Since seismic loads induce severe inelastic cycles at relatively large ductilities, the concept of using low-cycle fatigue theories to model damage is logical.

Assuming linear damage accumulation, the total cyclic fatigue damage may be obtained using the principle formulated by Palmgren (1924) and Miner (1945). Damage functions due to each individual cycle are summed until fracture occurs. Failure is assumed to occur when these damage functions sum up to or exceed unity:

$$D = \sum \frac{n_i}{N_f} \geq 1 \tag{12}$$

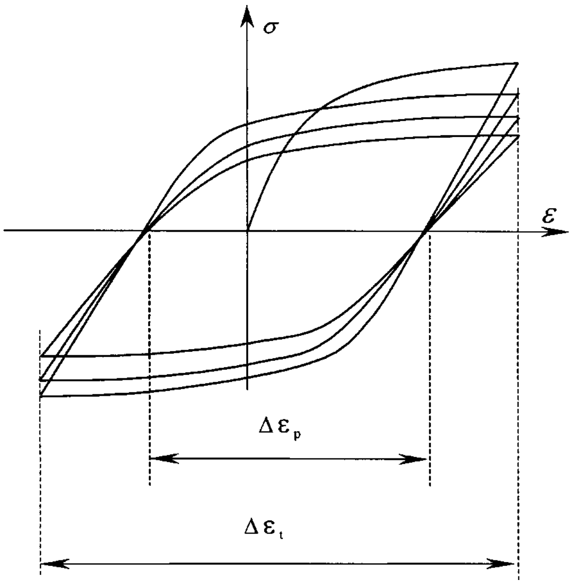

Fig. 3 Total and plastic strain amplitude

where n_i is the number of cycles for the current amplitude and N_f is the number of cycles to failure for this amplitude.

The quantification of the number of cycles to failure N_f is usually performed through the Manson-Coffin relationship (1953):

$$N_f = C(\Delta\varepsilon^p)^K \tag{13}$$

where $\Delta\varepsilon^p$ is the plastic strain amplitude of the hysteretic cycles (Fig. 3) and C and K are parameters depending on the materials which are usually determined through experimental tests. Some authors (Carnicero *et al.* 1998, Kunnath *et al.* 1997, Koh and Stephen 1991) suggested that total strain amplitude could be used instead of plastic strain. However, in seismic response evaluations the inelastic deformation ranges are usually of considerable magnitude which justifies the omission of elastic deformations in damage evaluation.

Then, the procedure of assesment the damage is as follows: First, for every load step the plastic strain increment is determined using Eq. (11) and enforcing the plastic consistency condition. With the plastic strain amplitude obtained and using Eqs. (12) and (13) the progressive damage increments are calculated.

Therefore, there is the need to determine the Manson-Coffin law in order to complete the model. Then, one of the key points of the cumulative damage law is related to the identification of the structural damage parameters C and K appearing in Eq. (13).

Usually, through experimental tests performed on beams made of different profiles some results are obtained to calibrate the Manson-Coffin relationship (Ballio & Castiglioni 1994). The specimens were subjected to displacement cycles of constant amplitude up to collapse. The results obtained allow the definition of a relationship between the amplitude of the displacement cycles imposed and the number of cycles performed to reach the failure (N_f). Performing the tests for different amplitudes a linear relationship amplitude N_f is obtained on a log-log scale which allows the determination of the parameters C and K.

These results, combined with the Miner law, may be useful as a criterion to predict the failure of structural elements. However, in the model proposed in Section 2, the damage is defined as an internal variable affecting the mechanical behavior and, basically, incorporating the gradual loss of stiffness. Therefore, the limiting value $d = 1.0$ of the damage variable may be identified with complete loss of stiffness. Due to it, in the definition of the parameters appearing in the Manson-Coffin law and, therefore, in the damage evaluation would be more convenient to keep the consistency with the definition of the damage index in the model as a variable measuring the progressive loss of stiffness. Experimental tests where the stiffness deterioration is measured are very important to develop a consistent damage model.

