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Collapse mechanism for deep tunnel subjected to seepage force in layered soils

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Abstract. The prediction of impending collapse of deep tunnel is one of the most difficult problems. Collapse mechanism of deep tunnel in layered soils is derived using a new curved failure mechanism within the framework of upper bound theorem, and effects of seepage forces are considered. Nonlinear failure criterion is adopted in the present analysis, and the possible collapse shape of deep tunnel in the layered soils is discussed in this paper. In the layered soils, the internal energy dissipations along velocity discontinuity are calculated, and the external work rates are produced by weight, seepage forces and supporting pressure. With upper bound theorem of limit analysis, two different curve functions are proposed for the two different soil stratums. The specific shape of collapse surface is discussed, using the proposed curve functions. Effects of nonlinear coefficient, initial cohesion, pore water pressure and unit weight on potential collapse are analyzed. According to the numerical results, with the nonlinear coefficient increase, the shape of collapse block will increase. With initial cohesion of the upper soil increase, the shape of failure block will be flat, and with the lower soil improving, the size of collapsing will be large. Furthermore, the shape of collapsing will decrease with the unit weight decrease.

Keywords: collapse mechanism; layered soils; nonlinear criterion; seepage force; upper bound

1. Introduction

There are several approaches to estimate the face tunnel stability problem which is the classical issue investigated by a number of scholars. Apart from limit equilibrium, numerical simulation and experimental method, limit analysis approach is widely adopted to evaluate the face and crown stability in natural cavities and tunnels, and more rigorous results can be derived with no assumption and the extreme simplicity of the calculations needed. By constructing a kinematically admissible failure mechanism and an admissible stress field, Leca and Dormieux (1990) established three dimensional failure modes, which proved to be rational through strict theoretical calculation and model test. On the basis of results obtained by Leca and Dormieux (1990), Soubra (2000) improved the failure mode and used the improved mode to analyze the face stability of the shallow tunnel. And compared with the results obtained by Leca and Dormieux (1990), Soubra (2000) found that the improved calculation result is superior to the existing results. Later, combined with reliability analysis and limit analysis upper bound theorem, Mollon *et al.* (2010)

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analyzed the three dimensional face stability of the shallow buried circular tunnel and found that reliability of overall system depends on the serviceability limit states. In addition, using limit analysis upper bound method to study three dimensional face stability of shallow tunnel, Subrin and Wong (2002) put forward a kind of failure mode and proved the validity of this mechanism.

As a matter of fact, a multitude of mountainous tunnels in deep stratum have been excavated, and the roof failure often occurs in deep tunnels. In previous work, some scholars shift their attention to the study of roof stability though it is indeed a rather tough problem owing to the random variability of mechanical properties of the rock as well as other affecting factors, such as the presence of cracks and fractures in the rock masses, for instance, Indraratna *et al*. (2010) proposed a joint material mode to describe the mechanical characteristics of the discontinuous rock masses instead of equivalent continuum-based method which is considered to be unsuitable and soil-infilled joint model was then employed to estimate the rock wedge in the crown of cavities. Later, employing nonlinear yield criterion, Fraldi and Guarracino (2009, 2010, 2011, 2012) put forward a curved collapse mechanism on the crown of deep tunnels proposed through practical engineering projects and obtained analytic solutions of collapse dimensions with the help of variational principle in the realm of plasticity theory. However, notice that the results of actual roof collapse suffer from the fact that the collapsing block presents the characteristics of three dimensions rather than simple two dimensions.

In tunnel engineering, it is generally acknowledged that ground water has an adverse influence on the stability of underground structures, thus the corresponding effect should not be ignored and more attention need to be fixed on such an issue. Huang *et al*. (2010) used the approach of conformal mapping to gain the analytic findings of steady ground water flowing into a horizontal tunnel. Feng *et al*. (2012) evaluated the influence of surrounding rock deterioration on segmental lining structure for underwater shield tunnel with large profile by model test and the calculated result shows that the crown and bottom of tunnel are liable to collapse. Wang *et al*. (2008) carried out the theoretical and experimental study of external water pressure on tunnel lining in controlled drainage under high water level and the solutions indicate that there exists an optimum size for grouting zone to supporting pressure. To investigate the impact of ground water exerted on the potential failure, the pore water pressure was regarded as external force to discuss the upper bound solutions in the framework of upper bound theorem, and then some examples were made to evaluate the dimensions of collapse.

With limit analysis method, the above works mentioned are based on the fact that the tunnel is excavated in single stratum. However, tunnels are often excavated in layered stratum described by different material parameters in practices, and subjected to seepage force. To solve the problem, this paper proposes new failure mechanism which satisfies the kinematically admissible conditions. The seepage forces are obtained from the gradient of excess pore pressure distribution, and are taken as external loading in the upper bound analysis. Then, the pore pressure is calculated by using pore pressure coefficient. The tunnel roof is located in the two layered stratums. The new failure mechanism consists of two different functions. By considering the falling block to be symmetrical with respect to the y-axis and adopting Greenberg minimum principle, a smooth and continuous curve function is established, as shown in Fig. 1. The coordinate system is shown, and it can be considered that the failure mechanism is made up of two curves, $y = f(x)$ and $y = g(x)$ in the coordinate system. c_0, σ_t, m and ρ are geotechnical parameters. L_1 and L_2 are the widths of the collapse block respectively at the layered position and at the tunnel roof. h_1 is the height of the collapse block in the upper soil, h_2 is the height between the tunnel roof and the layered position, and Δh is the height between the origin of coordinate and roof. q is the supporting force on the tunnel. Based on the new failure mechanism and nonlinear failure criterion, the internal energy dissipation and external work rates are calculated, and the collapse mechanisms of deep tunnel are obtained by optimization. A study is conducted to investigate the effects of the parameters in the nonlinear failure criterion and pore water coefficient on collapsing tunnel in the layered stratum.

