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Theoretical analysis of transient wave propagation in the band gap of phononic system

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Abstract. Phononic system composed of periodical elastic structures exhibit band gap phenomenon, and all elastic wave cannot propagate within the band gap. In this article, we consider one-dimensional binary materials which are periodically arranged as a 20-layered medium instead of infinite layered system for phononic system. The layered medium with finite dimension is subjected to a uniformly distributed sinusoidal loading at the upper surface, and the bottom surface is assumed to be traction free. The transient wave propagation in the 20-layered medium is analyzed by Laplace transform technique. The analytical solutions are presented in the transform domain and the numerical Laplace inversion (Durbin's formula) is performed to obtain the transient response in time domain. The numerical results show that when a sinusoidal loading with a specific frequency within band gap is applied, stress response will be significantly decayed if the receiver is away from the source. However, when a sinusoidal force with frequency is out of band gap, the attenuation of the stress response is not obvious as that in the band gap.

Keywords: phononic system; band gap; multilayered; transient wave; laplace transform; durbin

1. Introduction

Wave propagation in multilayered media has long been an interesting subject due to its significance and a large number of applications in aerospace, electronic engineering, mechanical engineering, oceanography, and earthquake engineering. For example, the coated-layer materials are very important in electronic engineering, and to avoid the delamination of the interface due to dynamic loadings is an important topic. Many studies in earthquake engineering focus on calculating the response of multilayered medium subjected to a sudden disturbance which is located either on the surface or inside the medium.

The transient response induced by a dynamic load applied on the surface of a half space was analyzed by Lamb (1904) by integral transform technique followed by the analytical evaluation of the inversion integrals. The wave propagation in a generalized thermo elastic plate embedded in an elastic medium was studied based on the generalized two dimensional theory of thermo elasticity by Ponnusamy and Selvamani (2012). The propagation of waves in a micropolar transversely isotropic half space in the theory of thermoelasticity without energy dissipation were discussed by Kumar and Gupta (2010). Theory and analysis of elastic wave in

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a stratified medium were studied in some details in the books by Ewing (1957) and Brekhovskikh (1980). A transfer matrix formalism to determine the unknown coefficients from the continuity conditions at the interfaces of multilayered media was introduced by Thomson (1950), and improved by Haskell (1953). In earth geophysics and earthquake engineering, this matrix method was widely used to determine the dispersion relation of surface waves in a layered half-space case.

Pekeris *et al.* (1965) proposed a transient wave solution for one layer overlaying the half-space. The propagation of transient waves was represented by a series, with each term indicating a wave propagating in the medium. The series expansion required the evaluation of a 4×4 determinant for the plate and a 6×6 determinant for the two-layered medium. Ma and Huang (1996) derived the transfer relation as the general representations of the responses between each layer, instead of the displacement-traction vector, to investigate the transient wave propagating in a multilayered medium. The theoretical, numerical and experimental results of transient responses in a layered medium subjected to in-plane loadings were presented by Ma and Lee (1999). The dynamic response of a layered medium subjected to anti-plane loadings was presented by Ma *et al.* (2001).

In addition to the analytical treatment, there was a different computational approach based on the numerical inversion of Laplace transforms. Narayanan and Beskos (1982) systematically proposed eight algorithms of numerical inversion of the Laplace transform which were compared with each other, with respect to their accuracy and computational efficiency. They found that the most accurate algorithm, but requiring more computational time, was the method proposed by Durbin (1974). Manolis and Beskos (1981) compared the algorithms proposed by Durbin and Papoulis (1957) for the numerical Laplace inversion. They found that Durbin’s algorithm was more time consuming than Papoulis’ but the accuracy was very high even for long-time calculation. More details about Durbin’s method and its applications to beam dynamic response were investigated by Manolis and Beskos (1980), Beskos and Narayanan (1983) and Providakis and Beskos (1986).

Analytical solutions of transient wave propagation in multilayered medium have been presented in the literature for 1-D, 2-D (Su *et al.* 2002), or 3-D (Ma and Lee 2006) problems. For one-dimensional problem of plane wave propagation in the direction normal to the layered medium, Sun *et al.* (1968) have presented continuum theory instead of “effective modulus theory” for determining dispersion relation. Black *et al.* (1960) proposed a characteristics method for wave propagation in a two-layered medium. Lundergan and Drumheller (1971) numerically simulated the response in a multilayered system with varying thickness, and their results were in excellent agreement with experiment. Harmonic waves in composites with isotropic layers were studied by Stern *et al.* (1971), Hegemier and Nayfeh (1973). Transient plane waves propagating in a periodical layered elastic medium were examined by Ting and Mukunoki (1979, 1980), and Tang and Ting (1985). Recently, Chen *et al.* (2004) developed an analytical solution based on Floquet’s theory to solve the problem of plate impact in layered heterogeneous material systems, and the comparison between analytical results and experiment data was very good. It is worthy to note that all the numerical calculations presented in above mentioned papers were limited to a layered half-space case, or fewer layers with early-time response. Lin and Ma (2011) proposed an analytical-numerical method to analyze the transient wave propagation, matrix-form solution and numerical Laplace inversion method make it feasible to calculate the long-time response for complex structures.

