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Comparing the generalized Hoek-Brown and Mohr-Coulomb failure criteria for stress analysis on the rocks failure plane

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Abstract. Determination of mobilized shear strength parameters (that identify stresses on the failure plane) is required for analyzing the stability by limit equilibrium method. Generalized Hoek–Brown (GHB) and Mohr-Coulomb (MC) failure criteria are usually used for obtaining stresses on the plane of failure. In the present paper, the applicability of these criteria for determining the stresses on failure plane is investigated. The comparison is based on stresses on the real failure plane which are obtained from the Mohr stress circle. To do so, 18 sets of data (consist of principal stresses and angle of failure plane) presented in the literature are used. In addition, the values account for (*VAF*) and the root mean square error (*RMSE*) indices were calculated to check the determination performance of the obtained results. Values of *VAF* and *RMSE* for the normal stresses on the failure plane evaluated from MC are 49% and 31.5 where for GHB are 55% and 30.5, respectively. Also, for the shear stresses on failure plane, they are 74% and 36 for MC, 76% and 34.5 for GHB. Results show that the obtained stresses and angles of failure plane for each criterion differ from real ones, but GHB results are closer to the empirical results. Also, it is inferred that results are affected by the failure envelope not real failure plane. Therefore, obtained shear strength parameters are not mobilized. Finally, a multivariable regressed relation is presented for determining the stresses on the failure plane.

Keywords: stresses on failure plane; Mohr-Coulomb failure criterion; Hoek-Brown failure criterion; Mohr stress circle

1. Introduction

Stability analysis of rock structures such as slopes and tunnels by the method of limited equilibrium is important in rock mechanics and rock engineering. In the limit equilibrium method, determination of mobilized shear strength parameters is important. These parameters, present normal or shear stresses at the verge of failure.

Different researches for determining stresses on failure plane have been carried. Hence some methods have been proposed for obtaining mobilized shear strength parameters. These methods are classified into two main categories. First, researches based on Mohr Coulomb (MC). Coulomb (1776) proposed a theory for estimating the mobilized shear strength parameters. Latter, Coulomb theory was modified and became more applicable by using Mohr stress circle (Mohr 1900) which leads to defining Mohr-Coulomb theory or Mohr-Coulomb criterion. This theory is widely utilized in geotechnics (Parry 2005). A method of deducing the best fit envelope from experimental data

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using the least squares method was described by Balmer (1952). Statistical treatment of various failure criterions applied to experiments on intact rock can be found in the literature (Colmenares and Zoback 2002, Hoek *et al*. 2002, Pincus 2000, Al-Ajmi and Zimmerman 2005, Pariseau 2007, Benz and Schwab 2008, Das and Basudhar 2009).

Hoek-Brown criterion is one of the most important and basis criteria in rock mechanics that was suggested by Hoek and Brown (1980). Hoek *et al*. (2002) presented the latest version of GHB which is used in experiments for finding the shear strength parameters on the failure plane (Jimenez *et al*. 2008, Yang and Yin 2010, Fu and Liao 2010, Shen *et al*. 2012).

Therefore using mentioned method, stresses on the failure plane are estimated and used for calculating the mobilized shear strength parameters. It is expected that resulted stresses based on the mentioned methods are equivalent to the stresses on the real plane of failure which leads to the mobilized shear strength parameters. In the present paper an investigation and comparison is done between two mentioned criteria for determining stresses on the failure plane. Study is done using the available results in the literature. Statistical methods such as values account for (*VAF*) and root mean square error (*RMSE*) are used for the study. Finally, the efficiency of two criteria is investigated based on the stresses on the failure plane.

2. Determination of stress on the failure plane based on Mohr-Coulomb failure criterion

Mohr-Coulomb (MC) failure criterion is one of the most applicable criteria of rocks' failure which is used for design and analysis of structures and continuum. MC advantages are its mathematical simplicity, clear physical meaning of the material parameters and general level of acceptance (Labuz and Zang 2012).

A linear relation between normal and shear stresses on the plane of failure, was suggested by Coulomb (Heyman 1972) which is

$$\beta = \frac{\pi}{4} + \frac{\varphi}{2} \tag{1}$$

where c and φ are shear strength parameters of intact rock and are called cohesion and angle of internal friction, respectively (Fig. 1).