The results in the paper can provide a rational failure mechanism for tunnel in layered soils, which can be used to predict the range of failure for tunnel roof. So that engineers choose more reliable support for tunnel in layered soils.

2. Upper bound theorem and nonlinear Power-Law criterion

2.1 Upper bound theorem

The upper bound theorem can be depicted as: when the velocity boundary conditions and strain compatibility condition are satisfied, the load calculated by equating the external rate of work with the rates of the energy dissipation in any kinematically admissible velocity field is no less than the actual load, which can be written as follows

$$\int_V \sigma_{ij} \cdot \dot{\varepsilon}_{ij} dV \geq \int_S T_i \cdot v_i dS + \int_V X_i \cdot v_i dV + \int_V -grad\ u \cdot v_i dV \tag{1}$$

in which σ_{ij} is the stress tensor in the kinematically admissible velocity field respectively, $\dot{\varepsilon}_{ij}$ is the strain rate, T_i is the limit load on the boundary surface, S is the length of velocity discontinuity, X_i is the body force of the mechanism which is caused by weight, $-grad\ u$ is excess pore pressure, V is the volume of the plastic zone, v_i is the velocity along the velocity discontinuity surface.

2.2 Nonlinear power-law criterion

In many practical problems, a substantial amount of experimental evidence suggests that the failure envelop of many soils is not linear in the $\sigma_n - \tau_n$ stress space. This departure from linearity is significant for stability calculation (Sun and Qin 2014, Zhang and Wang 2015, Zhu *et al*. 2010, 2012). Let the nonlinear Power-Law failure criterion be written in the plane $\sigma_n - \tau_n$. Generally, the nonlinear criterion can be expressed as

$$\tau_n = c_0 \left(1 + \frac{\sigma_n}{\sigma_t} \right)^{\frac{1}{m}} \tag{2}$$

where σ_n and τ_n are the normal stress and shear stress on the failure surface respectively, c_0 is the initial cohesion, σ_t is the axial tensile stress and m is the nonlinear coefficient. The values of parameters c_0, σ_t and m can be determined from test results.

3. Upper bounder analysis and variational approach

Before the analysis of collapsing mechanism of deep tunnel subjected to seepage forces in layered soils, it must point out that all the required conditions should be fulfilled. The behavior of the soil is ideally plastic, that is, all stress points don't exceed the yield surface. The yield surface is convex and the rates of plastic deformation can be obtained from the yield function through flow rule.

When the soils obey to associated flow rule, that is, the plastic potential surface is coincident with the yield surface, the two plastic potential functions of two soil strata are

$$\Im_1 = \tau_{n1} - c_{01}\left(1+\frac{\sigma_{n1}}{\sigma_{t1}}\right)^{\frac{1}{m_1}} \tag{3}$$

$$\Im_2 = \tau_{n2} - c_{02}\left(1+\frac{\sigma_{n2}}{\sigma_{t2}}\right)^{\frac{1}{m_2}} \tag{4}$$

where 1 and 2 in the subscript are the upper soil and the lower soil, respectively. According to the flow rule, plastic strain increment is proportional to the gradient of the plastic potential function. Through calculating, the normal stress of any point on the velocity discontinuity can be expressed as follows

$$\sigma_{n1} = -\sigma_{t1} + \sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}} f'(x)^{\frac{m_1}{(m_1-1)}} \tag{5}$$

$$\sigma_{n2} = -\sigma_{t2} + \sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}} g'(x)^{\frac{m_2}{(m_2-1)}} \tag{6}$$

where $f(x)$ is the function of velocity discontinuity surface in the upper soil and $f'(x)$ is the first derivative of $f(x)$, and $g(x)$ is the equation of velocity discontinuity surface in the lower soil and $g'(x)$ is the first derivative of $g(x)$.

Then at impending collapse, the dissipation densities of the internal forces on the detaching surface, $\dot{D}_{i1}$ and $\dot{D}_{i2}$, are

$$\dot{D}_{i1} = \sigma_{n1}\dot{\varepsilon}_{n1} + \tau_{n1}\dot{\gamma}_{n1} = \frac{v}{w}\left[1+f'(x)^2\right]^{-\frac{1}{2}}\left[-\sigma_{t1} + \sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}}(1-m_1)f'(x)^{\frac{m_1}{(m_1-1)}}\right] \tag{7}$$

$$\dot{D}_{i2} = \sigma_{n2}\dot{\varepsilon}_{n2} + \tau_{n2}\dot{\gamma}_{n2} = \frac{v}{w}\left[1+g'(x)^2\right]^{-\frac{1}{2}}\left[-\sigma_{t2} + \sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}}(1-m_2)g'(x)^{\frac{m_2}{(m_2-1)}}\right] \tag{8}$$

where $\dot{\varepsilon}_{n1}$ and $\dot{\varepsilon}_{n2}$ are normal plastic strain rates respectively, $\dot{\gamma}_{n1}$ and $\dot{\gamma}_{n2}$ are shear plastic strain rates respectively, w is the thickness of the plastic detaching zone, and v is the velocity of the collapsing block.

