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Partial safety factors for retaining walls and slopes: A reliability based approach

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Abstract. Uncertainties in design variables and design equations have a significant impact on the safety of geotechnical structures like retaining walls and slopes. This paper presents a possible framework for obtaining the partial safety factors based on reliability approach for different random variables affecting the stability of a reinforced concrete cantilever retaining wall and a slope under static loading conditions. Reliability analysis is carried out by Mean First Order Second Moment Method, Point Estimate Method, Monte Carlo Simulation and Response Surface Methodology. A target reliability index β = 3 is set and partial safety factors for each random variable are calculated based on different coefficient of variations of the random variables. The study shows that although deterministic analysis reveals a safety factor greater than 1.5 which is considered to be safe in conventional approach, reliability analysis indicates quite high failure probability due to variation of soil properties. The results also reveal that a higher factor of safety is required for internal friction angle φ, while almost negligible values of safety factors are required for soil unit weight γ in case of cantilever retaining wall and soil unit weight γ and cohesion c in case of slope. Importance of partial safety factors is shown by analyzing two simple geotechnical structures. However, it can be applied for any complex system to achieve economization.

Keywords: retaining wall; slope stability; uncertainty; reliability analysis; partial safety factors

1. Introduction

In deterministic approach, factor of safety (*FS*) is chosen based on past experience and outcome of failure. Uncertainties in any of the input parameters (e.g., cohesion, angle of internal friction, unit weight, pore pressure parameters etc.) remain unaddressed in this approach. As a result, their computations provide a single unique value of *FS*. The load and resistance factor design (*LRFD*) concept takes into account these effects by using degraded strength parameters in the resistance calculation. In this approach, separate partial safety factors are assigned for loads and resistance. Taylor (1948) was the first person to suggest different FS_p for soil strength. Several limit state codes have been developed, such as Eurocode 7 (Simpson *et al*. 1997, CEN 2001), Geo-Code 21 of Japan (Honjo and Kusakabe 2002), the Load and Resistance Factor Design (LRFD) bridge design specifications (AASHTO 1997), National Building Code of Canada (Becker 1996). The

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partial *FS* approach has also been suggested by different researchers like Meyerhof (1970, 1982). Eurocode 7 (1997) suggests a *FS* of 1.25 for 5-15% variation of friction (tan φ) and a *FS* of 1.4-1.6 for 20-50% variation of cohesion (c_u, c'). But this concept works well only if the input parameter data available from site are well engineered and accurate. A probabilistic framework may take into account the effect of these uncertainties. Reliability analysis is therefore used to assess uncertainties in engineering variables in terms of the reliability index (β). Such uncertainty is usually assessed by different approaches such as the First-Order Second-Moment Method (*FOSM*), Point Estimate Method (*PEM*), Hasofer-Lind Method, Response Surface Methodology (*RSM*), Monte Carlo Simulation Method (*MCS*) etc.

Past research works demonstrate that the consideration of variability in the input soil properties may result in high probability of failure (P_f) in spite of having high deterministic *FS* (Hoeg and Murarka 1974). Wu and Kraft (1970), Tang *et al*. (1976), Venmarcke (1977) applied a probabilistic approach in analyzing slope stability using *FOSM* and concluded that probabilistic approach is much economic than conventional approach. Christian *et al*. (1994) used the mean-first order reliability method, which is a simplification of the more general first order reliability method. Cherubini (2000) discussed both deterministic and probabilistic designs of an anchored sheet pile wall. Devaraj *et al*. (2004) performed a reliability analysis on a concrete gravity dam for the probability of failure due to the maximum compressive stresses developed at toe for sliding, overturning and bearing capacity of soil by using *FOSM* and *PEM*.

Bhattacharya *et al*. (2003) presented a numerical procedure for locating the surface of minimum reliability index $\beta_{\min}$ for earth slopes. The advantage of the procedure is that the critical probabilistic surface can be located by utilizing an existing deterministic slope stability algorithm with the addition of a simple module for the calculation of β. Castillo *et al*. (2004) found that dealing with several modes of failure at the same time and calculating the global failure probability creates complexities which make failure surface boundary highly non-differentiable. To avoid this problem and make it possible for *FS* and P_f to co-exist, they suggested a method based on design of fixing bounds for the *FS* and the probabilities for each failure mode, instead of fixing a global P_f.

Chalermyanont and Benson (2004, 2005) used *MCS* to develop a probabilistic design method for internal and external stability of mechanically stabilized earth walls. Low (2005) illustrated different reliability-based design procedures for retaining walls and pointed out the differences between reliability-based design and partial safety factor approach. Sayed *et al*. (2008) performed reliability analysis of reinforced concrete retaining walls using *FOSM* and *PEM* and concluded that the friction angle of the soil, unit weight and interface friction angle between soil and reinforcement are the most sensitive parameters in the design.

Babu and Basha (2006) and Srivastava and Babu (2010) presented a parametric study on optimum design of cantilever and gravity retaining walls using target reliability approach and *RSM*. Xue and Gavin (2007) used a genetic algorithm approach for simultaneously locating the critical slip surface and determining β for slope stability problems. Guharay and Baidya (2011) presented a study on a cantilever retaining wall and presented a partial safety factor approach for random variables. The issue of dependency between failure modes had been observed by Biernatowski and Puła (1988) and Zevgolis and Bourdeau (2010). They computed the wall's external stability for static case and modeled it as a series system with correlated failure modes.

