Geomechanics and Engineering, *Vol. 9, No. 5 (2015) 631-644*
DOI: http://dx.doi.org/10.12989/gae.2015.9.5.631

Analytical solution of nonlinear cylindrical bending for functionally graded plates

Tahar Hassaine Daouadji and Lazreg Hadji*

Université Ibn Khaldoun, BP 78 Zaaroura, 14000 Tiaret, Algérie

(*Received March 09, 2015, Revised June 12, 2015, Accepted June 16, 2015*)

Abstract. This article considers the problems of cylindrical bending of functionally graded plates in which material properties vary through the thickness. The variation of the material properties follows two power-law distributions in terms of the volume fractions of constituents. In addition, this paper considers orthotropic materials rather than isotropic materials. The traction-free condition on the top surface is replaced with the condition of uniform load applied on the top surface. Numerical results are presented to show the effect of the material distribution on the deflections and stresses. Results show that, all other parameters remaining the same, the studied quantities (stress, deflection) of P-FGM and E-FGM plates are always proportional to those of homogeneous isotropic plates. Therefore, one can predict the behaviour of P-FGM and E-FGM plates knowing that of similar homogeneous plates.

Keywords: functionally graded plates; cylindrical bending; stress; elasticity solutions

1. Introduction

In conventional laminated composite materials, there is a high chance that debonding will occur at some extreme loading conditions. On the other hand, gradually varying the volume fraction of the constituents can resolve this problem. Functionally graded materials (FGMs) are composite materials which exhibit a progressive change in composition, structure, and properties as a function of spatial direction within the material. They are widely used in mechanical, aerospace, nuclear, and civil engineering. Understanding static and dynamic behaviour of FG structures is of increasing importance. There has been considerable research (Wetherhold *et al*. 1996, Benatta *et al*. 2008, Fekrar *et al*. 2014, Khalfi *et al*. 2014) on the behavior of structure made of FGMs. These studies are limited to functionally graded materials in which the power-law function is used to describe the volume fractions. However, in the case of adding an FGM of a single power-law function to the multi-layered composite, stress concentrations appear on one of the interfaces where the material is continuous but changes rapidly (Lee and Erdogan 1994, Bao and Wang 1995). Therefore, Chung and Chi (2001) and Sallai *et al*. (2009) defined the volume fraction using two power-law functions (simply called P-FGM and E-FGM) to ensure smooth distribution of stresses among all the interfaces.

The static bending problem is one of the basic and also most studied problems in the analysis of FGM plates. Kadoli *et al*. (2008) studied the static behaviour of functionally graded metal-ceramic

*Corresponding author, Ph.D., E-mail: had_laz@yahoo.fr

Copyright © 2015 Techno-Press, Ltd.
http://www.techno-press.org/?journal=gae&subpage=7 ISSN: 2005-307X (Print), 2092-6219 (Online)

(FGM) beams under ambient temperature using displacement field based on higher order shear deformation theory. Thai and Vo (2012) examined the bending and free vibration of FG beams using various higher-order shear deformation beam theories. Mantari and Guedes Soares (2012) studied the bending analysis of thick exponentially graded plates using a new trigonometric higher order shear deformation theory. Taj and Choi (2013) investigated a simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates. Thai *et al*. (2013) conducted static analysis of FG plates using higher order shear deformation theory. Transverse shear stresses are represented as quadratic through the thickness and hence it requires no shear correction factor. Tounsi *et al*. (2013) presented a refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates. Bouderba *et al* (2013) discussed the thermo-mechanical bending response of FGM thick plates resting on Winkler– Pasternak elastic foundations. Ould Larbi *et al*. (2013) developed an efficient shear deformation beam theory based on neutral surface position for bending and free vibration of FG beams. Ait Amar Meziane *et al*. (2014) examined the buckling and free vibration of exponentially graded sandwich plates under various boundary conditions. Bousahla *et al*. (2014) proposed a novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of FG thick plates. Belabed *et al*. (2014) presented an efficient and simple higher order shear and normal deformation theory for FG plates. Zidi *et al*. (2014) studied the bending analysis of FG plates under hygro-thermo-mechanical loading using a four variable refined plate theory. Hadji *et al*., (2014) developed a higher order shear deformation theory for static and free vibration of FG beam. Hebali *et al*. (2014) presented the new quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates. Zhang (2013) studied the nonlinear bending analysis of FGM beams based on physical neutral surface and high order shear deformation theory.

Recently, Merazi *et al*. (2015) used a new hyperbolic shear deformation plate theory for static analysis of FGM plate based on neutral surface position. Junga and Han (2015) studied the Static and eigenvalue problems of Sigmoid Functionally Graded Materials (S-FGM) micro-scale plates using the modified couple stress theory. Hamidi *et al*. (2015) presented a sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates. Mahi *et al*. (2015) developed a new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates. Bourada *et al*. (2015) proposed a new simple shear and normal deformations theory for FG beams. Ait Yahia *et al*. (2015) studied the wave propagation in FG plates with porosities using various higher-order shear deformation plate theories. Meradjah *et al*. (2015) presented a new higher order shear and normal deformation theory for FG beams.

