Ocean Systems Engineering, Vol. 2, No. 4 (2012) 257-269
DOI: http://dx.doi.org/10.12989/ose.2012.2.4.257

Scour around spherical bodies due to long-crested and short-crested nonlinear random waves

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(Received January 13, 2012, Revised September 25, 2012, Accepted December 8, 2012)

Abstract. This paper provides a practical stochastic method by which the maximum equilibrium scour depth around spherical bodies exposed to long-crested (2D) and short-crested (3D) nonlinear random waves can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D nonlinear random waves, and using the regular wave formulas for scour and self-burial depths by Truelsen *et al.* (2005). An example calculation is provided.

Keywords: scour depth; self-burial depth; shear stress; long-crested waves; short-crested waves; nonlinear random waves; stochastic method

1. Introduction

The present work addresses the scour around a spherical body and the self-burial of such a body exposed to nonlinear random waves. Examples of spherical bodies are stones in scour protection layers and sea mines on the seabed. Such bodies, which originally were installed e.g. on a plane bed, may experience a range of seabed conditions, i.e., the bed may be flat or rippled; they may be surrounded by a scour hollow, and they may be self-buried. This is caused by the complicated flow generated by the interaction between the incoming flow, the sphere and the seabed. The result will depend on the incoming flow velocity (e.g., the relative magnitude between waves and current), the geometry of the bed and the bed material, as well as the ratio between the near-bed oscillatory fluid particle excursion amplitude and the diameter of the sphere. Moreover, ocean surface waves show a complex three-dimensional irregular pattern where the sharpening of the wave crests manifests wave nonlinearity, complicating the problem. Thus the assessment of the scour around a spherical body should include the effect of random waves, which is of interest in the design of e.g. scour protection layers.

Additional details on the general background and complexity of scour in the marine environment as well as reviews of the problems are given in e.g., Whitehouse (1998), Sumer and Fredsøe (2002). The specifics related to scour around spherical bodies and self-burial of such bodies exposed to steady currents and regular waves are addressed in Truelsen *et al.* (2005). Myrhaug *et al.* (2007) provided results for the scour and self-buried depths around spherical bodies due to long-crested

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linear random waves. A review of the stochastic method is given in Myrhaug and Ong (2011a). This stochastic method has recently been extended to provide a practical method for the scour around vertical piles (Myrhaug and Ong 2012, Ong *et al.* 2012), scour below pipelines (Myrhaug and Ong 2011b), burial and scour of short cylinders (Myrhaug *et al.* 2012).

The purpose of this study is to provide a practical approach by which the maximum equilibrium scour depth around spherical bodies exposed to long-crested (2D) and short-crested (3D) nonlinear random waves can be derived. The approach is based on assuming the waves to be a stationary narrow-band random process, adopting the Forristall (2000) wave crest height distribution representing both 2D and 3D random waves, and using the regular wave formulas for scour and self-burial depths by Truelsen *et al.* (2005). An example is given to demonstrate the application of the method.

2. Scour and self-burial in regular waves

2.1 Scour

The scour around a spherical body in regular waves was investigated in laboratory tests by Truelsen *et al.* (2005). They obtained the following empirical formula for the equilibrium scour depth S around the spherical body with the diameter D (see Fig. 1)

$$\frac{S}{D} = 0.3\{1-exp[-0.3ln(KC)]\} \qquad \text{for } KC \geq 1 \tag{1}$$

The Keulegan-Carpenter number is defined as

$$KC = \frac{UT}{D} \tag{2}$$

where U is the undisturbed linear near-bed orbital velocity amplitude, and T is the wave period. Eq. (1) is valid for live-bed scour, for which $\theta > \theta_{cr}$, and θ is the undisturbed Shields parameter defined by

$$\theta = \frac{\tau_w}{\rho g(s-1)d_{50}} \tag{3}$$

where τ_w is the maximum bottom shear stress under the waves, ρ is the density of the fluid, g is

