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Optimal design of homogeneous earth dams by particle swarm optimization incorporating support vector machine approach

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Abstract. The main aim of this study is to introduce optimal design of homogeneous earth dams with oblique and horizontal drains based on particle swarm optimization (PSO) incorporating weighted least squares support vector machine (WLS-SVM). To achieve this purpose, the upstream and downstream slopes of earth dam, the length of oblique and horizontal drains and angle among the drains are considered as the design variables in the optimization problem of homogeneous earth dams. Furthermore, the seepage through dam body and the weight of dam as objective functions are minimized in the optimization process simultaneously. In the optimization procedure, the stability coefficient of the upstream and downstream slopes and the seepage through dam body as the hydraulic responses of homogeneous earth dam are required. Hence, the hydraulic responses are predicted using WLS-SVM approach. The optimal results of illustrative examples demonstrate the efficiency and computational advantages of PSO with WLS-SVM in the optimal design of homogeneous earth dams with drains.

Keywords: homogeneous earth dams; oblique and horizontal drains; particle swarm optimization; weighted least squares support vector machine

1. Introduction

Homogeneous earth dams have been constructed to optimally utilize local materials for control and surface runoff storage. These structures have been built whenever only one type of material has been economically available (Chahar 2004). The stability analysis of the upstream and downstream slopes and the seepage through earth dam body are considered as two important items in geotechnical engineering. The seepage through dam body should be effectively controlled to prevent structural damage or interference with normal operations. In order to control and safely discharge seepage, oblique and horizontal drains are designed, and employed in earth dam body. The design of these drains must also provide sufficient flow capacity to safely control seepage through potential cracks in the impervious zone of earth dam. Horizontal drain can control seepage

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through body of a homogeneous earth dam, or prevent excessive uplift pressures from foundation seepage. Furthermore, the oblique and horizontal drain can prevent seepage from emerging on the downstream slope of dam. The oblique drain is also useful to reduce pore water pressures both during construction and following rapid reservoir drawdown.

The economical and safe design of an earth dam as an important aspect of geotechnical engineering depends on an appropriate shape design of this structure and its drains. Hence, finding a proper shape design of an earth dam and its drains is considered as an important problem in design approach of these dams. In order to achieve this aim, several alternative schemes with various patterns are selected and modified to obtain a number of feasible shapes. Therefore, the proper shape of earth dam with drains considering the economy and safety of design, structural considerations, etc. is selected as the final shape. In order to reliably achieve an optimal shape for earth dams with drains instead of the traditional and time–consuming trial–and–error design procedure, optimization techniques can be effectively utilized. Therefore, these techniques can guarantee achieving the economy and safety of final design of earth dam.

During the last few years, a number of researchers have investigated optimization techniques in the optimal design field of earth dams. Xu *et al*. (2003) proposed the optimal hydraulic design of earth dam cross–section using saturated–unsaturated seepage flow model. In this study, the optimal hydraulic design problem regarding an earth dam cross–section was formulated as an inverse problem. Abdul Hussein *et al*. (2007) presented a multi–objective approach with variably saturated numerical model in the optimal design of earth dams. In their research, the objective function included dam section, wetted area, and flow rate and drainage section. Roshani and Farsadizadeh (2012) introduced a multi–optimization of clay core dimensions in earth fill dams using particle swarm algorithm. In this work, the water seepage volume through the dam core and the costs regarding to the volume of earth dams were considered as objective functions. Mohammadi *et al*. (2013) utilized simulated annealing to determine optimal dimensions of clay core in earth dams.

In this study, optimal design of homogeneous earth dams with oblique and horizontal drains is introduced based on particle swarm optimization (PSO) incorporating weighted least squares support vector machine approach (WLS–SVM). The upstream and downstream slopes of earth dam, the length of oblique and horizontal drains and angle among the drains are selected as the design variables of the optimization problem. Furthermore, the seepage through the dam body and the weight of homogeneous earth dam are simultaneously minimized in the optimization process. In this work, the factor of safety against sliding of a part of the upstream and downstream dam slopes is considered as the constraints of the optimization problem. Hence, the hydraulic responses of homogeneous earth dam including the stability coefficient of the upstream and downstream slopes and the seepage through dam body are required in the optimization procedure. In order to compute these responses in the optimization procedure, in this study, first, a database of homogeneous earth dams based on the random combination of the design variables is generated. The hydraulic responses of dams are obtained using SEEP/W and SLOPE/W as the sub–programs of Geo–Studio software that is one of the popular geotechnical programs based on the finite element method (FEM). Then, WLS–SVM regression model is trained using the combinations of the design variables and their hydraulic responses in database. Therefore, WLS–SVM model is employed to predict the hydraulic responses of homogeneous earth dams in the optimization procedure. It is noted that finite element analysis (FEA) of earth dams using SEEP/W and SLOPE/W cannot directly performed in optimization procedure. In order to demonstrate the effectiveness and the computational advantages of PSO incorporating WLS–SVM approach in the

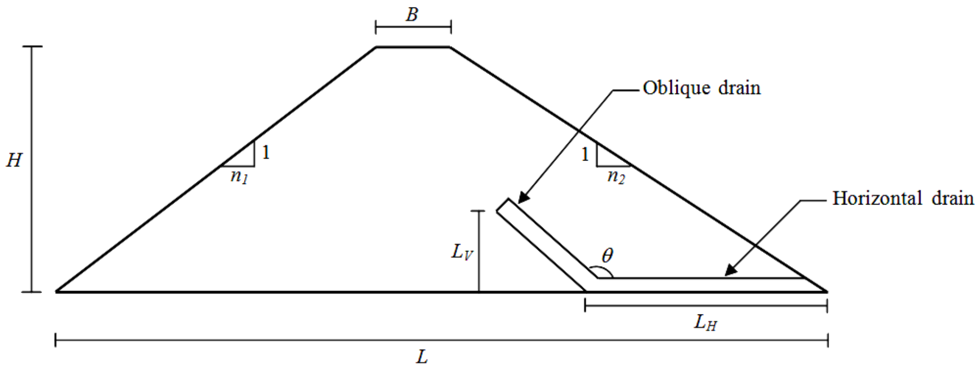


Fig. 1 A schematic cross–section of a homogeneous earth dam

optimal design of earth dams with drains, some homogenous earth dams with specific height are considered. The numerical results show that the proposed procedure can be efficiently utilized in the optimal design of homogeneous earth dams. Furthermore, the WLS–SVM regression model as an approximate analyzer with high accuracy can be reliably incorporated into the optimization process of homogeneous earth dams.