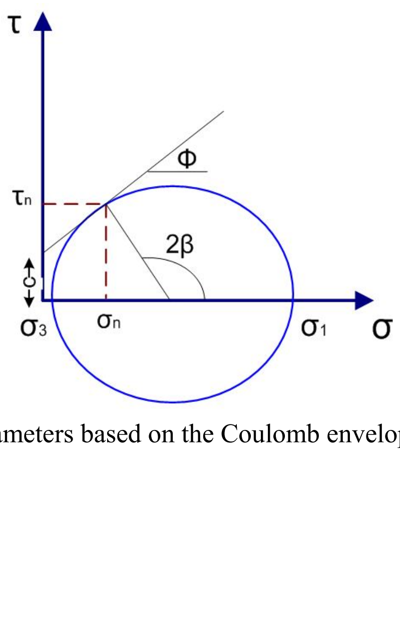


Fig. 1 Shear strength parameters based on the Coulomb envelope and Mohr stress circle

Also based on principle stresses, Coulomb equation is

$$\sigma_1 = k\sigma_3 + \sigma_c \tag{2}$$

In Eq. (2), parameters σ_c and k are uniaxial compressive strength and slope of line in the principal stresses coordinate, respectively which are determined using the following relations

$$k = \frac{1+\sin\varphi}{1-\sin\varphi} \tag{3}$$

$$\sigma_c = \frac{2c \times \cos\varphi}{1-\sin\varphi} \tag{4}$$

In order to determine shear strength parameters and stresses on the failure plane, Mohr stress circle is utilized.

When a sample is loaded triaxially so that $\sigma_1 > \sigma_2 = \sigma_3$, it breaks down on the plane with weakest molecular bond while reaching to the maximum value for deviatoric stresses. In the present state, this plane makes the angle β with the minimum stress (For more illustration, refer to Fig. 1).

Hence, according to the Mohr stress circle, normal and shear stresses are defined as

$$\sigma_n = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\beta \tag{5}$$

$$\tau = \frac{\sigma_1 - \sigma_3}{2}\sin 2\beta \tag{6}$$

3. Determination of stresses on failure plane based on the Generalized Hoek-Brown failure criterion

The empirical relation of GHB criterion is (Hoek *et al*. 2002)

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(\frac{m_b \sigma_3}{\sigma_{ci}} + s\right)^a \tag{7}$$

so that

$$m_b = m_i \times e^{\left(\frac{GSI-100}{28-14D}\right)} \tag{8}$$

$$s = e^{\left(\frac{GSI-100}{9-3D}\right)} \tag{9}$$

$$a = 0.5 + \frac{e^{\left(-\frac{GSI}{15}\right)} - e^{\left(-\frac{20}{3}\right)}}{6} \tag{10}$$

In the above relations, m_b,s and a are the input parameters of criterion that are depended on the degree of friability of mass rock and they are estimated from geological strength index (GSI). Also, σ_1 and σ_3 present maximum and minimum of principle stresses, respectively and σ_{ci} is the uniaxial compressive strength of intact rock. Parameter m_i is Hoek-Brown constant in intact rock and D is distributed factor. It must be noted that, this criterion can be represented based on the stresses on the failure plane.

Normal stress on the failure plane is related to the shear stress tangential to the failure plane by relation suggested by Balmer (1952)

$$\sigma'_n = \frac{\sigma'_1 + \sigma'_3}{2} - \frac{\sigma'_1 - \sigma'_3}{2} \cdot \frac{d\sigma'_1 / d\sigma'_3 - 1}{d\sigma'_1 / d\sigma'_3 + 1} \tag{11}$$

$$\tau = (\sigma'_1 - \sigma'_3) \frac{\sqrt{d\sigma'_1 / d\sigma'_3}}{d\sigma'_1 / d\sigma'_3 + 1} \tag{12}$$

Hence stresses on failure plane are determined by using Eqs. (7)-(11) and (12) as

$$\sigma'_n = \sigma'_3 + \frac{\sigma_{ci}}{2}(m_b \frac{\sigma'_3}{\sigma_{ci} + s})^a \left[1 - \frac{am_b(m_b \dfrac{\sigma'_3}{\sigma_{ci}} + s)^{a-1}}{2 + am_b(m_b \dfrac{\sigma'_3}{\sigma_{ci}} + s)^{a-1}} \right] \tag{13}$$

$$\tau = \sigma'_{ci}(m_b \frac{\sigma'_3}{\sigma_{ci}} + s)^a \frac{\sqrt{1 + am_b(m_b \dfrac{\sigma'_3}{\sigma_{ci}} + s)^{a-1}}}{2 + am_b(m_b \dfrac{\sigma'_3}{\sigma_{ci}} + s)^{a-1}} \tag{14}$$

Thus, using Eqs. (13)-(14) stresses on the failure plane can be estimated.