In this paper, the elasticity solutions are presented for P-FGM and E-FGM plates in cylindrical bending. Results show that, all other parameters remaining the same, the studied quantities (stress, deflection) of P-FGM and E-FGM plates are always proportional to those of homogeneous isotropic plates. Then, FGM beams behave like homogeneous beams which mean that no special techniques or software needs to be developed for their analysis.

2. Theoretical formulations

2.1 Properties of the FGM constituent materials

The functionally graded material (FGM) can be produced by continuously varying the constituents of multi-phase materials in a predetermined profile. The most distinct features of an FGM are the non-uniform microstructures with continuously graded macro properties. An FGM can be defined by the variation in the volume fractions. Most researchers use the power-law function or exponential function to describe the volume fractions. However, only a few studies used sigmoid function to describe the volume fractions. Therefore, FGM plates with sigmoid function will be considered in this paper in detail.

Consider an elastic rectangular FGM plate of uniform thickness h, which is made of a ceramic and metal, is considered in this study. The material properties, Young's modulus and the Poisson's ratio, on the upper and lower surfaces are different but are preassigned according to the performance demands. However, the Young's modulus and the Poisson's ratio of the plates vary continuously only in the thickness direction (y-axis) i.e., $E = E(y)$, $\upsilon = \upsilon(y)$. Delale and Erdogan (1983) indicated that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus. Thus, Poisson's ratio of the beams is assumed to be constant. However, the Young's modules in the thickness direction of the FGM plates vary with power-law functions (P-FGM) or with exponential functions (E-FGM).

2.1.1 The material properties of P-FGM plates

The volume fraction of the P-FGM is assumed to obey a power law function

$$g(y) = (\frac{y}{h} + \frac{1}{2})^p \tag{1}$$

where p is the material parameter that dictates the material variation profile through the thickness and h is the thickness of the plate. Once the local volume fraction $g(y)$ has been defined, the material properties of a P-FGM can be determined by the rule of mixture

$$E(y) = g(y)E_1 + (1 - g(y))E_2 \tag{2}$$

where E_1 and E_2 are the Young modules of the lowest ($y = h/2$) and top surfaces ($y = - h/2$) of the FGM plate, respectively. The variation of Young's modulus in the thickness direction of the

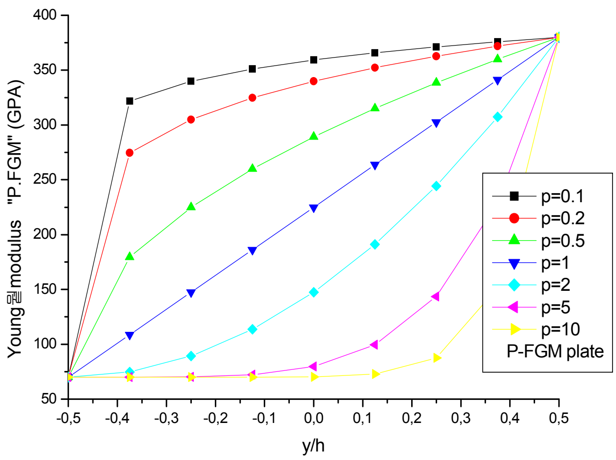

Fig. 1 The variation of Young's modulus in a P-FGM plate

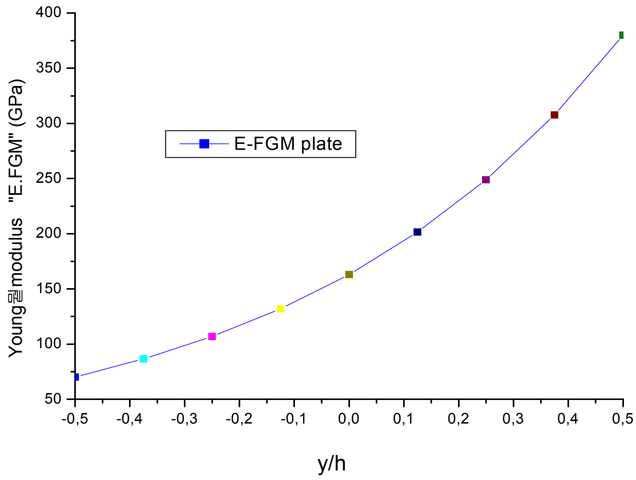

Fig. 2 The variation of Young's modulus in a E-FGM plate

P-FGM plate is depicted in Fig. 1, which shows that the Young's modulus changes rapidly near the lowest surface for $p > 1$, and increases quickly near the top surface for $p < 1$.

2.1.2 The material properties of E-FGM plates

Many researchers used the exponential function to describe the material properties of FGM as follows

$$E(z) = A.e^{B(y+\frac{h}{2})} \tag{3}$$

with

$$A = E_2 \quad \text{and} \quad B = \frac{1}{h}\ln(\frac{E_1}{E_2}) \tag{4}$$

The material distribution in the thickness direction of the EFGM plates is plotted in Fig. 2.

2.2 Fundamental equations for cylindrical bending of FGM plates

Consider an FGM plate of thickness h and length l, which is made of a ceramic at the upper surface and a metal at the lower surface, is considered in this study. The $x - z$ plane coincident with the mid-plane of the plate, occupying the region of $0 \le x \le l$, $-h/2 \le y \le h/2$, $-\infty \le z \le \infty$. Denote u and w as the displacement components in the x and z directions, respectively, σ_{xx}, τ_{xz} and σ_{zz} as the stress components respectively. The plate is subjected to uniform load on the top surface so that it is in a state of cylindrical bending, for which all displacements and stresses are independent of the coordinate y.