When the binary materials periodically arranged as an infinite layered system, this system is called “phononic systems”. The significant effect of phononic systems laid on the existence of

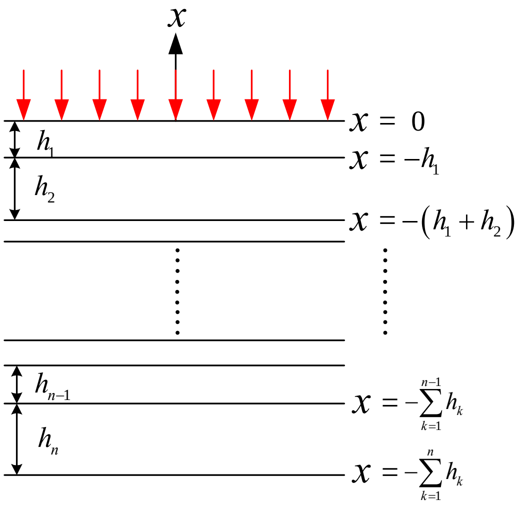

Fig. 1 Configuration and coordinate system of an *n*-layered medium subjected to uniform loadings

absolute band gap within which sound and vibrations are all forbidden. Many theoretical methods have been used to study the band gap phenomenon, such as, the transfer-matrix method (Munjal 1993, Sigalas and Soukoulis 1995), the multiple scattering theory (Liu *et al.* 2000, Psarobas and Sigalas 2002), the plane wave expansion method (Kushwaha and Djafari-Rouhani 1998, Wu *et al.* 2001, Wang *et al.* 2004), and the finite difference time domain method (Sigalas and Garcia 2000, Tanaka *et al.* 2000). However, above mentioned theoretical methods were based on the steady-state analysis. In this article, the transient responses of a multilayered medium with finite dimension subjected to a uniformly distributed loading applied on the top surface are analyzed based on the Laplace transform method. The interface and boundary conditions are used to construct the system of equations for determining the global field vector that is a stack of the field vectors in each layer. By using numerical Laplace inversion from Durbin's method, an analytical-numerical solution is obtained. The solution makes it possible to efficiently and accurately calculate the long-time transient responses for the multilayered medium. The technique of multilayered matrix-form solution and numerical Laplace inversion method presented in this study will be used to study the transient responses for phononic systems.

2. Transient elastic wave propagation in multilayered media

Consider an initially undisturbed multilayered medium consisting of *n* layers as shown in Fig. 1. Each layer is assumed to be elastic, homogeneous, isotropic, and perfectly bonded along the interface. The thickness and material constants of each layer are different. The stratified medium is subjected to uniformly distributed loadings applied on the top surface at $t = 0$. The quantities related to *i*th layer are suffixed by a superscript (*i*), and *n* stratified layers contain *n*+2 media including upper and lower half-spaces. In other words, (0) implies the upper half-space and (*n*+1) indicates the lower half-space.

We consider plane wave propagation in the x direction in which the only non-vanishing component of the displacement is in the x direction, and the 1-D longitudinal wave equation can be expressed as follows

$$\frac{\partial^2 u}{\partial x^2} = S_L^2 \frac{\partial^2 u}{\partial t^2}, \tag{1}$$

where $u(x,t)$ is the longitudinal displacement and S_L is the slowness of the longitudinal wave given by

$$S_L = \frac{1}{C_L} = \sqrt{\frac{\rho}{(\lambda + 2\mu)}} = \sqrt{\frac{\rho(1+\nu)(1-2\nu)}{E(1-\nu)}},$$

in which C_L, ρ, λ, μ, E and ν are the longitudinal wave velocity, mass density, Lamé constant, shear modulus, Young's modulus and Poisson's ratio, respectively. The boundary conditions on the top and bottom layers of the multilayered medium can be written as

$$\sigma_{xx}^{(1)}(0,t) = \sigma_0 \cdot f(t), \tag{2}$$

$$\sigma_{xx}^{(n)}\left(-\sum_{k=1}^{n} h_k, t\right) = 0, \tag{3}$$

where $f(t)$ is the traction function and σ_0 is a constant stress. The displacement and traction continuity conditions at the interface between two adjacent layers, i.e., i th layer and $(i+1)$ th layer, are expressed as follows