4. Stress analysis on the failure plane of some typical rocks

To have a case study, available data in the literature including principal maximum (σ_1) and minimum (σ_3) stresses and angle of failure plane with respect to the maximum stress (β) are used which are tabulated in Table 1.

To obtain the stresses on the failure plane in accordance with the real failure plane, Mohr stress circle is considered. Hence using Eqs. (5) and (6), normal and shear stresses are determined which are presented in Table 2.

Investigating Table 2 indicates that, as the differences of principal stresses increases, normal and shear stresses on the failure plane increases.

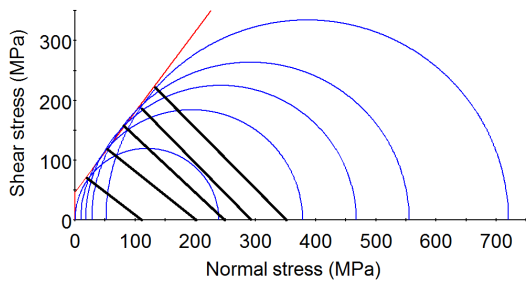
According to the date shown in Table 1, MC and GHB envelopes are depicted in Figs. 2 and 3, respectively. It is assumed the showed angle in the figures is equal to 2β. As seen from the figures, for some of the samples there is no tangential envelope on the Mohr stress circles (e.g., Fig. 2(d) or 3(a)). Therefore, the failure plane is not tangential to the circle.

Table 1 Minimum and maximum stresses (MPa) and also angle of failure plane (degree)

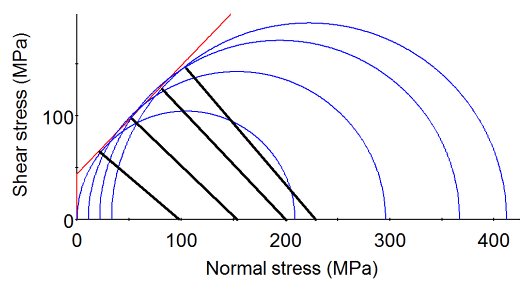
Darley Dale sandstone (Ramez 1967)			Typical rock (Liu *et al*. 2004)			Duhman dolomite (Mogi 1929)			Westerly granite (Mogi 1929)		
σ_3	σ_1	β	σ_3	σ_1	β	σ_3	σ_1	β	σ_3	σ_1	β
0.0	10.5	10	0.0	103.5	20	0.1	209	20	0.1	239	19
2.5	26.0	24	20	180.2	37	10.8	296	21	9.5	378	20
7.5	43.5	31	40	224.0	45	21.6	367	22	17.0	467	21
13.5	56.9	33	80	310.5	47	33.0	412	25	27.5	555	23
15.0	60.6	44	-	-	-	-	-	-	51.0	720	24

Table 2 Stresses (MPa) on the failure plane based on the Mohr stress circle

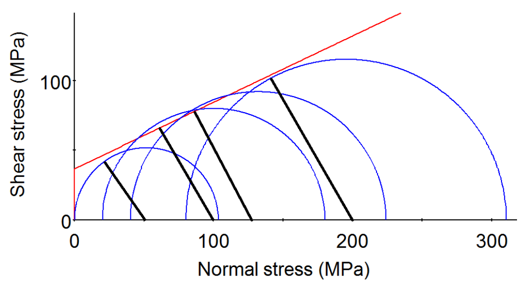
Westerly granite			Duhman dolomitec			Typical rock			Darley Dale sandstone		
$\sigma_1 - \sigma_3$	σ_n	τ	$\sigma_1 - \sigma_3$	σ_n	τ	$\sigma_1 - \sigma_3$	σ_n	τ	$\sigma_1 - \sigma_3$	σ_n	c
238.9	25.4	73.5	208.9	24.5	67.1	103.5	23.2	7.5	10.5	9.3	3.4
368.5	52.6	118.4	285.2	47.4	95.4	160.2	30.5	17.3	23.5	21.2	9.9
449.5	75.2	150.4	345.4	70.0	120.0	184.0	35.2	21.8	36.0	33.9	15.9
527.5	108.0	189.7	379.0	100.6	145.2	230.5	40.4	22.8	43.5	44.0	19.8
669.0	161.6	248.6					-	-	46.0	38.6	22.8