2. Design parameters of homogeneous earth dam

In order to design a homogeneous earth dam, the geometry of homogeneous earth dam can be defined using the upstream and downstream slopes with the oblique and horizontal drains shown in Fig. 1.

According to the model shown in Fig. 1, a schematic cross–section of a homogeneous earth dam with drains can be created by a vector $\boldsymbol{X}$ that has five components including length of drains with angle among them and shape parameters of a homogeneous earth dam as

$$\boldsymbol{X} = \{n_1 \;\; n_2 \;\; L_H \;\; L_V \;\; \theta\} \tag{1}$$

where n_1 and n_2 are the upstream and downstream slopes of the homogeneous earth dam, respectively. In this study, the values of the upstream and downstream slopes of the homogeneous earth dam are considered as equal. L_H and L_V are the length of horizontal and oblique drains, respectively. Furthermore, θ is considered as the angle between oblique and horizontal drains.

A proper design of a homogeneous earth dam based on these parameters has been considered as an important problem in dam engineering. An appropriate design of earth dam influences on the economy and safety of earth dam. Hence, in this study, the optimization technique is employed to find the optimal value of these parameters and obtain an optimal design of homogeneous earth dam.

3. Optimization problem of homogeneous earth dams

In this study, the optimal design of homogeneous earth dams is presented as a mutli–objective function. This optimization problem is defined as follows

$$\begin{aligned} &\text{Minimize:} \quad \text{Obj} = \{W(\boldsymbol{X}), Q(\boldsymbol{X})\}^T \\ &\text{subject to:} \quad g_j(\boldsymbol{X}) \geq \bar{g}_j \quad ; \quad j = 1, 2, ..., m \\ &\qquad \text{and} \quad \boldsymbol{X}^L \leq \boldsymbol{X} \leq \boldsymbol{X}^U \end{aligned} \tag{2}$$

where W and Q are the weight of dam and the seepage through the dam body, respectively. Also, g_j and $\bar{g}$ are jth the constraint and the allowable value of jth the constraint. X^L and X^U are the lower bound and the upper bound of the design variables, $\boldsymbol{X}$.

According to the geometrical model of the earth dam described in Section 2, the vector of design variables, $\boldsymbol{X}$, can be adopted from Eq. (1). In this optimization problem, the weight of an earth dam body is determined as

$$W(\boldsymbol{X}) = \frac{(B+L)}{2} H \gamma_d \tag{3}$$

where γ_d, B and L are the soil density, the crest width and the width of dam, respectively.

The stability of the upstream and downstream slopes is considered as extremely important in the design and construction of earth dams. Hence, in this study, the factor of safety against sliding of a part of the upstream and downstream slopes is selected as the constraint of the optimization problem. The allowable of this parameter for the downstream slope is usually taken between 1.25 and 1.5 (Ranjan and Rao 2000, US Army Corps 2003). Furthermore, the allowable of this parameter for the upstream slope is usually selected between 1.15 and 1.3 (Ranjan and Rao 2000, US Army Corps 2003). In this study, the allowable value of this constraint for both the upstream and downstream slope is taken as 1.25, i.e.

$$F_S \geq 1.25 \tag{4}$$

where F_S is the factor of safety against sliding.

3.1 Multi–objective optimization formulation

To solve a multi–objective optimization problem, the classical approach called weighted sum approach is to combine all objectives into a single objective (Rao 1996). The main advantage of the weighted sum approach is considered as a straightforward implementation. Since a single objective is used in fitness assignment, a single objective can be used with minimum modifications. Furthermore, this approach is computationally efficient. In this method, a weight, w_i, is assigned to each normalized objective function, $f_i'(X)$, so that the problem is converted to a single objective problem with a scalar objective function. Therefore, in the optimization problem of earth dams the scalar objective function is expressed as follows

$$\begin{aligned} &f(\boldsymbol{X}) = w_1 f_1'(\boldsymbol{X}) + w_2 f_2'(\boldsymbol{X}) \\ &w_1 + w_2 = 1 \quad and \quad w_1, w_2 \leq 1 \end{aligned} \tag{5}$$

Also, this approach is called as the priori approach since the user is expected to provide the weights. The main difficulty with this approach is the selection of a weight vector for each run. To solve this problem, the researchers proposed a weighted sum of multiple objective functions where a normalized weight vector is randomly generated for each solution in each iteration of optimization algorithm as follows (Rao 1996)

$$w_i = \frac{rand_i}{\sum_{j=1}^{2} rand_j} \quad ; \quad i = 1,2 \tag{6}$$

where $rand_i$ and $rand_j$ are non–negative random real numbers in [0, 1], respectively.

It is noted that the weight of each function can be considered as variable in optimization procedure (Rao 1996). Hence, in this study the weight of each function is selected as variable and optimized in the optimization procedure.

3.2 Constrain–handling technique

A number of constraint–handling techniques have been proposed in order to solve constrained optimization problems (Coello 2002). The penalty function method has been introduced as the most popular constraint-handling technique for the engineering problems because of its simple principle and ease of implementation (Rajeev and Krishnamoorthy 1992, Salajegheh *et al*. 2008, Salajegheh *et al*. 2009, Khatibinia *et al*. 2013b, Yazdani *et al*. 2013, Khatibinia and Naseralavi 2014). By using the concept of the penalty function, the objective function of constrained structural optimization problems is defined as follows

$$fit(\boldsymbol{X}) = \begin{cases} f(\boldsymbol{X}) & \text{if } \boldsymbol{X} \in R^d \\ f(\boldsymbol{X})\left(1 + r_p\, G(\boldsymbol{X})\right) & \text{otherwise} \end{cases} \tag{7}$$

where $fit(X)$ and r_p is the modified function (fitness function) and an adjusting coefficient, respectively; R^d denotes the feasible search space; and $G(X)$ is the penalization factor, which is defined as the sum of all active constraints violations, as indicated

$$G(\boldsymbol{X}) = \max\left[1 - \frac{F_S}{1.25}, 0.0\right]^2 \tag{8}$$

Hence, this formulation allows that for solutions with violated constraints, the objective function is always greater than the non–violated one (Khatibinia and Naseralavi 2014).