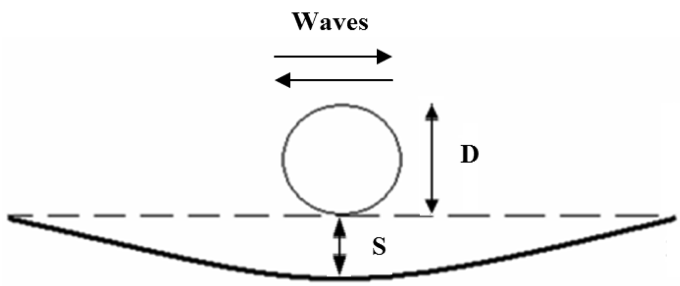

Fig. 1. Definition sketch of scour depth (S) around a spherical body (reproduced from Truelsen *et al*. (2005))

the acceleration due to gravity, $s = \rho_s / \rho$ is the sediment grain density to fluid density ratio, ρ_s is the sediment grain density, d_{50} is the median grain size diameter, and θ_{cr} is the critical value of the Shields parameter corresponding to the initiation of motion at the bed, i.e., $\theta_{cr} \approx 0.05$. One should note that the scour process attains its equilibrium stage through a transition period. Thus the approach is valid when it is assumed that the storm generating nonlinear random waves has lasted longer than the time-scale of the scour. The major flow structures that cause scour around a spherical body placed on the seabed are the lee-wake vortices governed by KC; these vortices act as a mechanism to transport the eroded sediments away from the body during each half cycle of the motion. Further details on the time-scale of the scour as well as on the mechanisms causing scour are given in Truelsen *et al.* (2005).

The maximum bottom shear stress within a wave-cycle is taken as

$$\frac{\tau_w}{\rho} = \frac{1}{2} f_w U^2 \tag{4}$$

where f_w is the friction factor, which here is taken from Myrhaug *et al.* (2001), valid for waves plus current for wave-dominated situations (see Myrhaug *et al.* 2001, Table 3)

$$f_w = c\left(\frac{A}{z_0}\right)^{-d} \tag{5}$$

$$(c, d) = (18, 1) \text{ for } 20 \lesssim A / z_0 \lesssim 200 \tag{6}$$

$$(c, d) = (1.39, 0.52) \text{ for } 200 \lesssim A / z_0 \lesssim 11000 \tag{7}$$

$$(c, d) = (0.112, 0.25) \text{ for } 11000 \lesssim A / z_0 \tag{8}$$

where $A = U/\omega$ is the near-bed orbital displacement amplitude, $\omega = 2\pi / T$ is the angular wave frequency, and $z_0 = d_{50} / 12$ is the bed roughness (see e.g., Soulsby (1997)). The advantage of using this friction factor for rough turbulent flow is that it is possible to derive the stochastic approach analytically. Note that Eq. (7) corresponds to the coefficients given by Soulsby (1997) obtained as a best fit to data for $10 \lesssim A / z_0 \lesssim 10^5$

One should note that the KC number alternatively can be expressed as

$$KC = \frac{2\pi A}{D} \tag{9}$$

Moreover, A is related to the linear wave amplitude a by

$$A = \frac{a}{\sinh kh} \tag{10}$$

where h is the water depth, and k is the wave number determined from the dispersion relationship $\omega^2 = gk \tanh kh$.

2.2 Self-burial

The self-burial of a spherical body in regular waves was also investigated in the laboratory tests by Truelsen *et al.* (2005). They obtained the following empirical formula for the equilibrium self-burial depth e of the spherical body with the diameter D (see Fig. 2)

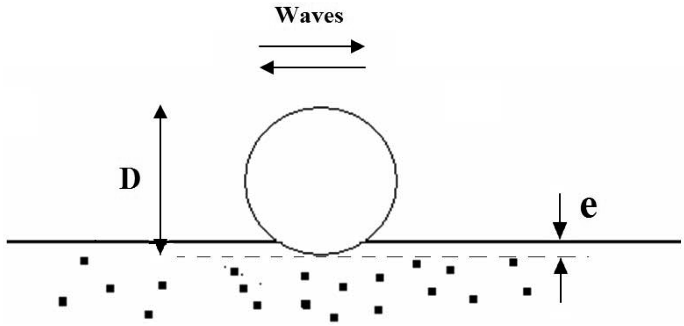

Fig. 2. Definition sketch of self-burial depth (e) around a spherical body (reproduced from Truelsen *et al.* (2005))

$$\frac{e}{D} = 0.5\{1 - exp[-q(KC - r)]\} \quad \text{for } KC \geq r \tag{11}$$

where q and r are coefficients given by the following values

$$(q, r) = (0.08, 1.4) \tag{12}$$

These results are valid for live-bed scour.