In the absence of body forces, the equations of equilibrium are

$$\frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} = 0, \qquad \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_y}{\partial y} = 0 \tag{5}$$

The strain components are given as

$$\varepsilon_x = \frac{\partial u}{\partial x}, \qquad \varepsilon_y = \frac{\partial v}{\partial y}, \qquad \gamma_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}, \tag{6}$$

u and v denote the displacement components.

The corresponding elastic constitutive law for the FGM plate expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \tag{7}$$

where Q_{ij} are the stiffness of the FGM plate and are given as

$$Q_{11} = Q_{22} = \frac{E(y)}{1-\nu^2}, \qquad Q_{12} = \frac{\nu\, E(y)}{1-\nu^2}, \qquad Q_{66} = \frac{E(y)}{2(1+\nu)} \tag{8}$$

We seek the following solutions of Eqs. (5), (6) and (7)

$$\begin{aligned} u(x,y) &= \bar{u}(x) + F\bar{u}_{,xx} + A\bar{v}_{,x} + B\bar{v}_{,xxx} \\ v(x,y) &= \bar{v}(x) + G\bar{u}_{,x} + C\bar{v}_{,xx} + D \end{aligned} \tag{9}$$

where $A = A(y)$, $B = B(y)$, $C = C(y)$, $D = D(y)$, $F = F(y)$ and $G = G(y)$ are undetermined functions. $\bar{u}$ and $\bar{v}$ are the mid-plane displacements.

Substituting Eq. (9) into Eqs. (6) and (7) gives

$$\begin{aligned} \sigma_x &= (Q_{11} + G'Q_{12})\bar{u}_{,x} + (AQ_{11} + C'Q_{12})\bar{v}_{,xx} + FQ_{11}\bar{u}_{,xxx} + BQ_{11}\bar{v}_{,xxxx} + D'Q_{12}, \\ \sigma_y &= (Q_{12} + G'Q_{22})\bar{u}_{,x} + (AQ_{12} + C'Q_{22})\bar{v}_{,xx} + FQ_{12}\bar{u}_{,xxx} + BQ_{12}\bar{v}_{,xxxx} + D'Q_{22}, \\ \tau_{xy} &= Q_{66}\left[(A'+1)\bar{v}_{,x} + (F'+G)\bar{u}_{,xx} + (B'+C)\bar{v}_{,xxx}\right] \end{aligned} \tag{10}$$

Substitute Eq. (10) into Eq. (5), and set

$$[Q_{66}(A'+1)]' = 0, \tag{11a}$$

$$[Q_{66}(A'+1)]' = 0, \tag{11b}$$

$$Q_{66}(A'+1) + (Q_{12}A + Q_{22}C')' = 0, \tag{11c}$$

$$Q_{11} + Q_{12}G' + [Q_{66}(F'+G)]' = Q_{66}k_1, \tag{11d}$$

$$Q_{11}A + Q_{12}C' + [Q_{66}(B'+C)]' = Q_{66}k_2, \tag{11e}$$

$$\bar{u}_{,xxx} = k_3, \tag{11f}$$

$$\bar{v}_{,xxxx} = k_4. \tag{11g}$$

where the prime denotes the derivative with respect to y, and k_i (i = 1, 2, 3, 4) are arbitrary constants. Therefore, Eq. (5) can be simplified as

$$k_1\bar{u}_{,xx} + k_2\bar{v}_{,xxx} = 0, \tag{12}$$

$$[Q_{66}(F'+G)+(Q_{12}F)']k_3 + [Q_{66}(B'+C)+(Q_{12}B)']k_4 + (Q_{22}D')' = 0. \tag{13}$$

By virtue of Eqs. (11f), (11g) and (12), we obtain

$$k_1k_3 + k_2k_4 = 0. \tag{14}$$

Integrating Eqs. (11g) and (12) yields

$$\bar{v}(x) = \frac{1}{24}k_4x^4 + \frac{1}{6}C_1x^3 + \frac{1}{2}C_2x^2 + C_3x + C_4 \tag{15}$$

$$\bar{u}(x) = \frac{1}{6}k_3x^3 - \frac{1}{2}\frac{k_2}{k_1}C_1x^2 + C_5x + C \tag{16}$$

where C_i (i = 1, 2, 3, 4, 5, 6) are integral constants which can be completely determined from the boundary conditions at cylindrical edges.

Now consider the boundary conditions on the top and bottom surfaces of the plate. We have $\tau_{xy} = 0$ at $y = \pm h/2$, $\sigma_y = -q$ at $y = -h/2$ and $\sigma_y = 0$ at $y = h/2$, where q is a constant.