4. Particle swarm optimization

In recent years, in engineering problem the successful applications of particle swarm optimization (PSO) as an optimization engine have been reported by Salajegheh *et al*. (2008), Salajegheh *et al*. (2009), Seyedpoor *et al*. (2012), Mahani *et al*. (2015) and Gharehbaghi and Khatibinia (2015). Hence, in this study, PSO is used to find the optimal values of the design variables. The PSO algorithm has been inspired by the social behavior of animals such as fish schooling, insects swarming and birds flocking. The standard PSO was introduced by Kennedy and Eberhart (2001) in the mid 1990s, while attempting to simulate the graceful motion of bird swarms as a part of a socio–cognitive study. It involves a number of particles, which are initialized randomly in the search space of an objective function. These particles are referred to as swarm. Each particle of the swarm represents a potential solution of the optimization problem. The *i*th particle in *t*th iteration is associated with a position vector, X_i^t, and a velocity vector, V_i^t, that shown as following

$$\begin{aligned} \boldsymbol{X}_i^t &= \{ x_{i1}^t, x_{i2}^t, \ldots, x_{iD}^t \} \\ \boldsymbol{V}_i^t &= \{ v_{i1}^t, v_{i2}^t, \ldots, v_{iD}^t \} \end{aligned} \tag{9}$$

where D is dimension of the solution space.

The particle fly through the solution space and its position is updated based on its velocity, the best position particle (***pbest***) and the global best position (***gbest***) that swarm has visited since the first iteration as

$$\boldsymbol{V}_i^{t+1} = \omega^t \boldsymbol{V}_i^t + c_1 r_1 (\boldsymbol{pbest}_i^t - \boldsymbol{X}_i^t) + c_2 r_2 (\boldsymbol{gbest}^t - \boldsymbol{X}_i^t) \tag{10}$$

$$\boldsymbol{X}_i^{t+1} = \boldsymbol{X}_i^t + \boldsymbol{V}_i^{t+1} \tag{11}$$

where r_1 and r_2 are two uniform random sequences generated from interval [0, 1]; c_1 and c_2 are the cognitive and social scaling parameters, respectively; and ω^t is the inertia weight used to discount the previous velocity of particle preserved.

Shi and Eberhart (1998) proposed that the cognitive and social scaling parameters c_1 and c_2 to be selected such that $c_1 = c_2 = 2$ to allow the product $c_1 r_1$ or $c_2 r_2$ to have a mean of 1. One of the main parameters affecting the performance of PSO is the inertia weight, ω, in achieving efficient search behaviour. A dynamic variation of inertia weight is proposed by linearly decreasing the inertia weight, ω, with each iteration algorithm as (Shi and Eberhart 1998)

$$\omega = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{t_{\max}} t \tag{12}$$

where $\omega_{\max}$ and $\omega_{\min}$ are the maximum and minimum values of ω, respectively. Also, $t_{\max}$ is the numbers of maximum iteration. It is noted that the linearly decreasing inertia weight of PSO has provided the better balance between the global search and local search (Shi *et al.* 2011).

5. Finite element model of homogeneous earth dam

In the present study, the seepage through the dam body and the stability coefficient of upstream and downstream slopes are considered as the hydraulic responses which required in the optimization procedure. To obtain these hydraulic responses, in this study the finite element idealization of homogeneous earth dam with oblique and horizontal drains is implemented in Geo–Studio software (Geo–Slope 1998). Geo–studio as one of the popular geotechnical programs is based on finite element method (FEM) that can be used to evaluate the performance of dams and levees with varying levels of complexity. SEEP/W and SLOPE/W as the sub–programs of Geo–Studio software are employed to achieve the seepage through the dam body and the factor of safety against sliding, respectively. In SEEP/W and SLOPE/W softwares, the dam body is modeled by using four–node isoparametric element and the foundation of the dam is also taken as being rigid. The geometry and finite element mesh of a homogeneous earth dam with oblique and

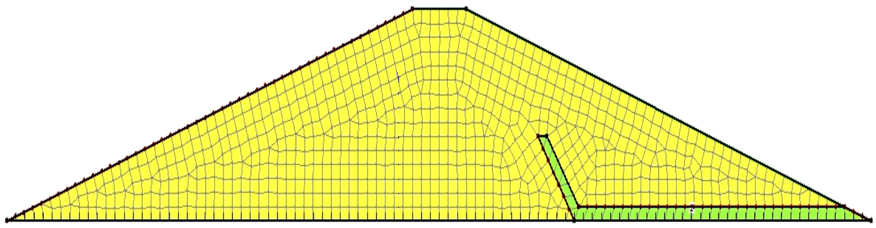


Fig. 2 Geometry and finite element mesh of homogeneous earth dam with oblique and horizontal drains

horizontal drains modeled in SEEP/W and SLOPE/W is shown in Fig. 2.

Sub–program SEEP/W calculates the leak using partial differential equations that make the water flow. The general differential equation governing for the in two–dimensional seepage can be expressed as follows

$$\frac{\partial}{\partial x}\left[K_x \frac{\partial h}{\partial x}\right]+\frac{\partial}{\partial y}\left[K_y \frac{\partial h}{\partial y}\right]=0 \tag{13}$$

where K_x and K_y are the coefficient of permeability in x and y directions, respectively. Also, h is the total head of water.