The main mechanism of self-burial of a spherical body is as follows: For a spherical body placed on the seabed the consequence of the developing scour is that the bearing area of the body is reduced causing an increased load on the soil; the bearing capacity of the soil will be exceeded, causing the soil to fail; the body will sink. The whole process will continue until the failure of the soil will stop, and consequently that the sinking ends. More details are given in Truelsen *et al.* (2005).

3. Scour and self-burial in random waves

3.1 Theoretical background

Under nonlinear waves the nonlinearity is primarily caused by the asymmetric wave velocity, i.e. that the near-bed orbital velocity is larger in the wave propagation direction than in the opposite direction. Catano-Lopera and Garcia (2007) addressed the effect of wave asymmetry on the scour depth around a finite length cylinder placed horizontally on a plane bed. In their experiments in regular waves plus currents they observed that normally the downstream length of the scour gap is larger than its upstream counterpart. Under waves alone this is primarily caused by the asymmetric wave velocity. Examples of wave asymmetry are shown in their Figs. 2(b) and 3(b). However, the effect of this asymmetry on the geometry of the scour hole was not elaborated further. In the present paper the effects of wave asymmetry are considered by using Stokes second-order wave theory.

For Stokes second-order waves the nonlinearity is primarily caused by the larger velocity under the wave crest (crest velocity) than under the wave trough (trough velocity). Based on the results by Catano-Lopera and Garcia (2007) referred to before, it seems that it is the largest velocity in the wave cycle (i.e., the crest velocity) which is responsible for the scour, rather than the mean of the crest velocity and the trough velocity (i.e., equal to the linear wave velocity). Thus the scour and self-burial depths for individual random Stokes second-order waves is obtained from Eqs. (1), (2) and (11) by replacing U with U_c, i.e., the maximum near-bed orbital velocity under the wave crest. This will be elaborated further in the forthcoming.

At a fixed point in a sea state with stationary narrow-band random waves consistent with Stokes second-order regular waves in finite water depth with a nonlinear crest height η_c, the non-dimensional nonlinear crest height, $w_c = \eta_c/a_{rms}$, and the non-dimensional nonlinear maximum horizontal particle velocity evaluated at the seabed, $\hat{U}_c = U_c/U_{rms}$, are (Dean and Dalrymple 1984)

$$w_c = \hat{a} + O(k_p a_{rms}) \tag{13}$$

$$\hat{U}_c = \hat{a} + O(k_p a_{rms}) \tag{14}$$

Here $\hat{a} = a/a_{rms}$ is the non-dimensional linear wave amplitude, where the linear wave amplitude a is made dimensionless with the *rms* (root-mean-square) value a_{rms}, and

$$U_{rms} = \frac{\omega_p a_{rms}}{sinhk_p h} \tag{15}$$

Moreover, $O(k_p a_{rms})$ denotes the second-order (nonlinear) terms which are proportional to the characteristic wave steepness of the sea state, $k_p a_{rms}$, where k_p is the wave number corresponding to ω_p(= the peak frequency of the wave spectrum) given by the dispersion relationship for linear waves (which is also valid for Stokes second-order waves)

$$\omega_p^2 = gk_p tanhk_p h \tag{16}$$

Now Eq. (13) can be inverted to give $\hat{a} = w_c - O(k_p a_{rms})$, which substituted in Eq. (14) gives $\hat{U}_c = w_c + O(k_p a_{rms})$. Thus it appears that $\hat{a}$ can be replaced by w_c in the linear term of $\hat{U}_c$, because the error involved is of second order. Consequently, by neglecting terms of $O(k_p a_{rms})$ the maximum near-bed orbital velocity under the wave crest in dimensional form can be taken as

$$U_c = \frac{\omega_p \eta_c}{sinhk_p h} \tag{17}$$

Moreover, $A_c = U_c / \omega_p$ is the maximum near-bed orbital displacement under the wave crest, $\hat{A}_c = A_c/A_{rms}$ is the non-dimensional maximum near-bed orbital displacement where

$$A_{rms} = \frac{a_{rms}}{sinhk_p h} \tag{18}$$

Furthermore

$$\omega_p = \frac{U_c}{A_c} = \frac{U_{rms}}{A_{rms}} \tag{19}$$

by combining Eqs. (15) and (18).

