

## Limit analysis of rectangular cavity subjected to seepage forces based on Hoek-Brown failure criterion

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**Abstract.** On the basis of Hoek–Brown failure criterion, a numerical solution for the shape of collapsing block in the rectangular cavity subjected to seepage forces is obtained by upper bound theorem of limit analysis. The seepage forces obtained from the gradient of excess pore pressure distribution are taken as external loadings in the limit analysis, and the pore pressure is easily calculated with pore pressure coefficient. Thus the seepage force is incorporated into the upper bound analysis as a work rate of external force. The upper solution of the shape of collapsing block is derived by virtue of variational calculation. In order to verify the validity of the method proposed in the paper, the result when the pore pressure coefficient equals zero, and only hydrostatic pressure is taken into consideration, is compared with that of previous work. The results show good effectiveness in calculating the collapsing block shape subjected to seepage forces. The influence of parameters on the failure mechanisms is investigated.

**Keywords:** limit analysis; cavity; Hoek-Brown criterion; seepage force; collapse

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### 1. Introduction

The potential collapse of a cavity roof is a practical problem in geotechnical and civil engineering. Due to the random variability of the mechanical properties of the in-situ rock and the presence of cracks and fractures in the rock banks, the complication of the problem increased, especially when many factors are taken into account, such as seepage forces and temperature. Recently, the stability of tunnel face, and other similar projects have been investigated by many scholars by evaluating their lower and upper bound solutions since limit analysis theory is first applied by Davis *et al.* (1980). Compared with slice method and limit equilibrium method, solutions obtained from limit analysis are more rigorous and the forces are considered without any assumptions (Chen 1975). Thus limit analysis method is widely used to analyze the stability of the front of cavity driven in deep or shallow strata owing to its advantages.

At the present time, the limit analysis method, as a complementary tool in tunnel engineering, is mainly used to analyze the stability of tunnel face excavated in shallow strata with linear Mohr-Coulomb criterion. However, the collapse mechanism of deep tunnel is a complicated nonlinear evolution process, and the characteristics of material are also nonlinear, which have been verified by many tests (Agar *et al.* 1985). That's to say, deep tunnel and shallow tunnel are of

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great difference on mechanical characteristics. Therefore, the nonlinear failure criterion, i.e., Hoek-Brown failure criterion, should be applied to solve the problem of roof collapsing for a deep-buried tunnel. And the Hoek-Brown failure criterion has been widely used in a variety of geotechnical engineering problems (Serrano and Olalla 1994, 1998, 1999, Maghous *et al.* 1998, Jimenez *et al.* 2008, Sofianos 2003, Sofianos and Halakatevakis 2002). Based on the generalized Hoek-Brown failure criterion, Merifield *et al.* (2006) derived the ultimate bearing capacity of a surface footing on a rock mass by using limit theorems, and the upper bound solutions are equal to lower ones of the bearing capacities precisely. In order to discuss the possible collapse of a rectangular cavern roof and tunnel with arbitrary sections based on the upper bound theorem of limit analysis and Hoek-Brown failure criterion, Fraldi and Guarracino (2009, 2010) obtained the exact solution of detaching profile.

As is known to all, water as well as other fluid has a significant impact on the stability of underground structure generally, thus it is of great importance to investigate its mechanism and treatment method. In previous study, Wang *et al.* (2012) used numerical simulation and upper bound analysis theorem to analyze the face stability of shield tunnel under seepage condition. Lee and Nam (2001) used the numerical method to study the seepage forces acting on the tunneling lining, and then the effect of seepage forces on the stability analysis was investigated by means of limit analysis. According to the study done by the above authors, the effect of seepage forces should be incorporated into the limit analysis of rectangular cavity which is in rich water stratum. When using the limit analysis theory to analyze the stability of rectangular cavity, the effect of seepage forces should be taken into consideration, and some methods should be also developed to calculate the rate of work done by seepage forces. The seepage forces obtained from the gradient of excess pore pressure distribution are taken as external loading in the limit analysis with referring to the study of Saada *et al.* (2012), and it is convenient to calculate the pore pressure with pore pressure coefficient.