By considering the falling block to be symmetrical with respect to the y-axis, the total dissipation at the right impending collapse results

$$\begin{aligned} W_D &= \int_0^{L_1} \dot{D}_{i1} w\sqrt{1+f'(x)^2}\,dx + \int_{L_1}^{L_2} \dot{D}_{i2} w\sqrt{1+g'(x)^2}\,dx \\ &= v\int_0^{L_1}\left[-\sigma_{t1} + \sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}}(1-m_1)f'(x)^{\frac{m_1}{(m_1-1)}}\right]dx \\ &= v\int_{L_1}^{L_2}\left[-\sigma_{t2} + \sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}}(1-m_2)g'(x)^{\frac{m_2}{(m_2-1)}}\right]dx \end{aligned} \tag{9}$$

The work rate of failure block produced by weight can be obtained by integral calculation

$$\begin{aligned} W_e &= v\left\{\rho_1'\int_0^{L_1}\left[f(x)-f(L_1)\right]dx + \rho_2'\int_0^{L_1}\left[f(L_1)-c(x)\right]dx + \rho_2'\int_{L_1}^{L_2}\left[g(x)-c(x)\right]dx\right\} \\ &= v\left\{\rho_1'\int_0^{L_1} f(x)\,dx + \rho_2'\int_{L_1}^{L_2} g(x)\,dx - \rho_2'\int_0^{L_2} c(x)\,dx + (\rho_2' - \rho_1')L_1 f(L_1)\right\} \end{aligned} \tag{10}$$

in which ρ'_1 and ρ'_2 are the buoyant weight per unit volume of the upper and lower soils, respectively. $\rho' = \rho - \rho_w$, in which ρ is the weight per unit volume of the soil, and ρ_w is the unit weight of water. The function $c(x)$ is the equation describing the circular tunnel profile which can be written as follows

$$c(x) = \sqrt{R^2 - L_2^2} - \sqrt{R^2 - x^2} \tag{11}$$

where R is the radius of the circular tunnel respectively.

According to the study of Saada *et al*. (2012), the distribution of excess pore pressure is defined as

$$u = p - p_w = p - \rho_w h \tag{12}$$

where p is the pore water pressure at the considered point which can be derived by an appropriate method $p = r_u\rho h$, r_u referring to pore pressure coefficient, and $p_w = \rho_w h$ is the hydrostatic distribution for pore pressure, h is the vertical distance between the roof of the tunnel and the top of the failure block. Then, $-grad\, u$ can be calculated by

$$-grad\, u = \rho_w - r_u\rho \tag{13}$$

Thus, the work rate produced by seepage forces along the velocity discontinuity surface is

$$\begin{aligned} W_u &= v\int_0^{L_1}(\rho_w - r_u\rho_1)\left[f(x)-f(L_1)\right]dx + v\int_0^{L_1}(\rho_w - r_u\rho_2)\left[f(L_1)-c(x)\right]dx \\ &\quad + v\int_{L_1}^{L_2}(\rho_w - r_u\rho_2)\left[g(x)-c(x)\right]dx \\ &= v\left\{\int_0^{L_1}(\rho_w - r_u\rho_1)f(x)\,dx + \int_{L_1}^{L_2}(\rho_w - r_u\rho_2)g(x)\,dx \right. \\ &\quad \left. - \int_0^{L_2}(\rho_w - r_u\rho_2)c(x)\,dx + r_u(\rho_1 - \rho_2)L_1 f(L_1)\right\} \end{aligned} \tag{14}$$

The power of supporting pressure of circular tunnel can be expressed as

$$W_q = Rqv \arcsin\frac{L_2}{R}\cos\pi \tag{15}$$

where q is the supporting pressure of the circular tunnel. Since the solution calculated by virtual work function is just an upper bound solution, moreover, according to the upper bound theorem, the solution is more close to real limit solution. Therefore, in order to obtain the optimum upper solution, it is necessary to construct an objective function ξ by the external rate of work and the rate of the energy dissipation

$$\xi = W_D - W_e - W_u - W_q \tag{16}$$

Substituting Eqs. (9), (10) , (14) and (15) into Eq. (16), the expression of objective function is given

$$\xi = v\left[\xi_1 + \xi_2 + (1-r_u)\rho_2\int_0^{L_2} c(x)dx - (1-r_u)(\rho_2-\rho_1)L_1 f(L_1) + Rq\arcsin\frac{L_2}{R}\right] \tag{17}$$

in which

$$\begin{aligned}\xi_1 &= \int_0^{L_1}\Lambda_1\left[f(x),\ f'(x),\ x\right]dx\\ &= \int_0^{L_1}\left[-\sigma_{t1}+\sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}}(1-m_1)f'(x)^{\frac{m_1}{(m_1-1)}}-(1-r_u)\rho_1 f(x)\right]dx\end{aligned} \tag{18}$$

$$\begin{aligned}\xi_2 &= \int_{L_1}^{L_2}\Lambda_2\left[g(x),\ g'(x),\ x\right]dx\\ &= \int_{L_1}^{L_2}\left[-\sigma_{t2}+\sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}}(1-m_2)g'(x)^{\frac{m_2}{(m_2-1)}}-(1-r_u)\rho_2 g(x)\right]dx\end{aligned} \tag{19}$$