Now the Forristall (2000) parametric crest height distribution based on simulations using second-order theory is adopted. The simulations were based on the Sharma and Dean (1981) theory; this model includes both sum-frequency and difference-frequency effects. The simulations were made both for 2D and 3D random waves. A two-parameter Weibull distribution with the cumulative distribution function (*cdf*) of the form

$$P(w_c) = 1 - exp\left[-\left(\frac{w_c}{\sqrt{8}\alpha}\right)^{\beta}\right]; \qquad w_c \geq 0 \tag{20}$$

was fitted to the simulated wave data. The Weibull parameters α and β were estimated from the fit to the simulated data, and are based on the wave steepness S_1 and the Ursell parameter U_R defined by

$$S_1 = \frac{2\pi}{g}\frac{H_s}{T_1^2} \tag{21}$$

and

$$U_R = \frac{H_s}{k_1^2 h^3} \tag{22}$$

Here H_s is the significant wave height, T_1 is the spectral mean wave period, and k_1 is the wave number corresponding to T_1. It should be noted that $H_s = 2\sqrt{2}a_{rms}$ when a is Rayleigh distributed. The wave steepness and the Ursell number characterize the degree of nonlinearity of the waves in finite water depth. At zero steepness and zero Ursell number the fits were forced to match the Rayleigh distribution, i.e., $\alpha = 1/\sqrt{8} \approx 0.3536$ and $\beta = 2$. Note that this is the case for both 2D and 3D linear waves. The resulting parameters for the 2D-model are

$$\begin{aligned} \alpha_{2D} &= 0.3536 + 0.2892 S_1 + 0.1060 U_R \\ \beta_{2D} &= 2 - 2.1597 S_1 + 0.0968 U_R^2 \end{aligned} \tag{23}$$

and for the 3D-model

$$\begin{aligned} \alpha_{3D} &= 0.3536 + 0.2568 S_1 + 0.0800 U_R \\ \beta_{3D} &= 2 - 1.7912 S_1 - 0.5302 U_R + 0.284 U_R^2 \end{aligned} \tag{24}$$

Forristall (2000) demonstrated that the wave setdown effects were smaller for short-crested than for long-crested waves, which is due to the fact that the second-order negative difference-frequency terms are smaller for 3D waves than for 2D waves. Consequently the wave crest heights are larger for 3D waves than for 2D waves (see Forristall (2000) for more details).

3.2 Outline of stochastic method

For scour below pipelines and around vertical piles in random waves Sumer and Fredsøe (1996, 2001) determined the statistical quantities of the wave height H and the wave period T to be used in the regular wave formulas for the scour depth and the scour width below pipelines and the scour depth around slender vertical piles. By trial and error they found that the use of H_{rms} (= the *rms* wave height) and T_p (= the peak period of the wave spectrum) gave the best agreement with data. To be more exact, Sumer and Fredsøe (1996, 2001) used U_{rms} and T_p in the calculation of the KC

number. However, here U_{rms} is related to $H_{rms} = 2a_{rms}$ by using Eq. (15). Here a tentative stochastic approach will be outlined. The highest among random waves in a stationary narrow-band sea-state is considered, as it is reasonable to assume that it is mainly the highest waves which are responsible for the scour response. It is also assumed that the sea-state has lasted long enough to develop the equilibrium scour depth. The highest waves considered here are those exceeding the probability $1/n$, $w_{c1/n}$ (i.e., $1-P(w_{c1/n}) = 1/n$). The quantity of interest is the expected (mean) value of the maximum equilibrium scour characteristics caused by the $(1/n)$th highest wave crests, which is given as

$$E[Y(w_c)|w_c > w_{c1/n}] = n \int_{w_{c1/n}}^{\infty} Y(w_c)p(w_c)\ dw_c \tag{25}$$

where Y represents the scour variable and $p(w_c)$ is the probability density function (*pdf*) of w_c. More specifically, the present approach is based on the following assumptions: (1) the free surface elevation is a stationary narrow-band process with zero expectation, and (2) the scour response formulas for regular waves given in the previous section (Eqs. (1), (2), (11) and (12)) are valid for irregular waves as well. These assumptions are essentially the same as those given in e.g. Myrhaug *et al.* (2009), where further details are found.