## 2. Upper bound with seepage forces

On the basis of upper bound theorem, the load obtained by equating the external rate of work to the rate of energy dissipation in any kinematically admissible velocity field is no less than the actual collapsing load when the velocity boundary condition is satisfied (Michalowski 1995). In order to take the effects of seepage forces into account in the realm of the upper bound theorem of limit analysis for slope stability, Saada *et al.* (2012) assumed that the work of seepage forces is equal to the sum of seepage forces regarded as external loading working on skeleton expansion. Thus the effect of seepage forces incorporated into the upper bound theorem can be written as follows

$$\int_{\Omega} \sigma_{ij} \cdot \varepsilon_{ij} d\Omega \geq \int_s T_i \cdot v ds + \int_{\Omega} X_i \cdot v d\Omega + \int_{\Omega} -grad u \cdot v d\Omega \quad (1)$$

where  $\sigma_{ij}$  and  $\varepsilon_{ij}$  are the stress tensor and strain rate in the kinematically admissible velocity field respectively,  $T_i$  is a surcharge load on boundary  $s$ ,  $X$  is the body force,  $\Omega$  is the volume of the collapse mechanism,  $v$  is the velocity along the velocity discontinuity surface,  $-grad u$  is excess pore pressure.

What's more, some other assumptions should be made: the material is perfectly plastic and follows an associated flow rule; the blocks bounded by the velocity discontinuity line and boundary surface are regarded as rigid materials.

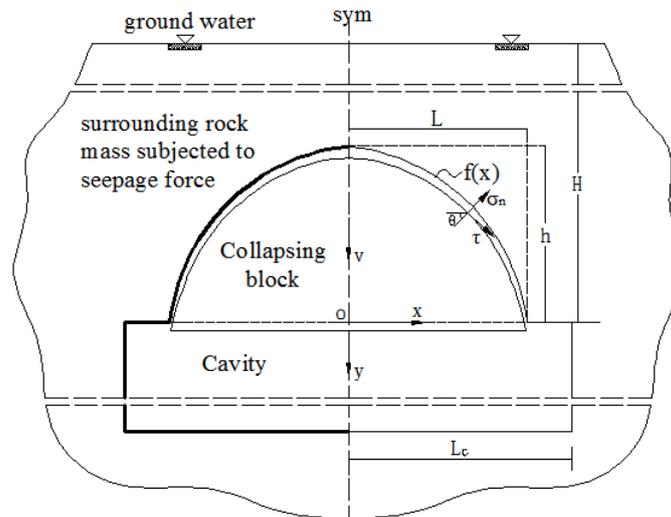


Fig. 1 Collapsing patterns for rectangular cavity subjected to seepage forces

### 3. Upper bound of rectangular cavity subjected to seepage forces

#### 3.1 Failure mechanism of rock mass for cavity

In regard to the upper bound theorem of limit analysis, how to construct a kinematically admissible failure mechanism is critical. On the basis of the actual mechanical characteristics of rock mass over the roof of a deep-buried tunnel, choosing an arched detaching curve  $f(x)$  to describe the velocity discontinuity surface is well coincident with the reality, which is presented in Fraldi and Guarracino (2009), as shown in Fig. 1. Owing to the presence of velocity discontinuity surface, it will induce the plastic flow. Thus, according to Hoek-Brown failure criterion and associated flow rule, the energy dissipation rate along the detaching surface can be worked out. By equating the rate of external work to the energy dissipation rate, the virtual work equation which meets the velocity boundary condition is derived. What's more, the variational calculation is applied to minimize the objective function so as to obtain the effective shape of the collapsing block in a limit state.

#### 3.2 Energy analysis with nonlinear failure criterion

Hoek–Brown failure criterion has two forms of expressions which are expressed by the major and minor principal stresses and the normal and shear stresses respectively (Hoek and Brown 1997). As the energy dissipation along the velocity discontinuity surface is caused by normal and shear stresses, it is convenient to use the later form of expression

$$\tau = A\sigma_c [(\sigma_n + \sigma_t)\sigma_c^{-1}]^B \tag{2}$$

where  $\sigma_n$  is the normal stress,  $\tau$  is the shear stress,  $A$  and  $B$  are physical parameters of the rock,  $\sigma_c$

and  $\sigma_t$  are the uniaxial compressive strength and the tensile strength of the rock mass, respectively.

On the basis of the Hoek-Brown failure criterion expressed in terms of normal and shear stresses and associated flow rule, the normal and shear stresses and strain on the velocity discontinuity surface can be worked out (Baker and Frydman 1983). Therefore, the energy dissipation rate determined by the internal forces on the velocity discontinuity surface can be expressed as (Fraldi and Guarracino 2009)

$$D_i = \sigma_n \dot{\varepsilon}_n + \tau_n \dot{\gamma}_n = \left\{ \left[ -\sigma_t + \sigma_c (AB)^{1/(1-B)} (1-B^{-1}) f'(x)^{1/(1-B)} \right] \left[ t \sqrt{1 + f'(x)^2} \right] \right\} \cdot v \quad (3)$$

in which  $\dot{\varepsilon}_n$  and  $\dot{\gamma}_n$  are normal and shear plastic strain rate respectively,  $f'(x)$  is the first derivative of  $f(x)$ ,  $t$  is the thickness of the velocity discontinuity surface. Thus the total energy dissipation rate determined by the internal forces on the detaching surface is

$$D = \int_0^s D_i ds = \int_0^L \left[ -\sigma_t + \sigma_c (AB)^{1/(1-B)} (1-B^{-1}) f'(x)^{1/(1-B)} \right] \cdot v dx \quad (4)$$