The difficulty is how to find the extremum of the objective function ξ. It can be found that the objective function ξ consists of two objective functions, ξ_1 and ξ_2. Thus, assuming that the objective function ξ_1 and ξ_2 obtain the extremum simultaneously, the extremum of the objective function ξ can be obtained. It also can be found that ξ_1 is determined by Λ_1 $[f(x), f'(x), x]$ and ξ_2 is determined by Λ_2 $[g(x), g'(x), x]$. Therefore, the calculation of upper solution of possible collapse is regarded as searching the minimum values of objective functions Λ_1 and Λ_2. The expressions of Λ_1 and $\Lambda_2$are functions which are turned into two Euler's equations by variational calculation. The variational equations of Λ_1 and Λ_2 on stationary conditions can be expressed as follows

$$\frac{\partial\Lambda_1}{\partial f(x)} - \frac{\partial}{\partial x}\left[\frac{\partial\Lambda_1}{\partial f'(x)}\right] = 0, \qquad \frac{\partial\Lambda_2}{\partial g(x)} - \frac{\partial}{\partial x}\left[\frac{\partial\Lambda_2}{\partial g'(x)}\right] = 0 \tag{20}$$

and from Eqs. (18) and (19), the explicit forms of the two Euler's equations for the problem are

$$-(1-r_u)\rho_1+\tfrac{m_1}{(m_1-1)}\sigma_{t1}\left(\frac{c_{01}}{m_1\sigma_{t1}}\right)^{\frac{m_1}{(m_1-1)}}f'(x)^{\frac{2-m_1}{m_1-1}}f''(x)=0 \tag{21}$$

$$-(1-r_u)\rho_2+\tfrac{m_2}{(m_2-1)}\sigma_{t2}\left(\frac{c_{02}}{m_2\sigma_{t2}}\right)^{\frac{m_2}{(m_2-1)}}g'(x)^{\frac{2-m_2}{m_2-1}}g''(x)=0 \tag{22}$$

It is obvious that Eqs. (21) and (22) are two nonlinear second-order homogeneous differential equations which can be solved by integral calculation. Thus the expressions of velocity discontinuity surface $f(x)$ and $g(x)$ can be obtained

$$f(x)=k_1(x+(1-r_u)^{-1}\rho_1^{-1}n_0)^{m_1}-n_1 \tag{23}$$

$$g(x)=k_2(x+(1-r_u)^{-1}\rho_2^{-1}n_2)^{m_2}-n_3 \tag{24}$$

in which

$$k_1=\frac{\sigma_{t1}}{(1-r_u)\rho_1}\left[\frac{(1-r_u)\rho_1}{c_{01}}\right]^{m_1},\qquad k_2=\frac{\sigma_{t2}}{(1-r_u)\rho_2}\left[\frac{(1-r_u)\rho_2}{c_{02}}\right]^{m_2} \tag{25}$$

n_0, n_1, n_2 and n_3 are integration constants, respectively, which can be determined by boundary condition. Since the detaching curves are supposed symmetric with respect to the *y*-axis, the equilibrium of the stresses on the plane of symmetry $x = 0$ requires that the shear component of stress vanishes on this plane, that is

$$\tau_{xy}(x=0,y=-(h_1+h_2+\Delta h))=0 \tag{26}$$

Furthermore, it can be found from Fig. 1 that there are three geometric equations to be satisfied

$$g(x=L_2)=0 \tag{27}$$

$$f(x=L_1)=-(h_2+\Delta h) \tag{28}$$

$$g(x=L_1)=-(h_2+\Delta h) \tag{29}$$

In order to keep the whole curve smooth, an equation should be satisfied as follows

$$f'(x=L_1)=g'(x=L_1) \tag{30}$$

Based on Eqs. (26), (27) and (28), the values of constants n_0, n_1, n_2 and n_3 can be determined. Then, the $f(x)$ and $g(x)$ can be expressed as

$$f(x)=k_1x^{m_1}-(h_1+h_2+\Delta h) \tag{31}$$

$$g(x)=k_2(x+Z)^{m_2}-k_2(L_2+Z)^{m_2} \tag{32}$$

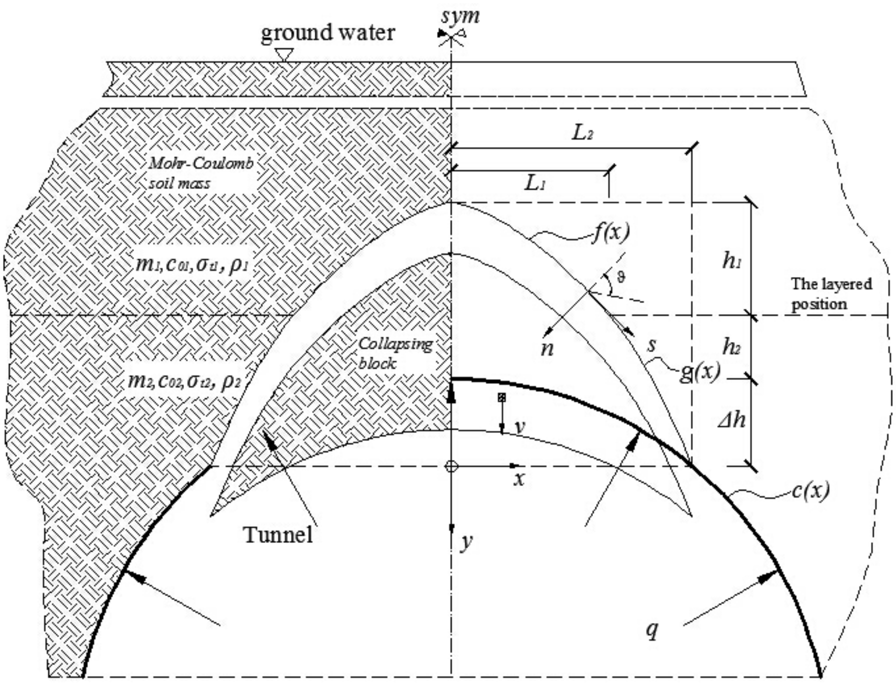


Fig. 1 Curved failure mechanism of circular tunnels excavated in the layered stratums

where

$$Z = \left[\frac{m_1 k_1}{m_2 k_2}\right]^{\frac{1}{m_2-1}} L_1^{\frac{m_1-1}{m_2-1}} - L_1 \tag{33}$$