3.3 Scour depth

For a narrow-band process $T = T_p$ where $T_p = 2\pi/\omega_p = 2\pi A_{rms}/U_{\mathrm{rms}}$ and where Eq. (19) has been used. By taking $U = U_{\mathrm{c}}$, $A = A_{\mathrm{c}}$ and substituting this in Eq. (1) using Eq. (2), and using from Eq. (19) that $A_c/A_{\mathrm{rms}} = U_c/U_{rms}$, Eq. (1) can be re-arranged to give the equilibrium scour depth due to individual narrow-band random Stokes second-order waves as

$$\hat{s} \equiv \frac{S}{0.3D} = 1-exp[-0.3ln(KC_{rms}\cdot \hat{U}_c)] \text{ for } \hat{U}_c \geq \hat{U}_{c1} = \frac{1}{KC_{rms}} \tag{26}$$

where

$$KC_{rms} = \frac{U_{rms}T_p}{D} = \frac{2\pi A_{rms}}{D} \tag{27}$$

Let Y denote $\hat{s}$. According to the previous discussion, $\hat{U}_c = w_c$ by neglecting terms of $O(k_p a_{rms})$, and consequently

$$Y = 1-exp[-0.3ln(KC_{rms}w_c)] \text{ for } w_c \geq w_{c1} = \frac{1}{KC_{rms}} \tag{28}$$

where the *cdf* of w_c is given in Eq. (20). However, since Eq. (28) is valid in a finite interval, i.e. for $w_c \geq w_{c1}$, then w_c follows the truncated Weibull distribution give by the cdf

$$P(w_c) = \frac{exp\left[-\left(\frac{w_{c1}}{\sqrt{8}\alpha}\right)^{\beta}\right]-exp\left[-\left(\frac{w_c}{\sqrt{8}\alpha}\right)^{\beta}\right]}{exp\left[-\left(\frac{w_{c1}}{\sqrt{8}\alpha}\right)^{\beta}\right]};\ w_c \geq w_{c1} \tag{29}$$

Now the mean of the maximum equilibrium scour depth caused by the (1/n)th highest wave crests follows from Eqs. (25) and (28), where $p(w_c) = dP(w_c) / dw_c$ with $P(w_c)$ as given in Eq. (29), and by using that (i.e., determined from $1 - P(w_{c1/n}) = 1/n$)

$$w_{c1/n} = \sqrt{8}\alpha\left[\ln n + \left(\frac{w_{c1}}{\sqrt{8}\alpha}\right)^{\beta}\right]^{1/\beta} \tag{30}$$

3.4 Self-burial depth

Similarly, by substitution in Eq. (11), using Eqs. (2) and (19), Eq. (11) can be re-arranged to give the equilibrium self-burial depth due to individual narrow-band random Stokes second-order waves as

$$\hat{e} \equiv \frac{e}{0.5D} = 1 - \exp[-q(KC_{rms} \cdot \hat{U}_c - r)] \text{ for } \hat{U}_c \geq \hat{U}_{c1} = \frac{r}{KC_{rms}} \tag{31}$$

Let Y denote $\hat{e}$. According to the previous discussion, $\hat{U}_c = w_c$ by neglecting terms of $O(k_p, a_{rms})$, and consequently

$$Y = 1 - \exp[-q(KC_{rms} \cdot w_c - r)] \text{ for } w_c \geq w_{c1} = \frac{r}{KC_{rms}} \tag{32}$$

where the *cdf* of w_c is given in Eq. (29).

Now the mean of the maximum self-burial depth caused by the $(1/n)$th highest wave crests follows from Eqs. (25) and (32) by using Eq. (30).

4. Results and discussion

For the random wave-induced scour and self-burial around spherical bodies due to nonlinear long-crested and short-crested waves no data exist in the open literature. Therefore the results in this section should be taken as tentative, and data for comparisons are required before any conclusion can be made regarding the validity of the approach. However, the authors' previous studies on random wave-induced scour characteristics around marine structures including comparison with data from random wave-induced scour experiments support that the method should be useful as an engineering approach (Myrhaug and Ong 2011a). First, the appropriate Shields parameter to use to determine the conditions corresponding to live-bed scour for 2D and 3D nonlinear random waves is discussed. Second, an example is given to demonstrate the use of the method. Finally, some comments to the method are given.

4.1 Shields parameter

For random waves it is not obvious which value of the Shields parameter to use to determine the conditions corresponding to live-bed scour. However, it seems to be consistent to use corresponding statistical values of the scour depth and the Shields parameter. That is, if e.g. the value $E[Y(w_c)|w_c > w_{c1/n}]$ of the scour depth is considered, then the corresponding value of the Shields parameter should be used. Some details of how this value of the Shields parameter for 2D and 3D nonlinear random waves can be calculated will be elaborated in the following.