Most of the past investigations are based on application of different methodologies to different geotechnical problems. Little work has been done on what safety factors should be assigned for design purpose based on probabilistic approach.

2. Objectives of the present study

This paper analyses the effect of variability of soil properties on the stability of two important geotechnical structures viz. a cantilever retaining wall and a slope under static loading conditions. A parametric study is carried out on the cantilever retaining wall against four modes of failure viz. overturning, sliding, eccentricity and bearing capacity failure. An attempt has also been made to obtain the optimal proportions of the retaining wall by varying the toe and heel length corresponding to different coefficient of variation (*COV*) of φ (5% and 10%) and γ (5%) and target reliability index, β in the range 3-3.2 for all failure modes. The slope stability problem is also addressed by probabilistic approach.

Partial safety factors for the random variables for these two structures are back-calculated by target reliability approach method for different variations of these soil properties corresponding to a target failure probability of 0.00135 (β = 3) as recommended by USACE (1997). Analysis shows that the same partial safety factor (FS_P) can have different levels of risk depending on the degree of uncertainty of the mean value of friction angle of the soil. These calculated FS_P values may be useful in design under static loading. This can be proved to be cost effective by optimizing the structure for specific site conditions.

3. Reliability analysis

3.1 Basic theory

Safety is a measure of how reliable a system is for some specified period under stated conditions, and is expressed in terms of the reliability index (β). The limit state function for the resistance (R load-carrying capacity) and the load effect (Q) can be defined as follows:

$Q > R$, the structure has no ability to fulfill its design purpose, failure.

$R > Q$, the structure has the ability to fulfill its design purpose, no failure or safe.

For uncorrelated normally distributed R and Q, β is calculated from Eq. (1) (Baecher and Christian 2003)

$$\beta = \frac{\mu_R - \mu_Q}{\sqrt{Q_R^2 + \sigma_Q^2}} \tag{1}$$

USACE (1997) stated that for good performance of a geotechnical system, the calculated β should be at least 3.0. Furthermore, P_f can be estimated from β, using the established equation P_f = 1-$\Phi(\beta) = \Phi(-\beta)$, where Φ is the cumulative distribution function (*CDF*) of the standard normal variate.

3.1.1 Mean First Order Second Moment Method (*MFOSM*)

In this method, only the first order terms of a Taylor's series expansion of the performance function are considered to estimate the mean (μ) and variance (σ^2) of the function. Considering uncorrelated variables, β is calculated from Eq. (2a)

$$\beta = \frac{\mu(x_i)}{\sqrt{\sum_{i-1}^{n}\left(\frac{\partial g}{\partial x_i}\right)^2 \sigma^2(x_i)}} \tag{2a}$$

The partial derivatives of *FS* with respect to each soil parameter are calculated numerically using Eq. (2b)

$$\frac{\partial g}{\partial x_i} = \frac{g^+ - g^-}{2m\sigma(x_i)} \tag{2b}$$

where g^+ and g^- are respectively values of *FS* obtained by using parameter values greater than and less than μ by an increment $m\sigma(x_i)$. Based on a detailed study of several problems, the value of m is taken as 1.0 in the present study, as recommended by Hassan and Wolff (1999).

3.1.2 Point Estimate Method (*PEM*)

In this method as proposed by Rosenblueth (1975) and also by Harr (1987), discrete values of the performance function are evaluated at the mean values (μ) of the basic variables, at values (μ + 1σ)and (μ – 1σ) for uncorrelated variables, where σ is the standard deviation. If n is the number of variables, in general 2^n terms are to be added.

3.1.3 Monte Carlo Simulation (*MCS*)

In this approach, a large number of realizations of the basic random variables X, i.e. x_j, j = 1,2,3...N are simulated and for each of the outcomes x_j, it is checked whether or not the limit state function taken in x_j is positive. All the simulations for which this is not the case are counted (n_f) and after N simulations, P_f may be estimated through $P_f = n_f / N$. In fact for $N \to \infty$, it exactly estimates the value of P_f. In the present paper, 10,00,000 sample points are generated by an algorithm coded in commercially available software MATLAB, which minimizes the effect of variation of the number of sample point generation.

3.1.4 Response Surface Metodology (*RSM*)

Response Surface Methodology (*RSM*) establishes an approximate explicit functional relationship between input variables (x_1, x_2, x_3…) and output response (y) through regression analysis based on least square error (Eq. (3a))

$$y = f(x_1, x_2, x_3, \ldots) + e \tag{3a}$$

e represents other sources of uncertainty not accounted for in 'f' with mean 0 and variance σ^2.