Sub–program SLOPE/W carries out limit equilibrium slope stability analysis of existing natural slopes, unreinforced man–made slopes, or slopes with soil reinforcement. The program uses many methods such as: Bishop's Modified method, Janbu's Simplified method, Spencer method, Morgenstern–Price method and others. SLOPE/W allows these methods to be applied to circular, composite, and non–circular surfaces (Geo–Slope 1998). In this study, the Bishop's simplified method incorporated into the Mohr–Coulomb's failure criterion is utilized to determine the stability coefficient. A simple form of the Bishop's simplified factor of safety equation in the absence of any pore water pressure is

$$F_S=\frac{1}{\Sigma W \sin\alpha}\Sigma\left[\frac{c\beta+W\tan\phi-\frac{c\beta}{F_S}\sin\alpha\tan\phi}{m_a}\right] \tag{14}$$

where W is the weight of the slice; c and ϕ are the effective cohesion and friction angle of soil, respectively. α is the inclination angle of the slice base. The equation is not unlike the ordinary factor of safety equation except for the m_a term, which is defined as

$$m_a=\cos\alpha+\frac{\sin\alpha\tan\phi}{F_S} \tag{15}$$

6. Prediction of the hydraulic responses using WLS–SVM

To obtain the hydraulic responses of earth dams in the optimization procedure, sub–programs SEEP/W and SLOPE/W cannot be used directly in the optimization procedure. Hence, WLS–SVM as regression model is employed in the optimization procedure to predict the hydraulic responses of earth dams. Recently, support vector machines (SVMs) have been successfully developed and used as an excellent machine learning algorithm to many engineering problems and have yielded encouraging results (Khatibinia *et al*. 2013a, Khatibinia and Khosravi 2014, Mahani *et al*. 2015, Gharehbaghi and Khatibinia 2015, Khatibinia *et al*. 2015). The SVM regression model was developed based on the structural risk minimization (SRM), and can escape from several drawbacks, such as local minimum and the necessity of a large number of controlling parameters in artificial neural networks (ANNs). As a simplification of traditional of SVM, the WLS–SVM model was introduced by Suykens *et al*. (2002) to decrease the training computational effort of SVM in the large–scale problem.

6.1 WLS–SVM regression model

WLS–SVM is described as the following optimization problem in primal weight space (Suykens *et al*. 2002)

$$\min J(\boldsymbol{w},\boldsymbol{e}) = \frac{1}{2}\|\boldsymbol{w}\|^2 + \frac{1}{2}\gamma\sum_{i=1}^{n}\bar{v}_i e_i^2 \tag{16}$$

subject to the following equality constraints

$$y_i = \boldsymbol{w}^T\varphi(\boldsymbol{x}_i) + b + e_i \ ; \qquad i = 1,2,\ldots,n \tag{17}$$

with $\{x_i, y_i\}_{i=1}^n$ a training data set, input data $x_i \in R^n$ and output data $y_i \in R$. $\varphi(.) : R^n \rightarrow R^d$ is a function which maps the input space into a higher dimensional space. The vector $w \in R^d$ represents weight vector in primal weight space. The symbols $e_i \in R$ and $b \in R$ represent error variable and bias term, respectively. By the optimization problem (17) and the training set, the model of WLS–SVM is defined as follows

$$y(\boldsymbol{x}) = \boldsymbol{w}^T\varphi(\boldsymbol{x}) + b \tag{18}$$

It is impossible to indirectly calculate w from (16), because the structure of the function $\varphi(x)$ is unknown in general. Therefore, the dual problem shown in Eq. (18) is minimized by the Lagrange multiplier method as follows

$$L(\boldsymbol{w},b,\boldsymbol{e};\boldsymbol{x}) = J(\boldsymbol{w},\boldsymbol{e}) - \sum_{i=1}^{n}\alpha_i(\boldsymbol{w}^T\varphi(\boldsymbol{x}_i) + b + e_i - y_i) \tag{19}$$

According to the Karush–Khun–Tucker (KKT) conditions, eliminating w and e the solution is given by the following set of linear equation

$$\begin{bmatrix} \Omega + \boldsymbol{V}_\gamma & \boldsymbol{1}_n^T \\ \boldsymbol{1}_n & \boldsymbol{0} \end{bmatrix}\begin{bmatrix} \boldsymbol{\alpha} \\ b \end{bmatrix} = \begin{bmatrix} \boldsymbol{y} \\ 0 \end{bmatrix} \tag{20}$$

$$\begin{gathered} \boldsymbol{V}_\gamma = diag\{1/\gamma\bar{v}_1,\ldots,1/\gamma\bar{v}_n\} \ ; \ \Omega_{i,j} = \left\langle\varphi(\boldsymbol{x}_i),\varphi(\boldsymbol{x}_j)\right\rangle_H \quad i,j = 1,\ldots,n \\ \boldsymbol{y} = [y_1,\ldots,y_n]^T ; \ \boldsymbol{1}_n^T = [1,\ldots,1] \ ; \ \boldsymbol{\alpha} = [\alpha_1,\ldots,\alpha_n] \end{gathered} \tag{21}$$

According to Mercer's condition, a kernel $K(.,.)$ is selected, such that

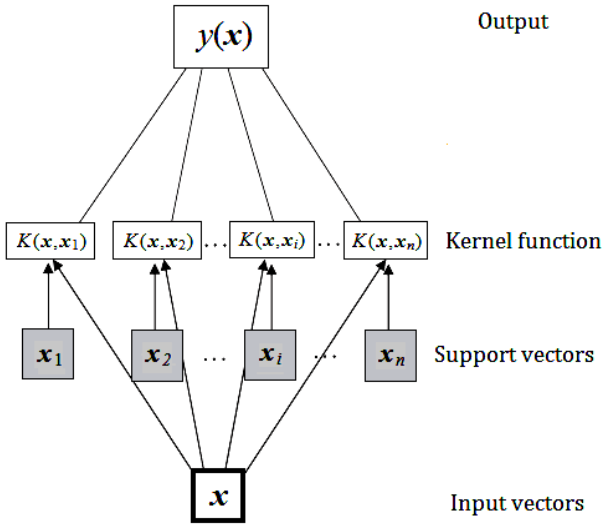


Fig. 3 The structure of WLS–SVM model

$$K(\boldsymbol{x},\bar{\boldsymbol{x}}) = \langle \varphi(\boldsymbol{x}), \varphi(\bar{\boldsymbol{x}}) \rangle_H \tag{22}$$

So, the resulting WLS–SVM model for the prediction of functions becomes

$$y(\boldsymbol{x}) = \sum_{i=1}^{n} \alpha_i \, K(\boldsymbol{x}_i, \boldsymbol{x}) + b \tag{23}$$

In this study, radial basis function (RBF) is selected as the kernel function of WLS–SVM. The kernel function is defined as follows

$$K(\boldsymbol{x}_i, \boldsymbol{x}_j) = \exp\left(-\frac{\left\|\boldsymbol{x}_i - \boldsymbol{x}_j\right\|^2}{2\sigma^2}\right) \tag{24}$$

where σ is positive real constant, and is usually so–called kernel width. The structure of WLS–SVM is shown in Fig. 3.