First, it is noted that the non-dimensional maximum Shields parameter under the wave crest for individual random waves, $\theta_c = \theta_m/\theta_{rms}$, is equal to the non-dimensional maximum bottom shear stress under the wave crest for individual random waves, $\tau_c = \tau_m / \tau_{rms}$. Here θ_{rms} is defined as

$$\theta_{rms} = \frac{\tau_{rms}/\rho}{g(s-1)d_{50}} \tag{33}$$

where, by definition

$$\frac{\tau_{rms}}{\rho} = \frac{1}{2}c\left(\frac{A_{rms}}{z_0}\right)^{-d} U_{rms}^2 \tag{34}$$

Moreover, θ_m is defined in Eq. (3) by replacing τ_w with τ_m, i.e., the maximum bottom shear stress under the wave crest for individual random waves. By using this and following Myrhaug and Holmedal (2011), θ_c is given as

$$\theta_c = w_c^{2-d} \tag{35}$$

Then the Shields parameter of interest in the present context is obtained by using the result in the Appendix as

$$E[\theta_c(w_c)|w_c > w_{c1/n}] = n(\sqrt{8}\alpha)^{2-d}\,\Gamma\left(1+\frac{2-d}{\beta},\ lnn\right) \tag{36}$$

For linear waves ($\alpha = 1/\sqrt{8}$, $\beta = 2$) the result is

$$E[\theta_c(w_c)|w_c > w_{c1/n}] = n\Gamma\left(2-\frac{d}{2},\ lnn\right) \tag{37}$$

More discussion of the bottom friction beneath 2D and 3D nonlinear random waves are given in Myrhaug and Holmedal (2011).

4.2 Example calculation

This example is included to demonstrate the application of the method. The given flow conditions are :
Significant wave height, $H_s = 3$ m
Spectral peak period, $T_p = 7.9$ s, corresponding to $\omega_p = 0.795$ rad/s
Water depth, $h = 10$ m
Median grain diameter (coarse sand according to Soulsby 1997, Fig. 4) $d_{50} = 1$ mm
s = 2.65 (as for quartz sand)
Diameter, $D = 1.0$ m

The calculated quantities are given in Table 1. It should be noted that S_1 and U_R are obtained by replacing T_1 and k_1 with T_P and k_p, respectively, since the wave process is assumed to be narrow-banded. Myrhaug and Rue (2003) found that the maximum equilibrium scour depth and the maximum equilibrium scour width below pipelines caused by the (1/10)th highest waves represent

Table 1. Example calculation with $h = 10$ m, $d_{50} = 1$ mm, $D = 1.0$ m, $n = 10$

a_{rms} (m)	1.06
k_p (rad/m)	0.0900
S_1	0.0308
U_R	0.370
α_{2D}, β_{2D} (Eqs. (23), (24))	0.4017, 1.9467
α_{3D}, β_{3D} (Eqs. (23), (24))	0.3911, 1.7875
A_{rms} (m)	1.03
U_{rms} (m/s)	0.819
A_{rms}/z_0	12360
c,d , Eq. (8)	0.112, 0.25
KC_{rms}, Eq. (27)	6.5
θ_{rms}, Eqs. (33) and (34)	0.220
Scour depth (Eqs. (25) to (30))	
S_{lin} (m)	0.520
$S_{nonlin,\ 2D}$ (m)	0.540
$S_{nonlin,\ 3D}$ (m)	0.544
Self-burial depth (Eqs. (25), (31), (32))	
e_{lin} (m)	0.559
$e_{nonlin,\ 2D}$ (m)	0.617
$e_{nonlin,\ 3D}$ (m)	0.629
Shields parameter; $\theta_m = \theta_c \cdot \theta_{rms}$	
θ_{clin}, Eq. (37)	2.83
θ_{mlin}	0.623
$\theta_{\mathrm{cnonlin,\ 2D}}$, Eq. (36)	3.65
$\theta_{\mathrm{cnonlin,\ 3D}}$, Eq. (36)	3.84
$\theta_{\mathrm{mnonlin,\ 2D}}$	0.803
$\theta_{\mathrm{mnonlin,\ 3D}}$	0.845

the upper values of the random wave-induced scour data, and thus suggesting that these values can be used for design purposes. Thus the results are exemplified for $n = 10$. Here the *rms* wave amplitude is given according to the Rayleigh distribution by $a_{rms} = H_s/2\sqrt{2}$. Now A_{rms}/z_0 (i.e., $z_0 = d_{50}/12$) exceeds 11000; thus $(c, d) = (0.112, 0.25)$. Moreover, θ_{rms} exceeds the critical Shields parameter $\theta_{cr} \approx 0.05$, i.e., live-bed conditions.