$$u^{(i)}\left(-\sum_{k=1}^{i} h_k, t\right) = u^{(i+1)}\left(-\sum_{k=1}^{i} h_k, t\right), \qquad \text{for } i = 1,2,3,\ldots,n-1, \tag{4}$$

$$\sigma_{xx}^{(i)}\left(-\sum_{k=1}^{i} h_k, t\right) = \sigma_{xx}^{(i+1)}\left(-\sum_{k=1}^{i} h_k, t\right), \qquad \text{for } i = 1,2,3,\ldots,n-1, \tag{5}$$

where the superscripts i in parentheses indicate the field quantities in the ith layer. For instance, $\square^{(i)}$ and $\square^{(i+1)}$ denote the displacement or stress fields in the ith layer and the (i+1)th layer, respectively. The boundary value problem and continuity conditions described above are solved by applying Laplace transform over time t with transform parameter p. The transform pair of the Laplace transform for a function $u(x,t)$ are given by

$$\hat{u}(x;p) = \int_0^{\infty} u(x,t)\, e^{-pt} dt, \tag{6a}$$

$$u(x,t) = \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} \hat{u}(x;p)\, e^{pt} dp. \tag{6b}$$

Apply the Laplace transform on Eq. (1), the general solution of displacement field in the transform domain can be presented as

$$\hat{u}\left(x;p\right) = u_{-}(p)e^{+pS_L x} + u_{+}(p)e^{-pS_L x}, \tag{7}$$

and the stress field is obtained follows Hooke's law

$$\hat{\sigma}_{xx}\left(x;p\right) = \rho C_L p u_{-}(p)e^{+pS_L x} - \rho C_L p u_{+}(p)e^{-pS_L x}. \tag{8}$$

Hence, we can rewrite these field quantities in transform domain as the displacement-traction matrix

$$\begin{bmatrix} \hat{u}\left(x;p\right) \\ \hat{\sigma}_{xx}\left(x;p\right) \end{bmatrix} = \begin{bmatrix} M_{11}(x;p) & M_{12}(x;p) \\ M_{21}(x;p) & M_{22}(x;p) \end{bmatrix} \begin{bmatrix} u_{-}(p) \\ u_{+}(p) \end{bmatrix}, \tag{9}$$

where

$$M_{11}(x;p) = e^{+pS_L x}, \tag{10}$$

$$M_{12}(x;p) = e^{-pS_L x}, \tag{11}$$

$$M_{21}(x;p) = \rho C_L p e^{+pS_L x}, \tag{12}$$

$$M_{22}(x;p) = -\rho C_L p e^{-pS_L x}, \tag{13}$$

are phase-related receiver elements. In order to avoid complicated mathematical expressions, boundary and interface continuity conditions can be represented as follows

$$\begin{bmatrix}
M_{21}^{(1)}(0) & M_{22}^{(1)}(0) & 0 & 0 & 0 & \cdots & \cdots & 0 \\
M_{11}^{(1)}(-h_1) & M_{12}^{(1)}(-h_1) & -M_{11}^{(2)}(-h_1) & -M_{12}^{(2)}(-h_1) & 0 & \cdots & \cdots & 0 \\
M_{21}^{(1)}(-h_1) & M_{22}^{(1)}(-h_1) & -M_{21}^{(2)}(-h_1) & -M_{22}^{(2)}(-h_1) & \vdots & \cdots & \cdots & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & & & \vdots \\
\vdots & \vdots & \ddots & \ddots & \ddots & & & \vdots \\
0 & \cdots & M_{11}^{(n-1)}\left(-\sum_{k=1}^{n-1} h_k\right) & M_{12}^{(n-1)}\left(-\sum_{k=1}^{n-1} h_k\right) & -M_{11}^{(n)}\left(-\sum_{k=1}^{n-1} h_k\right) & -M_{12}^{(n)}\left(-\sum_{k=1}^{n-1} h_k\right) & 0 & 0 \\
0 & \cdots & M_{21}^{(n-1)}\left(-\sum_{k=1}^{n-1} h_k\right) & M_{22}^{(n-1)}\left(-\sum_{k=1}^{n-1} h_k\right) & -M_{21}^{(n)}\left(-\sum_{k=1}^{n-1} h_k\right) & -M_{22}^{(n)}\left(-\sum_{k=1}^{n-1} h_k\right) & 0 & 0 \\
0 & \cdots & \cdots & \cdots & \cdots & 0 & -M_{21}^{(n)}\left(-\sum_{k=1}^{n} h_k\right) & -M_{22}^{(n)}\left(-\sum_{k=1}^{n} h_k\right)
\end{bmatrix} \cdot \begin{bmatrix} u_{-}^{(1)} \\ u_{+}^{(1)} \\ u_{-}^{(2)} \\ u_{+}^{(2)} \\ \vdots \\ \vdots \\ u_{-}^{(n)} \\ u_{+}^{(n)} \end{bmatrix} = \begin{bmatrix} \sigma_0 \hat{f}(p) \\ 0 \\ 0 \\ \vdots \\ \vdots \\ \vdots \\ 0 \\ 0 \end{bmatrix}. \tag{14}$$