The original performance function $G(x)$ is replaced by an equivalent function $R(x)$.

$$R(x) = a + \sum_{i=1}^{n} b_i x_i + \sum_{i=1}^{n} c_i x_i^2 \tag{3b}$$

The selection of sample points are based on combination of upper ($\mu + m\sigma$) and lower ($\mu - m\sigma$) limit value of each input parameters. Becker (1996) and Orr (2000) suggested that the value of m should be taken as 1.65 with the assumption that the input soil parameters follow normal distribution and upper and lower limit values have probability of 5% and 95% being exceeded.

A non-dimensional quantity R^2 (called coefficient of multiple determinations) and adjusted R^2 (R^2_{adj}) is calculated by Eq. (3c)

$$R^2 = \frac{\sum_{i=1}^{n} \left(\hat{y}_i - \bar{y}\right)^2}{\sum_{i=1}^{n} \left(\hat{y}_i - \bar{y}\right)^2} \quad ; \quad R_{adj}^2 = 1 - \frac{k-1}{k-p}\left(1 - R^2\right) \tag{3c}$$

where $\bar{y}$, y_i and $\hat{y}_i$ are the estimated mean value as well as the actual and the predicted value of the output response, respectively. k is the total number of observations and p is the number of regression coefficients. The value of R^2 lies between 0 and 1 and a value very close to 1 indicates that most of the variability in y is explained by regression model. For a good-fitted model, the difference between R^2 and R^2_{adj} should be small.

4. Parametric study I: Cantilever retaining wall

4.1 Deterministic analysis

A cantilever retaining wall is one of the most widely used earth retaining systems among various categories of retaining walls in civil engineering practice. A cantilever retaining wall with height H, as shown in Fig. 1, is analyzed.

The specific case of a two-dimensional cohesionless drained soil mass forming a horizontal backfill and retained by a vertical frictionless wall is considered. The wall is assumed to be able to rotate away from the soil a sufficient distance to mobilise the frictional resistance of the soil. Under such conditions, the active earth pressure coefficient as proposed by Rankine (1857) is given by Eq. (4)

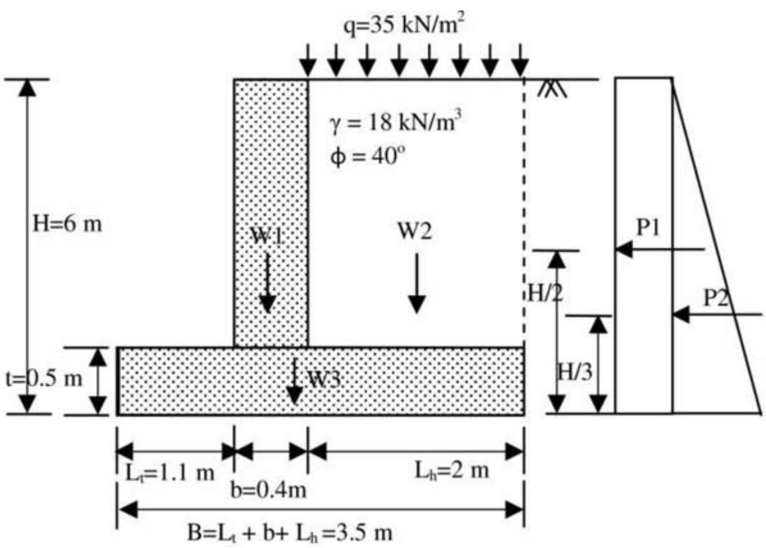


Fig. 1 Cantilever retaining wall

$$K_a = \frac{1-\sin\phi}{1+\sin\phi} \tag{4}$$

Traditional theories assume that the internal angle of friction (φ) and unit weight (γ) are spatially constant, so that the total active lateral earth force P_2 acting at height $H/3$ is given by Eq. (5)

$$P_2 = \frac{1}{2}K_a\gamma H^2 \tag{5}$$

Expressions for FS are given in Eqs. 6(a)-(d)

$$(FS)_1 = \frac{\sum M_R}{\sum M_o} \tag{6a}$$

$$(FS)_2 = \frac{\sum F_R}{\sum F_D} = \frac{(W_1+W_2+W_3+W')\tan\delta}{P_1+P_2} \tag{6b}$$

$$(FS)_3 = \frac{B}{6e} \tag{6c}$$

$$(FS)_4 = \frac{q_{ult}}{q_{\max}} \tag{6d}$$

where H, B, W_1, W_2 and W_3 are defined in Fig. 1.

W' is the total load due to surcharge acting per unit length of the backfill ($= q \times L_h$)

P_1 = horizontal force due to surcharge = $q \times H \times K_a$

δ = friction angle between the foundation soil and base of the wall $\left(=\frac{2}{3}\phi\right)$

M_R is the sum of moments about the heel due to weights W_1, W_2, W_3 and q' respectively. M_o is the sum of moments about heel due to horizontal component of active earth pressure.

e = eccentricity of the resultant force

q_{ult} = ultimate bearing capacity of a shallow foundation below the base slab of retaining wall (250 kN/m^2).

$$q_{\max} = \left[\frac{\sum V}{B}\left(1+\frac{6e}{B}\right)\right]$$

The deterministic safety factors obtained for the retaining wall are 1.65, 1.57, 2.12 and 1.64 for overturning, sliding, eccentricity and bearing modes of failure respectively.