Then the objective function ξ can be obtained, which is

$$\begin{aligned}\xi = v\Bigg\{ & Rq\arcsin\frac{L_2}{R} + \left[(1-r_u)\rho_1(h_1+h_2+\Delta h) - \sigma_{t1}\right]L_1 - \left[\frac{m_1}{(m_1+1)}\right]\sigma_{t1}\left[\frac{(1-r_u)\rho_1}{c_{01}}\right]^{m_1} L_1^{m_1+1} \\ & + \left[(1-r_u)\rho_2 k_2 (L_2+Z)^{m_2} - \sigma_{t2}\right](L_2 - L_1) \\ & - \left[\frac{m_2}{(m_2+1)}\right]\sigma_{t2}\left[\frac{(1-r_u)\rho_2}{c_{02}}\right]^{m_2}\left[(L_2+Z)^{m_2+1} - (L_1+Z)^{m_2+1}\right] \\ & + (1-r_u)(\rho_2-\rho_1)(h_2+\Delta h)L_1 + \left(\frac{1}{2}\right)(1-r_u)\rho_2\left[L_2\sqrt{R_2 - L_2^2} - R^2\arcsin\frac{L_2}{R}\right]\Bigg\}\end{aligned} \tag{34}$$

For the purpose of determining the values of L_1, L_2 and h_1, it is necessary to seek three linearly independent equations which include the three unknown constants. By equating external rate of work and the internal energy dissipation, an equation can be obtained

$$
\begin{aligned}
\xi = v\Bigg\{ & Rq\arcsin\frac{L_2}{R} + \left[(1-r_u)\rho_1(h_1+h_2+\Delta h)-\sigma_{t1}\right]L_1 - \left[\frac{m_1}{(m_1+1)}\right]\sigma_{t1}\left[\frac{(1-r_u)\rho_1}{c_{01}}\right]^{m_1} L_1^{m_1+1} \\
& + \left[(1-r_u)\rho_2 k_2 (L_2+Z)^{m_2} - \sigma_{t2}\right](L_2-L_1) \\
& - \left[\frac{m_2}{(m_2+1)}\right]\sigma_{t2}\left[\frac{(1-r_u)\rho_2}{c_{02}}\right]^{m_2}\left[(L_2+Z)^{m_2+1}-(L_1+Z)^{m_2+1}\right] \\
& + (1-r_u)(\rho_2-\rho_1)(h_2+\Delta h)L_1 + \left(\frac{1}{2}\right)(1-r_u)\rho_2\left[L_2\sqrt{R_2-L_2^2} - R^2\arcsin\frac{L_2}{R}\right]\Bigg\} \\
= 0 &
\end{aligned} \tag{35}
$$

Then, by combining and solving Eqs. (28), (29) and (35), the values of L_1, L_2 and h_1 can be obtained. Based on L_1, L_2 and h_1, the final forms of detaching curve consisting of $f(x)$ and $g(x)$ can be obtained, and the shape of failure surface can be drawn by Eqs. (31) and (32).

For one layered soil, Fraldi and Guarracino (2009) put forward a curved collapse mechanism on the crown of deep tunnels, and obtained analytical solutions of collapse dimension with the help of variational principle in the realm of plasticity theory. For two layered soils, when the properties of the two layered soils are the same, the present figures are same as the figures of Fraldi and Guarracino (2009). In order to avoid the same figures appearance, figures are not plotted for comparison.

In the preceding discussion, above equations are obtained under the condition of two layered soils are different. When $m_1 = m_2$, $c_{01} = c_{02}$, $\rho_1 = \rho_2$, and $\sigma_{t1} = \sigma_{t2}$, the properties of two layered soils are same. In the case, the present solutions agree well with Fraldi and Guarracino (2009), which proves that the analytical model of two layered soils is effective for predicting the collapse mechanism of tunnel roof.

4. Numerical results and discussions

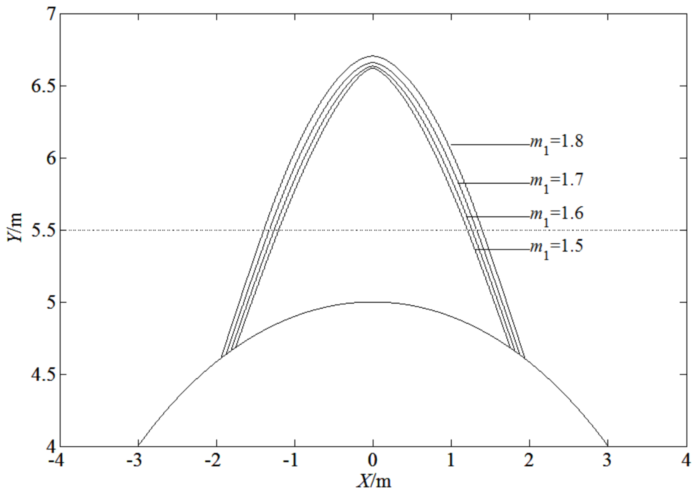
Based on the analytical solutions of velocity discontinuity surfaces $f(x)$ and $g(x)$ expressed in Eqs. (31) and (32), the explicit failure surfaces of circular tunnel in layered soils can be drawn for different parameters. Then, it is necessary to investigate the effect of parameters m, c_0, r_u and ρ on the shape of failure block. In generally, with the increase of buried depth, the nature of the soil is gradually getting better. And in order to distinguish the property of the two soils, then, in the process of calculation, the relationships of size among m, c_0, r_u and ρ are $m_1 > m_2$, $c_{01} \leq c_{02}$ and $\rho_1 < \rho_2$.