Based on the abovementioned experssions, the WLS–SVM model involves equality instead of inequality constraints and works with a least squares cost function (Suykens *et al*. 2002). Hence, the solution of the WLS–SVM is obtained based on a linear Karush–Kuhn–Tucker system instead of a quadratic programming problem. Furthermore, WLS–SVM can predict functions more robust and precise by assigning weights, and performance generality of WLS–SVM is better than that of least squares version of SVM (LS–SVM) (Suykens *et al*. 2002, Quan *et al*. 2010).

6.2 Predicting the hydraulic responses of homogeneous earth dams

To find the optimal design of a homogeneous earth dam, the hydraulic responses of homogeneous earth dams are required in the optimization procedure. The finite element analysis (FEA) of homogeneous earth dams with drains cannot be performed using sub–programs SEEP/W and SLOPE/W in the optimization procedure. Due to this problem, in the optimization procedure the hydraulic responses of dams are predicted using the WLS–SVM regression model instead of directly performing FEA of dams. To achieve this purpose, the WLS–SVM regression model is trained by using a randomly generated database which consists on the combinations of the design variables and their hydraulic responses. In the WLS–SVM model the 10–fold cross–validation (CV) is employed to find the optimal values of γ and σ for training the WLS–SVM model. Therefore, for predicting the hydraulic responses of earth dam training the WLS–SVM regression model based on RBF kernel function is performed by using the following procedure:

(1) A database for training and testing the WLS–SVM model is randomly generated. The design variable vector of earth dam defined in Eq. (1) is considered as the inputs of WLS–SVM, and the seepage through the dam body and the factor of safety (as the hydraulic responses of earth dam) are taken as the outputs of WLS–SVM.
(2) For each earth dam corresponding to each design variable vector in database FEA is performed by using sub–programs SEEP/W and SLOPE/W, and the hydraulic responses of earth dam are obtained.
(3) The provided database is divided to training and testing sets on a random basis.
(4) Two WLS–SVM models are assigned to the seepage through dam body and the stability coefficient. These models are trained and tested based on the generated sets.

In order to evaluate the performance of WLS–SVM, mean absolute percentage error (*MAPE*), relative root–mean–squared error (*RRMSE*) and absolute fraction of variance (R^2) are used as

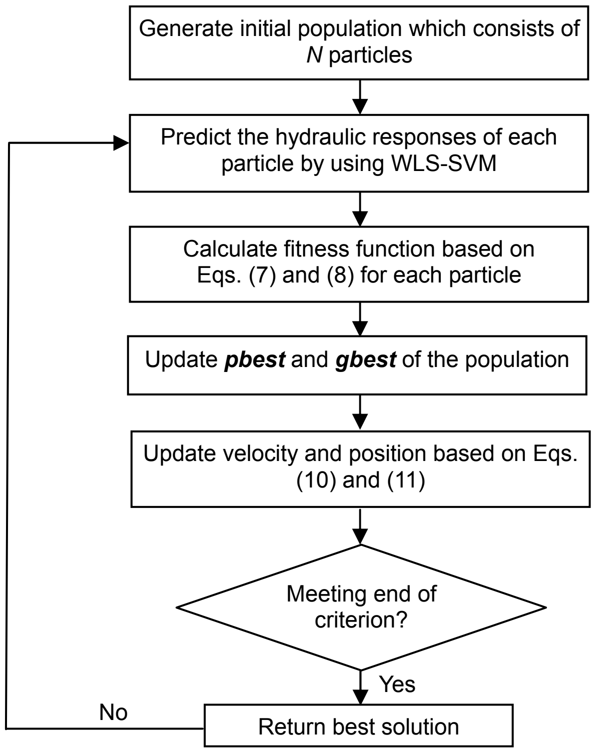


Fig. 4 Flowchart of PSO incorporating WLS–SVM

$$MAPE = \frac{1}{n_t}\sum_{i=1}^{n_t} 100 \times \left| \frac{y_i - \bar{y}_i}{y_i} \right| \tag{25}$$

$$RRMSE = \sqrt{\frac{n_t \sum_{i=1}^{n_t} (y_i - \bar{y}_i)^2}{(n_t - 1)\sum_{i=1}^{n_t} y_i^2}} \tag{26}$$

$$R^2 = 1 - \left(\frac{\sum_{i=1}^{n_t} (y_i - \bar{y}_i)^2}{\sum_{i=1}^{n_t} \bar{y}_i^2} \right) \tag{27}$$

where y and $\bar{y}$ are actual value and predicted value, respectively; and n_t is the number of testing samples. The smaller *RRMSE* and *MAPE* and the larger R^2 are indicative of better performance generality (Khatibinia *et al*. 2013a). Therefore, the framework of PSO incorporating WLS–SVM is depicted in Fig. 4.

7. Numerical examples

In order to investigate the computational efficiency of PSO incorporating WLS–SVM for the optimal design of homogeneous earth dams, earth dams with height 10, 15, 20, 25 and 30 m are considered. In this study, GC and GW are considered as type soil of dam body for earth dams and drainage systems. The width of dam crest depends on the nature of embankment materials, height and importance of structure, possible roadways requirements, and practicability of construction. In this study, the empirical relation recommended by USBR (2003) is used as follows

$$B = \frac{H}{5.0} + 3.0 \tag{28}$$

where B is the width of the dam crest shown in Fig. 1.