4.2 Probabilistic analysis

4.2.1 Application of MFOSM, PEM and MCS

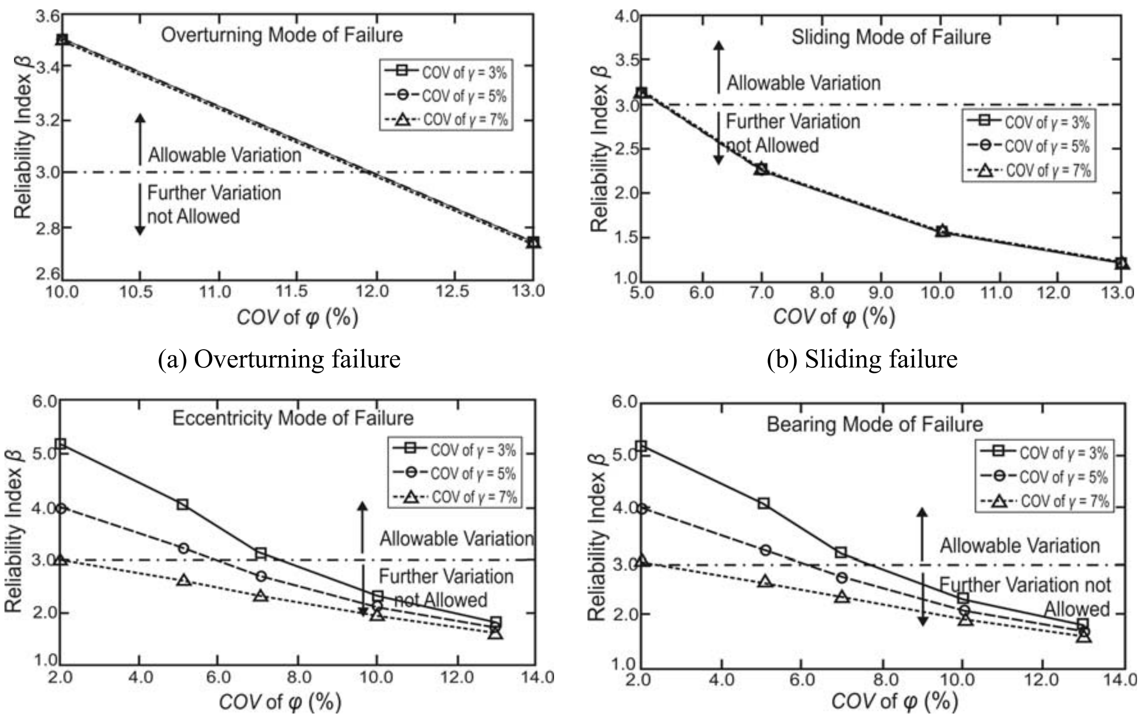
In this study, the backfill soil properties γ and φ are considered as random variables. It is assumed that γ follows normal distribution with mean of 18 kN/m^3 and COV of 3-7% and φ

follows normal distribution with mean of 40° and *COV* of 2-13%. These values of coefficient of variation are assumed in absence of any specific site data, according to Harr (1984) and Kulhawy (1992). For the probabilistic analysis of the retaining wall, the performance function can be defined as $g_i(x) = (FS)_i - 1$, where $(FS)_i$ denotes the factor of safety of the retaining wall for different modes of failure i.e., (i) $(FS)_1$ for overturning failure, (ii) $(FS)_2$ for sliding failure, (iii) $(FS)_3$ for eccentricity failure, and (iv) $(FS)_4$ for bearing capacity failure. Reliability index satisfying all the constraints in the form of performance function is achieved. The process is continued until the target reliability criterion is met.

The termination tolerance for the convergence of P_f is taken as 10^{-3} (corresponding $\beta > 3$), as recommended by USACE (1997). This target reliability index may vary with the importance of the structure. However, in geotechnical engineering, a target reliability index of 3 ($P_f = 0.00135$) is widely used (Foye *et al*. 2006, Ellingwood *et al*. 1980, Paikowsky 2004). *LRFD* bridge design specification (AASHTO 2007) specifies target $\beta > 3.5$ for critical structures and < 3.5 for less critical structures, although the criteria to establish whether a given structure is critical or non-critical are left to the designer.

Figs. 2(a)-(d) illustrates the role of random variables φ and γ on P_f calculated by *MCS*.

It can be observed that when both φ and γ are considered as random variables, in sliding and overturning modes of failure, the variation of β with variation of φ follows almost the same pattern for all *COV* of γ. On the other hand, for eccentricity and bearing modes, for smaller *COV* of φ, the



(a) Overturning failure

(b) Sliding failure

(c) Eccentricity failure

(d) Bearing failure

Fig. 2 Variation of β for different *COV* of φ and γ for different failure modes of retaining wall