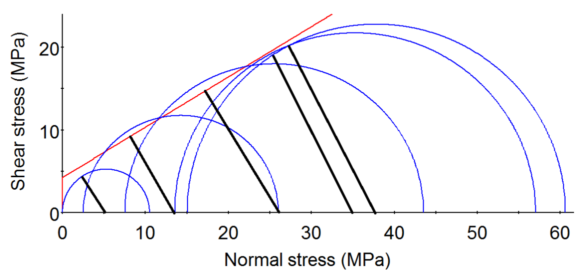
(a) Westerly granite



(b) Duhman dolomite



(c) Typical rock

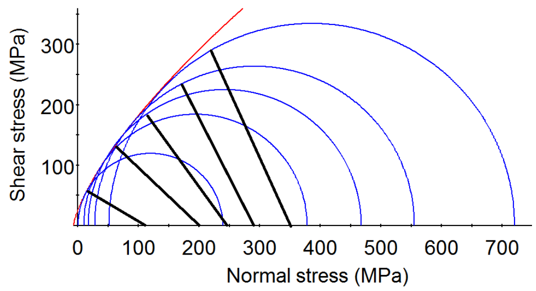


(d) Darley Dale sandstone

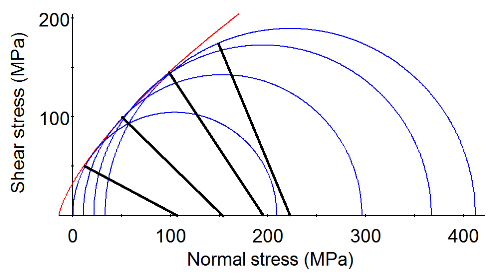
Fig. 2 Mohr-Coulomb envelope

Table 3 Stresses (MPa) on the failure plane and angle of failure plane (degree) based on the Mohr-Coulomb criterion

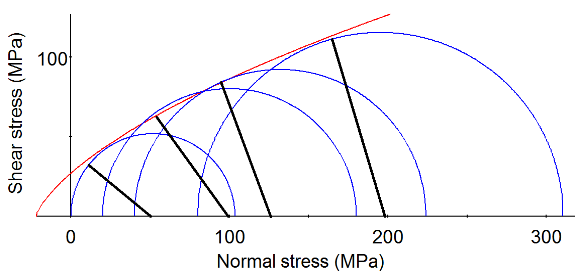
(a) Westerly granite				(b) Duhman dolomite			
$\sigma_1 - \sigma_3$	σ_n	τ	β	$\sigma_1 - \sigma_3$	σ_n	τ	β
238.9	0.0	52.0	20	208.9	0.0	28	24
368.5	51.59	112.1	20	285.2	38.9	64	24
449.5	64.0	128.2	20	345.4	59.0	82	24
527.5	115.7	189.2	20	379.0	76.0	98	24
669.0	123.6	198.3	20	-	-	-	-
(c) Typical rock				(d) Darley Dale sandstone			
$\sigma_1 - \sigma_3$	σ_n	τ	β	$\sigma_1 - \sigma_3$	σ_n	τ	β
103.5	0.0	36.9	30	10.5	0.0	4.3	30
160.2	61.2	66.4	30	23.5	7.5	8.8	30
184.0	89.5	79.5	30	36.0	15.0	13.3	30
230.5	147.7	107.1	30	43.5	24.0	19.0	30
-	-	-	-	46.0	26.6	20.0	30



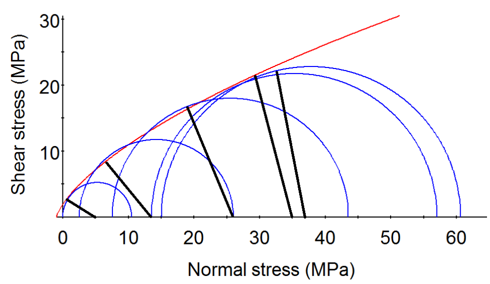
(a) Westerly granite



(b) Duhman dolomite



(c) Typical rock



(d) Darley Dale sandstone

Fig. 3 Hoek-Brown envelope

Table 4 Stresses (MPa) on the failure plane and angle of failure plane (degree) based on the Hoek-Brown criterion