By substituting the expressions of σ_y and τ_{xy} into the above boundary conditions at $y = \pm h/2$, we obtain.

$$A'(\pm h/2) + 1 = 0, \tag{17a}$$

$$Q_{12}(\pm h/2) + Q_{22}(\pm h/2)\,G'(\pm h/2) = 0, \tag{17b}$$

$$F'(\pm h/2) + G(\pm h/2) = 0, \tag{17c}$$

$$B'(\pm h/2) + C(\pm h/2) = 0, \tag{17d}$$

$$Q_{12}(\pm h/2)A(\pm h/2) + Q_{22}(\pm h/2)\,\mathrm{C}'(\pm h/2) = 0, \tag{17e}$$

$$Q_{12}(-h/2)[k_3F(-h/2) + k_4B(-h/2)] + Q_{22}(-h/2)D'(-h/2) = -q, \tag{17f}$$

$$Q_{12}(h/2)[k_3F(h/2) + k_4B(h/2)] + Q_{22}(h/2)D'(h/2) = 0, \tag{17g}$$

Given that $\bar{u}$ and $\bar{v}$ are the mid-plane displacements, namely, $\bar{u}(x) = u(x,0)$, $\bar{v}(x) = v(x,0)$.

We can deduce from Eq. (9) that

$$A(0) = 0\,, \quad B(0) = 0\,, \quad C(0) = 0\,, \quad D(0) = 0\,, \quad F(0) = 0\,, \quad G(0) = 0\,. \tag{18}$$

Integrating Eqs. (11a)-(11e) and (13) and making use of Eqs. (17)-(18) and (14), functions $A(y)$, $B(y)$, $C(y)$, $D(y)$, $F(y)$, $G(y)$ and the constant k_i (i = 1, 2, 3, 4) can be completely determined

2.3 Resultant force, bending moment, shear force and cylindrical boundary conditions

By virtue of Eq. (10) and $A(y) = -y$, the expressions for the resultant force Nx, bending moment Mx and shear force Qx, can be determined accordingly

$$N_x = \int_{-h/2}^{h/2} \sigma_{xx} dy = N_1 \bar{u}_{,x} + N_3 \bar{v}_{,xx} + N_5 \bar{u}_{,xxx} + N_7 \bar{v}_{,xxxx} + N_0 \tag{19}$$

$$M_x = \int_{-h/2}^{h/2} \sigma_{xx} y dy = M_1 \bar{u}_{,x} + M_3 \bar{v}_{,xx} + M_5 \bar{u}_{,xxx} + M_7 \bar{v}_{,xxxx} + M_0 \tag{20}$$

$$Q_x = \int_{-h/2}^{h/2} \tau_{xx} dy = Q_1 \bar{u}_{,xx} + Q_2 \bar{v}_{,xxx} \tag{21}$$

where

$$N_1 = \int_{-h/2}^{h/2} (Q_{11} + Q_{12} G') dy ; \qquad M_1 = \int_{-h/2}^{h/2} y (Q_{11} + Q_{12} G') dy \tag{22a}$$

$$N_3 = \int_{-h/2}^{h/2} (Q_{11} A + c_{12} C') dy ; \qquad M_3 = \int_{-h/2}^{h/2} y (Q_{11} A + Q_{12} C') dy \tag{22b}$$

$$N_5 = \int_{-h/2}^{h/2} Q_{11} F dy ; \qquad M_5 = \int_{-h/2}^{h/2} Q_{11} y F dy \tag{22c}$$

$$N_7 = \int_{-h/2}^{h/2} Q_{11} B dy ; \qquad M_7 = \int_{-h/2}^{h/2} Q_{11} y B dy \tag{22d}$$

$$N_0 = \int_{-h/2}^{h/2} Q_{12} D' dy ; \qquad M_0 = \int_{-h/2}^{h/2} Q_{12} D' y dy \tag{22e}$$

$$Q_1 = \int_{-h/2}^{h/2} Q_{66} (F' + G) dy ; \qquad Q_2 = \int_{-h/2}^{h/2} Q_{66} (B' + C) dy \tag{22f}$$

There are four different boundary conditions at the cylindrical edges to be considered here. These include simply supported (S), clamped-1 (C1), clamped-2 (C2), and free (F) conditions, namely

$$\text{S} : \qquad \bar{u} = 0; \quad \bar{v} = 0; \quad M_x = 0; \tag{23}$$

$$\text{C1} : \qquad \bar{u} = 0; \quad \bar{v} = 0; \quad \bar{v}_{,x} = 0; \tag{24}$$

$$\text{C2} : \qquad \bar{u} = 0; \quad \bar{v} = 0; \quad \bar{u}_{,z}\big|_{z=0} = 0; \tag{25}$$

$$\text{F} : \qquad N_x = 0; \quad M_x = 0; \quad Q_x = 0; \tag{26}$$

3. Numerical results and discussion

3.1 Effect reinforcement

For convenience, the following dimensionless quantities are introduced

$$\overline{W} = \frac{\overline{w}E}{qh}; \qquad \overline{\sigma}_x = \frac{\sigma_x}{q}; \qquad \overline{\tau}_{yx} = \frac{\tau_{yx}}{q}; \tag{27}$$

Based on the derived formulation, a computer program is developed to study the behavior of FGM plates in cylindrical bending. The analysis is performed for pure materials and different values of material parameter, *p*, for aluminium–alumina FGM. The Young's modulus and Poisson's ratio (Sallai *et al*. 2009) for aluminium are: 70 GPa and 0.3 and for alumina: 380 GPa and 0.3, respectively. Delale and Erdogan (1983) indicated that the effect of Poisson's ratio on the deformation is much less than that of Young's modulus. Thus, Poisson's ratio of the plates is assumed to be constant (Delale and Erdogan 1983, Saidi *et al*. 2013 Houari *et al*. 2013, El Meiche *et al*. 2011, Bourada *et al*. 2012, Bouchafa *et al*. 2015, Benachour *et al*. 2011) and is chosen to be 0.3. Consider an PFGM and EFGM rectangular plate subject to a uniform load on the top surface with an infinite extent in the *z*-direction. We assume q = 1×106 N/m^2, l = 1 m and h/l = 0.15. In all cases, the lower surface of the plate is assumed to be metal (aluminium) rich and the upper surface is assumed to be pure ceramic (alumina).