Among others, Krawinkler & Zohrei (1983) performed several experimental tests of constant amplitude cyclic loading on steel cantilever specimens in order to characterize the cumulative damage. In the experimental work developed, they consider damage associated to several different phenomena such as strength deterioration, energy dissipation and, as in Continuum Damage Mechanics, stiffness deterioration. The constant amplitude tests of several wide flange shapes (W 6×9) of ASTM A36 steel provided the relationship between damage increment per reversal (in terms of stiffness deterioration), and plastic rotation range. This relation is assumed to be constant within a certain range of the number of reversals. For it, three deterioration ranges were identified according to the deterioration rate. In the first and third ranges, deterioration grows rapidly while in the second range deterioration proceeds at a slow and almost constant rate. More details about it can be found in Krawinkler & Zohrei (1983).

For each range, the rate of stiffness deterioration per reversal, Δd_k, for constant amplitude cycling is expressed by a function of the form:

$$\Delta d_k = A(\Delta\theta_p)^a \tag{14}$$

where A and a are determined through experimental tests and $\Delta\theta_p$ is the plastic rotation range. From Eq. (14), assuming linear damage accumulation for reversals with variable amplitude, the accumulated damage can be expressed as:

$$d = \sum_{i=1}^{n} (\Delta d_k)_i = A \sum_{i=1}^{n} (\Delta\theta_p)_i^a \tag{15}$$

where n is the number of reversals.

Denoting as K_o and K the undamaged and damaged stiffnesses, respectively, the rate Δd_k represented by Krawinkler & Zohrei (1983) corresponds to the relation . In order to employ Eq. (14) in the model presented in Section 2 the existing relationship between the rate Δd_k defined in this equation and the rate Δd corresponding to the model has to be deduced. After some calculations, the following expression for the damage variable in the numerical model is deduced for a cantilever beam:

$$d = \frac{4(1 - K/K_o)}{4 - K/K_o} \tag{16}$$

Taking Eq. (16) in a incremental way and comparing with the value $\Delta d_k = (K_o - K)/K_o$ measured experimentally the following relationship between Δd and Δd_k is obtained

$$\Delta d = \frac{(4-d)^2}{12}\Delta d_k \tag{17}$$

or, applying Eq. (14):

$$\Delta d = \frac{(4-d)^2}{12}A(\Delta\theta_p)^a \tag{18}$$

This expression may be employed in the proposed model to evaluate in a consistent way the damage rate per reversal.

Since the proposed model applies for a range of degradation from zero to one and failure is defined as attainment of this damage value. Then, the number of reversals to failure N_f for constant amplitude cycling is obtained as:

$$N_f = \frac{1}{\Delta d} = \frac{12}{(4-d)^2 A\Delta\theta_p^a} \tag{19}$$

being Δd the damage increment per reversal calculated according to Eq. (18).

Then, for a cycling loading of constant amplitude the following expression for damage evaluation is suggested:

$$d = \frac{\tilde{\theta}_p}{N_f(\Delta\theta_p)\Delta\theta_p} \tag{20}$$

where $\tilde{\theta}_p$ represents the plastic cumulative rotation and N_f $(\Delta\theta_p)$ the number of reversal to failure for the plastic increment $\Delta\theta_p$. This expression can be considered as a Miner relationship but with the advantage of considering also non-complete cycles since the damage assessment is performed using an energy approach.

One inconvenient appears when it is tried to apply the proposed model to assess the cumulative damage under seismic loading. This type of loading is non-harmonic and completely irregular. Because of it, in order to evaluate the cumulative damage the random loading history is usually converted into an equivalent sum of cycles by using cycle counting methods such as the rainflow or the range pair methods. In the presented method the application of cycle counting methods is avoided. However, it is necessary the recalculation of the plastic cumulative rotation when the amplitude of the loops change in order to avoid jumps and discontinuities in the damage assesment using Eq. (20). For this, the following condition must be satisfied when a change of the plastic rotation increment per reversal is produced:

$$d(\tilde{\theta}_p^{old}, \Delta\theta_p^{old}) = d(\tilde{\theta}_p^{new}, \Delta\theta_p^{new}) \tag{21}$$

From this, it can be deduced that

$$\tilde{\theta}_p^{new} = N_f^{new}\Delta\theta_p^{new}\left(\frac{\tilde{\theta}_p^{old}}{N_f^{old}\Delta\theta_p^{old}}\right) \tag{22}$$

The proposed model is checked through the comparison with numerical results in the following section.