Owing to the collapsing block is symmetrical with regard to the y-axis, the work rate of failure block produced by weight can be written as

$$P_{\gamma'} = \int_0^L \gamma' f(x) \cdot v dx \quad (5)$$

where  $\gamma'$  is the buoyant weight which can be obtained by  $\gamma' = \gamma - \gamma_w$ ,  $L$  is the half width of the failure block. According to the study of Saada *et al.* (2012), the distribution of excess pore pressure is defined as

$$u = p - p_w = p - \gamma_w h \quad (6)$$

where  $p$  is the pore water pressure at the considered point which can be derived by an appropriate method  $p = r_p \gamma_w h$ ,  $r_p$  referring to pore pressure coefficient, and  $p_w = \gamma_w h$  is the hydrostatic distribution for pore pressure,  $\gamma_w$  referring to the water unit weight,  $h$  is the vertical distance between the roof of the cavity and the top of the failure block.

Afterwards,  $-\text{grad } u$  can be calculated by

$$-\text{grad } u = du/dh = \gamma_w - r_p \gamma \quad (7)$$

Thus according to Eq. (1), the seepage forces producing work rate along the velocity discontinuity surface is

$$P_u = \int_{\Omega} -\text{grad } u \cdot v d\Omega = \int_0^L (\gamma_w - r_p \gamma) f(x) \cdot v dx \quad (8)$$

Therefore, an objective function which is composed of external rate of work and the internal energy rate of dissipation can be constructed as follows

$$\begin{aligned} \zeta[f(x), f'(x), x] &= \int_0^L D_i \cdot \sqrt{1 + f'(x)^2} t dx - \int_0^L \gamma' f(x) \cdot v dx - \int_0^L -\text{grad } u \cdot v dx \\ &= \int_0^L \psi[f(x), f'(x), x] dx \end{aligned} \quad (9)$$

where  $\psi[f(x), f'(x), x]$  is a function which can be written as

$$\psi[f(x), f'(x), x] = \left[ -\sigma_t + \sigma_c (AB)^{1/(1-B)} (1-B)^{-1} f'(x)^{1/(1-B)} - (1-r_p)\gamma f(x) \right] \cdot v \quad (10)$$

In order to make the integral  $\psi$  over the interval  $[0, L]$  to achieve an extremum, a typical problem encountered of the calculation of variations, i.e., to find a function,  $y = f(x)$ , under the customary regularity conditions, which makes the Eq. (9) a stationary value. The expression of  $\psi$  is a function which can be turned into an Euler's equation by variational calculation. So the first variation of the total dissipation  $\zeta$  can be written as follows

$$\delta\zeta = \delta\zeta[f(x), f'(x), x] = 0 \quad \Rightarrow \quad \frac{\partial\psi}{\partial f(x)} - \frac{\partial}{\partial x} \left[ \frac{\partial\psi}{\partial f'(x)} \right] = 0 \quad (11)$$

According to Eq. (10), it can be easily obtained as follows

$$\frac{\partial\psi}{\partial f(x)} = -(1-r_p)\gamma \cdot v \quad \frac{\partial\psi}{\partial f'(x)} = -\sigma_c (AB)^{1/(1-B)} B^{-1} f'(x)^{B/(1-B)} \cdot v \quad (12)$$

$$\frac{\partial}{\partial x} \left[ \frac{\partial\psi}{\partial f'(x)} \right] = -\sigma_c (AB)^{1/(1-B)} (1-B)^{-1} f'(x)^{(2B-1)/(1-B)} \cdot f''(x) \cdot v \quad (13)$$