Figs. 3, 5 and 7 shows the distributions of dimensionless normal stress $\overline{\sigma}_x$ along the thickness direction of the PFGM and EFGM plate for different p and $\frac{E_1}{E_2}$ at $x = l/2$. It is found that $\overline{\sigma}_x$ is almost linearly distributed for $p = 0$ and $\frac{E_1}{E_2} = 1$, and the maximum tensile (compressive) stress occurs at $y = h/2$. However, for $p \succ 0$ and $\frac{E_1}{E_2} \succ 1$, for which the location of the maximum stiffness is at $y = h/2$, the value of the maximum tensile stress occurs at $y = h/2$ and increases with p and $\frac{E_1}{E_2}$.

Figs. 4, 6 and 8 depicts the distributions of dimensionless shear stress $\overline{\tau}_{yx}$ along the thickness direction of the PFGM and EFGM plate for different values p and $\frac{E_1}{E_2}$ at $x = l/4$. It is found that $\overline{\tau}_{yx}$ has a parabolic distribution for $p = 0$ and $\frac{E_1}{E_2} = 1$, for which the maximum value occurs at $y = 0$ and the curve is also symmetrical about this point. For $p \succ 0$ and $\frac{E_1}{E_2} \succ 1$, the point at which the maximum shear stress is reached approaches the bottom surface $y = h/2$ and increases with p and $\frac{E_1}{E_2}$.

Tables 1 and 2 gives the dimensionless deflection $\overline{W}$ of the FGM rectangular plate for seven different kinds of boundary conditions and three different values for p and $\frac{E_1}{E_2}$ at $x = l/2$. It is found that the deflection decreases as p and $\frac{E_1}{E_2}$ increases, regardless of the boundary conditions. This is simply because the whole rigidity of the FGM plate increases with p and $\frac{E_1}{E_2}$. In addition, the dimensionless deflections of the C2-F and C1-C1 plates are the largest and smallest among the plates, respectively, with the seven different boundary conditions.

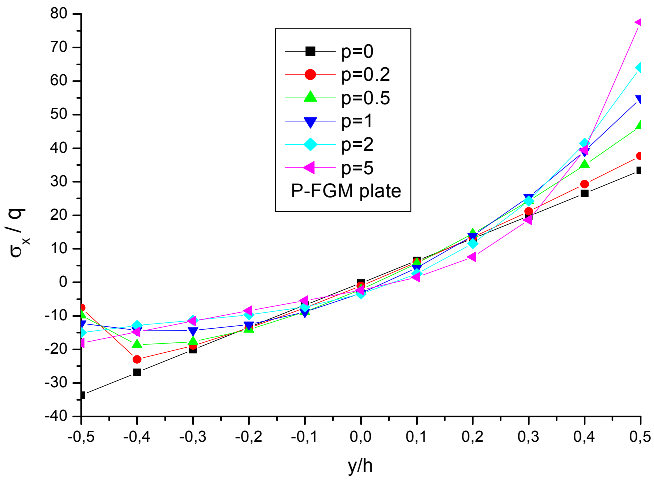

Fig. 3 Dimensionless normal stress of P-FGM plate en $x = L/2$ (\b(s-s))

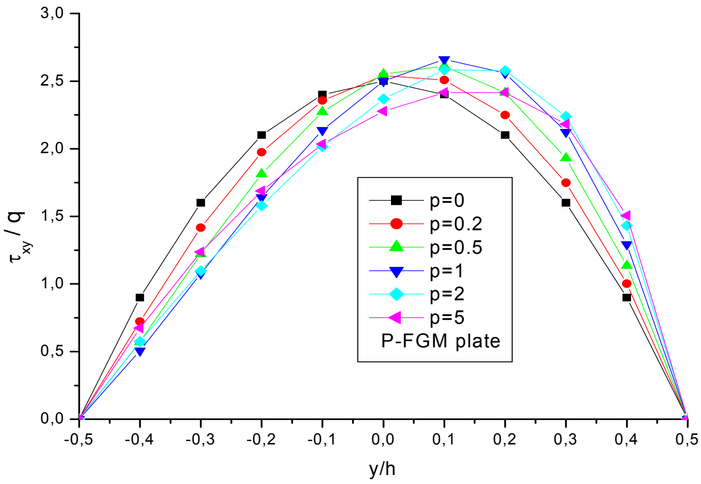

Fig. 4 Dimensionless shear stress of P-FGM plate en $x = l/4$ (\b(s-s))