4.1 Effects of nonlinear coefficients m on failure mechanisms

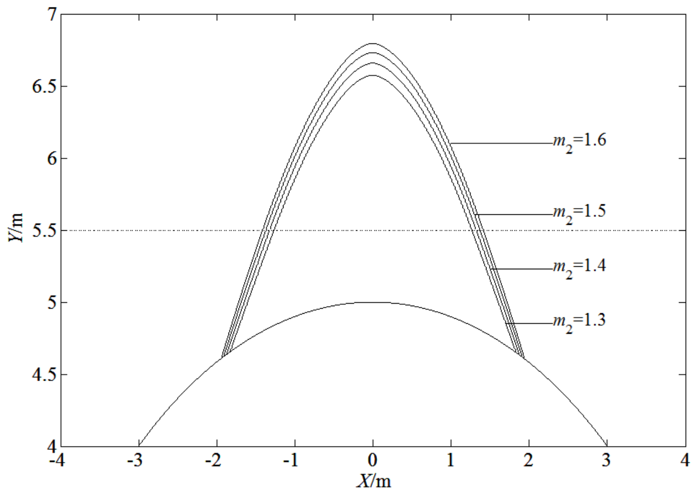
To investigate the effects of nonlinear coefficients m on the shape of failure surface, a deep tunnel roof located in two layered stratums is considered. Fig. 2 illustrates the failure mechanism of tunnel roof corresponding to $r_u = 0.1$, $c_{01} = 35$ kPa, $c_{02} = 50$ kPa, $\sigma_{t1} = 45$ kPa, $\sigma_{t2} = 60$ kPa, $\rho_1 = 17$ kN/m^3, $\rho_2 = 21$ kN/m^3, $q = 40$ kPa, $R = 5$ m and $h_2 = 0.5$ m. From Fig. 2, it is found that the widths L_1, L_2 and the height h_1 increase with the values of m_1 and m_2 increase. From the perspective of engineering, with the increase of nonlinear coefficient m, the nature of the soil is gradually getting worse, and the size of failure block is larger. This means that the circular tunnel excavated in the layered soils with low values of nonlinear coefficients m will contribute to controlling the size of failure block.

4.2 Effects of initial cohesions c_0 on failure mechanism

To investigate the effects of initial cohesions c_0 on the shape of failure surface, numerical results corresponding to $m_1 = 1.7$, $m_2 = 1.5$, $r_u = 0.1$, $\sigma_{t1} = 45$ kPa, $\sigma_{t2} = 60$ kPa, $\rho_1 = 17$ kN/m^3, $\rho_2 = 21$ kN/m^3, $q = 40$ kPa, $R = 5$ m and $h_2 = 0.5$ m are illustrated in Fig. 3. It can be seen that the shape of the collapsing block will become gradually flat with the parameter c_{01} increase. It also can be

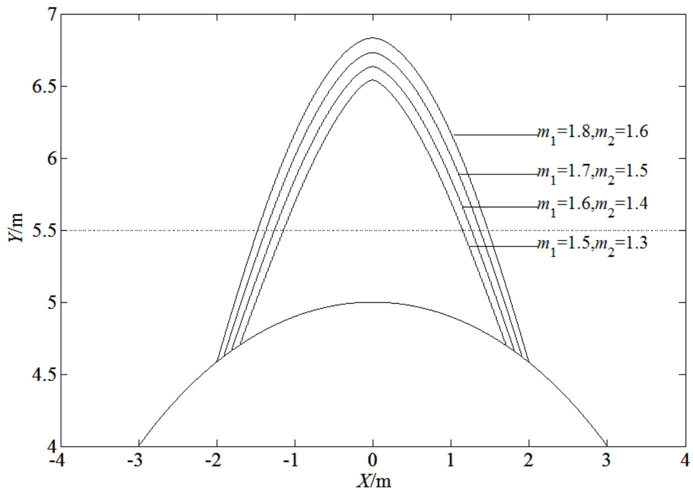


(a) Effects of nonlinear coefficient m_1 on failure mechanisms ($m_2 = 1.4$)



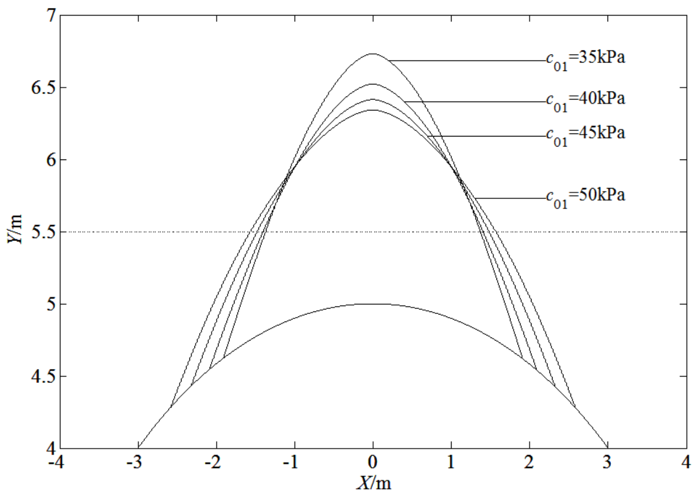
(b) Effects of nonlinear coefficient m_2 on failure mechanisms ($m_1 = 1.7$)