By substituting Eqs. (12) and (13) into Eq. (11), the Euler equation of the problem is obtained

$$\left[ \eta \cdot f'(x)^{(2B-1)/(1-B)} \cdot f''(x) - (1-r_p)\gamma \right] \cdot v = 0 \quad \forall v \in V \quad (14)$$

where

$$\eta = \sigma_c (AB)^{1/(1-B)} (1-B)^{-1} \quad (15)$$

$V$  represents the space of all admissible velocity fields. Eq. (14) is a nonlinear second-order homogeneous differential equation which can be calculated by the method of integral calculation. After the first integration, it can be written as

$$\left[ \eta(1-B)^{-1} \right] \cdot f'(x)^{B/(1-B)} - (1-r_p)\gamma x - \tau_0 = 0 \quad (16)$$

where  $\tau_0$  is a constant calculated by the geometrical condition, and it should satisfy  $f'(x=0) = 0$  for the detaching curve  $f(x)$  is symmetrical with regard to  $y$ -axis. Thus, the expression of velocity discontinuity surface  $f(x)$  can be derived as follows

$$\begin{aligned} f(x) &= B \left[ \frac{(1-r_p)\gamma B}{\eta(1-B)} \right]^{(1-B)/B} \cdot \left[ x + \frac{\tau_0}{(1-r_p)\gamma} \right]^{1/B} - h \\ &= A^{-1/B} \cdot \left[ \frac{(1-r_p)\gamma}{\sigma_c} \right]^{(1-B)/B} \cdot \left[ x + \frac{\tau_0}{(1-r_p)\gamma} \right]^{-1/B} - h \end{aligned} \quad (17)$$

where  $h$  stands for integration constant which can be obtained with the derivation of detaching curve  $f(x)$ .

By substituting Eq. (17) into Eq. (10),  $\psi$  is obtained as

$$\begin{aligned} \psi[f(x), f'(x), x] &= \left\{ \begin{aligned} & -\sigma_t + \sigma_c (AB)^{1/(1-B)} (1-B^{-1}) \left[ \frac{(1-r_p)\gamma B}{\eta(1-B)} \right]^{1/B} \cdot \left[ x + \frac{\tau_0}{(1-r_p)\gamma} \right]^{1/B} \\ & - (1-r_p)\gamma \cdot A^{-1/B} \cdot \left[ \frac{(1-r_p)\gamma}{\sigma_c} \right]^{(1-B)/B} \cdot \left[ x + \frac{\tau_0}{(1-r_p)\gamma} \right]^{1/B} + (1-r_p)\gamma \cdot h \end{aligned} \right\} \cdot v \\ &= \left\{ (1-r_p)\gamma \cdot h - \sigma_t - \frac{1}{B} \cdot A^{-\frac{1}{B}} \cdot \sigma_c^{\left(\frac{B-1}{B}\right)} \cdot [(1-r_p)\gamma]^{\frac{1}{B}} \cdot \left[ x + \frac{\tau_0}{(1-r_p)\gamma} \right]^{1/B} \right\} \cdot v \end{aligned} \quad (18)$$

By virtue of calculating the integral of  $\psi$  over the interval  $[0, L]$  according to Eq. (18), the expression of  $\zeta$  presents below

$$\begin{aligned} \zeta[f(x), f'(x), x] &= \int_0^L \psi[f(x), f'(x), x] dx \\ &= \left\{ [(1-r_p)\gamma \cdot h - \sigma_t] \cdot L - A^{-1/B} \cdot (1+B)^{-1} \cdot \sigma_c^{(B-1)/B} \cdot [(1-r_p)\gamma]^{1/B} \cdot \left[ L + \frac{\tau_0}{(1-r_p)\gamma} \right]^{(1+B)/B} \right\} \cdot v \end{aligned} \quad (19)$$

Indeed, the value of  $\tau_0$  can be easily obtained. On the basis of symmetrical characteristics of the detaching curve  $f(x)$ , it indicates that the value of  $\tau_0$  is equal to zero on the basis of  $f'(x=0) = 0$ , so  $f(x)$  can be simplified as follows

$$f(x) = A^{-1/B} \cdot \left[ \frac{(1-r_p)\gamma}{\sigma_c} \right]^{(1-B)/B} \cdot x^{1/B} - h \quad (20)$$

On the other hand, there is an implicitly condition of  $f(x=L) = 0$ , i.e.,

$$f(x=L) = 0 \quad \Rightarrow \quad h = A^{-1/B} \cdot \left[ \frac{(1-r_p)\gamma}{\sigma_c} \right]^{(1-B)/B} \cdot L^{1/B} \quad (21)$$

At last, according to upper bond theory of limit analysis, it can be calculated easily by virtue of equating the rate of the energy dissipation to the external rate of work, i.e.,

$$\zeta[f(x), f'(x), x] = 0 \quad \Rightarrow \quad \left[ (1-r_p)\gamma \cdot h - \sigma_t \right] \cdot L - A^{-1/B} \cdot (1+B)^{-1} \cdot \sigma_c^{(B-1)/B} \cdot [(1-r_p)\gamma]^{1/B} \cdot L^{(1+B)/B} = 0 \quad (22)$$