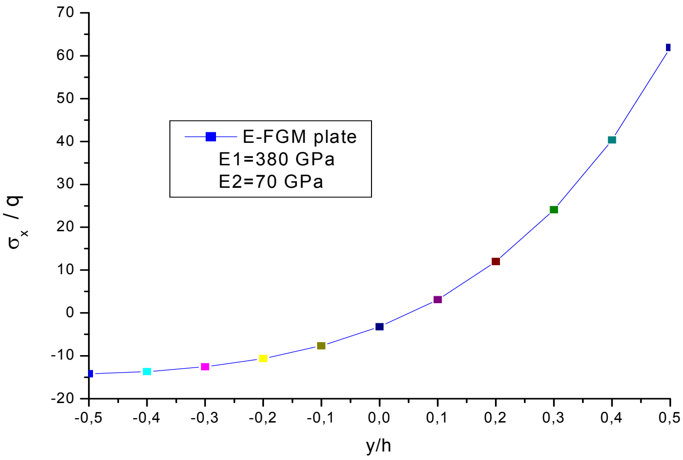

Fig. 5 Dimensionless normal stress of E-FGM plate en $x = l/2$ (\b(s-s))

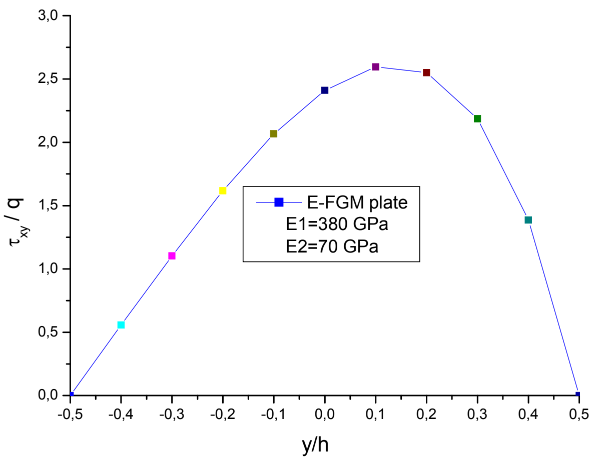

Fig. 6 Dimensionless shear stress of E-FGM plate en $x = l/4$ (\b(s-s))

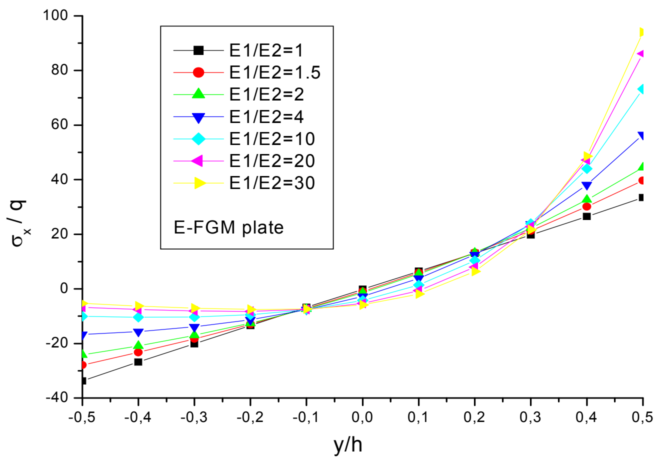

Fig. 7 Dimensionless normal stress of E-FGM plate en $x = l/2$ (\b(s-s)) for different ratio of E1/E2

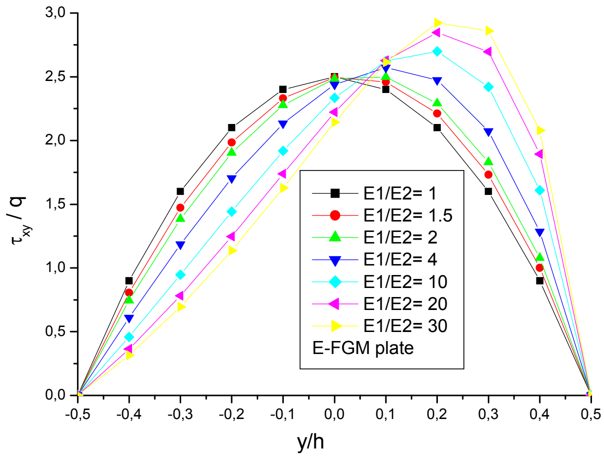

Fig. 8 Dimensionless shear stress of E-FGM plate en $x = l/4$ (\b(s-s)) for different ratio of E1/E2

Table 1 Dimensionless deflection $\overline{W}$ of the PFGM rectangular plate for seven different kinds of boundary conditions and three different values for p at $x = l/2$

Boundary conditions	$P = 0$	$P = 0.5$	$P = 5$
S-S	324.47	358.12	176.168
C1-C1	60.74	25.08	13.16
C2-C2	82.494	58.46	23.31
C1-S	125.90	71.31	38.72
C2-S	147.41	93.52	58.55
C1-F	1811.08	2143.08	1129.13
C2-F	1893.98	2227.26	1205.15

Table 2 Dimensionless deflection $\overline{W}$ of the EFGM rectangular plate for seven different kinds of boundary conditions and three different values for $\frac{E_1}{E_2}$ at $x = l/2$