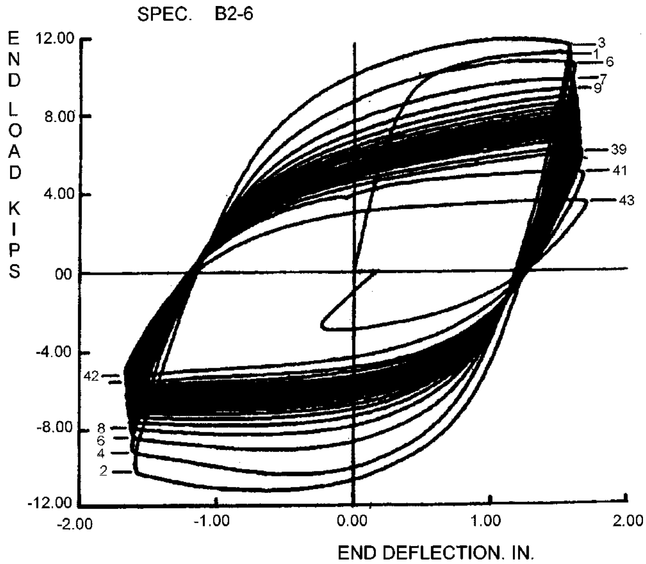

Fig. 4 Experimental results for constant amplitude

4. Numerical simulation

As an example, the proposed model is hereafter applied to the evaluation of the response of a steel specimen with W6×9 section subjected to a cyclic loading of constant amplitude equal to 1.7 in (0.043 m).

The experimental results for the specimen can be found in Krawinkler and Zorei (1983) and are shown in Fig. 4. Such as it was commented in the last section, three ranges of response were obtained according to the rate of deterioration. Ranges I and III develop with a rapid deterioration while range II proceeds at a slow and almost constant rate. Thus, a single cumulative damage model cannot be used for the full range of interest and, then, three different damage models have to be used.

For purposes of comparison, the prediction based on range II of deterioration is adopted and, therefore, some discrepancy in the results can be expected. Range II includes the intermediate cycles of the loading process. From the tests, in this range the following rate of deterioration per reversal has been obtained

$$\Delta d_k^{II} = 0.446[\Delta\theta^p]^{1.415} \tag{23}$$

This rate has been employed in Eq. (17) to evaluate the damage increment in the numerical simulation.

In the same way, the following values have been employed to define completely the plastic dissipative potential corresponding to Eq. (9): $M_y = 23$ kN m; $X_\infty = 20$ kN m; $\alpha = 150$.

Fig. 5 shows the results obtained in the numerical simulation. As it has been commented before, only the function corresponding to the wider range of number of reversals (range II) has been used, which implies a certain deviation from the experimental results for the first and the last cycles but, in any case, a good agreement can be deduced through the comparison of Figs. 4 and 5.