The maximum equilibrium scour depth and the maximum equilibrium self-burial depth around the sphere are considered; the effect of nonlinearity is to increase the scour and the self-burial depths; the nonlinear to linear ratios for 2D and 3D waves are: 1.038 and 1.046, respectively, for the scour depth; 1.104 and 1.125, respectively, for the self-burial depth. Consequently, short-crested waves give slightly larger values than long-crested waves; the ratio between the scour depth for 3D waves and the scour depth for 2D waves is 1.007, while the 3D to 2D ratio for the self-burial depth is 1.019. This is a result of the smaller wave setdown effects for 3D than for 2D waves in finite water depth, as discussed before in the paragraph after Eq. (24).

Finally, for the Shields parameter it is noted that for both linear and nonlinear waves θ_m exceeds θ_{cr} implying live-bed conditions. Short-crested waves give a slightly larger value than long-crested waves.

4.3 Discussion

All the formulas used in this paper for the proposed approach were determined on the basis of small-scale tests with waves alone and one body surface roughness. Various scale effects such as the effects of the Shields parameter, the Reynolds number (*Re*), the body surface roughness, the Froude number (*F*), and the influence of the presence of the small-scale ripples in the laboratory experiments on the end results were discussed in Truelsen *et al.* (2005). They concluded that the effect of the Shields parameter is insignificant for the case of live-bed scour; the effects of *Re*, surface roughness and *F* on scour are also insignificant; ripples are not essential in the scour process.

These results indicating no scale effects might be the case as long as the bedload mode is considered. However, in full scale, when the full scale transport mode is suspension, this is not necessarily the case. However, it is believed that the proposed approach can be applied when formulas for scour for the suspension mode are available.

Although simple, the present approach should be useful as a first approximation to represent the stochastic properties of the maximum equilibrium scour depth around a spherical body and the maximum equilibrium self-burial depth of such a body exposed to random waves. However, comparisons with data are required before a conclusion regarding the validity of this approach can be given. In the meantime the method should be useful as an engineering tool for design purposes.

5. Conclusions

A practical stochastic approach for estimating the maximum equilibrium scour depth around a spherical body and the maximum equilibrium self-burial depth of such a body due to long-crested (2D) and short-crested (3D) nonlinear random waves is given.

An example calculation demonstrates the effects of nonlinear waves. The scour and the self-burial depths are only slightly larger beneath 3D nonlinear waves than beneath 2D nonlinear waves. This behaviour is attributed to the smaller wave setdown effects for 3D than for 2D waves in finite water depth.

Although simple, the present approach should be useful as a first approximation to represent the stochastic properties of the maximum equilibrium scour and self-burial depths around spherical bodies exposed to 2D and 3D nonlinear random waves. However, comparisons with data are required before a conclusion regarding the validity of this approach can be given. In the meantime the method should be useful as an engineering tool for the assessment of scour and in scour protection work.

References

Abramowitz, M. and Stegun, I.A. (1972), *Handbook of mathematical functions*, Dover, New York.

Catano-Lopera, Y.A., Garcia, M.H. (2007), "Geometry of scour hole around, and the influence of the angle of attack on the burial of finite length cylinders under combined flows", *Ocean Eng*., **34**(5-6), 856-869.

Dean, R.G. and Dalrymple, R.A. (1984), *Water wave mechanics for engineers and scientists*, Prentice-Hall, Inc., New Jersey, USA.

Forristall, G.Z. (2000), "Wave crest distributions: observations and second-order theory", *J. Phys. Oceanogr*., **30**, 1931-1943.

Myrhaug, D., Føien, H. and Rue, H. (2007), "Tentative engineering approach to scour around spherical bodies in random waves", *Appl. Ocean Res*., **29**(1-2), 80-85.

Myrhaug, D. and Holmedal, L.E. (2011), "Bottom friction and erosion beneath long-crested and short-crested nonlinear random waves", *Ocean Eng*., **38**(17-18), 2015-2022.