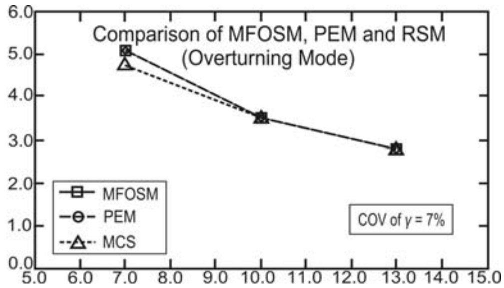


Fig. 3 Comparison among values of β obtained from *MFOSM*, *PEM* and *MCS*

variation of β is different for different *COV* of γ. But this difference gradually dies out as *COV* of φ increases. So it can be concluded that γ has almost negligible effect on sliding and overturning modes of failure compared to φ. It may be said that it is necessary to assess the value of φ with greater accuracy during site investigation. Although the deterministic analysis of the retaining wall with the mean values of φ and γ yields *FS* of 1.57, which is conventionally considered to be safe, but sensitivity analysis reveals that for a site having *COV* of $\varphi > 5\%$, the structure of the retaining wall has to be modified to bring down β under tolerable limit.

From Fig. 3, it can be observed that for *COV* of $\gamma = 7\%$ in case of overturning mode of failure, the values of β obtained by *MFOSM* and *PEM* compares almost exactly with each other, while *MCS* is little on the conservative side. The highly non-linear formulation of *FS* makes it impossible to evaluate first order derivatives, therefore numerical approximation was used; whereas *MCS* does not require such derivative and this may explain the discrepancy.

4.2.2 Application of RSM

In the reliability analysis of the retaining wall, it is assumed that input parameters φ and γ are uncorrelated normally distributed and *COV* of these parameters are 10% and 7% respectively. The $(\mu + m\sigma)$ and $(\mu - m\sigma)$ values for φ are 46.6° and 33.4° respectively, while that for γ are 20.08 and 15.92 kN/m^2 respectively.

The functional relationships between two input variables φ and γ and output responses i.e., FS_1, FS_2, FS_3 and FS_4 can be replaced by simple functional relationship between input and output variables using *RSM*. For the generation of response surface model, pseudo-static *FS* is calculated for the four combinations of input parameters (i.e., sample points) as indicated in Table 1.

Table 1 Output Response for Each Combination of Input for RSM of Retaining Wall

Sr. No.	φ	γ	φ	γ	FS_1	FS_2	FS_3	FS_4
1	+	+	46.6	20.08	2.24	2.57	11.63	2.09
2	+	–	46.6	15.91	2.28	2.61	6.59	2.24
3	–	+	33.4	20.08	1.23	0.96	1.14	1.21
4	–	–	33.4	15.91	1.25	0.97	1.05	1.32

Using these data, regression analysis is performed by MS-Excel Data Analysis (Regression Analysis) to obtain a linear response surface model as given in Eqs. 7(a)-(d)

$$FS_1 = 0.0778\phi - 0.00725\gamma - 1.2317$$
$$R^2 = 0.9999,\ R^2_{adj} = 0.9998 \tag{7a}$$

$$FS_2 = 0.122765\phi - 0.005636\gamma - 3.033945$$
$$R^2 = 0.9999,\ R^2_{adj} = 0.9998 \tag{7b}$$

$$FS_3 = 0.607083\phi - 0.61403\gamma - 30.2325$$
$$R^2 = 0.92043,\ R^2_{adj} = 0.7613 \tag{7c}$$

$$FS_4 = 0.68144\phi - 0.3106\gamma - .45367$$
$$R^2 = 0.99967,\ R^2_{adj} = 0.99901 \tag{7d}$$

It can be observed that values of R^2 and R^2_{adj} are close to 1.0 (Eq. (3c)) and hence, the developed surface model is adequate. Since it is assumed that input parameters are uncorrelated normally distributed and also it is linearly related to the output response, μ and σ^2 of normally distributed FS_2 (for sliding) can be easily obtained as 1.78 and 0.49, respectively. Therefore, β = [(1.78 – 1.0) / 0.49] = 1.58.

This value can be well compared with those obtained from the previously mentioned methods. The marginal difference between *RSM* and *MFOSM* may be attributed to the fact that in *MFOSM*, the analysis was carried out without linearising the function, although φ and γ are non-linearly varying with *FS*. The advantage of *RSM* is that it is computationally less demanding.

4.2.3 Variation in the values of L_t / H and L_h / H for COV of φ = 5% and 10% and COV of γ = 5%

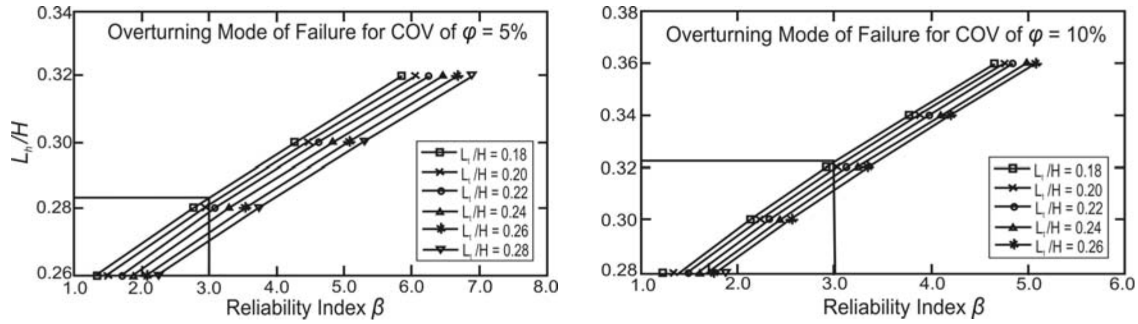
In this case, the toe length L_t and heel length L_h of the retaining wall are considered as variables along with φ and γ. In Figs. 4(a)-(b), the ratio L_t / H is varied from 0.16 to 0.24, while the ratio L_h / H is varied from 0.24 to 0.4 for *COV* of φ = 5% and 10% and *COV* of γ = 5%, thus trying to optimize the wall dimensions for the desired value of β.