Combining Eqs. (21) and (22), the explicit form of  $L$  and  $h$  have been derived

$$h = (1+B)/B \cdot [(1-r_p)\gamma]^{-1} \cdot \sigma_t \quad \text{and} \quad L = A \cdot \sigma_c^{(1-B)} \cdot [(1-r_p)\gamma]^{(B-1)} \cdot h^B \quad (23)$$

Consequently, the expression of  $f(x)$  is

$$f(x) = A^{-1/B} \cdot \left[ \frac{(1-r_p)\gamma}{\sigma_c} \right]^{(1-B)/B} \cdot x^{1-B} - (1+B)/B \cdot [(1-r_p)\gamma]^{-1} \cdot \sigma_t \quad (24)$$

#### 4. Analytical results and discussions

On the basis of the work of Saada *et al.* (2012) which investigated the stability of rock slopes subjected to seepage forces with upper bound theorem, the corresponding method is applied to derive the shape of potential collapsing block in rectangular cavity in this paper and the results has been well validated by comparing with the previous work.

##### 4.1 Comparison

In the paper, the shape of detaching curve is obtained by limit analysis based on Hoek–Brown criterion. In order to make a comparison with the previous work which has been obtained on the basis of Mohr-Coulomb criterion, it is necessary to make some transformations. When  $B \rightarrow 1$ , the Hoek-Brown criterion reduces to the Mohr-Coulomb criterion, so the expression of velocity discontinuity surface  $f(x)$  turns out to be

$$f^{(M-C)} = \lim_{B \rightarrow 1} f(x) = A^{-1} \cdot x - 2\sigma_t \cdot [(1-r_p)\gamma]^{-1} = x \cot \phi - 2c \cdot \cot \phi \cdot [(1-r_p)\gamma]^{-1} \quad (25)$$

Based on the relationship between the Hoek–Brown generalized criterion and the Mohr-Coulomb criterion, in which the coefficient of  $A$  and  $B$  as well as  $\sigma_t$  are equal to

$$B = 1, \quad A = \tan \phi, \quad \sigma_t = c \cdot \cot \phi, \quad \Rightarrow \quad \tau_n^{(M-C)} = \sigma_n \cdot \tan \phi + c \quad (26)$$

The  $h$  and  $L$  become

$$h^{(M-C)} = 2c \cdot \cot \phi \cdot [(1-r_p)\gamma]^{-1} \quad \text{and} \quad L^{(M-C)} = 2c \cdot [(1-r_p)\gamma]^{-1} \quad (27)$$

The result is coincident with that of Fraldi and Guarracino (2009) when the pore pressure coefficient  $r_p = 0$ , which in turn validates the correctness of the result. Meanwhile, through the analysis, the unstable shows if the inequality below is met

$$L_c \geq L = A \cdot \sigma_c^{(1-B)} \cdot [(1-r_p)\gamma]^{B-1} \cdot h^B \quad (28)$$

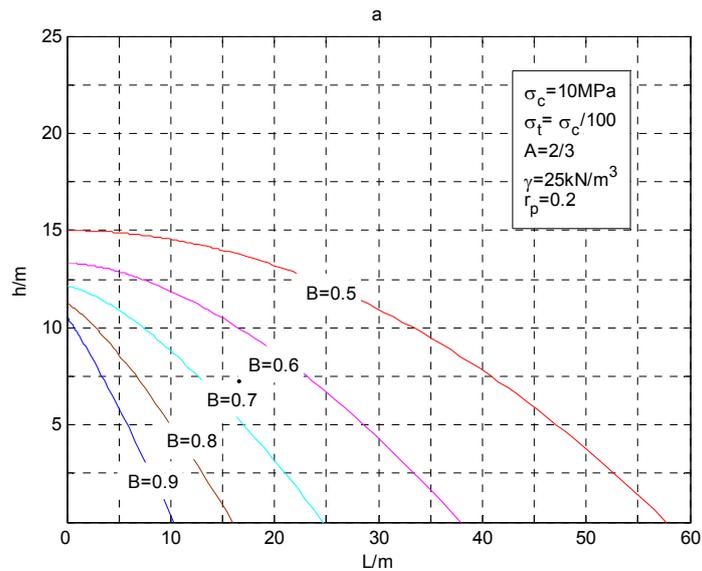
$$H \gg h = (1+B)/B \cdot [(1-r_p)\gamma]^{-1} \cdot \sigma_t \quad (29)$$

From Eq. (29), it is obvious that the deep cavity considered as the ‘deep’ not only is because of its actual depth but also depends on some physical properties of the rock mass, i.e., the parameters of  $A$ ,  $B$  as well as  $\sigma_c$ ,  $\sigma_t$ ,  $\gamma$  and  $r_p$ .