The height of water in the reservoir of dam is assumed to be 80% of the dam height. Permeability of the body material and the drainage materials are considered to be equal to 10^{-7} and 10^{-3} m/sec, respectively. The soil cohesion of the body materials is chosen to be 15 kN/m, and the angle of the internal friction angle is equal with 25 degrees. The soil cohesion of the drain

Table 1 The parameter of PSO method

Parameter	Value
Numbers of maximum iteration	200
Population size	50
$\omega_{\min}$	0.4
$\omega_{\max}$	0.9

materials is considered zero and the angle of the internal friction is 40 degrees. Furthermore, the parameters of PSO based on the authors' experience are given in Table 1.

In order to consider the stochastic nature of the optimization process, the twenty independent optimization runs are performed for each example and the best solutions are reported.

7.1 The lower and upper bounds of design variables

The lower and upper bounds of design variables required for the optimization process are considered as follows:

Length of horizontal drains: The minimum length of the horizontal drain ($L_{H,\mathrm{min}}$) which can keep the phreatic line just within the dam body can be calculated as follows (Chahar 2004)

$$\begin{gathered} L_{\mathrm{H,min}^*} = \frac{1+\lambda^2 n_2^2}{2\lambda^2 n_2^2}\{0.3n_1 + n_2 + F_{B^*}(n_1+n_2) + B_* - \sqrt{[0.3n_1 + n_2 + F_{B^*}(n_1+n_2)+B_*]^2 - n_2^2}\} \\ h = H - F_B\,;\quad L_{\mathrm{H,min}^*} = L_{\mathrm{H,min}}/h;\quad F_{B^*} = F_B/h;\quad B_* = B/h;\quad \lambda = K_x/K_y \end{gathered} \tag{29}$$

Furthermore, the maximum length of the horizontal drain ($L_{H,\mathrm{max}}$) is expressed as follows (Chahar 2004)

$$L_{\mathrm{H,max}^*} = F_{B^*}(n_1+n_2) + B_* + \frac{1+\lambda^2 n_2^2}{2\lambda^2 n_2^2}[0.3n_1 + n_2 - \sqrt{(0.3n_1+n_2)^2 - n_2^2}\,]\,;\ L_{\mathrm{H,max}^*} = L_{\mathrm{H,max}}/h \tag{30}$$

Length of oblique drains: In this study, the water normal level dam reservoir is considered as the maximum length of the oblique drain ($L_{V,\mathrm{max}}$). Also, as the minimum length of the oblique drain ($L_{V,\mathrm{min}}$) is considered as $L_{V,\mathrm{max}}/2$.

Angle between the oblique and horizontal drains: The discrete values of 80, 85, 90, 95, 100, 110, 115, 125 and 135 degrees are considered as angle between the oblique and horizontal drains.

Upstream and downstream slopes: In this study, the discrete values of 2, 2.25, 2.5, 2.75 and 3 are considered as the upstream and downstream slopes.

7.2 Training and testing WLS–SVM

To generate a database for training and testing WLS–SVM, the design variable vector of earth dam with the specific height, defined in Eq. (1), is considered as the input vector of WLS–SVM, and the factor of safety against sliding and the seepage through the dam body are taken as the output of WLS–SVM. For achieving this purpose, first, design of computer experiments is employed by generating a set of combinations of the design variables. This set is spread in the entire design variables by design of computer experiments. In this study, Latin Hypercube Design (LHD) proposed for computer experiments by McKay *et al*. (1979) is used for generating 200 earth dam samples with the specific height. The hydraulic responses of all earth dam samples are obtained using FEA in sub–programs SEEP/W and SLOPE/W. Then, the samples are selected on a random basis and from which 70% and 30% samples are employed to train and test the performance generality of WLS–SVM. In WLS–SVM, the 10–fold cross–validation proposed by Suykens *et al*. (2002) is used for finding the parameters γ, σ and training the WLS–SVM model. The statistical values and the optimal values of γ and σ for predicting the hydraulic responses of dam samples found from testing modes are obtained. Therefore, the results of testing the performance generality of the WLS–SVM based on the statistical values of *MAPE*, *RRMSE* and R^2 are given in Tables 2 and 3.

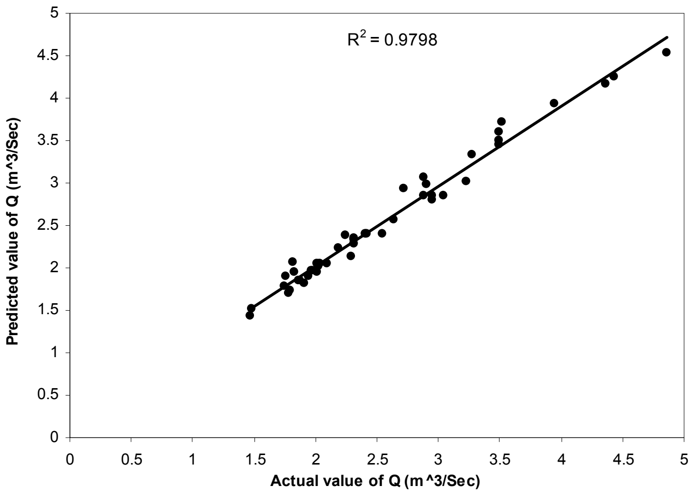
All of the statistical values in Tables 2 and 3 demonstrate that the WLS–SVM models achieve a good performance generality in predicting the hydraulic responses of earth dam samples. Also, a comparison between the predicted data and the actual data is performed to assess the prediction

Table 2 Statistical values based on the optimal values of WLS–SVM in testing mode for the seepage

Hight of dam (m)	Optimal values		Statistical values		
	γ	σ	*MAPE*	*RRMSE*	R^2
10	1364.151	80.397	7.55	0.1258	0.9842
15	489.439	32.856	3.35	0.0446	0.9980
20	784.824	64.832	8.81	0.1248	0.9835
25	672.732	98.332	2.83	0.0371	0.9986
30	495.931	74.903	4.85	0.0701	0.9950