In compact notation, the previous equation is written as

$$\mathbf{Mc} = \hat{\mathbf{t}}, \tag{15}$$

where

$$\mathbf{c} = \begin{pmatrix} u_{-}^{(1)} & u_{+}^{(1)} & u_{-}^{(2)} & u_{+}^{(2)} & \cdots & u_{-}^{(n)} & u_{+}^{(n)} \end{pmatrix}^T, \tag{16}$$

and

$$\hat{\mathbf{t}} = \left(\sigma_0 \hat{f}(p) \quad 0 \quad \cdots \quad \cdots \quad 0\right)^T, \tag{17}$$

the coefficient matrix $\mathbf{M}$ is a $2n \times 2n$ matrix. Subsequently, Eq. (15) can be solved directly by

$$\mathbf{c} = \mathbf{M}^{-1}\hat{\mathbf{t}}. \tag{18}$$

Once the global field vector $\mathbf{c}$ is obtained, the response functions in each layer can be determined. Furthermore, we can relate the response vector $\mathbf{b}$ to the global field vector $\mathbf{c}$ with a phase-related receiver matrix $\mathbf{R_{cv}}$ by arranging the response functions in each layer into this response vector

$$\mathbf{b}(x;p) = \mathbf{R_{cv}}(x;p) \cdot \mathbf{c}, \tag{19}$$

where the phase-related receiver matrix is given by

$$\mathbf{R_{cv}}(x;p) = \left[\begin{array}{cc:cc:c:cc} M_{11}^{(1)}(x;p) & M_{12}^{(1)}(x;p) & & & & & \\ M_{21}^{(1)}(x;p) & M_{22}^{(1)}(x;p) & & & & & \\ \hdashline & & M_{11}^{(2)}(x;p) & M_{12}^{(2)}(x;p) & & & \\ & & M_{21}^{(2)}(x;p) & M_{22}^{(2)}(x;p) & & & \\ \hdashline & & & & \ddots & & \\ \hdashline & & & & & M_{11}^{(n)}(x;p) & M_{12}^{(n)}(x;p) \\ & & & & & M_{21}^{(n)}(x;p) & M_{22}^{(n)}(x;p) \end{array}\right]. \tag{20}$$

It is noted that $\mathbf{b}$ is the response vector, which represents the solutions of displacement and normal stress in each layer of the multilayered medium in the transform domain. With the transformed solution at hand, the inverse transform is performed to obtain the transient solution in time domain. We use numerical inversion of the Laplace transform from the well-known Durbin's method, which is a combination of finite Fourier sine and cosine transforms, and will be briefly described in the next section.

3. Numerical laplace inversion

In the procedure for executing the traditional inversion of Laplace transformation by analytical approach, branch cuts or residues are usually needed in the complex plane of p. If boundary conditions become complicated, traditional analytical Laplace inversion is too difficult to be used. Hence, in order to reduce the difficulty, numerical inversion methods were proposed by different researchers in the literature, i.e., Durbin (1974), Papoulis (1957), Narayanan and Beskos (1982). In this paper, the method proposed by Durbin, which is an accurate and efficient method for numerically inverting Laplace-transformed functions is used. In Durbin's method, the inverse

Laplace transformation of a function $\hat{f}(p)$ is expressed as the following series

$$f(t)=\frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty}\hat{f}(p)\,e^{pt}dp$$
$$=\frac{2e^{\alpha t}}{T}\left\{-\frac{1}{2}\mathrm{Re}\left[\hat{f}(\alpha)\right]+\sum_{k=0}^{N}\mathrm{Re}\left[\hat{f}\left(\alpha+i\frac{2k\pi}{T}\right)\right]\cos\left(\frac{2k\pi t}{T}\right)-\mathrm{Im}\left[\hat{f}\left(\alpha+i\frac{2k\pi}{T}\right)\right]\sin\left(\frac{2k\pi t}{T}\right)\right\}. \quad (21)$$

Note that the infinite series involved can only be summed up to N terms, and the transform parameter p is composed of real part α and imaginary part $\frac{2k\pi}{T}$

$$p=\alpha+ik\frac{2\pi}{T} \qquad \text{for } k=0, 1, 2, 3, \ldots, N, \quad (22)$$

in which T is the total time interval of interest, and the number of equidistant points, N, is a finite positive integer. It is suggested that $\alpha T=5$ to 10 can be used for good results. The matrix-form solution, i.e., Eq. (19), is substituted into Durbin's formula Eq. (21) to obtain the transient response in time domain.