Boundary conditions	$\frac{E_1}{E_2} = 1$	$\frac{E_1}{E_2} = 2$	$\frac{E_1}{E_2} = 10$
S-S	324.47	223.72	79.21
C1-C1	60.74	43.18	19.90
C2-C2	82.494	55.51	23.99
C1-S	125.90	56.28	24.30
C2-S	147.41	70.30	25.62
C1-F	1811.08	1303.73	607.77
C2-F	1893.98	1350.48	624.32

4. Conclusions

The elasticity solutions for functionally graded plates in cylindrical bending are obtained by extending the FGM plate theory suggested by Mian and Spencer. The material coefficients can vary arbitrarily with the thickness-coordinate. The numerical results show that boundary conditions and material in homogeneity have obvious effects on the response of the FGM rectangular plates in cylindrical bending. Especially, it is easy to cause stress concentration near the location of the maximum stiffness when the material in homogeneity change greatly. Therefore, the mechanical behavior of FGM rectangular plates in cylindrical bending can be optimized by properly adjusting the factors mentioned above in engineering applications.

Because no simplifying hypotheses about the stress and displacement fields are introduced, the proposed elasticity solution can serve as a benchmark for accessing validity of various approximate plate theories or numerical methods that may be used in the analysis of such plates.

References

Ait Amar Meziane, M., Abdelaziz, H.H. and Tounsi, A. (2014), "An efficient and simple refined theory for buckling and free vibration of exponentially graded sandwich plates under various boundary conditions", *J. Sandw. Struct. Mater.*, **16**(3), 293-318.

Ait Yahia, S., Ait Atmane, H., Houari, M.S.A. and Tounsi, A. (2015), "Wave propagation in functionally graded plates with porosities using various higher-order shear deformation plate theories", *Struct. Eng. Mech.*, *Int. J.*, **53**(6), 1143-1165.

Bao, G. and Wang, L. (1995), "Multiple cracking in functionally graded ceramic/metal coatings", *Int. J. Solid. Struct.*, **32**(19), 2853-2871.

Belabed, Z., Houari, M.S.A., Tounsi, A., Mahmoud, S.R. and Anwar Bég, O. (2014), "An efficient and simple higher order shear and normal deformation theory for functionally graded material (FGM) plates", *Compos.*: *Part B*, **60**, 274-283.

Benachour, A., Daouadji, H.T., Ait Atmane, H., Tounsi, A. and Meftah, S.A. (2011), "A four variable refined plate theory for free vibrations of functionally graded plates with arbitrary gradient", *Compos. Part B*, **42**(6), 1386-1394.

Benatta, M.A., Mechab, I., Tounsi, A. and Adda bedia, E.A. (2008), "Static analysis of functionally graded short beams including warping and shear deformation effects", *Computat. Mater. Sci.*, **44**(2), 765-773.

Bouchafa, A., Bachir Bouiadjra, M., Houari, M.S.A. and Tounsi, A. (2015), "Thermal stresses and deflections of functionally graded sandwich plates using a new refined hyperbolic shear deformation theory", *Steel Compos. Struct.*, *Int. J.*, **18**(6), 1493-1515.

Bouderba, B., Houari, M.S.A. and Tounsi, A. (2013), "Thermomechanical bending response of FGM thick plates resting on Winkler–Pasternak elastic foundations", *Steel Compos. Struct.*, *Int. J.*, **14**(1), 85-104.

Bourada, M., Tounsi, A., Houari, M.S.A. and Adda Bedia, E.A. (2012), "A new four-variable refined plate theory for thermal buckling analysis of functionally graded sandwich plates", *J. Sandw. Struct. Mater.*, **14**(1), 5-33.

Bourada, M., Kaci, A., Houari, M.S.A. and Tounsi, A. (2015), "A new simple shear and normal deformations theory for functionally graded beams", *Steel Compos. Struct.*, *Int. J.*, **18**(2), 409-423.

Bousahla, A.A., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2014), "A novel higher order shear and normal deformation theory based on neutral surface position for bending analysis of advanced composite plates", *Int. J. Computat. Method.*, **11**(6), 1350082.

Chung, Y.L. and Chi, S.H. (2001), "The residual stress of functionally graded materials", *J. Chinese Inst. Civil Hydraul. Eng.*, **13**, 1-9.

Delale, F. and Erdogan, F. (1983), "The crack problem for a nonhomogeneous plane", *ASME J. Appl. Mech.*, **50**(3), 609-614.

El Meiche, N., Tounsi, A., Ziane, N., Mechab, I. and Adda Bedia, E.A. (2011), "A new hyperbolic shear deformation theory for buckling and vibration of functionally graded sandwich plate", *Int. J. Mech. Sci.*, **53**(4), 237-247.

Fekrar, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2014), "A new five-unknown refined theory based on neutral surface position for bending analysis of exponential graded plates", *Meccanica*, **49**(4), 795-810.

Hadji, L., Daouadji, T.H., Tounsi, A. and Bedia, E.A. (2014), "A higher order shear deformation theory for static and free vibration of FGM beam", *Steel Compos. Struct.*, *Int. J.*, **16**(5), 507-519.