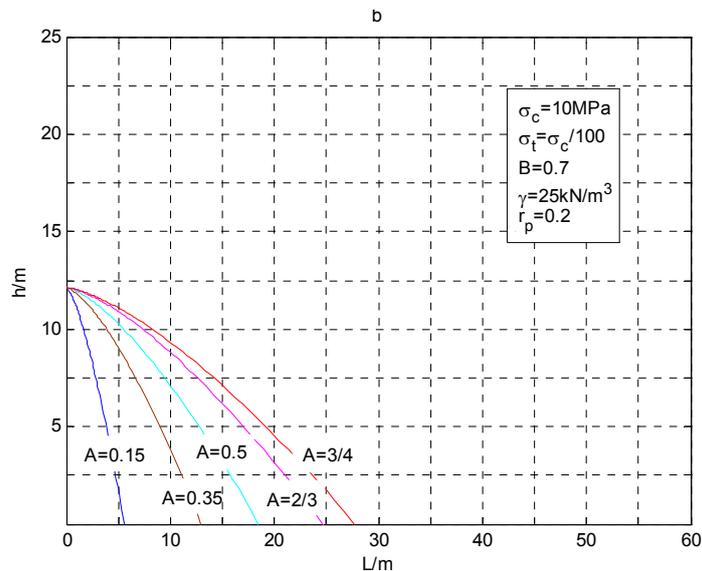
##### 4.2 Discussions of numerical results

In order to discuss the impact of the pore pressure coefficient and different rock parameters

over the shape of collapsing block for rectangular cavity, different shapes of collapsing blocks on the condition of seepage forces based on Hoek-Brown failure criterion are plotted in Figs. 2 and 3. According to the work done by Fraldi and Guarracino (2009), the rock mass parameter  $B$  varies from  $3/4$  to  $1$ ,  $A$  varies from  $0.15$  to  $3/4$ ,  $\sigma_t/\sigma_c$  varies from  $1/300$  to  $1/50$ , and  $\gamma$  varies from  $15 \text{ kN}\cdot\text{m}^{-3}$