Table 1 Material constants of Pb and Epoxy used in 20-layered medium

Material	Density (kg/m^3)	Longitudinal Wave Velocity (m/s)	Impedance	Thickness in Each Layer (cm)
Pb	11600	2050	23780000	2
Epoxy	1180	2535	2991300	1

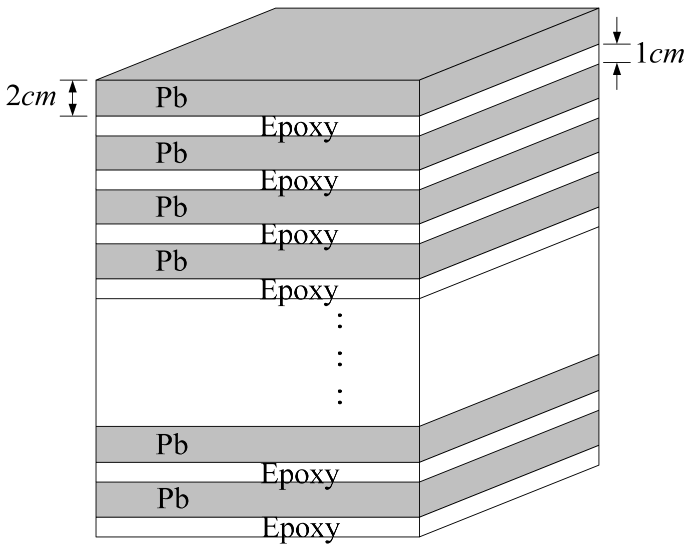

Fig. 2 One-dimensional binary materials (Pb-Epoxy) periodically arranged as a 20-layered medium

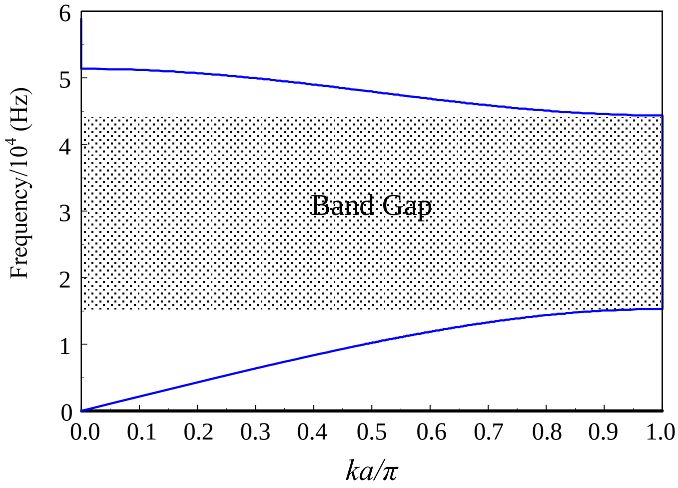

Fig. 3 The band structure of Pb-Epoxy phononic system

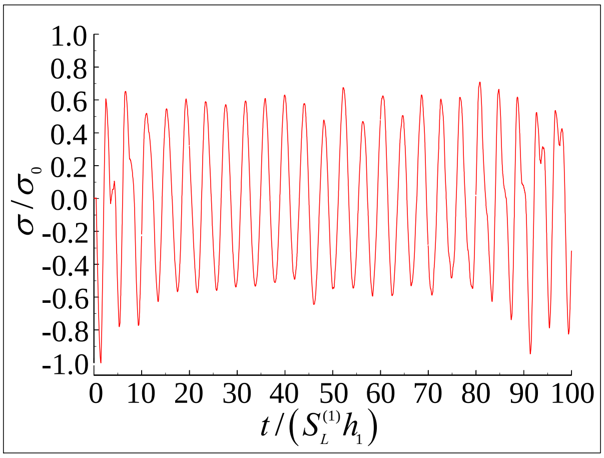

Fig. 4 Transient stress response at the midpoint of the first layer ($\omega = 25000$ Hz)