Hamidi, A., Houari, M.S.A., Mahmoud, S.R. and Tounsi, A. (2015), "A sinusoidal plate theory with 5-unknowns and stretching effect for thermomechanical bending of functionally graded sandwich plates", *Steel Compos. Struct.*, *Int. J.*, **18**(1), 235-253.

Hebali, H., Tounsi, A., Houari, M.S.A., Bessaim, A. and Adda Bedia, E.A. (2014), "New quasi-3D hyperbolic shear deformation theory for the static and free vibration analysis of functionally graded plates", *J. Eng. Mech*, **140**(2), 374-383.

Houari, M.S.A., Tounsi, A. and Anwar Bég, O. (2013), "Thermoelastic bending analysis of functionally graded sandwich plates using a new higher order shear and normal deformation theory", *Int. J. Mech. Sci.*, **76**, 467-479.

Junga, W.Y. and Han, S.C. (2015), "Static and eigenvalue problems of Sigmoid Functionally Graded Materials (S-FGM) micro-scale plates using the modified couple stress theory", *Appl. Math. Model.*, **39**(12), 3506-3524.

Kadoli, R., Akhtar, K. and Ganesan, N. (2008), "Static analysis of functionally graded beams using higher order shear deformation theory", *Appl. Math. Model.*, **32**(12), 2509-2525.

Khalfi, Y., Houari, M.S.A. and Tounsi, A. (2014), "A refined and simple shear deformation theory for thermal buckling of solar functionally graded plates on elastic foundation", *Int. J. Computat. Method.*, **11**(5), 135007.

Lee, Y.D. and Erdogan, F. (1994), "Residual/thermal stress in FGM and laminated thermal barrier coatings", *Int. J. Fract.*, **69**(2), 145-165.

Mahi, A., Adda Bedia, E.A. and Tounsi, A. (2015), "A new hyperbolic shear deformation theory for bending and free vibration analysis of isotropic, functionally graded, sandwich and laminated composite plates", *Appl. Math. Model.*, **39**(9), 2489-2508.

Mantari, J.L. and Guedes Soares, C. (2012), "Bending analysis of thick exponentially graded plates using a new trigonometric higher order shear deformation theory", *Compos. Struct.*, **94**(6), 1991-2000.

Meradjah, M., Kaci, A., Houari, M.S.A., Tounsi, A. and Mahmoud, S.R. (2015), "A new higher order shear and normal deformation theory for functionally graded beams", *Steel Compos. Struct.*, *Int. J.*, **18**(3), 793-809.

Merazi, M., Hadji, L., Daouadji, T.H., Tounsi, A. and Adda Bedia, E.A. (2015), "A new hyperbolic shear deformation plate theory for static analysis of FGM plate based on neutral surface position", *Geomech. Eng.*, *Int. J.*, **8**(3), 305-321.

Ould Larbi, L., Kaci, A., Houari, M.S.A. and Tounsi, A. (2013), "An efficient shear deformation beam theory based on neutral surface position for bending and free vibration of functionally graded beams", *Mech. Based Des. Struct. Mach.*, **41**(4), 421-433.

Saidi, H., Houari, M.S.A., Tounsi, A. and Adda Bedia, E.A. (2013), "Thermo-mechanical bending response with stretching effect of functionally graded sandwich plates using a novel shear deformation theory", *Steel Compos. Struct.*, *Int. J.*, **15**, 221-245.

Sallai, B.-O., Tounsi, A., Mechab, I., Bachir Bouiadjra, M., Meradjah, M. and Adda Bedia, E.A. (2009), "A theoretical analysis of flexional bending of Al/Al_2O_3 S-FGM thick beams", *Computat. Mater. Sci.*, **44**(4), 1344-1350.

Taj, M.N.A.G., Chakrabarti, A. and Sheikh, A.H. (2013), "Analysis of functionally graded plates using higher order shear deformation theory", *Appl. Math. Model.*, **37**(18-19), 8484-8494.

Thai, H.T. and Choi, D.H. (2013), "A simple first-order shear deformation theory for the bending and free vibration analysis of functionally graded plates", *Compos. Struct.*, **101**, 332-340.

Thai, H.T. and Vo, T.P. (2012), “Bending and free vibration of functionally graded beams using various higher-order shear deformation beam theories”, *Int*. *J*. *Mech*. *Sci*., **62**(1), 57-66.

Tounsi, A., Houari, M.S.A., Benyoucef, S. and Adda Bedia, E.A. (2013), “A refined trigonometric shear deformation theory for thermoelastic bending of functionally graded sandwich plates”, *Aerosp*. *Sci*. *Technol*., **24**(1), 209-220.

Wetherhold, R.C., Seelman, S. and Wang, J. (1996), “The use of functionally graded materials to eliminate or control thermal deformation”, *Compos*. *Sci*. *Technol*., **56**(9), 1099-1104.

Zhang, D.G. (2013), “ Nonlinear bending analysis of FGM beams based on physical neutral surface and high order shear deformation theory”, *Compos*. *Struct.*, **100**, 121-126.

Zidi, M., Tounsi, A., Houari, M.S.A., Adda Bedia, E.A. and Anwar Bég, O. (2014), “Bending analysis of FGM plates under hygro-thermo-mechanical loading using a four variable refined plate theory”, *Aerosp*. *Sci*. *Technol*., **34**, 24-34.

CC