It can be observed from Figs. 4(a)-(b), that for overturning mode of failure as the *COV* of φ increases from 5% to 10%, to keep β value at 4 and for L_t/H = 0.18, one requires L_h/H = 0.31 for *COV* of φ = 5% and L_h/H = 0.345 for *COV* of φ = 10%.

Therefore, the magnitudes of L_h and L_t is to be increased proportionally to maintain the same level of reliability. Similar observations are noted for other modes of failure also. So, depending upon importance of the structure and what degree of safety is required, the structure can be optimised.

4.2 Probabilistic Partial Factor of Safety (FS_P)

Partial safety factors for different random variables are calculated by target reliability index approach depending upon the importance of the structure. A target probability of failure (P_f = 0.00135, corresponding reliability index β = 3) is first set.



(a) *COV* of φ = 5% (b) *COV* of φ = 10%

Fig. 4 Variation of L_h / H with β for different values of L_t / H against overturning failure

Considering *COV* of φ =13%,
Performance Function $g = R - Q = \sum F_V \tan\delta - \sum F_H$

$$\mu_R = F_V \tan\left(\frac{2}{3}\mu_\phi\right) = F_V \tan\left(\frac{2}{3}\times\frac{40}{FS_P}\right)$$

$$\mu_Q = F_H \times \mu\left(1-\sin\phi\right)\times\mu\left(\frac{1}{1+\sin\phi}\right) = F_H \times \mu\left(1-\sin\phi\right)\times\mu\left(\sin^2\phi+\sin\phi+2\right)$$

by Binomial Expansion, neglecting higher order terms

$$= F_H \times\left(1-\sin\frac{40}{FS_P}\right)\times\left(\sin^2\frac{40}{FS_P}+\sin\frac{40}{FS_P}+2\right)$$

$$\sigma_R^2 = Var\left(R\right) = F_V{}^2\times\tan\left(\frac{2^2}{3^2}\times\sigma_\phi^2\right) = F_V{}^2\times\tan\left(\frac{2^2}{3^2}\times\frac{27.04}{FS_P{}^2}\right)$$

$$\text{Now, } Var\left(XY\right) = \mu_X^2.Var(Y) + \mu_Y^2.Var(X) + Var(X).Var(Y)$$

Thus, variance of Q can also be calculated similarly by Binomial Expansion of the function.
Now corresponding to a target probability of failure P_f = 0.00135 (β = 3), one can back-calculate the values of FS_P (presented in Table 2) from the equation $\beta = \mu_g / \sigma_g$.

Table 2 Probabilistic partial Factor of Safety (FS_P) of Retaining Wall

COV of φ (%)	2	5	7	10	13
FS_P required	1.06	1.55	1.89	2.41	2.94

It can be noted that for *COV* of φ above 5%, factor of safety should be increased to have target $P_f < 10^{-4}$. Thus instead of using a global value of *FS* for the retaining wall, FS_P for different random variables may be used for design purpose for more economic design of the structure.

It may be noted here that although *LRFD* (AASHTO 2007) suggests load factors and resistance factors for different structures, the method is only fruitful when the site data obtained is consistent and accurate. This uncertainty is accounted for in this partial safety factor approach.

5. Parametric study II: Slope stability

5.1 Deterministic analysis

Slopes may be artificial, i.e., man-made, as in embankments for highways and railroads, earth dams etc. or natural as in hillside and valleys, coastal and river cliffs etc. The stability analysis of embankments and fills usually involve less uncertainty than natural slopes and cuts, because fill materials are preselected and processed.