(a) Westerly granite				(b) Duhman dolomite			
$\sigma_1 - \sigma_3$	σ_n	τ	β	$\sigma_1 - \sigma_3$	σ_n	τ	β
238.9	20.69	70.3	15.0	208.9	14.5	39.5	15
368.5	42.3	102.3	22.5	285.2	37.5	64.0	22
449.5	61.3	127.2	30.0	345.4	62.0	86.0	29
527.5	130.5	205.8	32.5	379.0	84.0	104.0	32.5
669.0	139.4	214.8	35.0	-	-	-	-
(c) Typical rock				**(d) Darley Dale sandstone**			
$\sigma_1 - \sigma_3$	σ_n	τ	β	$\sigma_1 - \sigma_3$	σ_n	τ	β
103.5	25.4	46.2	22.5	10.5	1.0	3.4	20.0
160.2	59.4	66.2	29.0	23.5	6.8	8.6	30.0
184.0	92.6	82.8	34.0	36.0	16.0	14.7	33.0
230.5	154.5	108.3	35.5	43.5	29.3	20.1	35.0
-	-	-	-	46.0	30.5	21.0	37.5

Also using these figures, stresses and angle of failure plane for the samples are shown in Tables 3 and 4 based on the MC and GHB criteria, respectively. As seen from Table 3, normal stresses on the plane of failure in uniaxial loading are zero.

5. Results and discussion

In this section, a comparison is done between the performance of MC and GHB criteria for determining the stresses and angle of failure plane. Hence, performance indices consist of values account for (*VAF*) and root mean square error (*RMSE*) are calculated.

$$VAF = \left(1 - \frac{\mathrm{var}(y_i - \hat{y}_i)}{\mathrm{var}(y)}\right) \times 100\% \tag{15}$$

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(y_i - \hat{y}_i)^2} \tag{16}$$

In the above equation, y_i indicates stresses determined based on the Mohr stress circle and $\hat{y}_i$ shows stresses based on the two criterions. Also, N is number of samples.

It is clear that, higher values for performance index *VAF* correspond to the better and more accurate results. In the other words, if the stresses determined from Mohr stress circle and two criteria are exactly the same, *VAF* will be equal to 100%.

Another performance index is the root mean square error *RMSE*. It most be noted, that values close to zero corresponds to the more accurate analysis.

In the following two sub-sections are considered. In the first section a comparison is done for the stresses on the failure plane obtained by MC, GHB envelopes and those stresses on the real plane of failure. In the second part, the angle of failure plane obtained from two criterions is compared with the real angle of failure plane.

Using Eqs. (15)-(16), the calculated *VAF* and *RMSE* performance indices are presented in Table 5.

Index of *VAF* for normal stresses based on the Mohr stress circle and MC criterion is 49% and for the Mohr stress circle and GHB criterion is estimated as 55% which is not a desirable performance index. Also, for the case of shear stress, *VAF* is 74% for MC criterion and 76% for GHB criterion.

Also as Table 5 indicates, index *RMSE* obtained from GHB is more accurate in comparison with the MC criterion. Therefore study of two indices show that results from GHB criterion is more acceptable in comparison with the MC criterion.

In Fig. 4, a relative comparison for the angle of failure plane is done based on the results from the MC and GHB envelopes (Tables 3 and 4) and experimental results (Table 1). This figure is plotted for 18 data sets.

It is clear that, there are differences between the angles of failure plane resulted from each envelope and real analysis. Also in each case, the angle obtained from MC envelope is equal which is not accurate and differ from real ones.