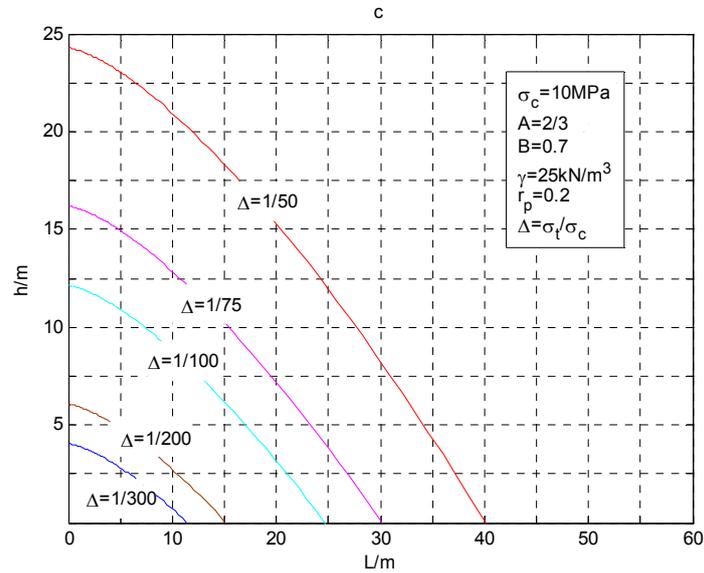


(a) Influences of Parameter  $B$  on collapse mechanisms

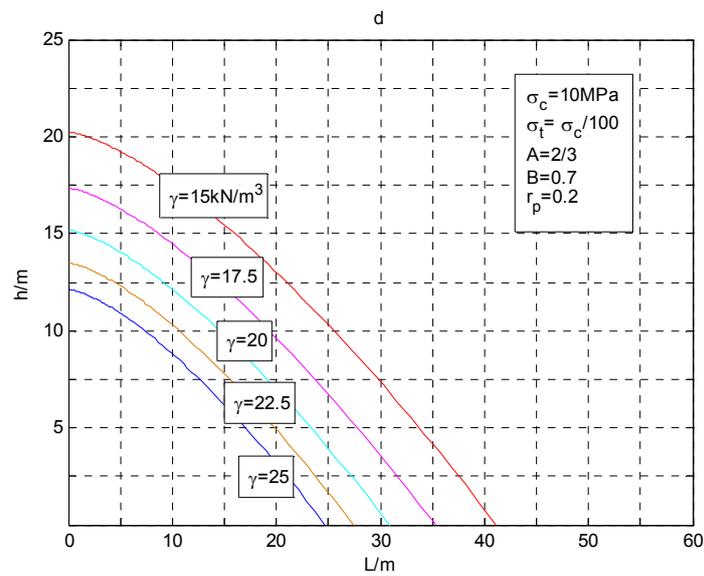


(b) Influences of Parameter  $A$  on collapse mechanisms

Fig. 2 Shape of collapsing block with regard to different rock parameters



(c) Influences of parameter  $\Delta$  on collapse mechanisms



(d) Influences of parameter  $\gamma$  on collapse mechanisms

Fig. 2 Continued

to  $25 \text{ kN}\cdot\text{m}^{-3}$ , and the corresponding plots are drawn in Fig. 2, respectively. Meanwhile, on the basis of the study of Saada *et al.* (2012), the value of  $r_p$  is generally below 0.3, so the analysis of seepage forces estimated by pore pressure coefficient  $r_p$  ranging from 0 to 0.4 is conducted as shown in Fig. 3.

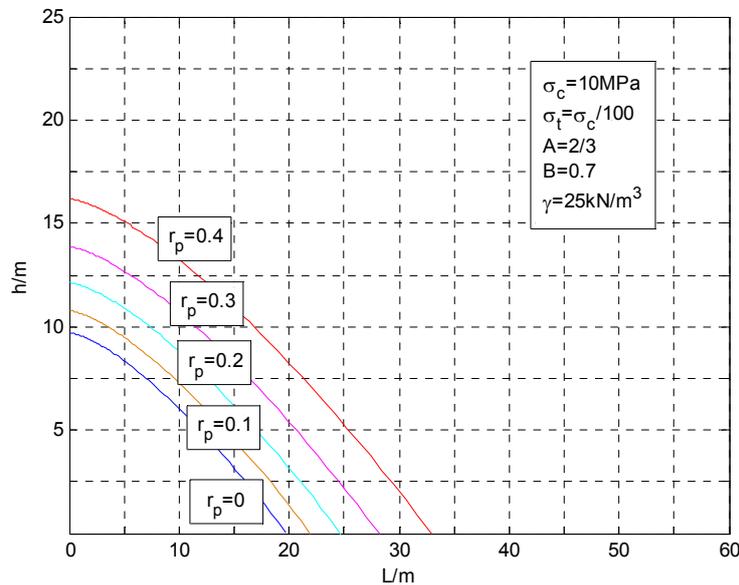


Fig. 3 Shape of collapsing block with respect to different pore pressure coefficients

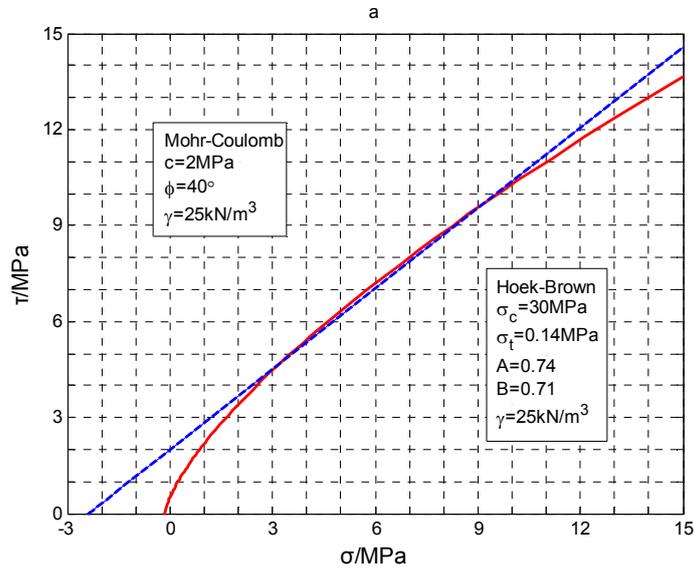
Table 1 Numerical solution of the width and height of the collapsing block with regard to different parameters

Parameters maximum value	$A = 3/4$	$B = 0.5$	$\sigma_t / \sigma_c = 1/50$	$\gamma = 15 \text{ kN}\cdot\text{m}^{-3}$	$r_p = 0.4$	Referring value
L/m	27.7829	57.7350	40.1186	41.1598	32.9278	24.6959
h/m	12.1429	15.0000	24.2857	20.2381	16.1905	12.1429