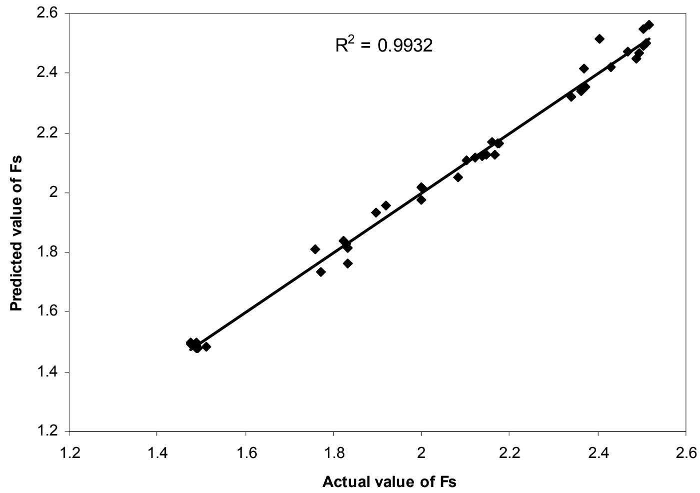
Table 3 Statistical values based on the optimal values of the WLS–SVM in testing mode for stability coefficient

Hight of dam (m)	Optimal values		Statistical values		
	γ	σ	*MAPE*	*RRMSE*	R^2
10	760.445	45.85	0.955	0.0173	0.9997
15	389.439	26.85	1.080	0.0148	0.9998
20	409.79	28.93	1.251	0.0233	0.9997
25	590.46	34.75	1.302	0.0178	0.9995
30	650.34	47.87	0.926	0.0123	0.9998



(a) The seepage through the body dam

Fig. 5 The actual versus predicted values for the hydraulic responses of earth dam with H = 15 m



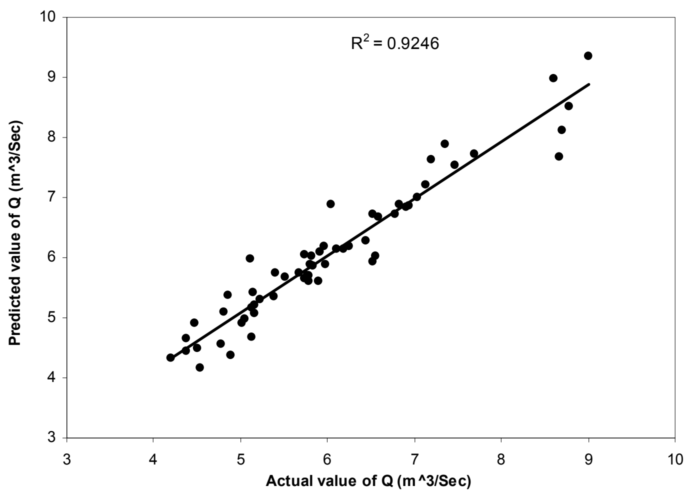
(b) The stability coefficient

Fig. 5 Continued

performance of WWL–SVM. For this purpose, the plots of the actual versus predicted values for the seepage through the dam body and the factor of safety against sliding, for H=15 *m* and 30 *m*, are also depicted in Figs. 5 and 6.

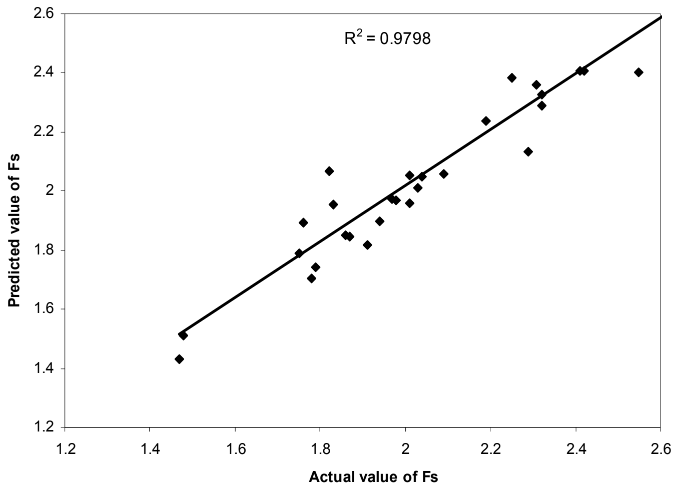
7.3 Results of optimization

In order to consider the stochastic nature of the optimization process, the twenty independent optimization runs are performed for each dam with specific height and the best solutions are reported. For example, the optimal results for earth dam with H = 15 m are given in Table 4.



(a) The seepage through the body dam

Fig. 6 The actual versus predicted values for the hydraulic responses of earth dam with H = 30 m



(b) The stability coefficient

Fig. 6 Continued

Table 4 The optimal results of the twenty independent optimization runs for H = 15 m

No.	w_1	w_2	n_1 (n_2)	θ°	L_H (m)	L_V (m)	F_s	$Q\times 10^{-7}$ (m^3/Sec)	W (kg)
1	0.0909	0.9091	2.75	105	22	9	2.1469	1.2749	9922.5
2	0.0909	0.9091	2.75	110	22	9	2.1671	1.297	9922.5
3	0.0909	0.9091	2.75	120	22	10	2.178	1.3111	9922.5
4	0.9090	0.9091	2.75	120	22	10	2.178	1.3111	9922.5
5	0.0935	0.9065	2	130	12	9	1.6725	1.3577	7560
6	0.0930	0.907	2	115	14	9	1.6963	1.3807	7560
7	0.0728	0.9272	2.75	110	22	8	2.2032	1.3811	9922.5
8	0.0909	0.9091	2	120	12	6	1.751	1.4675	7560
9	0.0217	0.9783	2.75	90	22	6	2.2253	1.546	9922.5
10	0.0179	0.9821	2.5	130	19	11	2.0006	1.5931	9135
11	0.0909	0.9091	2.75	135	22	9	2.2835	1.6275	9922.5
12	0.0229	0.9771	3	110	27	7	2.455	1.7325	10710
13	0.0221	0.9779	2.75	105	27	8	2.3083	1.7351	9922.5
14	0.1055	0.8945	2.75	80	25	6	2.3082	1.7978	9922.5
15	0.0277	0.9723	2.75	135	25	10	2.3188	1.8195	9922.5
16	0.0108	0.9892	2.75	90	33	10	2.3413	1.9413	9922.5
17	0.0792	0.9208	2.75	105	31	9	2.3372	1.9709	9922.5
18	0.0289	0.9711	3	135	29	10	2.5107	2.0111	10710
19	0.0281	0.9719	3	125	31	8	2.5142	2.1919	10710
20	0.1265	0.8735	3	80	36	6	2.5954	2.5329	10710