Table 5 The values account for (*VAF*) and the root mean square error (*RMSE*) indices

Model	Predicted parameter	*VAF*	*RMSE*
Mohr-Coulomb criterion	σ_n	0.49	31.5
	τ	0.74	36
Hoek-Brown criterion	σ_n	0.55	30.5
	τ	0.76	34.7

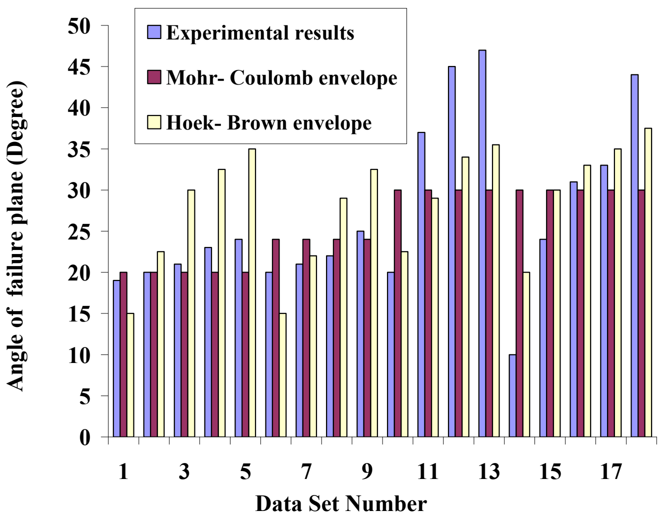
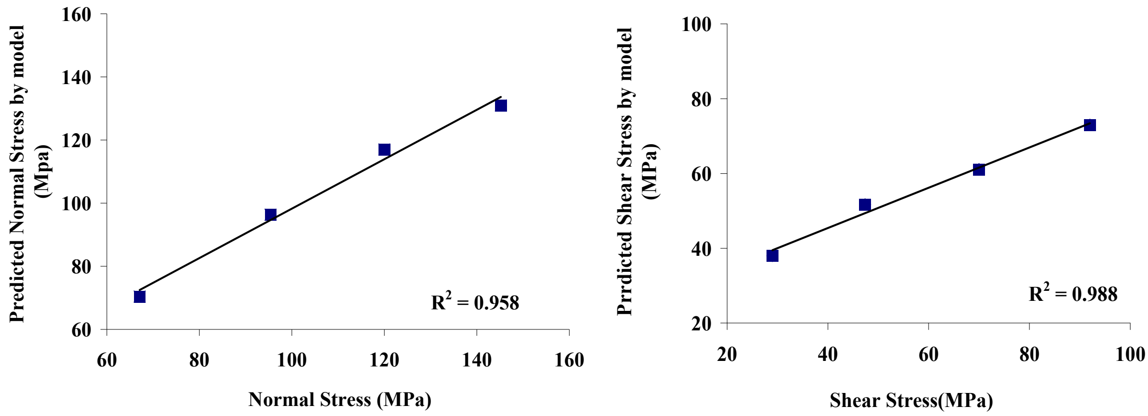


Fig. 4 Variation of angle of failure plane versus different envelopes



(a) Correlation for normal stress

(b) Correlation for shear stress

Fig. 5 Correlation between the real stresses on the failure plane and predicted stresses based on the suggested model for Duhman dolomite samples

In order to have a more accurate model for predicting the stresses on the failure plane, a multivariable regression analysis is used. Differences of principal stresses ($\sigma_1 - \sigma_3$) and angle of failure plane (β) are considered as independent variables. For proposing the relation, some data sets consist of Westerly granite, Darley Dale sandstone and Typical rock are used. Therefore, the regressed relations the normal and shear stresses are

$$\sigma_n = 0.131(\sigma_1 - \sigma_3) + 0.338\beta + 7.3 \tag{17a}$$

$$\tau = 0.329(\sigma_1 - \sigma_3) + 0.923\beta - 16.8 \tag{17b}$$

The correlation coefficients for the normal and shear stresses are 0.73 and 0.91, respectively. To validate the proposed relations, Duhman dolomite data sets are used. Results of validation are shown in Fig. 5.

As figure shows, the predicted results by the suggested model are close to those of empirical values.

6. Conclusions

In the present paper, efficiency of Mohr-Coulomb (MC) and Generalized Hoek-Brown (GHB) criteria for determining the stresses on the plane of failure was investigated. To verify the results, two performance indices (*VAF* and *RMSE*) are determined. Comparison of these indices shows that, there are differences between the results obtained from criteria and Mohr stress circle. Hence, stresses and angle of failure plane are inaccurate which leads to overestimate determination of mobilized shear strength parameters. Also, results show that GHB criterion is more exact in comparison with the MC criterion.

In addition, for obtaining the stresses on the failure plane, a multivariable regressed relation is proposed based on the stresses on the real plane of failure.

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