Fig. 2 Effects of nonlinear coefficients on collapse mechanisms of circular tunnel in the layered soils



(c) Effects of nonlinear coefficients m_1 and m_2 on failure mechanisms

Fig. 2 Continued



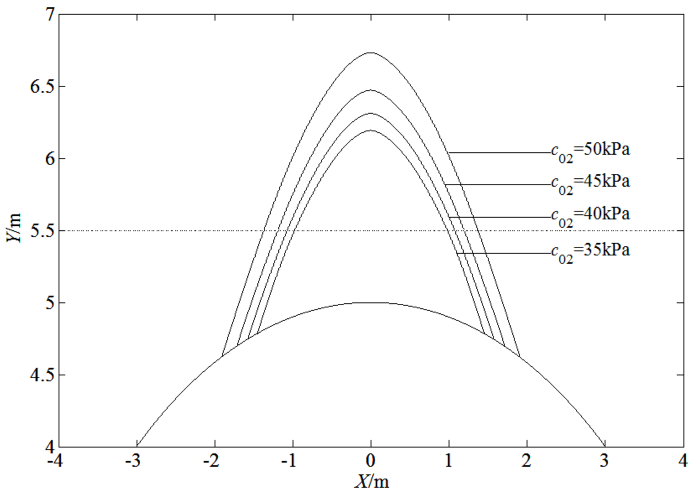
(a) Effects of initial cohesion c_{01} on failure mechanisms (c_{02} = 50 kPa)

Fig. 3 Effects of initial cohesions on collapse mechanisms of circular tunnel excavated in the layered soils

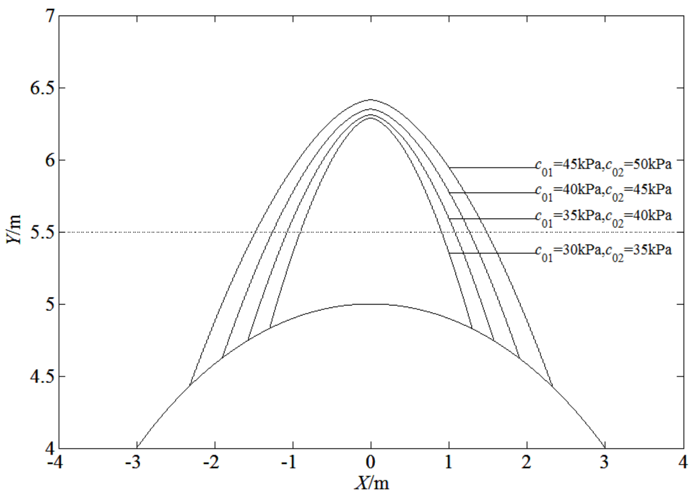
seen that the size of the collapsing block will improve no matter when only c_{02} increase or c_{01} and c_{02} increase simultaneously.

4.3 Effects of pore pressure coefficient r_u on failure mechanisms

Seepage forces have unfavorable effects of increasing collapse area when a linear yield criterion is used. To investigate how the collapse mechanisms are effected when a nonlinear yield



(b) Effects of initial cohesion c_{02} on failure mechanisms (c_{01} = 35 kPa)



(c) Effects of initial cohesions c_{01} and c_{02} on failure mechanisms

Fig. 3 Continued

criterion is used. Numerical results corresponding to m_1 = 1.6, m_2 = 1.4, c_{01} = 35 kPa, c_{02} = 50 kPa, σ_{t1} = 45 kPa, σ_{t2} = 60 kPa, ρ_1 = 17 kN/m^3, ρ_2 = 21 kN/m^3, q = 40 kPa, R = 5 m and h_2 = 0.5 m are illustrated in Fig. 4, with the varying from 0.0 to 0.3. From Fig. 4, it can be found that the collapsing area of tunnel roof will improve with the pore pressure coefficient increase.

4.4 Effects of unit weights ρ on failure mechanisms

To investigate the effects of unit weights on the shape of failure surface, the numerical results corresponding to m_1 = 1.7, m_2 = 1.5, σ_{t1} = 45 kPa, σ_{t2} = 60 kPa, c_{01} = 35 kPa, c_{02} = 50 kPa, q = 40 kPa, R = 5 m and h_2 = 0.5 m are represented in Fig. 5. It can be seen from Fig. 5 that the size of the collapsing block will decrease with the unit weight of the soil increase. From the perspective of energy analysis, this means that in order to set the equation $\xi = 0$ to be satisfied, the size of collapse block will be large when the values of unit weights is low. From the perspective of engineering, this means that the circular tunnel in layered soils with high values of unit weights will contribute to controlling the size of collapse block.