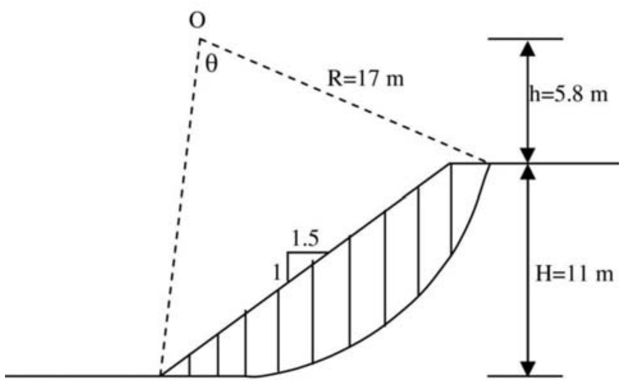


Fig. 5 Geometry of slope

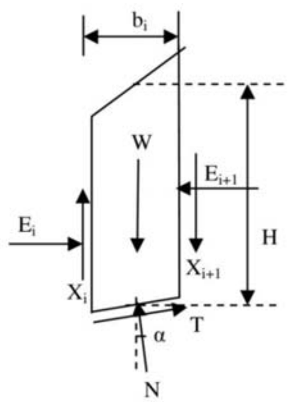


Fig. 6 Forces acting on a typical slice

A finite slope having height H = 11 m and slope 1 V : 1.5 H, as shown in Fig. 5, is considered for analysis. The critical slip circle has a radius of 17 m. Friction angle (φ), cohesion (c) and unit weight (γ) of backfill soil are considered as random variables for the analysis. Water table is assumed to be at a great depth from the ground surface, thus not affecting the stability of the slope. The slope is analyzed by Fellenius (1936) and Bishop's Method (1955) of analysis.

In order to formulate the algorithm to solve for FS based on the above-mentioned methods, it is essential to consider the forces acting on a typical slice as shown in Fig. 6.

5.1.1 Ordinary method of slices or Fellenius Method (1936)

The Ordinary method of slices assumes that the inter-slice forces are parallel to the base of each slice, thus they can be neglected and FS is given by Eq. (8)

$$FS = \frac{c' L + \tan\phi . \sum W \cos\alpha}{\sum W \sin\alpha} \tag{8}$$

where H, b_i, R, θ and W are defined in Fig. 5.

L is the length of the slip surface = $R\theta$

α is the angle the normal acting on the slice makes with vertical.

For probabilistic analysis, expression for margin of safety M for Fellenius Method is given by Eq. (9)

$$\begin{aligned} M &= \left(c' L + \tan\phi . \sum W \cos\alpha\right) - \left(\sum W \sin\alpha\right) \\ &= C - D \end{aligned} \tag{9}$$

5.1.2 Bishop's Method (1955)

In Bishop's method the factor of safety is determined using an iterative process, since FS appears in both sides of the equation. The inter-slice shear forces are neglected, and only the normal forces are used to define the inter-slice forces. The factor of safety is given by Eq. (10)

$$FS = \frac{\sum \left[\frac{c' b + W \tan\phi}{\Psi} \right]}{\sum W \sin\alpha} \qquad \text{where } \Psi = \cos\alpha + \frac{\sin\alpha . \tan\phi}{FS} \tag{10}$$

For probabilistic analysis, expression for margin of safety M for Bishop Method is given by Eq. (11)

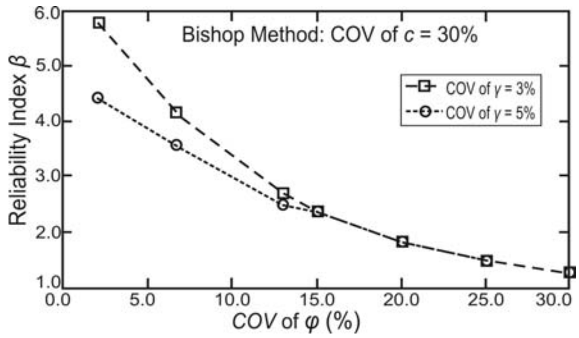
$$M = \frac{C}{D} - 1 = FS - 1 \tag{11}$$

The deterministic FS obtained are 1.55 and 1.52 by Fellenius and Bishop's Method respectively.

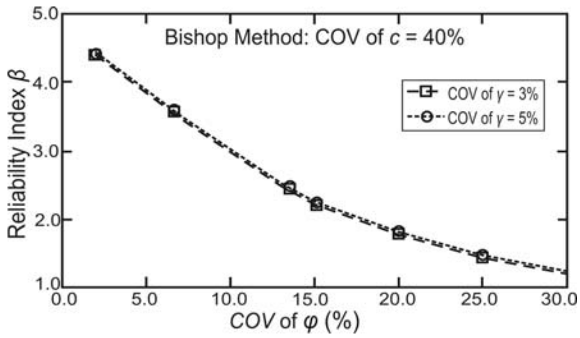
5.2 Probabilistic analysis

5.2.1 Application of MFOSM, PEM and MCS

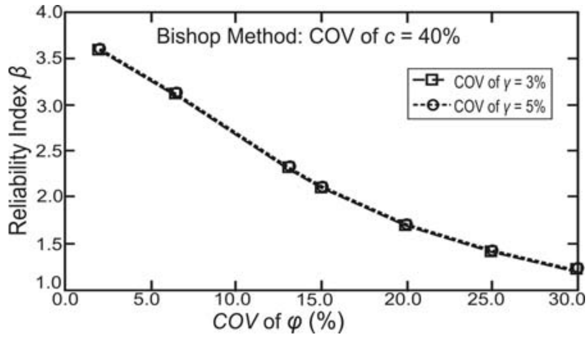
The soil properties φ, γ and c are considered as random variables. It is assumed that φ follows



(a) *COV* of *c* = 30%



(b) *COV* of *c* = 40%



(c) *COV* of *c* = 50%

Fig. 7 Variation of β for different values of *COV* of φ and γ for Slope

normal distribution with mean of 32° and *COV* of 2-30%, γ follows normal distribution with mean of 20 kN/m^3 and *COV* of 3-7% and c follows normal distribution with mean of 10 kN/m^2 and *COV* of 30-50%.