4. Numerical results of the transient responses and discussion

Consider a one-dimensional binary materials (Pb and Epoxy) are periodically arranged as a 20-layered medium as shown in Fig. 2. The material constants are indicated in Table 1. The thickness of Pb and Epoxy is 2 cm and 1 cm, respectively. A dynamic sinusoidal loading is applied on the top surface of the 20-layered medium. The function of sinusoidal loading is expressed as follows

$$f(t) = \sin(\omega t), \tag{23}$$

where ω is the circular frequency. This function expressed in the transform domain is

$$\hat{f}(p)=\frac{\omega}{p^2+\omega^2}. \tag{24}$$

Substitute Eq. (24) into the response vector, i.e., Eqs. (17)-(19), the response vector $\mathbf{b}$ is a 40×1 vector, the phase-related receiver matrix $\mathbf{R_{cv}}$ is a 40×40 matrix, and the global field vector $\mathbf{c}$ is a 40×1 vector. Therefore, the matrix-form formulation indicated in Eq. (19) can be worked out, the displacement and stress fields in the transform domain can be determined subsequently. The numerical Laplace inversion (Durbin's method) can be performed to obtain the transient response. For the numerical calculation, $\alpha T=5$ and $N=10000$ are used in the Durbin's method.

The band structure of Pb-Epoxy phononic system obtained by plane wave expansion method is shown in Fig. 3. The band gap exists for frequencies between 16000 Hz and 44000 Hz. Next, we will calculate and discuss the transient responses for three kinds of source frequency, i.e., 10000 Hz, 25000 Hz, and 50000 Hz, for different receivers. We begin with the frequency $\omega = 25000\,\text{Hz}$ which is in the band gap. At the midpoint of the first layer (Pb), the stress response for the

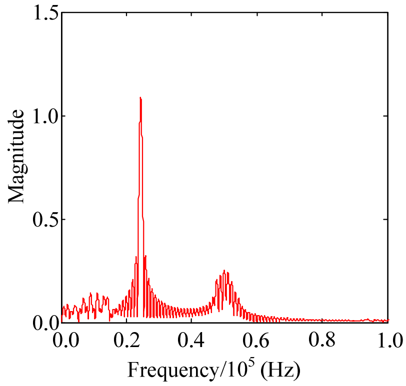

Fig. 5 Frequency spectrum at the midpoint of the first layer

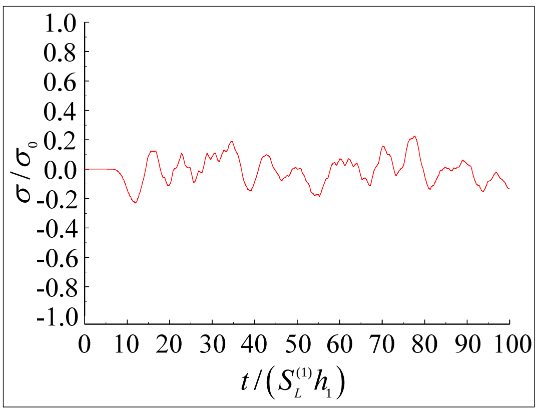

Fig. 6 Transient stress response at the midpoint of the 10th layer ($\omega = 25000\,\text{Hz}$)

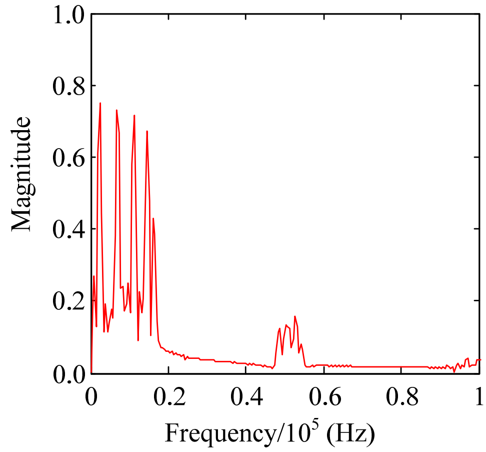

Fig. 7 Frequency spectrum at the midpoint of the 10th layer

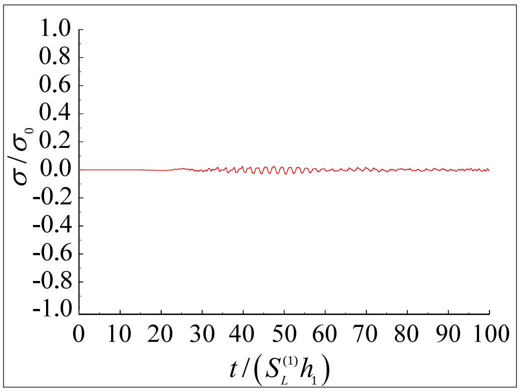

Fig. 8 Transient stress response at the midpoint of the 20th (last) layer ($\omega = 25000$ Hz)