Myrhaug, D., Holmedal, L.E., Simons, R.R. and MacIver, R.D. (2001), "Bottom friction in random waves plus current flow", *Coastal Eng*., **43**(2), 75-92.

Myrhaug, D. and Ong M.C. (2011a), *Random wave-induced scour around marine structures using a stochastic method. In: marine technology and engineering*, (Eds., Guedes Soares, C. *et al*.), **1**, 537-562. CRC Press/ Balkema,Taylor & Francis Group, London, UK.

Myrhaug, D. and Ong M.C. (2012), "Scour around vertical pile foundations for offshore wind turbines due to long-crested and short-crested nonlinear random waves", *J. Offshore Mech. Arct. Eng.,* DOI:10.1115/1.4007048, in press.

Myrhaug, D. and Ong, M.C. (2011b), "Long- and short-crested random wave-induced scour below pipelines", *Proceedings of the ICE - Maritime Eng*., **164**(4), 173-184.

Myrhaug, D., Ong, M.C., Føien, H., Gjengedal, C. and Leira, B.J. (2009), "Scour below pipelines and around vertical piles due to second-order-random waves plus a current", *Ocean Eng*., **36**(8), 605-616.

Myrhaug, D. and Rue, H. (2003), "Scour below pipelines and around vertical piles in random waves", *Coastal Eng*., **48**(4), 227-242.

Myrhaug, D., Ong, M.C. and Hesten, P. (2012), "Burial and scour of short cylinders due to long-crested and short-crested nonlinear random waves", *Proceedings of the 6th Int. Conf. on Scour and Erosion*, Paris, France, 783-790.

Ong, M.C., Myrhaug, D. and Hesten, P. (2012), "Scour around vertical piles due to long-crested and short-crested nonlinear random waves plus a current", *Coastal Eng*., http://dx.doi.org/10.1016/j.coastaleng.2012.10.005, in press.

Sharma, J.N. and Dean, R.G. (1981), "Second-order directional seas and associated wave forces", *J. Soc. Petrol. Eng*., **21**(1), 129-140.

Soulsby, R.L. (1997), *Dynamics of marine sands. A manual for practical applications*, Thomas Telford, London, UK.

Sumer, B.M. and Fredsøe, J. (1996), "Scour below pipelines in combined waves and current", *Proceedings of the15th OMAE Conf*., Florence, Italy, **5**, 595-602.

Sumer, B.M. and Fredsøe, J. (2001), "Scour around pile in combined waves and current", *J. Hydraul. Eng. – ASCE*, **127**(5), 403-411.

Sumer, B.M. and Fredsøe, J. (2002), *The mechanics of scour in the marine environment*, World Scientific, Singapore.

Truelsen, C., Sumer, B.M. and Fredsøe, J. (2005), "Scour around spherical bodies and self-burial", *J. Waterw, Port Coastal and Ocean Eng.- ASCE*. **131**(1), 1-13

Whitehouse, R.J.S. (1998), *Scour at mar, ine structures. A manual for practical applications*. Thomas Telford, London, UK.

Appendix

Let x be Weibull distributed with the *pdf*

$$p(x) = \beta x^{\beta-1} exp(-x^{\beta});\ x \geq 0,\ \beta > 0 \tag{A1}$$

The expected value of the $(1/n)$th largest values of x is given as

$$E[x_{1/n}] = n \int_{x_{1/n}}^{\infty} xp(x)dx \tag{A2}$$

where $x_{1/n}$ is the value of x which is exceeded by the probability $1/n$.

Moreover, from Abramowitz and Stegun (1972, Ch. 6.5, Eq. (6.5.3)) it is given that

$$\Gamma(a, x) = \int_{x}^{\infty} e^{-t} t^{a-1} dt \tag{A3}$$

By utilizing this, the following result is obtained

$$\int_{x_1}^{\infty} x^m p(x)dx = \int_{x_1}^{\infty} x^m \beta x^{\beta-1} exp(-x^{\beta})dx = \Gamma\left(1 + \frac{m}{\beta}, x_1^{\beta}\right) \tag{A4}$$

by using Eq. (A1), and where $\Gamma(\square,\square)$ is the incomplete gamma function: $\Gamma(x, 0) = \Gamma(x)$ where Γ is the gamma function. The result in Eq. (A4) is obtained by substituting $t = x^{\beta}$ in the second integral in Eq. (A4) and using Eq. (A3).