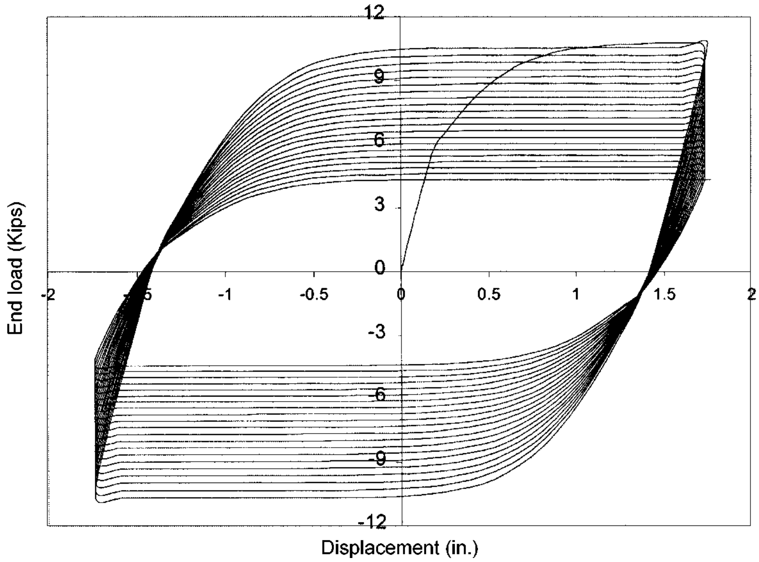

Fig. 5 Numerical results for constant amplitude

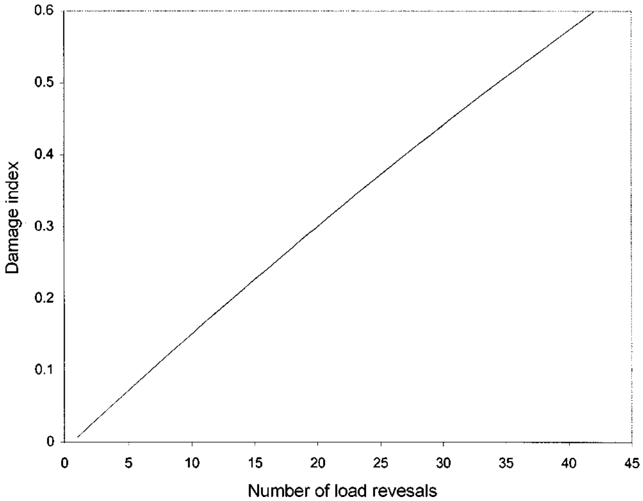

Fig. 6 Damage evolution

In Fig. 6, the damage evolution through the number of cycles obtained numerically is represented. The last numerical value (d = 0.6) can be compared with the last experimental value (d = 0.65) which has been obtained through Eq. (16) measuring the relation K/K_o in the last cycle of range II. As it is logical, the numerical value is a little smaller than the experimental value since in the numerical results the range I, for which the deterioration proceeds at high rate, has not been considered.

In Fig. 7, the results for the same specimen subjected to a lateral cyclic load history of increasing amplitude are shown

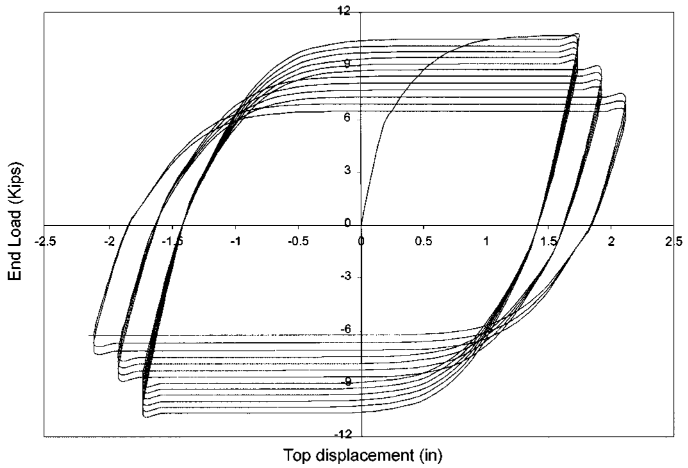

Fig. 7 Numerical results for increasing amplitude

5. Conclusions

In this paper a method was presented for the assesment of the structural response based on a cumulative damage index coupled to the mechanical response. With the proposed method, it is possible to account for the actual damage accumulated at the end sections of each structural member, which is dependent on the maximum plastic excursion but also on the absorbed energy and the loading history. Besides, these damage variables are more representative of the predicted state of the structure than the general damage indexes used in the uncoupled models.

The results obtained are very hopeful. The model performs very well under cyclic loading of constant amplitude. The model appears to be very interesting since it applies the concepts of the CDM in a simplified way to simulate the cumulative damage.

The approach presented is amenable of further generalizations and it would be convenient to obtain experimental results for more complex loading histories (cyclic loading with variable deflection amplitudes, seismic loading) in order to check the efficiency of the model in more realistic loading cases.

In the model proposed, damage is related to the stiffness degradation. A very interesting possibility for the future research would be to try performing through some experimental tests a calibration of this damage index in order to formulate a repairability or failure criterion of the structure with the purpose of performing a possible future intervention of seismic retrofitting.

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