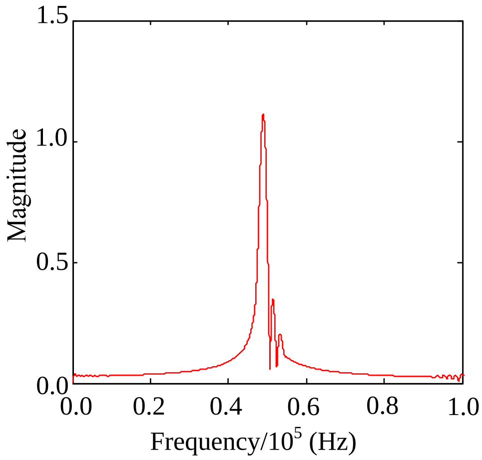

Fig. 9 Frequency spectrum at the midpoint of the 20th (last) layer

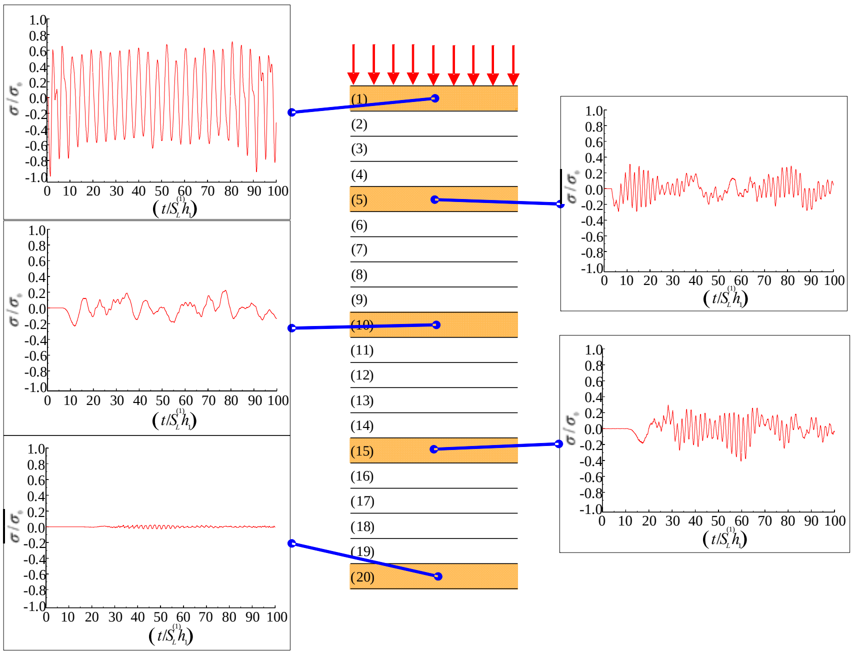

Fig. 10 Stress responses for the 1st, 5th, 10th, 15th, and 20th layer when a sinusoidal loading ($\omega = 25000$ Hz) applied on the upper surface

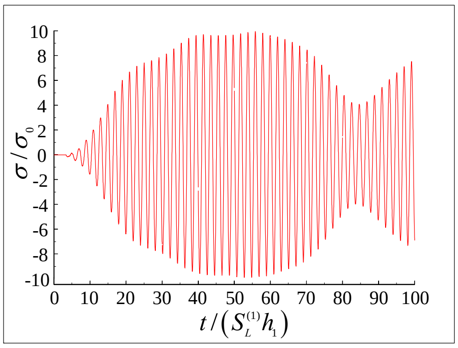

Fig. 11 Transient stress response at the midpoint of the 5th layer ($\omega = 50000$ Hz)

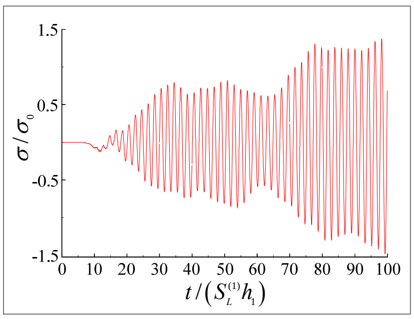

Fig. 12 Transient stress response at the midpoint of the 10th layer ($\omega = 50000$ Hz)

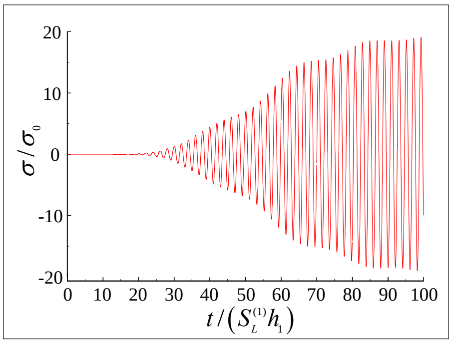

Fig. 13 Transient stress response at the midpoint of the 15th layer ($\omega = 50000$ Hz)