In the preceding analysis of tunnel failure mechanism, two layered soils under seepage force are considered. The case of the multi layered soils is not considered in this paper. However, there is no difficulty in principle in extending this present method to deal with the problem of multi

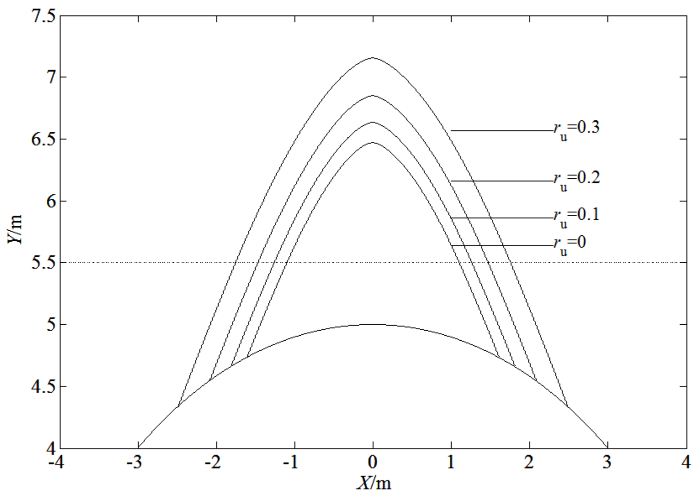
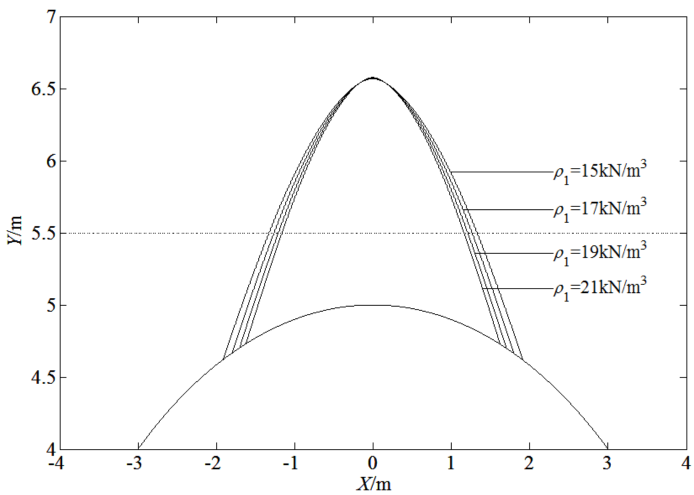
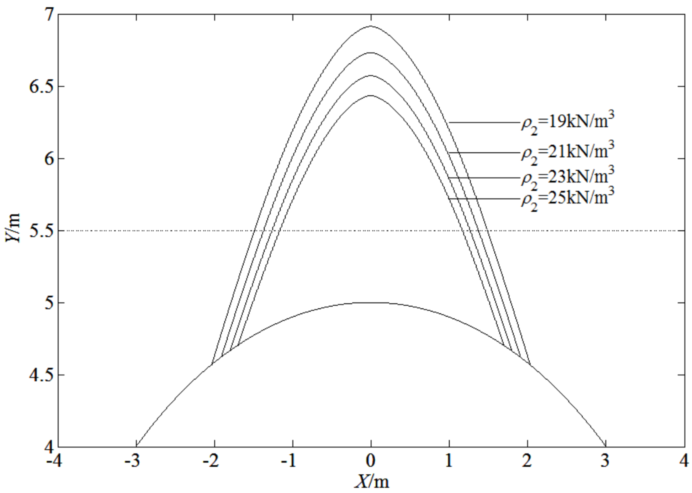


Fig. 4 Effects of pore pressure coefficient on collapse mechanisms of circular tunnel in the layered soils

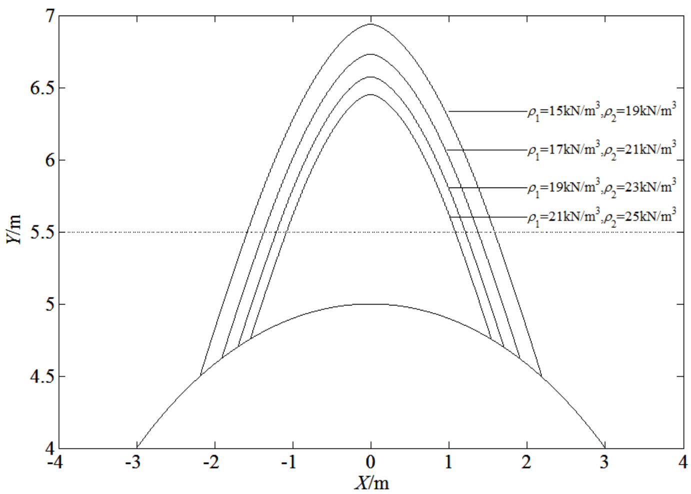


(a) Effects of unit weight ρ_1 on failure mechanisms (ρ_2 = 23 kN/m^3)

Fig. 5 Effects of unit weights on collapse mechanisms of circular tunnel in the layered soils



(b) Effects of unit weight ρ_2 on failure mechanisms (ρ_1 = 17 kN/m^3)



(c) Effects of unit weights ρ_1 and ρ_2 on failure mechanisms

Fig. 5 Continued

layered soils. Moreover, the proposed method, being simple and rigorous, may be an attractive alternative to other existing solutions, and can be easily extended to other stability problems such as calculation of the safety factor of slope and bearing capacity of foundation in engineering.

5. Conclusions

Considering the fact that almost all geomaterials have the nature of nonlinearity over wide range of normal stresses, this paper has focused the efforts on the collapse mechanism of circular tunnel subjected to seepage force in layered soils, with a nonlinear failure criterion. The numerical solution for the shape of collapse mechanism is obtained by upper bound theorem of limit analysis. Based on the discussion above, some conclusions are drawn:

(1) With nonlinear coefficients increase, the possible collapsing area of the failure block increases. When only initial cohesion of upper soil increases, the possible collapsing shape of the failure block will be flat. When only initial cohesion of lower soil improves, the possible size of the failure block will increase. When initial cohesions of upper and lower soils increase simultaneously, the possible shape of the collapsing block will be large and flat.

(2) With pore pressure coefficient improving, the possible size of collapsing will develop gradually. The possible collapsing shape of the failure block will decrease with unit weight of soil increase.

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