The values of β showed marginal difference when calculated by *MFOSM*, *PEM* and *MCS*, similar to the retaining wall. Since *MCS* considers nonlinear terms, so the variation of β, calculated by *MCS*, with variation of the random variables γ, φ and c are presented in Figs. 7(a)-(c). It is observed from the figures that variation of γ has a negligible effect on the stability of the slope, especially when *COV* of c exceeds 30%. But when the *COV* of φ exceeds approximately 10%, the design has to be modified to bring down β under tolerable limit. Similarly, when *COV* of c exceeds approximately 40%, necessary precautions should be adopted. From Figs. 7(a)-(c), it can be concluded that φ is the most important parameter to be assessed during field investigation than c or γ. It is also suggested that a greater FS_P should be assigned to φ than to γ or c, rather than applying an overall *FS* to the design, thereby economizing the structure.

5.2.2 Application of RSM

In the reliability analysis of the slope, it is assumed that input parameters c, φ and γ are uncorrelated normally distributed and coefficients of variation of these parameters are 50%, 20% and 7% respectively. The ($\mu + m\sigma$) and ($\mu - m\sigma$) values for φ are 42.56° and 21.44° respectively,

Table 3 Output response for each combination of input for *RSM* of slope

Sr. No.	c	φ	γ	c	φ	γ	FS
1	+	+	+	18.25	42.56	22.31	2.35
2	+	+	–	18.25	42.56	17.69	2.52
3	+	–	+	18.25	21.44	22.31	1.38
4	+	–	–	18.25	21.44	17.69	1.55
5	–	+	+	1.75	42.56	22.31	1.75
6	–	+	–	1.75	42.56	17.69	1.77
7	–	–	+	1.75	21.44	22.31	0.79
8	–	–	–	1.75	21.44	17.69	0.80

while that for γ are 22.31 and 17.69 kN/m^2 respectively and that for c are 18.25 and 1.75 respectively. For the generation of response surface model, static *FS* is calculated for the 8 combinations of input parameters (i.e., sample points) as indicated in Table 3.

Using these data, regression analysis is performed to obtain a linear response surface model as given in Eq. (12)

$$\begin{gathered} FS = 0.040576c + 0.04581f - 0.02035\gamma + 0.14751 \\ R^2 = 0.99576,\ R^2_{adj} = 0.992583 \end{gathered} \tag{12}$$

It can be observed that values of R^2 and $R^2{}_{adj}$ are close to 1.0 and hence, the developed surface model is adequate. Using the above response surface models, for the given mean and variance of the input parameters, μ and σ^2 are evaluated using simple statistical calculations.

5.3 Probabilistic Partial Factor of Safety (FS_P)

FS_P for different random variables are calculated by target reliability index approach. A target probability failure (P_f = 0.00135, corresponding reliability index β = 3) is first set.

Considering *COV* of c = 50%

$$\mu_C = 23.736\mu_C + 681.53 = 23.736 \times \left(\frac{10}{FS_P}\right) + 681.53$$

$$\mu_D = 592.68$$

$$\sigma_C^2 = Var(C) = 23.736^2 \times Var(c) = 23.736^2 \times \left(\frac{25}{FS_P{}^2}\right)$$

$$\sigma_D^2 = Var(D) = 0$$

The suggested FS_P values back-calculated by probabilistic approach for stability of the slope corresponding to a target reliability index β = 3 (corresponding to P_f = 0.00135) is presented in Table 4.

Table 4 Probabilistic Partial Factor of Safety (FS_P) of slope

COV of ϕ (%)	FS_P required
2	–
7	–
10	–
13	1.15
15	1.34
20	1.77
25	2.35
30	2.76

It is observed that although the deterministic *FS* was 1.55, yet for variations of φ above 20%, one require a *FS* greater than that determined by conventional method for the safety of the structure.

6. Conclusions

Reliability theory provides a rational and efficient means of characterizing the uncertainty which is prevalent in geotechnical engineering. P_f is not viewed as a replacement for *FS*, but as a supplement. The major conclusions which can be drawn from the present study are summarized as follows:

- The results of reliability analysis of both retaining wall and slope show that the reliability index is very sensitive to the uncertainty in φ than γ in case of cantilever retaining wall and γ and c in case of slope. This indicates that β provides more meaningful information than the deterministic factor of safety.
- A greater FS_P is desirable for φ, while much smaller value of FS_P is desirable for γ and c. So, safety factors may be recommended based on the site conditions and to what degree of safety is required for the particular structure, rather than applying a global safety factor for the entire structure.
- A cantilever retaining wall and a slope have been used in the present study to highlight the applicability and effectiveness of reliability theory in the design of structures. The suggested values of FS_P are pertinent to the worst condition by varying the parameters within their applicable ranges and corresponding to target reliability index $\beta = 3$.
- Importance of FS_P concept is highlighted using two simple geotechnical structures and this concept can easily be applied to solve any complex geotechnical problems. In fact, it may become more beneficial for more complex or large size project where economy plays a vital role.

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