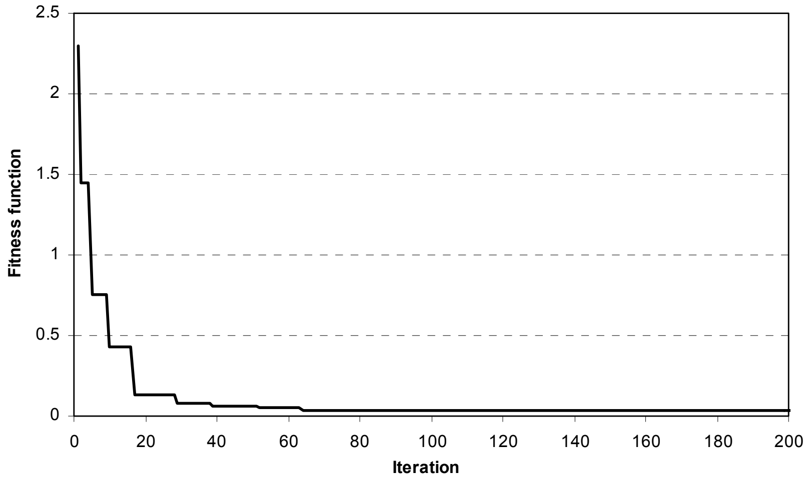


Fig. 7 The Convergence history of the best solution for H = 15 m

According to the optimal results shown in Table 4, it is observed that in 90% of the optimal cases the weight w_2 corresponding to the seepage through of the dam body is higher than 0.9. Therefore, the seepage through of dam body as objective function is more important than the weight of earth dam.

Also, the convergence history of the best solutions of PSO for case of H = 15 m is shown in Fig. 7.

The best optimal results of earth dam for different height of earth dam are expressed in Table 5. The results of Table 5 indicate that the seepage through of the dam body as objective function is more important than the weight of earth dam.

As shown in Table 5, in all the obtained optimal results the optimal angle between the oblique and horizontal drains is greater than 100 degrees. In the most of the optimal cases, the optimal upstream and downstream slopes of earth dams are equal to 2.75. In order to assess the performance of WLS–SVM in the optimization procedure, the best optimal design of earth dams are also analyzed by an accurate FEA in sub–programs SEEP/W and SLOPE/W, and then the actual values of the hydraulic responses are determined. In Table 6, the actual values of the hydraulic responses are compared with their values predicted by WLS–SVM model.

All the comparisons shown in Table 6 reveal that the WLS–SVM model is reliably employed in the optimization procedure instead of sub–programs SEEP/W and SLOPE/W and can accurately

Table 5 The best optimal results of earth dams for different height of dams

H (m)	w_1	w_2	n_1 (n_2)	θ°	L_H (m)	L_V (m)	F_s	$Q\times 10^{-7}$ (m^3/Sec)	W (kg)
10	0.0909	0.9091	2.75	105	16	9	2.5761	1.0017	4550
15	0.0909	0.9091	2.75	120	22	9	2.178	1.3111	9922
20	0.0909	0.9091	2.5	125	22	10	1.9345	1.9586	15960
25	0.0914	0.9086	2.75	120	34	10	1.984	2.0005	26863
30	0.0909	0.9091	2.75	105	43	9	1.8061	4.1317	38430

Table 6 Comparison of the actual value of hydraulic responses with their values predicted by WLS–SVM

H (m)	FEA		WLS–SVM		Error (%)	
	$Q\times 10^{-7}$ (m^3/Sec)	F_s	$Q\times 10^{-7}$ (m^3/Sec)	F_s	Q	F_s
10	1.02	2.438	1.00	2.5761	2.082	5.664
15	1.25	2.151	1.31	2.1780	4.888	1.255
20	2.2	1.747	1.95	1.9345	10.972	10.732
25	1.93	1.831	2.00	1.9840	3.652	8.356
30	4.10	1.754	4.13	1.8061	0.773	2.970

predict the hydraulic responses of earth dams in the optimization procedure. Furthermore, the performance generality and the optimum design obtained by PSO incorporating WLS–SVM are significantly good. In other words, the best solution has been attained by using the WLS–SVM model, in terms of the accuracy and degree of feasibility of the solutions and therefore it can be reliably incorporated into the optimization process of homogeneous earth dams with drains.

8. Conclusions

In this study, the optimal design of homogeneous earth dams with drains is introduced based on particle swarm optimization (PSO) incorporating weighted least squares support vector machine (WLS–SVM) approach to find the optimal shape of dam and effective parameters of drains. To achieve this purpose, the upstream and downstream slopes of earth dam, oblique and horizontal drains and angle among the drains are considered as the design variables of the optimization problem. Furthermore, the seepage through dam body and the weight of homogeneous earth dam are minimized in the optimization process. The WLS–SVM regression model is utilized to predict the hydraulic responses of the earth dams instead of directly performing finite element analysis (FEA) in optimization procedure. In order to assess the merits of PSO with WLS–SVM, a number of earth dams with the specific height are considered and optimized.

Numerical results show that the proper optimal design can be achieved for homogeneous earth dams with drains. Furthermore, in optimization procedure, the FEA can be efficiently replaced by WLS–SVM as approximate analysis. The results indicate that the seepage through of the dam body as objective function is more important than the weight of earth dam. As obvious from the FEA of the optimum design, WLS–SVM can accurately approximate the hydraulic responses of earth dam, and the error between the actual and predicted values of the hydraulic responses is small. Therefore, the PSO algorithm incorporating WLS–SVM can be reliably employed to find the optimal design of homogeneous earth dams with drains.

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