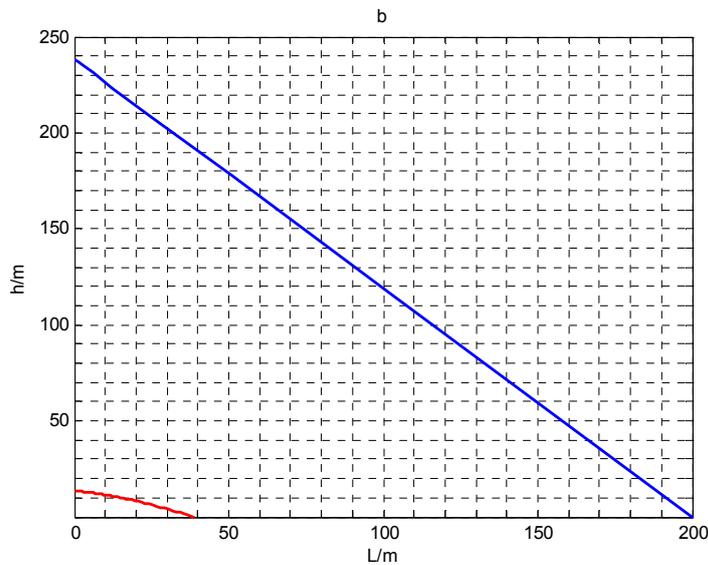
It is found from the figures that, with the increase of  $B$  and  $\gamma$ , the width and height of the collapsing block both decrease. While with the increase of  $\sigma_t$  and  $r_p$ , the width and height of the collapsing block increase. In particular, when  $A$  becomes larger, the width of the collapsing block increases, but the corresponding height stays the same. For the condition of pore pressure coefficient  $r_p = 0$ , it means the impact of seepage forces is not taken into consideration, and the shape of collapsing block is exactly the same as the work of Fraldi and Guarracino (2009), which shows the validity of the method presented in the paper for calculating seepage forces.

In comparison with the result of Fraldi and Guarracino (2009), it can be obviously found from Fig. 3 that the width and height of the collapsing block on the condition of seepage forces are larger than that ignoring the seepage forces. It can also explain that the effect of water has a negative influence on the stability of cavity. What's more, the corresponding values are determined by the pore pressure coefficient to a large extent.

The maximum value of the collapsing width  $L$  and height  $h$  corresponding to the parameters with reference to the work of Fraldi and Guarracino (2009) can be derived by numerical software, and its values are presented in Table 1. It can be found that each of the rock parameters has an impact on the scale of the collapsing block to some extent. The referring value of parameters in Table 1 is corresponding to  $A = 2/3$ ,  $B = 0.7$ ,  $\sigma_t / \sigma_c = 1/100$ ,  $\gamma = 25 \text{ kN}\cdot\text{m}^{-3}$ , and  $r_p = 0.2$ .



(a) Equivalence relationship between Hoek-Brown failure criterion and Mohr-Coulomb one



(b) Comparison of collapse mechanisms

Fig. 4 Comparison between results from classical Hoek-Brown and equivalent Mohr-Coulomb parameters

In order to make a comparison on the results based on Hoek-Brown failure criterion and Mohr-Coulomb one, the relationship of two criterions above should be built firstly. According to the work of Hoek and Brown (1997) and Fraldi and Guarracino (2009), this relationship can be established by means of linear regression which is shown in Fig. 4(a).

Though the comparison is on the basis of the equivalence between Hoek-Brown failure criterion and Mohr-Coulomb one, but each of the collapsing block presents different scale, as shown in Fig. 4(b). It is obvious that the width and height of collapsing block with Hoek-Brown criterion are smaller than that with the latter one, from which the more effective collapsing scale is obtained with the nonlinear properties of rock taken into consideration.

## 5. Conclusions

By using the upper bound theorem of limit analysis, the exact solution describing the shape of collapsing block in rectangular cavity subjected to seepage forces has been derived based on Hoek–Brown failure criterion and the method of the calculus of variations. The effect of seepage forces is taken as a work rate of external force incorporated to upper bound theorem and it has a bad influence on the stability of cavity. The result obtained under the condition of zero pore pressure coefficient is exactly the same with the work of Fraldi and Guarracino (2009), which shows the validity of the method proposed in the paper for calculating seepage forces. The paper can be regarded as an extended work of the study of Fraldi and Guarracino (2009), but Fraldi and Guarracino (2009) didn't take the effect of seepage forces on the prediction of collapsing mechanism into account. According to the results discussed above, it can be noticed that the rock parameters with a high value of  $B$ , as well as a low value of the tensile strength and seepage forces can lead to a smaller collapsing block from the point of engineering.

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