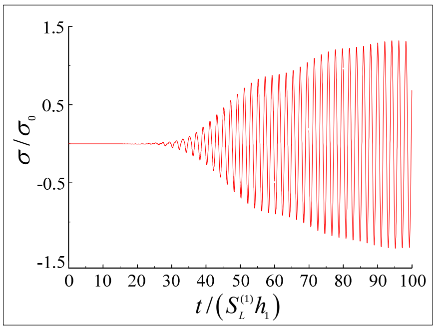

Fig. 14 Transient stress response at the midpoint of the 20th layer ($\omega = 50000$ Hz)

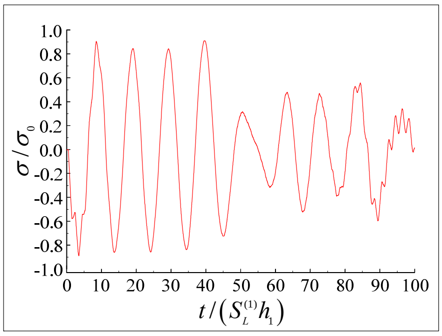

Fig. 15 Transient stress response at the midpoint of the first layer ($\omega = 10000$ Hz)

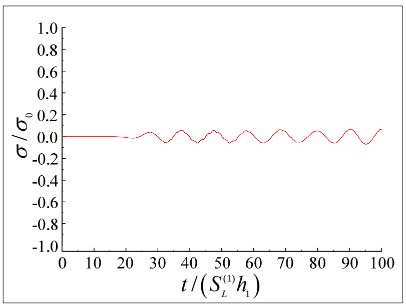

Fig. 16 Transient stress response at the midpoint of the 20th layer ($\omega = 10000$ Hz)

20-layered medium is shown in Fig. 4. The source wave generated from the upper surface arrives at the normalized time $t/\left(S_L^{(1)}h_1\right)=0.5$, and the transient response for stress oscillates between $\sigma/\sigma_0=1$ and -1 after $t/\left(S_L^{(1)}h_1\right)=0.5$. Figure 5 shows the frequency spectrum for Fig. 4 and indicates that the dominated frequency is 25000 Hz which is the same as the source function. In Fig. 6, when the receiver is located at the midpoint of the 10th layer (Epoxy), the magnitude of the stress is significantly reduced and oscillates between $\sigma/\sigma_0=0.2$ and -0.2. The frequency spectrum is indicated in Fig. 7 which shows the dominated frequency is below 20000 Hz. In Fig. 8, transient response at the midpoint of the 20th layer (Epoxy) is observed with extremely small magnitude. The FFT is applied on the transient response (Fig. 8) to obtain the frequency spectrum, and the result is shown in Fig. 9. As shown in Fig. 9, source frequency 25000 Hz disappear which indicates that the source frequency 25000 Hz can not propagate inside the periodic medium. Stress responses for the midpoint of 1st, 5th, 10th, 15th and 20th are expressed in Fig. 10. It is concluded that when a sinusoidal loading with frequency within the band gap applied on the upper surface of the medium, stress response will decay significantly as the receiver is away from the source.

For the source frequency $\omega=50000\,\mathrm{Hz}$ (out of band gap), transient stress responses at the midpoint of the 5th, 10th, 15th and 20th are shown in Figs. 11-14, respectively. It is observed from Figs. 11-14 that stress responses are divergent within normalized time $t/\left(S_L^{(1)}h_1\right)=100$. For the source frequency $\omega=10000\,\mathrm{Hz}$ (out of band gap), transient stress responses at the midpoint of the 1st and 20th are shown in Figs. 15-16, respectively. It is observed that the magnitude of the stress response is reduced to $0.2\sigma_0$.

5. Conclusions

In this study, we analyze the band gap phenomenon of phononic system from the viewpoint of transient wave-propagation. The transient response induced by wave propagation in a multilayered medium is determined by the Laplace transform technique. The analytical solution in the transform domain is obtained and the transient response in time domain is constructed by an numerical Laplace inversion method (Durbin's formula). A one-dimensional phononic system is simulated by a 20-layered binary medium. When a sinusoidal loading with frequency within the band gap applied on the upper surface of the medium, stress response will decay significantly if the receiver is away from the source. For source frequency $\omega=25000\,\mathrm{Hz}$ (within the band gap), the magnitude of stress response at the last layer is almost vanished. However, for source frequency (10000 Hz or 50000 Hz) is out of the band gap, the magnitude of stress responses at the last layer of the 20-layered medium will not be small.

Acknowledgments

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